ADVANCING CONTINUUM AND DISCONTINUUM MODELS OF BRITTLE ROCK
DAMAGE AND ROCK-SUPPORT INTERACTION

by
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ABSTRACT

Stress-induced damage and spalling in underground mines continue to remain a major hazard to mining personnel. In order to reduce worker injuries caused by falls of ground, it is necessary to develop techniques through which the mechanisms of pillar damage can be accurately interpreted and supports can be designed to control the anticipated displacements along pillar boundaries. With advancement in numerical modeling techniques, it is now possible to study such rock mechanics problems in detail, but there is a need for improvement in the available approaches before they can be utilized as tools for the design of ground support schemes. The goal of this research is, therefore, to advance continuum and discontinuum models to better reproduce both observed pillar damage mechanisms and the interaction between supports and (unsupported) ground. The associated findings contribute significantly towards improving our understanding of the phenomenological capabilities of continuum and discontinuum modeling approaches and their applications in different mining scenarios.

The contents of this thesis can be broadly sub-divided into two sections – continuum (using Itasca’s FLAC\textsuperscript{3D} software) and discontinuum (using Itasca’s UDEC software) analyses of pillar damage and rock-support interaction. In the first section, a rock yield criterion is developed that considers both brittle fracturing at low confinement and shearing at higher confinement. When implemented in FLAC\textsuperscript{3D} pillar models, results consistent with the empirical trend of pillar strength as a function of width to height ratio for granite, conglomerate, and coal were obtained. In terms of site-specific case studies, pillar displacement and stress data from two different sites could be reproduced using this yield criterion. Subsequent investigations on rock-support interaction revealed that continuum models tend to underestimate the effect of support on
otherwise unsupported ground, and accordingly is limited in its potential application as a support design tool.

In the second portion of this thesis, the Voronoi Bonded Block Modeling (BBM) approach is employed, which represents a material domain by an aggregate of polygonal blocks. Laboratory-scale modeling of a granitic rock was pursued to understand how decisions related to model setup affect the ability of such grain-based models to reproduce various deformation mechanisms. Ultimately, a BBM representation utilizing different elastic properties for the different mineral grains along with inelastic grain properties was necessary to match the pre- and post-peak attributes of the granite under consideration. The input properties from the laboratory-scale models, however, were found to not be directly applicable to pillar-scale simulations because much larger blocks sizes were used in the latter case and blocks sizes are known to exert a significant influence on the macroscopic behavior of BBMs. Accordingly, pillar-scale BBMs were calibrated independently, although the findings regarding model setup decisions from the grain-based BBM are, in part, transferable across scales. For example, the pillar BBMs required an inelastic block representation to simulate the damage process within the confined sections of the pillars, similarly to how the laboratory-scale model also required inelastic grains to replicate the high confinement attributes observed in laboratory triaxial tests.

The analysis of rock-support interaction was conducted by comparing the lateral displacements along the pillar edges without support and with various support patterns for both the polygonal and the triangular (also called Trigon) block geometries. The polygonal BBM produced behavioral differences that were closer to empirical field-data assembled from hard rock mines in comparison to the Trigon models. Coal pillars were simulated with elongated, inelastic Voronoi blocks to account for the anisotropic cleating of coal. For the case of the West
Cliff longwall mine, this model representation was able to reproduce the pillar displacements at two neighboring sites that had different support patterns by modifying the support in a BBM calibrated to one site to match the support at the adjacent site. This is perhaps the first study to quantitatively demonstrate that BBMs can replicate the influence of rock reinforcement on ground behavior in rock undergoing spalling.

Lastly, as BBMs are computationally intensive and cannot easily be applied at the mine-scale in 3D, an integrated modeling approach was established using two different mining case studies. In this approach, the larger-scale stress distribution was assessed using FLAC^3D^, and the deformation behavior near excavation boundaries (with and without support) under the expected loading path was estimated using a BBM. This study, as a whole, has demonstrated the capabilities of continuum and discontinuum modeling approaches under a variety of conditions, and the proposed approach for studying ground-support interaction has the potential for practical application in the context of site-specific support design.
CO-AUTHORSHIP

The thesis “Advancing continuum and discontinuum models of brittle rock damage and rock-support interaction” is the product of research conducted by the author, Sankhaneel Sinha. The individuals (co-authors) as mentioned below, also had a significant contribution in this research in terms of scientific and editorial feedback. The author has permission from all the co-authors to use the materials (data, figures, text, etc.) as presented in Appendix O.

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DEDICATION

To the person who has sacrificed a lot for me,

To the most industrious person, the epitome of patience,

To my Mom, for making me what I am today.
1.1 Introduction and Motivation

With advances in technology and continued depletion of near-surface resources, it is becoming both possible and essential to exploit increasingly deep ore deposits. Present day metal and non-metal mines have descended to depths beyond 1 km, where the magnitudes of pre-mining stresses are very high (He et al., 2005; Yu et al., 2012; Snelling et al., 2013; Yang et al., 2017; Walton et al., 2018b). In such highly stressed ground, mining systems rely primarily on unmined pillars for maintaining the functional integrity of underground openings and roadways. Because of their significance to mine stability, pillars have been the focus of a large body of rock mechanics and ground control research, with many significant advancements in fundamental knowledge and pillar design approaches occurring during the latter half of the 20th century.

Despite the indisputable contribution of empirical (Salamon and Munro, 1967; Bieniawski and Heerden, 1975; Wilson, 1983; Mark, 1987; Potvin et al., 1989; Lunder and Pakalnis, 1997; Martin and Maybee, 2000), laboratory (Das, 1986; Medhurst, 1998; Li et al., 2011; Walton et al., 2015a; Zhou et al., 2017; Zhou et al., 2018a) and numerical (Esterhuizen, 2006; Diederichs, 2007; Mortazavi et al., 2009; Walton et al., 2015b; Mohamed et al., 2015, 2016a, 2016b) studies towards advancing the state-of-the-art with respect to pillar mechanics and pillar design, worker injuries and fatalities have continued to occur. For example, between 1995-2009 (Figure 1.1a), more than half of all groundfall fatalities in US coal mines were directly related to pillar mechanics in some way (“retreat mining”, “rib falls”, “coal bursts”, and “major roof falls”). Furthermore, over forty injuries associated with falls of “face”, “rib”, or “pillar”
occurred in 2017 (see Figure 1.1b). Fracturing and collapse of the outermost portion of pillars also continues to be a major problem in deep metal mines (Hudyma, 1986; Kaiser et al., 1996; Krauland et al., 2001; Wagner, 2019). It is clear that our fundamental understanding of pillar and pillar support mechanics must advance to achieve significant further reductions in groundfall injuries and fatalities associated with pillar and pillar support problems.

Figure 1.1 (a) Classification of ground fatalities between 1995-2009 (Mark et al., 2011a), (b) Injuries due to “fall of face, rib or pillar” from 2013-2017 (MSHA, 2019).

With the advent of advanced numerical modeling methods, it is possible to study complex rock mechanics problems that are otherwise difficult to investigate using experimental or analytical approaches. Broadly speaking, there are two high-level classifications for numerical modeling methods used in rock mechanics: continuum and discontinuum. Continuum methods consider the ground as an equivalent continuous material governed by constitutive laws that account for the evolution of elastic and inelastic strains in response to loading (Fairhurst and Pei, 1990; Jing and Hudson, 2002). Discontinuum methods, on the other hand, explicitly model pre-existing and/or stress-induced fractures that may range in size from millimeter-scale grain-boundary contacts to large meter-scale joints, including their ability to fully separate (Jing and
Hudson, 2002; Jing and Stephansson, 2007). In terms of scales of applicability, continuum models are better suited for analyzing large-scale problems (e.g. Esterhuizen et al., 2010; Walton et al., 2015; Tulu et al., 2017), such as in the context of mine-wide stress redistribution analysis, while discontinuum approaches are typically limited to laboratory-scale simulations of fracturing processes (e.g. Lan et al., 2010; Ghazvinian et al., 2014; Farahmand and Diederichs, 2015) or investigations of individual excavations (Gao et al., 2014; Garza-Cruz et al., 2014; Oliviera and Diederichs, 2017; Abousleiman et al., 2019).

Notwithstanding the many valuable capabilities of continuum models, as a consequence of their formulation, such models are unable to reproduce large deformation processes such as stress-induced rockmass bulking, and tend to underestimate the effect of support elements on the behavior of otherwise unsupported ground (Lorig and Varona, 2013; Bahrani and Hadjigeorgiou, 2018; Sinha and Walton, 2019b). Additionally, the behavior of such models is heavily dependent on the constitutive model employed, but there is no well-defined yield criterion in the literature that is appropriate for simulating both the low confinement extensile cracking (Martin and Chandler, 1994; Diederichs, 2003) and confined shearing (Diederichs, 2003) mechanisms that occur in mine pillars. Discontinuum modeling is a useful alternative, as it can reproduce the rock fracturing process reasonably well (Lan et al., 2010; Ghazvinian et al., 2014; Bewick et al., 2014; Bahrani et al., 2014; Farahmand and Diederichs, 2015; Zhou et al., 2019), but it is not known how reliably this modeling approach can simulate the aforementioned transition in damage mode and the interaction between support and the ground. With all this in mind, the overarching motivation of this thesis is to advance the application of continuum and discontinuum models to simulate the stress-driven brittle rock damage process and associated rock-support interaction. This will enhance our understanding of how the underlying damage mechanisms can be
represented in numerical models, and will ultimately allow for the development of effective support layouts, which can improve worker safety in underground mines.

1.2 Research Needs and Objectives

Continuum models are now widely applied in rock engineering (Shabanimashcool and Li, 2012; Walton et al., 2016; Feng et al., 2019; Azarfar et al., 2019), but the results from these models are heavily governed by the selection of a yield criterion and associated input parameters (Hajiabdolmajid et al., 2002; Edelbro, 2010; Walton et al., 2015). The selection of a yield criterion for a particular set of conditions should ideally be based on knowledge of the micro-mechanical damage processes that ultimately control the global failure of the system. Previous pillar modeling attempts using Hoek-Brown (Martin and Maybee, 2000), strain-softening Mohr-Coulomb (Iannacchione, 1999; Mortazavi et al., 2009) and S-shaped yield criteria (Kaiser et al., 2011) have predicted a near-exponential increase in hard rock pillar strength with width to height (W/H) ratio, which contradicts the convex or flattening shape of widely accepted empirical pillar strength equations (Hedley and Grant, 1972; Lunder and Pakalnis, 1997). Note that “hard rock” in this context refers to brittle rockmasses that are relatively massive in nature (Geological Strength Index > ~65; Carter et al., 2008). The differences in observed behavior are likely related to the inability of these three criteria to appropriately capture the transition in damage mechanism in rock pillars from low-confinement brittle fracturing to shearing under high confinement. It follows that there is a need to develop an improved rock yield criterion that properly accounts for these fundamental aspects of the damage process.

Several authors have also attempted to model coal pillars using Hoek-Brown (Jaiswal and Shrivastava, 2009; Esterhuizen et al., 2010) and strain-softening Mohr-Coulomb constitutive
models (Shabanimashcool and Li, 2012 and 2013; Li et al., 2015; Zhang et al., 2015). These studies investigated pillar behavior only from a macroscopic standpoint (i.e. matching the peak strength to empirical strength equations, matching bolt loads in individual excavations, etc.), but did not examine the progressive fracturing process within the pillars. As coal is highly brittle in nature (Mishra and Nie, 2013; Kim et al., 2018), a yield criterion that is based on the principles of brittle rock damage is likely to be applicable to the simulation of coal damage as well.

Considering the wide application of continuum models, a natural inclination is to employ this modeling method for support design, but the ability of these models to capture rock-support interaction must first be verified. Previous studies have reported a negligible change in ground behavior when support elements were explicitly considered within continuum models (Edelbro, 2009; Lorig and Varona, 2013; Walton et al., 2016; Renani et al., 2016; Bahrani and Hadjigeorgiou, 2018); this clearly contradicts the field observations of Kaiser et al. (1996) and Colwell (2006). It seems that such models are not capable of reproducing the support effect and hence might not be usable in many support design scenarios. In contrast, discontinuum models have been reported to exhibit large reductions in displacement due to support installation for mining case studies (Gao et al., 2015; Kang et al., 2015; Yang et al., 2018; Bai and Tu, 2020), which is more consistent with what is expected based on in-situ observations (Kaiser et al., 1996). Despite this anecdotal evidence, in order to establish this modeling technique as a design tool, the simulated rock-support interaction behavior in discontinuum models needs to be more thoroughly validated.

Bonded Block Modeling (BBM) is an increasingly common application of the discontinuum modeling approach and is the primary focus of this study. This approach represents a material domain as an aggregate of detachable polygonal/triangular blocks. The emergent
macroscopic behavior of BBMs is controlled primarily by the geometry of the blocks (the
smallest unbreakable elements in the model) and the properties assigned to their two components
- contacts between blocks and zones (triangular mesh) within blocks. While the majority of
former BBM studies have focused on laboratory-scale rock fracturing process (Lan et al., 2010;
Gao and Stead, 2014; Ghazvinian et al., 2014; Farahmand and Diederichs, 2015; Gao et al.,
2016; Mayer and Stead, 2017), there has been some limited success in simulating field-scale
behavior as well (Christianson et al., 2006; Coggan et al., 2012; Bai et al., 2016; Sinha and
Walton, 2018b). Intuitively, if a small-scale model calibrated to various laboratory attributes is
appropriately up-scaled, one would expect such a model to exhibit realistic excavation-scale
behavior. However, this is not necessarily the case, as much larger blocks have to be used in the
large-scale models due to computational limitations, and block size is known to significantly
affect the macroscopic response of BBMs (Ghazvinian et al., 2014; Fabjan et al., 2015; Azocar,
2016; Insana et al., 2016). If meaningful relationships between input parameters and model
representations across the range of relevant scales are to be developed, it is imperative that our
understanding of these systems first advances considerably.

With respect to prior small-scale modeling attempts, it is interesting to note the various
representations of zones, blocks, and contacts that have been employed to model the same rock
type. For example, Lac du Bonnet granite was modeled using BBMs with fully homogeneous
properties and an elastic constitutive model for the zones by Ghazvinian et al. (2014), fully
homogeneous properties and an inelastic constitutive model for the zones by Noorani and Cai
(2015) and mineralogically heterogeneous (blocks and contacts) BBMs with elastic zones by Lan
et al. (2010), Chen and Konietzky (2014) and Farahmand and Diederichs (2015). The lack of
consistency in the model representations used highlights the gap in our understanding of BBM
behavior in general. In the context of large-scale applications, most previous studies have employed elastic blocks, and hence the relationship between decisions regarding model representation and the resulting model capabilities is largely unexplored.

The input parameters in BBMs are constrained by matching the macroscopic behavior of the model with those measured in the laboratory or field. Prior to testing the capability of BBMs to capture the support effect in mining structures (i.e. pillars), two major issues need to be addressed first: (1) Three-dimensional BBMs are computationally intensive, and it is difficult to apply a realistic 3D load path to an underground mine pillar being modeled in a 2D software, and (2) The availability of strength and deformational properties representing the behavior of unsupported ground (i.e. pillars) that are necessary for constraining unsupported BBM input parameters is limited. Additional data corresponding to supported and unsupported ground conditions (e.g. large-scale laboratory testing, extensometer data from the field) are therefore required for such studies. Incorporation of further data into calibration efforts will allow for the development of well-constrained BBMs that can ultimately be used for predictive purposes and for the development of new design tools or guidelines.

Besides understanding the rock-support interaction mechanism, the stress redistribution process at a mine-wide scale and its effect on the local behavior of pillars must be understood before reliable field-scale models can be developed as first approximations. There are two major approaches for characterizing this load redistribution process: (1) Empirical methods (Carr and Wilson, 1982; Mark, 1987; Mark and Iannacchione, 1992; Colwell et al., 1999) that rely on rock pressure data measured in the field. The spatial resolution of such data is typically low due to the high cost of stress measurement devices. (2) Continuum numerical models that are well suited for two- or three-dimensional mining problems but require calibration to field measurements.
They can represent both the larger-scale stress perturbations and the associated local damage processes, but are often not appropriate for designing support systems (as previously discussed).

It therefore follows that if BBMs can be shown to realistically reproduce the effect of ground support on rockmass behavior, then an integrated approach could be developed, where the mine-wide stress distribution can be assessed using the continuum method and the deformation behavior near excavation boundaries (with and without support) under the expected stress paths can be estimated using a 2D discontinuum model. The application of a load path from a 3D model to a 2D model (Shabanimashcool and Li, 2012) and coupling between continuum and discontinuum software tools (PFC$^{2D}$-FLAC$^{2D}$; Potyondy and Cundall, 2004; Cai et al., 2007; Saiang, 2010; Song and Hong, 2012; Zhang et al., 2017a; Jia et al., 2018; Zhang et al., 2019; PFC$^{3D}$-FLAC$^{3D}$: Khazaei et al., 2015; Zhao et al., 2018; Qu et al., 2019) have been implemented previously, and the approach discussed above simply represents an alternative to directly coupled methods (e.g. PFC$^{2D}$-FLAC$^{2D}$ or PFC$^{3D}$-FLAC$^{3D}$).

With all that in mind, the following research objectives were pursued in this study to advance numerical models to better reproduce the brittle rock damage process and rock-support interaction mechanism:

- Objective 1 – Develop a rock yield criterion that is based on the fundamental fracturing process of brittle rocks

- Objective 2 – Test whether the rock yield criterion developed from Objective 1 is capable of simulating the progressive damage in coal pillars through back-analysis of a case study
• Objective 3 – Compare the capabilities of continuum and discontinuum models (BBM) with respect to replication of realistic rock-support interaction

• Objective 4 – Develop relationships between model representations/inputs and model capabilities in laboratory-scale and field-scale BBMs

• Objective 5 – Demonstrate an integrated continuum-discontinuum modeling approach for support design through back-analysis of a historical case study

• Objective 6 – Analyze loading and damage processes in pillars using geotechnical data as well as numerical models (integrated approach) calibrated to these data from specific mining sites analyzed for the first time in this work.

Given that this thesis is broadly focused on the scientific advancement of numerical approaches to simulate the brittle rock damage process, both hard rock and coal pillars were considered. Hard rocks were considered because the concepts of brittle rock damage were developed largely for massive igneous rocks with unconfined compressive strengths (UCS) of more than 180 MPa (Martin, 1993; Diederichs, 1999; Andersson et al., 2009). Coal was considered for two main reasons: from a practical standpoint, coal mines have high injury/fatality rates relative to other types of mines, meaning that extending the findings of this study to consider coal will have a greater potential impact on worker health and safety; from a scientific perspective, the coal mine case studies provide a unique opportunity to test elongated BBMs as means to capture the anisotropic behavior of coalmasses (this is discussed later in Section 1.5 under Objectives 3 and 6).
Figure 1.2 Infographic showing how the different research objectives are related to one another.
To clearly portray how each of the different objectives are related to one another, a flowchart is presented (Figure 1.2). In this flowchart, the main goal of developing a support-design tool is first sub-divided into “rockmass representation” and “rock-support interaction”; the “rockmass representation” deals with the replication of the macro-mechanical properties of the rock/system (in this case, a pillar) as well the underlying micro-mechanical processes, while the “rock-support interaction” relates to changes in the model behavior when support is considered. In this thesis, rockbolts (reinforce) as well as mesh and shotcrete (retain and reinforce) have been considered and consequently the broader term ‘support’ has been employed (Kaiser et al., 1996). The “rock-support interaction” analysis was naturally conducted after an appropriate “rockmass representation” was achieved. Based on the results of the initial analyses regarding “rockmass representation” and “rock-support interaction”, a modeling approach for the development of a support design tool was selected. The answers to the research questions in Figure 1.2 were developed throughout the thesis and are shown here only to elucidate the thought process of the author. It should be noted that this flowchart is highly simplified, in that the research questions and the associated answers are much more nuanced than what is presented in Figure 1.2.

1.3 Review of Pertinent Literature

This section presents a brief overview of the relevant literature that was reviewed as part of the development of the research objectives presented in Section 1.2. First, brittle rock damage mechanics and associated numerical modeling attempts are summarized, followed by discussions on rock-support interaction as represented in continuum models and finally the applications of
BBMs at small and large-scales are examined. More detailed literature reviews are provided in Chapters 2-12 as relevant to each specific topic considered.

1.3.1 Brittle rock damage process and pillar damage mechanics

The first series of systematic studies on the in-situ damage processes in crystalline rocks was conducted at Canada’s Underground Research Laboratory (URL) and Sweden’s Aspö URL. These studies documented the development of extensile fractures in massive granitic rocks and provided a better understanding of brittle rockmass behavior (Martin, 1997; Diederichs, 2003; Reed, 2004; Diederichs, 2007; Andersson and Martin, 2009; Andersson et al., 2009). The surface-parallel fracturing (or “spalling”) process is primarily a cohesion-loss process that is followed by mobilization of frictional strength (Martin and Chandler, 1994; Martin et al., 1999; Hajiabdolmajid et al., 2003). In other words, the frictional strength is mobilized only when shearing occurs between rock blocks that are formed by intact rock fracturing (cohesion loss).

Based on laboratory testing (Martin, 1993; Martin and Chandler, 1994) and field observations (Martin, 1997; Martin et al., 1999; Diederichs, 2007; Andersson et al., 2009; Cai, 2010), it is now known that the Crack Initiation (CI) threshold and Crack Damage (CD) thresholds represent the lower and upper bound in-situ strengths of massive brittle rockmasses, respectively (Diederichs and Martin, 2010). CI and CD can be determined from laboratory compression tests as the point of crack volumetric strain reversal (which is equivalent to the point of non-linearity in the lateral strain-axial stress curve) and the point of non-linearity in the axial strain-axial stress curve, respectively (Ghazvinian, 2010; Diederichs and Martin, 2010). Other techniques such as acoustic emission monitoring (e.g. Eberhardt et al., 1998; Chang and Lee, 2004; Ghazvinian, 2011; Zhao et al., 2013) or ultrasonic monitoring (e.g. Modiriasari et al.,
2017; Barnhoorn et al., 2018; Shirole et al., 2019; Shirole et al., 2020) can also be used to estimate these thresholds.

Besides being important in-situ strength indices, CI and CD also represent the onset of certain structural changes in an intact rock specimen that is being loaded in compression. In particular, CI corresponds to the stress level when extensile microcracks orientated sub-parallel or parallel to the direction of major principal stress start forming within the specimen (Diederichs, 1999; Cai, 2010). CD, on the other hand, marks the onset of microcrack interaction and coalescence, and is more broadly known as the yield point of the specimen. As loading progresses, the cracks continue to interact and propagate, ultimately leading to the formation of a shear plane at peak strength (Bieniawski, 1967). It is important to note here that unlike CI and CD, which are material-specific stress thresholds, peak strength is dependent on the testing system properties and the constraints imposed by the testing configuration, and as such is not an inherent rock property (Diederichs, 2007; Diederichs and Martin, 2010).

In an underground mining setting, the development of microscopic damage due to excavation-induced stress changes ultimately results in the formation of macroscopic spalling fractures. These fractures, typical in massive to sparsely fractured rockmasses under high stress (Carter et al., 2008), are tensile in nature and are caused by high tangential stresses close to the excavation boundary, where confining stresses are low. At the immediate boundary, spalling occurs at the CI threshold, as there is no geometric restriction to prevent the propagation of nascent microcracks (Diederichs, 1999). However, as one moves away from the excavation boundary and the confining stresses increase, microcrack propagation is impeded, and a larger major principal stress is therefore required to cause spalling. In the principal stress space ($\sigma_1$-$\sigma_3$), the elevated stress levels fall on a line known as “Spalling limit”: $\sigma_1/\sigma_3=k$, where $k=7-10$ for
heterogeneous rocks (Diederichs, 2007) and >10 for homogeneous rocks. Even further from the excavation boundary (higher confinement), the “Spalling limit” lies above the CD threshold, and failure at these locations occurs via shearing (Diederichs, 1999; Kaiser et al., 2010). The majority of previous studies have focused on the peripheral spalling process, as this directly influences the serviceability of the excavations and support requirements (Martin, 1997; Kaiser et al., 2000; Andersson et al., 2009). It is, however, important to also account for the confined shear damage mechanism if the focus of a study is on higher \( W/H \) (i.e. “squatter”) pillars that develop a confined ‘core’ at their centers. Generally speaking, the above principles that were developed based on the study of hard rock damage processes apply to coal as well, as coal is brittle in nature (Mishra and Nie, 2013; Kim et al., 2018) and exhibits mechanical characteristics that are similar to those of hard rocks. For example, the Hoek-Brown constant \( m_i \), which is an indicator of brittleness (Cai, 2010), could be as high as 40.9 for some types of coal (Kim et al., 2020) and is generally >15 for hard rockmasses that exhibit spalling (Carter et al., 2008; Diederichs and Martin, 2010).

Besides being important in-situ strength indices, CI and CD also represent the onset of certain structural changes in an intact rock specimen that is being loaded in compression. In particular, CI corresponds to the stress level when extensile microcracks orientated sub-parallel or parallel to the direction of major principal stress start forming within the volume of the specimen (Diederichs, 1999; Cai, 2010). CD, on the other hand, marks the onset of microcrack interaction and coalescence, and is more broadly known as the yield point of the rock specimen. As loading progresses, the cracks continue to interact and propagate, ultimately leading to the formation of a shear plane at peak strength (Bieniawski, 1967). It is important to note here that unlike CI and CD, which are material-specific stress thresholds, peak strength is dependent on
the testing system properties and the constraints imposed by the testing configuration, and as such is not an inherent property of rocks (Diederichs, 2007; Diederichs and Martin, 2010).

A pillar subjected to increasing load (e.g. during sequential excavation of stopes or longwall face advance) generally exhibits six failure stages, as defined by Krauland and Soder (1987): (1) No fractures, (2) Slight spalling of pillar corners and pillar walls, (3) One or a few fractures near the pillar surface with distinct spalling, (4) Fracturing in central parts of the pillars, (5) One or a few fractures occurring through central parts of the pillar, dividing it into two or several parts, and, (6) Disintegration of the pillar, forming a well-developed hour-glass shape with central part completely crushed. These six stages of failure were suggested on the basis of field observations and are similar to the pillar classification categories proposed by Pritchard and Hedley (1993). Wagner (1980) showed from in-situ testing that when spalling initiates along the periphery, the excess stress is transferred through the center of the pillar, creating a highly confined ‘core’. It is in this ‘core’ that failure occurs in a shear mode rather than in a tensile mode.

Current pillar design methodologies are based only on the amount of load being applied to the pillar and the peak strength, which is a function of W/H (Salamon and Munro, 1967; Obert and Duvall, 1967; Hedley and Grant, 1972; Bieniawski and Van Heerden, 1975; Lunder and Pakalnis, 1997), and essentially neglects any load-carrying capacity of pillars post-peak. From the same in-situ tests discussed above, Wagner (1980) illustrated how the strength of the core increases and the post-peak behavior becomes more ductile with an increase in W/H (Van Heerden, 1975). A direct implication is that a pillar designed based on conventional approaches may be capable of contributing to global stability, even in the post-peak portion of its stress-strain curve, if it is relatively ductile after the attainment of peak-strength. Based on this, the
post-peak behavior of brittle rock needs to be further studied and incorporated into the pillar design process. If numerical models are employed for this purpose, then first an improved constitutive model must be developed such that the entire range of damage mechanisms relevant to rock pillars can be reproduced.

1.3.2 Continuum constitutive models and associated numerical simulations

Previous attempts to numerically simulate the brittle rock damage process using conventional yield criteria (i.e. Hoek-Brown (HB) and Mohr-Coulomb (MC)) have not been successful (Pelli et al., 1991; Martin, 1997; Martin et al., 1999; Hajiabdolmajid et al., 2002; Walton et al., 2016). This is not surprising, as the aforementioned yield criteria were based on laboratory tests or field observations where the mode of failure was primarily shear (Martin, 1997; Kaiser and Kim, 2015). A failure criterion is not expected to reproduce multiple failure modes unless the underlying physical mechanisms are similar (Patterson and Wong, 2005).

The inability of shear yield criteria to model the brittle failure behavior of intact rock led to the development of a Cohesion-Weakening-Frictional-Strengthening (CWFS) model (Hajiabdolmajid et al., 2002; Hajiabdolmajid et al., 2003). This model, originally conceived by Schmertmann and Osterberg (1960) for soils, essentially degrades the cohesive strength and mobilizes the frictional strength simultaneously or non-simultaneously as a function of the plastic shear strain ($\varepsilon_{ps}$), where $\varepsilon_{ps}$ is a well-accepted proxy for damage in the field of rock mechanics (Hajiabdolmajid et al., 2002; Alejano and Alonso, 2005; Diederichs, 2007; Zhao et al., 2010; Walton et al., 2015). In relation to the brittle damage mechanisms discussed previously, the initial cohesive strength of the CWFS model is defined by the CI threshold, while the mobilized frictional strength is prescribed by the “Spalling limit” (Figure 1.3). The CI
threshold is less sensitive to changes in confining stress (Martin, 1997; Diederichs, 1999) and is exceeded by the “Spalling limit” in the $\sigma_1$-$\sigma_3$ space beyond a certain point (black dotted line in Figure 1.3). To the left of this point, the strength degrades from the CI threshold to the “Spalling limit” (rapid propagation of fractures) while to the right, there is some hardening from the CI threshold to the “Spalling limit” (a stress increase is needed to induce the interaction and coalescence of microcracks).

Figure 1.3 Components of the CWFS strength model.

Since its development, the CWFS model has been successfully used in replicating the formation of stress-induced notch around the periphery of tunnels (Hajiabdolmajid et al., 2002; Edelbro, 2009; Edelbro, 2010; Zhao et al., 2010; Lee et al., 2012; Walton et al., 2014; Renani and Martin, 2018a; Dadashzadeh, 2020). This model is functionally similar to the Damage-Initiation-Spalling-Limit (DISL) model of Diederichs (2007), with the only difference being that the CWFS model is based on the linear MC framework, while the DISL model uses the non-
linear HB framework. For simplicity, this thesis only directly considers the CWFS model, although the conclusions drawn should also be generally applicable to the DISL model.

Since the CWFS strength model is most representative of brittle rock damage under low confinement, its application should be restricted only to the surficial portions of a pillar. This is because as the rock at the excavation boundary dilates, it increases the confining stresses within the pillar (Walton et al., 2016), which in turn suppresses the formation of extensile cracks (Diederichs, 2003). It follows that a CWFS model alone is not sufficient in describing the failure behavior of a rock pillar over the entire range of confinement likely to be experienced by the rockmass. A comprehensive yield criterion must reflect the brittle failure mechanism at low confinement and shear failure mechanism at higher confinement within the pillar (Chen, 1993; Esterhuizen and Ellenberger, 2007; Kang et al., 2015; Bai et al., 2019; Li et al., 2019).

In keeping with the idea of a complete yield criterion, a tri-linear/S-shaped criterion was conceptualized by Diederichs (2007) and formalized Kaiser et al. (2011) to model rock pillars of various W/H. The S-shaped envelope considers tensile fracturing/spalling at low confinement and shear at high confinement (Diederichs, 2007), thereby providing realistic strength estimates for a rockmass subject to a wide variety of confining stresses. The strength model of Kaiser et al. (2011) is referred to as the “ultimate S-shaped criterion” in this thesis, as it is a representation of the ultimate strength envelope, rather than the strength envelope that exists at any given state during the damage process; in contrast, the model developed in this thesis is referred to as the “progressive S-shaped criterion” (see Chapter 2). In other words, this criterion only defines the upper bound strength for all confining stress conditions and thereby neglects the gradual evolution of damage that occurs during loading. It is perhaps for this reason that an exponential
increase in strength with W/H that contradicts the empirically estimated trend of pillar strength was obtained using this criterion (Kaiser et al., 2011).

1.3.3 Rock-support interaction and limitations of continuum models in reproducing support effects

The design of support for underground structures has been a topic of extensive research in the field of rock mechanics. Support is an integral part of underground mines, as it helps to maintain the structural integrity of excavations. In particular, mesh and bolts are commonly used to support the roof and walls of underground excavations in rock. Despite its importance, support design is commonly based on empirical approaches or site-specific experience (Larson and Dunford, 1996; Colwell, 2006; Mohamed et al., 2016a). The primary reason is the complexity of the interaction between support elements and the rockmass that makes it difficult to characterize analytically. While numerous studies have focused on developing analytical models for bolt/cable loading mechanics (Hyett et al., 1996; Li and Stillborg, 1999; Ma et al., 2004), these were limited to rather simplified geological conditions (for example, assuming only a single rock fracture, an elastic rock material, etc.). Other authors like Serbousek and Signer (1987) and Grasselli (2005) have conducted laboratory testing, but these studies were again limited by a simplified consideration of the host material and loading conditions.

The functions of support in rocks can be broadly classified as follows (Kaiser et al., 1996): (1) Reinforce: strengthen the rockmass thereby enabling it to support itself; (2) Retain: provide areal coverage of broken rocks to prevent unraveling; (3) Hold: tie retaining elements of the support system to stable rock. While numerical models are a valuable tool for investigation of rock-support interaction mechanisms, there have only been a handful of studies exploring this
topic. Further study is needed to identify if continuum and/or discontinuum modeling methods are capable of realistically reproducing the reinforcement effect of support elements.

Perhaps the most relevant study in this regard is by Bahrani and Hadjigeorgiou (2018), who simulated a drift in the George Fisher mine (Queensland, Australia) using continuum and discontinuum modeling approaches. Support (cable bolts and a shotcrete liner) was found to be significantly more effective in reducing drift convergence in a discontinuum model than in a continuum model. Almost no difference was noted in drift convergence with and without support elements in the continuum model. A similar observation was made by Sinha and Walton (2017b) when modeling rockbolts in FLAC\textsuperscript{3D} granite pillar models. The authors hypothesized that continuum models might be implicitly accounting for the support effect due to their inability to allow discrete block separation during the stress-induced damage formation process. In other words, when an element on the surface of a pillar yields, the model’s inherent formulation does not allow it to be dislodged from the surface and thus maintains a continuous strain-distribution among its elements (termed as ‘strain-continuity’). A direct consequence is the generation of confinement in elements further from the excavation wall due to the yielded surficial elements acting as a boundary to the interior zones. While the function of bolts and mesh in a real scenario with stress-induced fracturing is to hold the damaged peripheral rock zone that is devoid of any self-supporting capacity in place and generate some confinement within the pillar, continuum models may simulate this phenomenon implicitly by sustaining the original geometry of the models.

The ‘strain-continuity’ hypothesis is supported by the results of Walton et al. (2015), who successfully replicated the brittle deformation and progressive damage (calibrated to data from two multi-point extensometers) in a hard rock pillar using FLAC\textsuperscript{3D}. The modeled mine used a
combination of bolt and mesh to provide support to the pillars, but the numerical model itself did not require any support to capture the overall trend in the extensometer measurements. Additionally, none of the continuum models presented by Edelbro (2009), Walton et al. (2014) or Renani et al. (2016) considered surficial supports, but all were able to replicate the rock damage process as observed in the field in supported cases. As stated by Renani et al. (2016): “Sensitivity analyses showed that the shotcrete support and final concrete lining [in the model] did not influence the radial displacements and the extent of plastic zone. Hence, the support was excluded from the back analyses to increase the computational efficiency”.

Lorig and Varona (2013), while analyzing the effect of supports in continuum models, note that “Numerical models, especially continuum models, are not usually capable of realistically simulating the disaggregation of the rockmass as it deforms. The primary use of numerical models, therefore, is to demonstrate strain compatibility between the selected support and the deformations resulting after the support is installed”. Implicit in this statement is an acknowledgement that many continuum models are effectively modeling the behavior of reinforced ground and can therefore be used to study strain compatibility with support. This compatibility does enable continuum models to correctly predict bolt loads as measured in the field for reinforced rockmasses, even if the bolt elements used in the models do not influence the modeled ground behavior in a realistic manner (Zhang et al., 2015; Raju et al., 2015).

It is evident that a different modeling approach is required to realistically capture the influence of support elements and a rockmass undergoing stress-induced progressive damage. BBM is considered a valid alternative for this purpose, as it allows for explicit separation of failed/fractured material. From a physical standpoint, such models should produce true unconfined surfaces as the blocks (representing intact rock) detach upon failure, permitting the
support to ‘pin’ them against the pillar surface. Gao et al. (2014), Kang et al. (2015), and Bai and Tu (2020) have reported large reductions in roof displacements due to installation of supports in excavation-scale BBMs.

1.3.4 Laboratory-scale Bonded Block Models (BBMs)

Grain-scale microstructures in crystalline rocks are known to control their emergent macroscopic mechanical response to loading (Wong et al., 2006; Lan et al., 2010; Farahmand et al., 2015). The microstructure of a rock is generally characterized by grain sizes, grain shapes, grain types, contact properties, elastic properties and crystallographic orientations (Fritzen, 2011). Lan et al. (2010) describe three sources of microstructural heterogeneity: (1) Geometric – due to variability in shape and size of the grains; (2) Elastic – due to stiffness contrast of constituent minerals; (3) Contact – due to variable length, orientation and mechanical properties of grain contacts. While the influence of geometric heterogeneity has been studied using experimental techniques in the past (Olsson, 1974; Eberhardt et al., 1999), recent years have seen a rapid increase in polygonal bonded-block and grain-based modeling studies investigating all aspects of heterogeneity (Lan et al., 2010; Ghazvinian et al., 2014; Farahmand and Diederichs, 2015; Chen et al., 2016; Park et al., 2017; Liu et al., 2018; Zhou et al., 2019). A major advantage of numerical approaches is the flexibility to investigate a wide range of microstructural variations, which are otherwise difficult (and often impossible) to study through laboratory testing.

So-called “grain-based” and “bonded-block” modeling approaches are subsets of the Discrete Element Method (DEM) (Cundall, 1971), as they represent a material space using aggregates of detachable blocks. Two important attributes that have led to their widespread
acceptance are the general resemblance in shape of the modeled blocks to actual mineral grains and the ability of the modeling methods to allow for realistic fracture formation and opening. Over the years, these modeling approaches have been successfully used in reproducing various features of the rock damage process (Kazerani and Zhou, 2010; Lan et al., 2010; Ghazvinian et al., 2014; Bewick et al., 2014; Bahrani et al., 2014; Fabjan et al., 2015; Farahmand and Diederichs, 2015; Chen et al., 2016; Mayer and Stead, 2017; Park et al., 2017; Li et al., 2019; Zhou et al., 2019).

The Grain-Based Modeling (GBM) method uses a combination of bonded circular/spherical particles (Potyondy and Cundall, 2004) in mineral grains and smooth-joints along the grain contacts (Mas Ivars et al., 2008). Alternatively, the Bonded Block Method (BBM) employs a continuum mesh within the blocks and a Coulomb-slip model for the block contacts (Itasca, 2014). BBMs have the potential advantage of scalability, where a laboratory-sized specimen can be easily up-scaled to simulate a field-scale structure (Insana et al., 2016; Farahmand et al., 2018), although the exact parameter scaling laws are not known yet. While both these approaches are capable of realistically replicating the rock fracturing process, the focus of this literature review will be on BBM, the approach used in this thesis.

In BBM, the individual blocks typically are either polygons with four or more sides (Voronoi) or triangles (Trigon). The Voronoi blocks are more representative of the petrographic characteristics of crystalline rocks, but there has been some success in modeling granites using Trigons as well (Gao and Stead, 2014; Gao et al., 2016). A key issue with the use of Trigons is the resulting predisposition towards shear fracturing due to the availability of linear failure pathways (Ghazvinian et al., 2014; Mayer and Stead, 2017; Sinha and Walton, 2019b). This is supported by the fact that in Gao and Stead (2014), fracturing at block boundaries initiated in
shear mode rather than in tensile mode, which is inconsistent with the prevailing understanding of microfracture nucleation in brittle rocks (Kranz, 1983; Diederichs, 2007). It follows that Voronoi BBM, owing to its interlocking block geometry, is better suited for studying the deformation and damage in brittle geomaterials.

There have been a large number of attempts to model the progressive fracturing process in rocks under compression using Voronoi BBM (Christianson et al., 2006; Lan et al., 2010; Kazerani and Zhou, 2010; Norouzi et al., 2013; Gao and Stead, 2014; Fabjan et al., 2015; Farahmand and Diederichs, 2015; Noorani and Cai, 2015; Chen and Konietzky, 2014; Tan et al., 2016; Gao et al., 2016; Park et al., 2017; Li et al., 2017; Bahaaddini and Rahimi, 2018; Stavrou and Murphy, 2018; Li et al., 2019). As previously noted, various representations have been employed in these studies to model the same rock type; those that employed more complex models were typically capable of reproducing a larger number of laboratory-derived attributes. There is, however, a lack of understanding in the general capabilities of the different model representations with respect to their ability to replicate specific rock mechanical attributes. Although a review of literature can provide some insight into the strengths and limitations of the different BBM representations, definite conclusions cannot be drawn due to differences in the specific methodologies employed in the prior studies (e.g. loading mechanism, boundary conditions, 2D versus 3D, different rocks and calibration targets).

In terms of micromechanical modeling of coal, two studies using Voronoi BBM were identified. Bai et al. (2016) and Zhu et al. (2020) used a homogeneous, elastic zone representation to replicate the Young’s modulus (E), Poisson’s ratio (v), UCS, and tensile strength of coals specimens from Chinese mines. There remain areas for improvement in small-
scale coal BBM models to address other aspects of its mechanical behavior, such as cleat-related strength anisotropy (Kim et al., 2018).

1.3.5 Field-scale Bonded Block Models (BBMs)

1.3.5.1 Trigons and Tetrahedral Blocks

Contrary to small-scale applications, a greater proportion of the large-scale BBM studies have been conducted using triangular blocks – Trigons in 2D and Tetrahedra in 3D (e.g. Gao et al., 2014a and b; Gao and Stead, 2014; Garza-Cruz et al., 2014, 2019a, b; Kang et al., 2015; Zhang et al., 2018; Wu et al., 2019). The likely reason is the lower computational burden of a Trigon/Tetrahedra block structure having the same number of potential fracture pathways as that of a corresponding Voronoi structure; this is because Voronoi blocks are polygonal in shape and therefore have a larger area than a trigon block with the same edge length (see Figure 1.4; Azocar, 2016). There is also some anecdotal evidence that suggests that in 3D models, the extra degrees of freedom with respect to block kinematics allow tetrahedral block geometries to replicate dilation in a realistic manner, whereas 3D Voronoi block structures tend to overestimate bulking (Azocar, 2016; Walton and Sinha, 2020).

With respect to the Trigon/Tetrahedra literature, Gao et al. (2014a), Kang et al. (2015) and Gao and Stead (2014) attempted to model failures in coal mine entries, with Gao et al. (2014b) focused specifically on longwall caving in a German coal mine. More recently, Wu et al. (2019) analyzed the damage evolution in different W/H coal pillars to identify an optimum width for stability. Some support guidelines were also presented, but they were mostly qualitative in nature (i.e. the supports were not tested in the models). Only Gao et al. (2014) and Kang et al.
Garza-Cruz et al. (2014) used elastic tetrahedral blocks to simulate the spalling process around a tunnel that was subjected to a hypothetical loading-unloading stress path. Garza-Cruz et al. (2019a and b) employed elastic tetrahedra to study the effect of shear loading on pillar behavior and the effect of four different support patterns on roof displacement at the Eleanore mine (Quebec, Canada), respectively. The latter study (Garza-Cruz et al., 2019b) was focused on understanding the effect of shearing along sub-horizontal roof structures on the integrity of the rockbolts. Although there have been quite a few attempts to model underground structures, none of them quantitatively assessed how well the bulking process is reproduced and whether the rock-support interaction behaviors are realistic.

Figure 1.4 Voronoi and Trigon block(s) with the same edge length.
1.3.5.2 Voronoi Blocks

The relevant mining-focused studies employing polygonal BBMs are those by Preston et al. (2013), Azocar (2016, Bai et al. (2016), Muaka et al. (2017), Ghazvinian et al. (2017), Farahmand et al. (2018), Zhang et al. (2018), Kaiser (2019), Wang et al. (2019) and Bai and Tu (2020). Preston et al. (2013) used Voronoi pillar models to investigate the effect of pillar height on its ultimate strength. This study also considered persistent and non-persistent (10% rock bridge) sub-vertical jointing, as is typically observed in Doe Run mines in Missouri. Muaka et al. (2017) utilized the Voronoi approach to develop a methodology for designing jointed hard rock pillars that were transected by clay-filled shear zones. Wang et al. (2019) studied the failure modes of coal pillars due to the excavation of an adjacent underlying coal seam in China using the Voronoi approach. Farahmand et al. (2018) introduced Discrete Fracture Networks (DFNs) in BBMs with heterogeneous properties and an elastic constitutive model in the zones to replicate the S-shape strength envelope of non-persistently jointed rockmasses and the depth of spalling around the URL test tunnel in Canada. Supports were not considered in any of these studies. In terms of 3D modeling, Ghazvinian et al. (2017) examined the stress distribution and displacements around a horse-shoe shaped tunnel housed in an anisotropic rock with elongated Voronoi blocks while Azocar (2016) attempted to simulate the notch formation around the Mine-By tunnel in URL, Canada.

Instabilities related to water-rich roof layers in a Chinese coal mine were analyzed using excavation-scale Voronoi models by Bai et al. (2016). The effect of support was not explored in detail, since supports were found to be inefficient in controlling the large displacements at the site. Zhang et al. (2018) investigated the effect of fault slippage on the failure of a coal mine entry and developed alternative support patterns to control the large deformations. Results
considering some limited comparisons between unsupported and supported BBMs were presented by Kaiser (2019), but a detailed analysis is not presented. Most recently, Bai and Tu (2020) studied the failure mechanisms of a laminated jointed roof and the effect of different support schemes (combinations of rockbolts, cables, and steel mesh) on the deformation and stress distribution around a coal mine entry. A comparison of the roof deflection without support and with the alternate support schemes was presented, but the results were not validated against any field data. From this discussion, it is apparent that there is a need for research focusing on assessment of intact rock bulking and rock-support interaction using Voronoi BBM.

1.4 Scope Limitations

A key caveat of this thesis is that it focuses on such geological conditions where the behavior of the system (i.e. pillar) is governed by damage to intact material rather than by deformation along distinct structural features. Accordingly, future studies should investigate the effect of structural features on the damage process (for example, fault-induced rockbursting) and how they affect the support requirements. Other limitations of this thesis are as follows: (1) The entire modeling work is based on FLAC$^{3D}$ and UDEC, both of which are Itasca Consulting software. Other continuum modeling approaches like FEM (e.g. RS2, ANSYS) and discontinuum modeling approaches like the combined Finite-Discrete Element Method (e.g. ELFEN, Irazu), Bonded Particle Modeling (e.g. PFC), Lattice Spring Method (e.g. SRMTools) etc. are commercially available and the conclusions drawn in this thesis might be equally applicable to those modeling techniques, but further studies are required to confirm this by reproducing the results shown here (or similar). (2) It might be possible to upscale small-scale BBMs to model large-scale scenarios following some specific rules (similar to those suggested
by Dadashzadeh, 2020), but the issue of upscaling was not addressed in detail in this study. (3) Since UDEC is a 2D software, any out-of-plane deformation and/or stress perturbation is effectively neglected in the BBM models presented in this thesis; in future, a similar study should be conducted using 3D BBMs.

1.5 Thesis Outline

This is a manuscript-based thesis where all but one (Chapter 12) of the main Chapters have been published, accepted, or are under review with peer-reviewed international journals as of the date of thesis publication. The main Chapters, as included in this thesis, are nearly identical to the originally published or submitted individual manuscripts. Each of the main Chapters has its own specific abstract, introduction, main body text, results, discussions, conclusions, and acknowledgements. The references cited in each individual manuscript are presented in a comprehensive list at the end of the thesis.

The thesis consists of thirteen Chapters in total, with the eleven main Chapters (Chapter 2 to Chapter 12) organized in the order of the thesis objectives as described in Section 1.2. There are multiple Chapters (and therefore journal papers) associated with each objective. To depict how each of the main Chapters (Chapters 2-12) relates to a particular objective, a table is presented that groups the different Chapters with respect to their parent objectives (Table 1.1). Some additional results are presented in the form of appendices (A-N) to elaborate on some analyses that help explain methodological details that are not fully described in the individual Chapters. The appendices are also listed in Table 1.1 alongside the Chapters in which they are referenced.
In Chapter 1 of the thesis, a brief review of the literature associated with the rock fracturing mechanisms in context of rock pillars and the numerical modeling methods for rock damage simulation is presented. Subsequently, the gaps associated with brittle rock damage modeling and rock-support interaction modeling are identified and used to formulate the research objectives of the thesis.

In Chapter 2 of the thesis, the progressive S-shaped yield criterion is developed by combining the CWFS strength model at low confinement and a shear yield model at higher confinement. The criterion is subsequently implemented in FLAC\textsuperscript{3D} to model granite and conglomerate pillars of different W/H. The strengths as a function of W/H are ultimately compared to the empirical pillar strength database from the literature to establish the reliability of the yield criterion.

In Chapter 3 of the thesis, a comparison is made between the pillar behaviors obtained from the progressive S-shaped yield criterion (from Chapter 2) and three other criteria from literature. The effects of using a constant dilation angle versus a mobilized dilation angle, incorporating heterogeneity in the strength properties and modifying the length to width ratio on the overall behavior of the pillar are also studied.

In Chapter 4 of the thesis, the progressive S-shaped yield criterion is employed for modeling the damage in a coal pillar rib at the West Cliff Mine in Australia. The model is calibrated against displacements and stress measurements from the field for various locations of the longwall face. The issue of parameter non-uniqueness is highlighted by presenting the results for a model that is calibrated only to the displacement measurements but not to the stress measurements.
Table 1.1 Relationship between main thesis chapters and research objectives identified in Section 1.2.

<table>
<thead>
<tr>
<th>Objective</th>
<th>Description of the Chapter</th>
<th>Chapter #</th>
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<tr>
<td><strong>Objective 1: Develop a rock yield criterion for brittle rocks</strong></td>
<td>Progressive S-shaped yield criterion is developed &amp; model results are compared to empirical data</td>
<td>Chapter 2</td>
<td>-</td>
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<tr>
<td></td>
<td>Other yield criteria are compared; effects of dilatancy, heterogeneity &amp; pillar length to width ratio are investigated</td>
<td>Chapter 3</td>
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<tr>
<td><strong>Objective 2: Test if the criterion is applicable to coal</strong></td>
<td>Continuum modeling of West Cliff Mine case study to reproduce field measurements</td>
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<tr>
<td><strong>Objective 3: Compare support effect in continuum models and BBMs</strong></td>
<td>Effect of support in continuum &amp; BBM granite pillar models and the continuum West Cliff coal model (Chapter 4) is studied</td>
<td>Chapter 5</td>
<td>A, B, C</td>
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<td>West Cliff Mine case study is modeled using BBM &amp; the support effect is tested against field data</td>
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<td></td>
<td>Model representations &amp; support influence are tested in pillar-scale BBMs</td>
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<td><strong>Objective 5: Demonstrate an integrated 3D continuum-2D discontinuum modeling approach</strong></td>
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<td><strong>Objective 6: Analyze damage process in pillars using geotechnical data and calibrated numerical models</strong></td>
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<td></td>
<td>Using the 3D-2D modeling approach to develop a site-specific model of a Western US coal mine, instrumented by the author</td>
<td>Chapter 12</td>
<td>M, N</td>
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In Chapter 5 of the thesis, the effects of support on ground behavior in continuum and discontinuum models are analyzed. For that purpose, supports of varying densities are incorporated in the FLAC$^{3D}$ granite pillar models, the West Cliff Mine model, and BBM granite pillar models. Both the Voronoi and the Trigon BBM block geometries are tested. To determine which of the models exhibit behaviors closer to reality, bulking factors are computed and contrasted with the empirical data of Kaiser et al. (1996).

In Chapter 6 of the thesis, the West Cliff Mine case study is modeled with elongated blocks to account for the anisotropy of coalmasses. This case study is unique in the sense that two adjacent pillars were instrumented, but the pillars had different rib support patterns. To assess whether BBMs have the capability of reproducing ground behaviors for varying support densities, the model is first calibrated to the field data from one of the pillars, followed by modification of support to match that of the adjacent pillar, and then the model displacements are compared to those measured in the field. The effect of block shape is also investigated by replacing the elongated blocks with isotropic polygonal blocks.

In Chapter 7 of the thesis, different block and contact (homogeneous/heterogeneous properties) and zone (elastic/inelastic constitutive model) representations are tested in laboratory-scale BBMs to understand the relationship between model complexity and the capabilities of these micromechanical models to replicate specific rock mechanical attributes. Subsequently, the calibration of the small-scale BBM is conducted independent of the large-scale models, to reproduce a large suite of attributes derived from laboratory data.

In Chapter 8 of the thesis, the representation of zone dilation angle is modified in the calibrated small-scale BBM from Chapter 7 and results are compared to post-peak behavior
observed in the laboratory data. The 2D Digital Image Correlation (2D-DIC) technique is implemented to quantify the strains induced by the explicit separation of the blocks. To date, the application of the 2D-DIC approach has been restricted only to real material testing; this study, therefore, is a methodological advancement to extend its applicability to discontinuum models.

In Chapter 9 of the thesis, a large-scale model complexity analysis (Chapter 7) is conducted by modifying the zone representation (elastic/inelastic) of a pillar BBM to match the stress-strain curves for pillars of varying W/H. The support effect for each W/H geometry is also quantified through the calculation of bulking factor. Lastly, a BBM with intra-block fracturing capability (each block composed of more blocks) is developed and the support analysis is repeated for this model.

In Chapter 10 of the thesis, an integrated 3D continuum – 2D BBM approach is developed to model the progressive damage process in a granite pillar located in Creighton Mine, Sudbury, Canada. In particular, a cross-section of the pillar is simulated using a BBM with a load path from a calibrated mine-scale FLAC\textsuperscript{3D} model. The model is calibrated by matching the displacements recorded at six points within the pillar by a multi-point borehole extensometer (MPBX) in the field. The calibrated model is subsequently used to understand how the support and load path influenced the damage evolution in the pillar.

In Chapter 11 of the thesis, a large stress cell data set (44 borehole pressure cells) from a Western US coal mine is analyzed to improve our understanding of the global stress redistribution process in longwall panels. The BPC measurements are converted into actual rock stress using the Bieniawski’s (1984) stress gradient equation. Various analyses are conducted, including but not limited to the following: determination of abutment angle, analysis of stress
changes in the yielded and intact portions of the coal pillars and visualization of the three-
dimensional stress patterns in the pillars and abutment for different locations of the longwall
face. Additionally, trends in the field data (stress changes in the rib with damage progression)
were compared to those observed in the West Cliff Mine model from Chapter 4.

In Chapter 12 of the thesis, displacement and subsidence measurements from a different
Western US mine, instrumented by the author, are used to calibrate a mine-scale FLAC\textsuperscript{3D} model.
Using the integrated modeling approach demonstrated in Chapter 10, a pillar-scale BBM is
generated with elongated block geometry (Chapter 6) and this model is also calibrated to the
displacement data. The effect of two different support schemes is subsequently tested. This
Chapter further advances the integrated modeling approach and provides an example as to how
to develop and use a site-specific local BBM for support design.

Chapter 13 presents a summary of the primary findings of this research along with a
discussion on the conclusions and contributions of this thesis.
CHAPTER 2

A PROGRESSIVE S-SHAPED YIELD CRITERION AND ITS APPLICATION TO ROCK PILLAR BEHAVIOR

This paper has been published in the journal *International Journal of Rock Mechanics and Mining Sciences* (Sinha and Walton, 2018a). It is reprinted with permission from Elsevier with some minor variations.

2.1 Abstract

Yielding of hard rock pillars under moderate to high stresses involves the formation of excavation-parallel extensile fractures. In recent decades, researchers have found that this behavior can be best replicated by a Cohesion-Weakening-Frictional-Strengthening (CWFS) model which captures the non-simultaneous mobilization of cohesion and friction; this is a mechanism that has been experimentally proven to occur in rocks undergoing brittle failure. In the context of rock pillars, the extensile fracturing process is limited only to the surficial portions. The inner core of rock pillars, on the other hand, fails through a shear mechanism. A realistic rock strength criterion must therefore account for the two different failure behaviors. To this end, this study introduces an improved yield criterion that represents small-scale damage processes (extensile cracking under low confinement and semi-brittle shear under higher confinement) while exhibiting an emergent pillar behavior consistent with what has been observed in the field. The failure criterion was implemented in the finite difference software FLAC$^3$D which was then used to investigate the effect of width to height ratio on the strength of pillars. The pillar strengths matched an empirical database from literature, which establishes the
capability of the new yield criterion to model brittle rock material behavior considering both
tensile and shear mechanisms.

2.2 Introduction

With advances in technology and gradual depletion of near-surface ores, it becomes
essential to exploit increasingly deeper deposits. Present day metal and non-metal mines have
descended to depths beyond 2 kms, where the magnitudes of pre-mining stresses are very high.
In such highly stressed ground, most mining systems rely primarily on unmined rock pillars for
maintaining the functional integrity of underground openings and roadways. Generally speaking,
a pillar is capable of supporting the overburden load as long as its strength exceeds the applied
stress. Two-dimensional and three-dimensional numerical models are often used to evaluate the
stability of mine structures, including pillars.

There are three broad classifications of numerical modeling methods: continuum,
discontinuum and hybrid. Each method has its own advantages and disadvantages in context of
ground control problems. The continuum method treats the ground as an equivalent continuous
material with properties that approximately reflect the effect of joints and discontinuities
(Fairhurst and Pei, 1990). Although this method lacks the ability to explicitly capture stress-
induced fracturing, it has been successfully used to model case studies under various geological
and mining conditions (Hajiabdolmajid et al., 2002; Shan et al., 2005; Edelbro, 2009; Zhao et al.,
2010a; Kang et al., 2014; Chugh and Sinha, 2015; Walton et al., 2015b; Walton and Diederichs,
2015b; Sinha and Chugh, 2016). For relatively competent rockmasses which deform primarily
through stress-induced damage to intact rock, continuum modeling represents a useful analysis
approach.
The accuracy of continuum model results is heavily governed by the selection of a proper yield criterion and associated input parameters. Selection of a yield criterion for a particular set of conditions is ideally based on knowledge of the micro-mechanical damage processes that ultimately control the global failure of the system. In this study, a new yield criterion for intact rock is proposed, and its ability to produce model results consistent with pillar behaviors observed in-situ is demonstrated. Previous modeling attempts using Hoek-Brown (Martin and Maybee, 2000), strain-softening Mohr-Coulomb (Iannacchione, 1999; Mortazavi et al., 2009) and S-shaped yield criterion (Kaiser et al., 2011) have predicted a near-exponential increase in strength with W/H ratio which contradicts the convex shape of empirical pillar strength equations (Hedley and Grant, 1972; Lunder and Pakalnis, 1997) as well as the trend obtained in this study. The differences in observed behavior is likely related to the inability of the three criteria to appropriately capture the damage mechanisms of intact rocks.

In-situ damage processes in crystalline rocks were first systematically analyzed at Canada’s Underground Research Laboratory (URL) and Sweden’s Aspo URL. These studies documented the development of brittle spalling and fracture generation in massive granitic rocks and provided a better understanding of brittle rockmass behavior (Martin, 1997; Reed, 2004). It was found that in massive to sparsely fractured rock under high stress conditions, damage near excavation boundary was dominated by extensile fractures, which macroscopically appeared as spalling (Hoek et al., 1995; Diederichs, 2003, 2007; Diederichs et al., 2003). The surface-parallel fracturing process was primarily a cohesion-loss process, followed by mobilization of friction (Martin and Chandler, 1994; Martin et al., 1999). Attempts to numerically simulate this behavior using shear yield models were not met with success (Pelli et al., 1991; Martin, 1997; Hajiabdolmajid et al. 2002). This is not surprising, since the yield criterions were based on
laboratory tests where the mode of failure was primarily shear (Martin, 1997; Kaiser and Kim, 2015). A failure criterion is not expected to reproduce multiple failure modes unless the underlying physical mechanisms are similar (Paterson and Wong, 2005).

The inability of shear yield criteria to model the brittle failure behavior sparked the development of a Cohesion-Weakening-Frictional-Strengthening (CWFS) model (Hajiabdolmajid et al., 2002). The CWFS model is a strain-dependent yield criterion which accounts for the non-simultaneous mobilization of friction and cohesion. The changes in these two strength components is controlled by plastic shear strain - a variable which is used as a proxy for damage (Hajiabdolmajid et al., 2002; Diederichs, 2007; Walton et al., 2015b). Since its development, the CWFS model has been successfully used in replicating the stress-induced notch formations around the periphery of tunnels (Hajiabdolmajid et al., 2002; Edelbro, 2009; Zhao et al., 2010a). However, most of the previous studies only focused on rockmass behavior under low confinement conditions. This study considers the behavior of rocks under low as well as high confinement, by using pillars as the study context due to their potential to experience wide ranges of confinement across their extents.

The applicability of the CWFS model should be restricted to the low-confinement surficial portions of a pillar because as the rock at the excavation boundary dilates, it increases the confining stresses within the pillar which in turn suppresses the formation of extensile cracks. This has been mathematically demonstrated using the concepts of fracture mechanics (Diederichs, 2003). It therefore follows that a CWFS model alone is not sufficient in describing the failure behavior of a rock pillar over the entire range of confinement likely to be experienced by the rockmass. A comprehensive yield criterion must reflect the brittle failure mechanism at low confinement and shear failure mechanism at higher confinement.
The authors suggest the need to use a progressive S-shaped strength envelope that can account for the different failure mechanisms in the different confinement regimes. In the last decade, an S-shaped failure envelope was conceived by Diederichs (2007) and formalized by Kaiser et al. (2011). This envelope has a strong theoretical basis and combines the CWFS strength model at low confinement and a conventional shear yield envelope at higher confinement. Nevertheless, these precursory studies only describe the final shape of the envelope, ignoring the evolutionary nature of the damage process. As will be shown later in this study, it is not sufficient to capture the shape of only the ultimate strength envelope; the complete strength envelope must be defined for all material states (i.e. with respect to material damage). The reason is the complex interrelationships between the mobilization of cohesion, friction, and dilation that ultimately control material behavior. Without accounting for the evolution of these parameters, small-scale damage and stress redistribution processes are not captured properly. In summary, a rock yield criterion must satisfy two important criteria: (a) account for the evolving nature of the damage process, and, (b) be consistent with the expected damage mechanisms for the entire range of expected confining stresses.

The progressive S-shaped yield criterion as proposed in this study satisfies both these criteria and when incorporated in the finite difference software, FLAC³D, it can be used to replicate some well-documented pillar behaviors seen in the field. This study utilizes the calibrated rockmass parameters of Walton et al. (2015b) and Walton and Diederichs (2015b) as a starting point for the pillar models. The choice of the two case studies was based on their disparate rock UCS value - the Creighton Granite (2015) has a UCS of 220 MPa while the studied conglomerate has a lower UCS of 95 MPa (2015a). The pillar model was used to investigate the effect of W/H and L/W ratios on the global strengths of pillars. The focus of this
study is to introduce the progressive S-shaped criterion and demonstrate its capabilities in reproducing observed pillar behaviors.

2.3 Development of a progressive S-shaped yield criterion

The progressive S-shaped yield criterion satisfies the two important and fundamental criteria of a realistic rock yield criterion. This section will elucidate how the two criteria are met and provide mechanistic interpretation regarding the different segments of the envelope. The development of the criterion was motivated by the need for an improved failure envelope for numerical modeling as evidenced by the inability of previous continuum modeling attempts to replicate observed rock pillar strength trends. Some recommendations for selection of material parameters for the criterion will also be provided.

The evolving nature of the damage process was captured by relating the different segments of the envelope to the incremental plastic parameter, mathematically defined by (Itasca, 2016a):

\[
\Delta \epsilon^{ps} = \frac{1}{\sqrt{2}} \sqrt{(\Delta \epsilon_1^p - \Delta \epsilon_m^p)^2 + (\Delta \epsilon_m^p)^2 + (\Delta \epsilon_3^p - \Delta \epsilon_m^p)^2}
\]  

\[
\Delta \epsilon_m^p = \frac{1}{2} (\Delta \epsilon_1^p + \Delta \epsilon_3^p)
\]

where, \(\Delta \epsilon^{ps}\) is the incremental plastic parameter, \(\Delta \epsilon_1^p\) and \(\Delta \epsilon_3^p\) are the plastic strain increments in the principal directions. There is no globally accepted indicator that can quantify system damage corresponding to a particular stress/strain level. As a solution, Itasca (2016a) developed the incremental plastic parameter and has been using it in FLAC and FLAC\(^{3D}\). This parameter has two major advantages: (a) Easy implementation in constant-strain quadrilateral (in FLAC\(^{3D}\)) and
triangular (in FLAC2D) zones, and, (b) Approximate linear relationship with maximum plastic shear strain \( (\gamma_p) \) given by \( \varepsilon^{ps} = \frac{1}{2} \gamma^p \) (Walton, 2014). The plastic shear strain corresponding to the degradation of cohesion and mobilization of friction for different rock types can be estimated from loading-unloading tests on laboratory scale samples.

The progressive S-shaped criterion has three basic envelopes: (a) Yield, (b) Peak, and, (c) Residual (Figure 2.1). Each of these envelopes corresponds to a particular cumulative plastic shear strain \( (\varepsilon^{ps} = \sum \Delta\varepsilon^{ps}) \); these values depend heavily on the rock being considered. The evolution of the yield into peak and residual envelope ensures that the evolving nature of the damage process is accounted for. The left and the right portion of the criterion, separated by a blue line in Figure 2.1, exhibit the characteristics of a CWFS and shear yield model, respectively. This transition from extensile failure to shear failure is generally believed to occur at \( \sigma_3 \approx 0.1 \) UCS (Valley et al., 2012); for a given rock type, more specific estimates can be obtained by identifying the point of intersection between the spalling limit and the crack damage thresholds (as discussed later). The bimodal nature of the envelope allows the criterion to represent both distinctive failure mechanisms.

The plastic shear strain corresponding to the degradation of cohesion \( (\varepsilon_c^{ps}) \) and mobilization of friction \( (\varepsilon_\phi^{ps}) \) may or may not be equal for a particular rock. For example, the Creighton Granite has \( \varepsilon_c^{ps} = 0.0025 \) and \( \varepsilon_\phi^{ps} = 0.0055 \) (Walton et al., 2015b) while the studied Conglomerate has \( \varepsilon_c^{ps} = \varepsilon_\phi^{ps} = 0.003 \) (Walton and Diederichs, 2015b). The change from one intermediate envelope to the next, as a function of the plastic shear strain, must account for the similar/dissimilar \( \varepsilon_c^{ps}, \varepsilon_\phi^{ps} \) values. To illustrate how this can be achieved within the framework of the progressive S-shaped criterion, consider Figure 2.1 corresponding to Creighton Granite. In
In this case, the cohesion degrades completely with partial friction mobilization (\(\varepsilon^{ps} = 0 - 0.0025\)), followed by constant cohesion (at its degraded level) and complete mobilization of friction (\(\varepsilon^{ps} = 0.0025 - 0.0055\)). In such a case, a fourth envelope has to be defined between the peak and yield envelope which corresponds to the degraded cohesion and semi-mobilized friction angle. The semi-mobilized friction angle can be computed by proportionally scaling it with plastic shear strain. In cases where \(\varepsilon^{ps}_c = \varepsilon^{ps}_\phi\), friction mobilizes and cohesion degrades simultaneously, eliminating the need for an intermediate envelope.

The diverging trend of the intact Hoek-Brown fit and Crack Damage (CD) threshold (right side of the peak envelope) in Figure 2.1 is consistent with the findings of Martin (1997).
Even though the rock analyzed by Martin (1997) is not mineralogically the same as Creighton Granite, the authors expect them to behave in a similar fashion, given their granitic composition, comparable strength and damage threshold levels. The CD threshold in Figure 2.1 was found to be 76% of the HB fit at $\sigma_3=0$, which is slightly higher than that obtained by Martin (1997). This discrepancy could be attributed to the choice of a linear CD representation in the progressive S-shaped criterion over the curved representation adopted by Martin (1997).

The progressive S-shaped criterion has 14 input parameters in total: 6 cohesions, 6 friction angles and 2 plastic shear strains to constrain the evolution of the yield surface. When developing this criterion, two assumptions were made: (a) the intersection point of the low and high confinement portion will be constant for all envelopes, and, (b) all envelopes will coincide for some value of confinement (the point at which deformation becomes perfectly plastic). These two constraints reduce the degrees of freedom such that the entire criterion can be defined by 11 principal parameters instead of 14.

The methodology recommended for generating a progressive S-shaped criterion for a given rock type is as follows: (a) First, the cohesion, friction angle and plastic shear strain for peak envelope is selected. (b) The confining stress which separates the high and low confinement portions is then computed. (c) Next, the cohesion of the left side and the friction angle for both sides of the Yield envelope is selected. The cohesion for the right side of the Yield envelope could then be calculated to ensure the same intersection point of the low and high confinement portions of the Yield envelope as for the Peak envelope. This then constrains the confining stress point at which all three envelopes would coincide. (e) Finally, the cohesion and friction angle of the left side of the residual envelope is selected. The right segment of the residual envelope is calculated based on the two constraints stated above.
2.3.1 Yield envelope

The left portion of the yield envelope corresponds to the Crack Initiation (CI) threshold while the right portion follows a modified form of the Mogi’s Line. CI marks the onset of random extensile cracking, with cracks mostly oriented along the direction of maximum principal stress ($\sigma_1$). For brittle cracking (eg. CWFS model), CI has been found to be an appropriate representation of the initial in-situ yield threshold (Martin, 1997; Diederichs, 2007; Walton et al., 2015b) and can be determined using acoustic emission techniques (Eberhardt et al, 1998), volumetric stress-strain curve (Brace et al., 1996) and/or axial/lateral stress-strain curves (Lajtai, 1974; Diederichs, 2007; Nicksiar and Martin, 2012).

In the initial phases of formulation, CI was considered as the first point of yield over the entire range of confinement, including for shear yield. It is unclear, however, what the general mechanistic significance of CI is for failure mechanisms which are not brittle in nature. Some recent laboratory tests on Indiana Limestone indicated that at higher confinement, when the failure mechanism transitions from brittle to semi-ductile, the initial yield is governed by a steeply inclined line (relative to CI) with a slope similar to that of Mogi’s line (Walton et al., 2017). The reason for such a behavior is intuitive; the increased confinement suppresses the formation of tensile cracks causing the initial strength to extend above CI. From the axial stress-strain, axial stress-volumetric strain and axial stress-circumferential strain plot of an Indiana Limestone sample tested at 50 MPa confinement (Figure 2.2a), it can be seen that although some axial strain non-linearity initiates at Point i, the volumetric strain reversal does not occur until the stress level is close to peak (Point ii). The high lateral confinement suppresses the lateral dilation forcing grain-scale rearrangement to occur between points i and ii.
Figure 2.2 (a) Axial, Volumetric and Circumferential strain versus minimum principal stress for an Indiana Limestone sample at $\sigma_3 = 50$ MPa, (b) Modified Boltzmann sigmoid curve fitted to 54 crack volumetric strain reversal data points (20 uniaxial and 34 triaxial).

For the same set of laboratory tests on Indiana Limestone, a modified Boltzmann sigmoid curve has been fit to 54 crack volumetric strain reversal data points (Figure 2.2b). The zone of transition, marked by a blue dotted line, has a negative intercept and a slope of about 5.5 which is significantly steeper than the Mogi’s line (Mogi, 1966). Such a gradient is not surprising, given the fact that the Mogi’s line is an overall fit to a curved envelope (Byerlee, 1968) over a confinement range of 0-600 MPa; the actual slope of the brittle-ductile transition line, even for non-carbonate rocks, can be significantly different at lower confinement levels. For a low porosity rock like Creighton Granite (<1%), where the grains have limited ability for rearrangement, the slope of the brittle ductile transition should be closer to 4.4. Over the course of model calibration, this was ultimately chosen to be 3.8 for this study.

The point of crack volumetric strain reversal is generally used as the in-situ yield strength even though it does not coincide with the point of axial strain non-linearity. For instance, in CWFS strength model, CI is used as the in-situ yield although yield initiates at CD in laboratory.
Under high confinement, however, axial strain non-linearity begins at an early stage (Point i; Figure 2.2a). The lateral strain curve remains linear up to total volumetric strain reversal (Point ii; Figure 2.2a) or crack volumetric strain reversal, both of which coincide beyond the brittle-ductile transition (Walton et al., 2017). In light of the previous discussion and to ensure consistency with the in-situ definition of crack volumetric strain reversal, the modified Mogi’s Line has been chosen as the right side of the Yield Envelope. To define the slope of the right portion of the yield envelope, the authors suggest conducting laboratory triaxial testing where possible. If it is not feasible to achieve the required confinement level, this parameter could be obtained from back analysis instead.

2.3.2 Peak envelope

The left portion of the peak envelope is coincident with the Spalling Limit while the right portion is defined by the Crack Damage (CD) threshold. The spalling limit describes the ultimate strength of spalled ground at low confinement (more specifically, it is the residual strength at very low confinement and mobilized strength at moderate confinement). In the principal stress space, it can be mathematically represented by \( \sigma_1/\sigma_3 = 10-20 \), corresponding to a friction angle of \( 55^\circ - 65^\circ \) (Kaiser et al., 2000; Diederichs, 2007). This relationship assumes zero residual cohesion; in reality, this may be non-zero depending on the structure and petrographic characteristics of the rock. Recently, Walton (2019) conducted a statistical study on back-analyzed CWFS input parameters available in literature. This study provides guidelines on selection of peak cohesion, peak friction and residual friction. In absence of any laboratory data, these could be used as a starting point for numerical models.
The slope of the spalling limit, referred to here as the mobilized friction angle, is related to the peak frictional component of strength and can be correlated with the \( m_i \) parameter of Hoek-Brown failure criterion. To estimate the slope of the spalling limit, the authors suggest determining \( m_i \) from tensile strength, uniaxial and triaxial strength data and then employing the relationship proposed by Walton (2019). An empirical estimate of \( m_i \) can also be obtained from tables such as those published by Marinos and Hoek (2000).

The Crack Damage threshold (CD) marks the onset of unstable crack interaction and is oftentimes referred to as long-term laboratory shear strength (Martin and Chandler, 1994). Prior to achieving CD, tensile microcracks remain randomly distributed throughout a sample. When a critical crack intensity is attained, these microcracks interact forming meso-cracks (Diederichs, 2003). Further interaction and propagation of these cracks lead to the formation of a shear failure plane.

Previous experimental observations have confirmed the peak strength to be a system-dependent variable (specifically, a function of specimen geometry and loading conditions; Hudson et al., 1972). Martin and Chandler (1994) identified the CD to be a true material property. Based on the fact that pillar loading is a slow phenomenon, CD threshold was chosen as the peak shear strength envelope instead of the short-term laboratory peak envelope. CD is generally associated with inelastic deformation in the axial direction (\( \sigma_1 \)) and can be determined as the point of non-linearity in the axial stress-strain curve or the point of reversal in the axial strain-volumetric strain curve obtained from laboratory compression testing (Diederichs and Martin, 2010). It can also be identified using acoustic emission techniques (Eberhardt et al., 1998; Diederichs and Martin, 2010; Ghazvinian, 2010).
2.3.3 Residual envelope

The residual envelope was derived through simultaneous degradation of both portions of the peak envelope. The reduction in the right portion of the envelope is analogous to the degradation from peak to residual in a conventional shear strength model. The rationale behind degrading the left portion is based on the experimental observations of Martin and Chandler (1994) who found that with increasing damage, friction reduces by 30-50% while cohesion remains constant. Again, due to absence of sufficient laboratory data, no general guideline for selecting this portion of the envelope could be established. Ultimately, it will be demonstrated later in this paper that the strength of pillars is not significantly affected by the residual envelope. This is because of the large plastic strain lag between the peak and the residual envelope used in the model. Based on the findings of Martin and Chandler (1994), a 30-50% reduction in friction angle of spalling limit over 4 times the plastic shear strain associated with the peak envelope was employed in this study.

2.4 Application of the progressive S-shape yield criterion in modeling pillars

The primary objective of this paper is to establish an improved rock yield criterion that can be easily implemented in modeling software. For the purposes of this study, the progressive S-shaped criterion was implemented in FLAC\textsuperscript{3D} (2016a). FLAC\textsuperscript{3D} is a three-dimensional finite difference code developed by Itasca that is used by researchers and consultancy firms worldwide. The bilinear strain-softening ubiquitous joint plastic constitutive model was selected for this study. This model was selected as it allowed for two yield surface segments to be used, as is required for the progressive S-shaped criterion, and it also allowed for each segment to evolve as a function of cumulative plastic shear strain.
The focus of this study is on those geologic conditions where the behavior of the system of interest (in this case, a pillar) is governed by damage to intact material rather than by deformation along distinct structural features. Since the effect of through-going joints or pre-defined planes of weakness is not relevant here, very high strength properties were assigned to the ubiquitous joints. Previous studies have found CI and CD threshold limits to be true material properties for intact rocks (Martin and Chandler, 1994; Diederichs and Martin, 2010). Since fractures were not considered in the current study, a laboratory to field scale degradation in parameters was not applicable.

2.4.1 Model development

An 8 m cubic pillar model with 0.166 m mesh elements was developed and loaded quasistatically through two elastic beams on either side (Figure 2.3). Roller boundaries were assigned to the sides of the beam while the top and the bottom surfaces were subjected to a constant velocity boundary condition. To eliminate the dynamic effects of loading, a very low velocity ($5 \times 10^{-8}$ m/step) was selected. This was based on a sensitivity analysis of the effect of loading rate on the pre- and post-peak portion of the stress-strain curve. The pillar corners were rounded with two rows of elements to ensure a smooth transfer of stress through the model. Other variations of rounding (i.e. non-rounded, 1, 2, 3 and 4 rows of elements) were tested as a part of model development. It was found that beyond 2-element rounding, the stress and displacement contours were relatively consistent. Additionally, the 2-element rounding geometry represents a reasonable approximation of the typical shape of pillars in the field.

The rockmass parameters of Creighton Granite (Walton et al., 2015b) were used as a starting point for the pillar models which were tested. During the calibration process, they were
adjusted to reproduce the empirical pillar strength curve, which will be discussed in Section 2.5.1. The Creighton mine case study was performed using the classical CWFS strength model; therefore, only some of the relevant parameters, like the cohesion and friction for the left portions of the Yield and Peak Envelope, were available. The outstanding parameters had to be estimated from uniaxial and triaxial datasets for similar rock type. Table 2.1 lists the calibrated parameters of the progressive S-shaped criterion in $\sigma_1$-$\sigma_3$ space and the plastic shear strains associated with them. The rockmass input parameters for the bilinear strain softening model was computed using Eqs 2.3 and 2.4 and are also listed in Table 2.1.

$$c = \frac{UCS \cdot (1 - \sin \phi)}{2 \cos \phi}\quad (2.3)$$

$$\phi = \sin^{-1}\left(\frac{k-1}{k+1}\right)\quad (2.4)$$

where, $c$ is cohesion, $\phi$ is friction angle, and $k$ is the slope of the envelope in $\sigma_1 - \sigma_3$ space.

Table 2.1 Thresholds and rockmass parameters relevant to the Granite pillar model.

<table>
<thead>
<tr>
<th>Segments of the S-shaped envelope</th>
<th>Threshold in $\sigma_1$-$\sigma_3$ space</th>
<th>Model Input Parameters</th>
<th>Plastic shear strain (milli)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield (Left portion)</td>
<td>$\sigma_1 - \sigma_3 = 81$</td>
<td>Cohesion (MPa)</td>
<td>40.5</td>
</tr>
<tr>
<td>Yield (Right portion)</td>
<td>$\sigma_1 - 3.8\sigma_3 = -31.2$</td>
<td>Friction (degrees)</td>
<td>-8.0</td>
</tr>
<tr>
<td>Intermediate (Left portion)</td>
<td>$\sigma_1 - 2.2\sigma_3 = 15.6$</td>
<td></td>
<td>5.3</td>
</tr>
<tr>
<td>Intermediate (Right portion)</td>
<td>$\sigma_1 = 2.6\sigma_3$</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Peak (Left portion)</td>
<td>$\sigma_1 - 5.6\sigma_3 = 24.7$</td>
<td></td>
<td>5.3</td>
</tr>
<tr>
<td>Peak (Right portion)</td>
<td>$\sigma_1 - 2.7\sigma_3 = 140.8$</td>
<td></td>
<td>43.2</td>
</tr>
<tr>
<td>Residual (Left portion)</td>
<td>$\sigma_1 - 3.9\sigma_3 = 0.4$</td>
<td></td>
<td>0.1</td>
</tr>
<tr>
<td>Residual (Right portion)</td>
<td>$\sigma_1 - 3.5\sigma_3 = 17.5$</td>
<td></td>
<td>4.7</td>
</tr>
</tbody>
</table>

50
Mechanistically speaking, the plastic shear strain corresponding to the degradation of cohesion and mobilization of friction is an inherent property of a particular rock. However, since the formulation of FLAC$^3$D requires partitioning the problem domain into constant-strain quadrilaterals, the dimension of the yield region is dictated by the zone edge length. As such, the plastic shear strain selected for different mesh sizes will generate different model results (in terms of the volume of failed rockmass and associated stress redistribution process). This has a serious implication on the applicability of laboratory derived plastic strain values to numerical
models. A viable solution, recommended by Itasca, is to recalibrate the model to field measurements every time the mesh dimension is modified (Itasca, 2016b). Itasca (2016b) also suggests a linear relationship between the plastic shear strain and dimension of the mesh.

Considering the hypothetical nature of the study, it was not possible to recalibrate the model parameters to extensometer measurements of Walton et al. (2015b). Instead, a linear scaling factor of 1.9 (mesh size in Walton et al. (2015b) was 0.3125 m in comparison to 0.166 m in this study) was chosen to approximately account for the grid dependency.

Dilation angle plays a major role in the damage evolution process within a pillar. With the onset of yield, inelastic lateral strains develop which typically increases the confinement in the system. This is a localized phenomenon but has global implications on pillar behavior. The minimum principal inelastic strain is mathematically related to the maximum principal inelastic strain through the dilation angle (Walton and Diederichs, 2015a). Several studies have found that the dilation angle is a function of the confining stress and the plastic shear strain and can be represented by a mathematical model (Alejano and Alonso, 2005; Zhao and Cai, 2010; Walton and Diederichs, 2015a; Walton et al., 2015a). Accounting for this mobilized nature of dilation angle is necessary to completely capture the micro-damage processes within a pillar. For that purpose, the Walton-Diederichs (WD) dilation angle model has been employed in this study (Walton and Diederichs, 2015a). It must be noted here that the particular dilation model used in this study is not too critical; the authors expect other mobilized dilation models to yield similar results. The relevant parameters for the pillar model were taken from Walton et al. (2015b) to reasonably capture the brittle dilation of Creighton Granite. The parameters represent a pre-peak dilation with a faster decay in the post-peak portion, consistent with brittle rock behavior. The
reader is referred to Walton and Diederichs (2015a) for details on selection of WD input parameters.

FLAC³D provides the flexibility of customizing the data-extraction process through the use of a built-in programming language called FISH. Three table functions were developed for this study which recorded the maximum principal stress, minimum principal stress and the cumulative plastic shear strain for every element in the pillar. Since this is a computationally intensive process, the three variables were recorded every 2000 solution steps. Another FISH function was defined to measure the average relative displacements of the top and bottom of the pillar. These relative displacements were later converted to strain by dividing by the pillar height.

A total of eight model geometries corresponding to width-to-height (W/H) ratios of 0.5, 1, 1.5, 2, 2.5, 3, 3.5 and 4 were set up. The width of all the models were kept constant at 8 m while the height was varied. To ensure homogeneity of the applied strain, the velocity was scaled with respect to pillar height and is represented by the following equation:

\[ V \left( m/\text{step} \right) = 5 \times 10^{-8} \frac{\text{Height}}{8} \]  

(2.5)

All 8 models were allowed to equilibrate until the global strengths were captured. Since the applied velocities were lower for squatter pillars and the failure transitioned from a brittle to ductile mechanism, more solution steps were required for larger W/H pillar models. The model was evaluated by comparison of outputs with empirical pillar strength curves, the hourglass shape of the pillar core (Krauland and Soder, 1987) and the development of stresses along the mid-section of the pillar (Wagner, 1974). The authors believe that the robustness of the developed yield criterion could be fairly substantiated on the basis of these assessments.
2.4.2 Model results

Figure 2.4 shows the stress-strain curves obtained for W/H ratios of 0.5, 1, 2, 3 and 4. In the models, the failure behavior transitioned from a brittle to ductile mechanism with increase in the W/H ratio. With increase in W/H ratio, the proportion of the outer shell/confined core (Valley et al., 2012) increases causing the strength to escalate. The rate of increase in strength is expected to decline with higher W/H ratio, which most of the previous continuum numerical models (Iannacchione, 1999; Martin and Maybee, 2000; Mortazavi et al., 2009; Kaiser et al., 2011) failed to capture. An exponential increase in strength beyond W/H of 1.5-2.0 as observed from previous continuum modeling attempts would imply that such pillars are practically indestructible.

The W/H ratio of 2 defines the transition from a brittle to ductile behavior. This is strictly dependent on the rock type under study and is a result of the complex interaction of the constitutive model, geomechanical parameters and dilation angle. There is a drastic increase in strength between W/H of 2 and 3. To perceive the underlying reason, the elemental stress states for W/H of 2 and 3 were plotted at their peak strength (Figure 2.5). It was found that the strength of W/H=2 pillar is majorly controlled by a combination of the spalling limit (left side of the peak envelope) and the CI threshold (left portion of the yield envelope) while for W/H=3, it is mainly the spalling limit and the modified Mogi’s line. A greater proportion of the stress states (% in Figure 2.5) exceeded $\sigma_3=40.5$ MPa line for W/H=3. Since the squatter pillar strengths were being controlled by the right side of the yield and peak envelope, a drastic jump is natural.

There is some amount of non-linearity in the stress-strain curve prior to achieving the peak strength. This hardening is more discernible for higher W/H ratios as it occurred over a
wider range of axial strain. In slender pillars, there is geometric freedom for formation of cross shear planes, causing a near-immediate collapse. This is illustrated through plastic shear strain plots (Figure 2.4) for different loading stages. The localization of plastic yield initiates at the corner followed by the formation of a through-going concave shear plane. The geometric constraint in squatter pillars prevents the formation of such through-going shear planes, restricting damage to the outer portions of the pillar. Since the evolution of the stress-strain curve is guided by the increasing yield in the system, squatter pillars showed enhanced hardening behavior.

![Figure 2.4 Average stress-strain curve for W/H of 0.5, 1, 2, 3 and 4 with shear band formation along the edges of the figure.](image)

Interestingly, pillars with W/H>1 continued to carry load over significant ranges of strain after achieving their peak strengths. This is significant from the perspective of pillar design, since most pillar design methodologies only consider the peak strength of pillars. For example, although pillars with W/H = 1 and W/H = 2 have nearly identical peak strengths, W/H = 2 may
be sufficient to ensure stability due to their continued load carrying capacity following the attainment of peak strength. Figure 2.6 shows the total plastic energy/m\(^3\) dissipated at the first major drop in the stress-strain curve. If the energy dissipation process of a pillar system can be controlled through use of appropriate support, then the overall pillar dimension can be properly optimized through an understanding of its energy dissipation capacity.

Figure 2.5 Elemental stress states colored by plastic shear strain for (a) W/H=2, (b) W/H=3.

The average stress-strain curve only provides a broad overview of the failure behavior of pillars. A complete understanding requires interpretation of both the micromechanical damage mechanisms and their emergent effect on the macroscopic behavior. To that end, the stress and plastic strain path of a central element (point iii, Figure 2.7a) for the W/H=3 model has been plotted (Figure 2.7b and c). Initially, with increase in applied strain, the stress in this sampled zone follows the yield envelope up to 57 MPa confinement without undergoing any yield (\(\varepsilon_p \sim 0\)). The increase in confinement and axial stress in this portion of the curve is related to the dilation of peripheral elements. Since damage propagates inward, an element located on the outer side of the sampled element will be slightly more damaged at any particular point in time. This is
displayed in Figure 2.7c where the plastic strain paths for the sampled element and an adjacent element are represented by solid and dotted lines, respectively.

![Graph showing plastic energy dissipation](image)

Figure 2.6 (a) Calculation of dissipated plastic energy per unit volume from stress-strain curve, (b) Total plastic energy per unit volume dissipated for different W/H models.

In an attempt to explain the series of complex interactions that occurred within the models, 8 points on the two plastic strain curves were chosen (N’, S’, N”, S” etc. in Figure 2.7c). With onset of inelastic deformation, the neighboring element experienced more dilation than the sampled element (S’, N’). The confining stress, as a result, reduced in the neighboring element but increased in the sampled zone. Beyond S’, sufficient damage nucleated in the sampled zone allowing it to dilate and subsequently raise the confinement in the neighboring element (note the confinement change from N’ to N” and S’ to S”). Such a loading-unloading process is typical and was noted in all the pillar models.

Both the elements continued to dilate and relax between N”, S” and N””, S”” leading to a further reduction in the load carrying capacity of the sampled zone. Note the lower confinement and higher plastic strain localization at N”” in comparison to S””. Late in the pillar damage
process, at N’’’ and S’’’, the sampled and the neighboring element possess the same level of plastic damage.

Figure 2.7 (a) Location of the four points on a horizontal section along the center of the W/H=3 pillar, (b) Stress-path for an element located at the center of W/H=3 pillar, (c) Plastic-shear strain path for the sampled point and an adjacent element, (d) Confinement path of Points i, ii, iii and iv as a function of step number.

The initial behavior of each element is localized and is majorly controlled by its immediate adjacent neighbors. However, with sufficient damage, the behavior transitions from a local to a global scale whereby a group of zones govern the deformation and stress pattern. For clarity of understanding, four points (namely i, ii and iv, in addition to Point iii; see Figure 2.7a) were selected. Their locations were chosen to ensure that the local behavior of the inner and
outer portions of the pillar could be adequately captured. This was necessary to understand how small-scale damage processes lead to a macroscopic pillar failure.

The evolution of confinement for the four points was plotted against the solution step number (Figure 2.7d). Some interesting observations and conclusions can be derived from this figure: (a) The step number corresponding to the first drop in confining stress is higher for points located deeper into the pillar. This is consistent with the fact that damage progresses from the outer to inner regions. (b) The segment bounded by green lines shows that the confinement reduces for Points i and ii but increases for iii and iv. Physically, the outer portions are dilating, resulting in a confinement increase in the pillar core. It is very clear that instead of local interactions, the behavior of the pillar elements is actually being governed by blocks/slabs whose thickness varies with progressive loading. (c) The section bounded by black lines also demonstrates an increase in confinement for points iii and iv and some relaxation for i and ii. The fall in the confinement stress path for point iii coincides with the axial stress reversal in Figure 2.7b and the reversal in Figure 2.7c (see between N''', S''' and N''''', S''''''). (d) The simultaneous rise in confinement for point ii, point iii & iv in the narrow region between the green and black demarcated zones in Figure 2.7d occurs due to dilation of the first 1-1.5 m of the pillar edge (refer to the continued fall in confinement of point i in this region).

A logical concern is that the stress path may follow the residual envelope producing pillars with infinite strength. Such a behavior has indeed been demonstrated in previous modeling studies (Martin and Maybee, 2000; Mortazavi et al., 2009; Kaiser et al., 2011). In this case, where the S-shaped criterion is used in conjunction with WD dilation model, the inverse relationship of dilation angle with plastic strain and confinement creates a feedback loop limiting the magnitude of confinement that could be generated in the core. Thus, with continued loading,
the core yields followed by a complete collapse. It is very important to realize here that the selection of a dilation model is equally important as is the selection of a yield criterion. Failure to account for both these aspects could lead to unrealistic results being obtained.

A pillar subjected to increasing load (e.g. during sequential excavation of stopes) exhibits six failure stages as defined by Krauland and Soder (1987): (a) No fractures, (b) Slight spalling of pillar corners and pillar walls, (c) One or few fractures near surface with distinct spalling, (d) Fracturing in central parts of the pillars, (e) One or a few fractures occurring through central parts of the pillar, dividing it into two or several parts, and, (f) Disintegration of the pillar forming a well-developed hour-glass shape with central part completely crushed. The six stages of failure were suggested by Krauland and Soder (1987) on the basis of field observations. A reliable pillar model should be able to capture most of these stages. An attempt is made here to perform a direct comparison to establish the credibility of the developed yield criterion. Specifically, two aspects of a failing pillar are targeted: (a) hourglass shape of the core, and, (b) progressive localization of stress along the mid height of the pillar.

Figure 2.8 (a, b) shows the stress concentration around the core of the pillar for W/H of 2 and 3. As a pillar is perpetually loaded, the sides spall off which channels the excess load though the core - the shape of this highly stressed volume is similar to an hourglass. The models developed in this study distinctly capture the hourglass shape for pillars with W/H>1. In slender pillars, the formation of through-going shear planes prevent the development of any confined core. The larger base of the hourglass for wider pillars reflects their higher load-carrying capacity.
Figure 2.8 (a, b) only illustrates the stresses corresponding to the solution step when the global peak strengths were achieved. It, however, does not show the evolution of these stresses as the model is subjected to increasing load. To that end, 6 stages were selected for the W/H=3 model that could adequately bracket the temporal variation of stresses within the pillar (Figure 2.8c-h). The plots provided show maximum principal stress ($\sigma_1$) along a horizontal plane passing through the pillar center. The results are similar to the boundary-relaxation-core-loading phenomenon observed by Wagner (1974) in the field. As spalling initiates, the peripheral elements start to yield, pushing the stresses deeper into the pillar. The excellent correspondence with the findings of Wagner (1974) is intuitively satisfying and serves to further support the robustness of the progressive S-shaped criterion.

Figure 2.8 Concentration of vertical stress around the core of the pillar with (a) W/H = 2, (b) W/H=3, (c-h) development of vertical stress along the mid-height of the pillar with increasing damage.
2.5 Comparison to empirical pillar strength database

2.5.1 Pillar database and progressive S-shaped criterion results

Over the years, there has been significant development in rockmass characterization
techniques and numerical modeling capabilities. Despite these advances, pillar design continues
to rely predominantly on empirical formulae and design charts. Both these approaches are based
on statistical analysis of field observations with empirical/numerical estimation of stresses. A
variety of methods ranging from tributary area theory (Hedley and Grant, 1972) to more complex
two/three dimensional finite element (Von Kimmelmann et al., 1984) and boundary element
modeling (Krauland and Soder, 1987; Potvin et al., 1989) were utilized in past for estimating the
average pillar stresses at the point of failure.

Most of the numerical modeling works considered the rockmass to be homogeneous,
isotropic and perfectly elastic. Although this may be an acceptable approximation in some cases,
it does not account for the loss in load carrying capacity due to plastic yield and subsequent
loading of the neighboring pillars. It also does not consider the fact that some of the failures may
be structure-driven rather than stress-driven, particularly for lower W/H pillars. In the room and
pillar mining method, where the pillars are equidimensional and located in a near-regular grid,
there could be multiple pillars subjected to elevated stress level rendering the selection of one
failed pillar practically impossible. Despite the limitations inherent in any individual study, the
empirical systems overall work well and represent a reliable starting point for design.

Most empirical formulae and charts relate the normalized strength to the W/H ratio of
pillars. In context of rock pillars, the well-known works are those of Lunder and Pakalnis (1997),
Krauland and Soder (1987), Potvin et al. (1989), Sjoberg (1992) and Hudyma (1988) with Hedley and Grant’s (1972) being the most commonly used pillar design formula. Despite its popularity, its formulation has been based on some radical assumptions that may or may not be applicable to all geo-mining conditions (Malan and Napier, 2011). Firstly, the power-law form of the equation, adopted from the Salamon and Munro (1967) formula, was developed for square pillars; the Uranium mine database that Hedley and Grant used consisted of long slender pillars. The inherent assumption then is that the strength of square pillars is more or less similar to long slender pillars. Secondly, the entire formulation was constrained by only three failed pillar cases in contrast to 27 failed cases in Salamon and Munro (1967). Thirdly, the simplistic tributary area concept, combined with horizontal stress estimates from two nearby mines, was used to compute the failure load of the inclined pillars. The question then remains is how has it been successfully implemented in pillar design all over the world? One possible reason could be that although the formulating database considered hard rocks (UCS=210 MPa), its successful application may have been restricted to moderate strength (UCS<200 MPa) rock pillars only.

A major concern in some of the listed studies is the fact that a variety of rocks with different mechanical characteristics were considered before segregating the datasets (Potvin et al., 1989; Lunder and Pakalnis, 1997). Even though the pillar strengths were normalized to UCS to account for the variability in geology, other factors (like brittleness, dilation etc.), which affect the W/H trend in strength, were neglected. In the absence of additional information on rock properties, UCS was chosen as a proxy for those other factors in this study. First, all available data points in literature were combined and then filtered into three cases: a) Soft: UCS in the range 70-140 MPa, b) Moderate: UCS in the range 140-200 MPa, and c) Hard: UCS in the range
200-300 MPa. Figure 2.9 (a-c) shows the three groups with a visually estimated line separating stable and failed case histories.

With increase in UCS, a rightward shift in the steep portion of the curve was noticed (Figure 2.9d). If the segregation was not performed and the entire dataset was used instead, then all three divisions would be expected to follow a single trend. The difference in the shape is not a mere coincidence; it is related to the brittleness and dilation characteristics of the rock type. The friction angle of the CI threshold has been found to decrease with increase in rock UCS (Walton, 2019). Stronger rocks typically exhibit a larger lag between the mobilization of friction and degradation of cohesion. The delay in strength mobilization prevents pillar-scale hardening behavior from occurring at moderate W/H ratios. Additionally, in context of dilatancy, stronger rocks tend to maintain their capacity to dilate under confined conditions more in the early stages of inelastic deformation (Walton and Diederichs, 2015a) which means that smaller pillar cores in moderate W/H ratio pillars can dilate and lose confinement more readily.

Figure 2.10a shows the model results obtained using parameters representative of Creighton Granite compared with the failed, unstable and stable case histories for the UCS group 200-300 MPa. The progressive S-shaped criterion model clearly delineates between the stable and failed case histories, and the overall convex shape is consistent with what has been historically associated with this type of relationship (Hedley and Grant, 1972; Lunder and Pakalnis, 1997). Up to a W/H of 1.5, the strength is practically constant while it increases drastically between W/H of 1.5 and 2.5. The increase in strength with W/H has been demonstrated extensively in the past; the shape, however, does not conform to Martin and Maybee (2000), Mortazavi et al. (2009) and Kaiser et al. (2011). Due to unavailability of
failed/unstable data points, the squatter portion of the curve could not be well constrained; the trend, nonetheless, is consistent with the expected convex shape.

Figure 2.9 Empirical strength database divided into (a) UCS: 70-140 MPa, (b) UCS: 140-200 MPa, (c) UCS: 200-300 MPa, (d) comparison of the strength envelopes for each UCS range, (e) Combined database, (f) comparison of the three division with the combined database.
For purposes of comparison, the model outputs have been overlaid with different pillar strength equations and the three previous modeling results (Figure 2.10b). Except for Krauland and Soder (1987), all the other equations seem to over-predict the strength of slender pillars. This is not unlikely given that Lunder and Pakalnis (1997) did not segregate the dataset while Krauland and Soder (1987) and Von Kimmelmann et al. (1984) developed the empirical curves for rocks with UCS of 94 MPa (sulphide ore in Limestone) and 100 MPa (metasediments), respectively. The issues related to the study by Hedley and Grant (1972) have been discussed previously. It becomes apparent that segregation of the dataset and understanding the domain of applicability of the empirical design methods are a necessity to correctly estimate pillar strengths in a particular geological setting.

Figure 2.10 (a) Stable, Unstable and failed cases (UCS>200 MPa) with black line indicating model results, (b) Comparison of progressive S-shaped results with empirical strength equations and previous modeling attempts.

Another interesting observation is that both Krauland and Soder (1987) and Lunder and Pakalnis (1997) underestimate the strength of squatter pillars. The development of these empirical methods required classifying pillars under stable, failed and unstable groups through
visual examination. Oftentimes, squat pillars exceed their serviceability limits through surficial spalling processes but their core remains intact. Hence, a pillar which may have been classified as ‘failed’ may actually be capable of carrying additional load. Misinterpretation of pillar states in the formulating database could be a probable reason for the observed strength discrepancy. In context of the numerical modeling results, the stark difference in trend is likely related to the inability of the three criteria to capture the damage mechanisms of intact rocks, augmented by an inaccurate representation of dilation angle. A detailed comparison of the four criteria is a topic for future research.

### 2.5.2 Model behavior for weaker rock

The grouping of the empirical database reveals the mechanistic differences in pillar failure mechanisms as a function of its constituent material. Indeed, the behavior of a softer pillar is different from a harder one – a fact that many previous studies have neglected. Figure 2.9 (e-f) compares the strength curves obtained from the three groups to that fitted to the entire database. Clearly, the W/H ratio corresponding to the steep increase in strength is overestimated for stronger rocks (UCS>140 MPa). For a W/H ratio of 1-2, the combined curve predicts a much higher strength than the 200-300 group. The 200-300 MPa group represents brittle rocks where the extent of spalling controls the peak strength.

To demonstrate the ability of the progressive S-shaped criterion to capture the varying trends in the strength curve, a weaker rock type (a conglomerate with a UCS of 95 MPa) was modeled based on the parameters provided by Walton and Diederichs (Walton and Diederichs, 2015b). To make a more focused comparison with the empirical database, only pillar case studies with UCS values between 75 MPa and 125 MPa were considered. Because the original study in
Walton and Diederichs (2015b) used the CWFS strength model, some of the outstanding parameters of the progressive S-shaped criterion had to be estimated on the basis of engineering judgement. The parameters were then adjusted to calibrate the model strength results to the empirical data. The same model geometry and loading conditions, as described in Section 2.4.1, were used. A total of 7 models were simulated with W/H of 0.5, 1, 1.5, 2, 2.5, 3 and 3.5.

Table 2.2 lists the rockmass parameters for the Conglomerate model; the post-yield dilation parameters were chosen from Walton and Diederichs (2015b) to represent a moderate strength rock with slower dilation decay in the post-peak portion of the model. Unlike Creighton Granite, the mobilization of friction and degradation of cohesion occurs for the same value of plastic shear strain. This is one of the main reasons, in addition to dilation angle model changes, for the disparity seen in the trend of the strength curves.

Table 2.2 Thresholds and rockmass parameters relevant to the Conglomerate pillar model.

<table>
<thead>
<tr>
<th>Segments of the S-shaped envelope</th>
<th>Threshold in $\sigma_1$-$\sigma_3$ space</th>
<th>Model Input Parameters</th>
<th>Plastic shear strain (milli)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield (Left portion)</td>
<td>$\sigma_1-1.8\sigma_3=42.5$</td>
<td>Cohesion (MPa) 16</td>
<td>Friction (degrees) 16</td>
</tr>
<tr>
<td>Yield (Right portion)</td>
<td>$\sigma_1-3.2\sigma_3=31.5$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peak (Left portion)</td>
<td>$\sigma_1-12.2\sigma_3=5.6$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peak (Right portion)</td>
<td>$\sigma_1-2.4\sigma_3=80$</td>
<td>Cohesion (MPa) 0.8</td>
<td>Friction (degrees) 58</td>
</tr>
<tr>
<td>Residual (Left portion)</td>
<td>$\sigma_1-3.8\sigma_3=2.9$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Residual (Right portion)</td>
<td>$\sigma_1-3.7\sigma_3=4.2$</td>
<td>Cohesion (MPa) 1.2</td>
<td>Friction (degrees) 34.8</td>
</tr>
</tbody>
</table>

The average stress-strain curves for W/H=1, 1.5, 2, 3 and the model peak strengths overlaid with the empirical database are shown in Figure 2.11 (a, b). The model results were able to replicate the increase in pillar strength between the W/H = 0.5 and W/H = 1.5 cases as shown
in the empirical data. The ability of the model results to separate the failed and stable case histories within the constrained domain for both stronger more brittle rock types (i.e. Creighton Granite) and rocks with a more moderate strength (i.e. conglomerate) suggests that the progressive S-shaped criterion has broad applicability.

Figure 2.11 (a) Average stress-strain curve for Conglomerate pillar models, (b) Stable, unstable and failed cases (75<UCS<125 MPa) with black line indicating model results.

2.6 Conclusion

This study has presented an improved rock yield criterion that has the capability of capturing small-scale damage processes while exhibiting an emergent pillar behavior consistent with what has been observed in the field. It accounts for extensile-spalling behavior in low confinement areas and shear-based failure in high confinement areas using plastic shear strain as a proxy to damage. Most of the defining parameters are related to damage threshold levels and can be obtained through laboratory rock testing. In absence of such testing, the reader can refer to the guidelines proposed by Walton (2019) as a starting point followed by calibration of the most uncertain parameters to achieve a better fit to field measurements.
Numerical models of pillar loading successfully demonstrated the hourglassing of the pillar core and the progressive localization of stress along the mid-section in agreement with the findings of Krauland and Soder (1987) and Wagner (1974). Some of the previous empirical studies on pillar strength considered a variety of rocks with significantly different UCS. Such a classification may not be accurate in explaining the behavior of pillars within a particular rock type. As a solution, this study has segregated the database by UCS and utilized two end-member groups to assess the validity of the numerical models. It was found that the progressive S-shaped criterion can successfully encompass the failure mechanisms in rock pillars composed of both very strong and moderate strength rocks.

The rock pillar model developed in this study was subsequently utilized in assessing the effect of W/H ratio on the pillar strength. The models predicted an overall convex trend which is consistent with empirical findings but in contradiction with previous modeling studies. The choice of yield criterion and representation of the dilation angle seemed to be the root cause for this disparity. Nonetheless, the criterion developed in this study was successful in representing the behavior of rock pillars. Further endeavors are being made to generalize the selection of the input parameters for different rock types.

2.7 Acknowledgements

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CHAPTER 3

NUMERICAL ANALYSES OF PILLAR BEHAVIOR WITH VARIATION IN YIELD CRITERION, DILATANCY, ROCK HETEROGENEITY AND LENGTH TO WIDTH RATIO

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3.1 Abstract

With recent advances in numerical modeling, design of underground structures increasingly relies on numerical modeling-based analysis approaches. While modeling tools like the discrete element method (DEM) and the combined finite-discrete element method (FDEM) are useful for investigating small-scale damage processes, continuum models remain the primary practical tool for most field-scale problems. The results obtained from such models are heavily dependent on the selection of an appropriate yield criterion and dilation angle. Towards improving its capabilities in handling mining-related problems, the authors have previously developed a new yield criterion (called progressive S-shaped criterion). The focus of the current study is to demonstrate its use in modeling rock pillars through a comparative analysis against four other yield criteria. In addition to the progressive S-shaped criterion, only one out of the four other criteria predicted a trend in strength consistent with an empirical pillar strength database compiled from the literature. Given the closely-knit relationship between yield criterions and dilation angle in controlling the overall damage process, a separate comparison was conducted using a mobilized dilation model, a zero degree dilation angle and a constant non-zero dilation angle. This study also investigates the impact of meso-scale heterogeneity in
mechanical properties on the overall model response by assigning probability distributions to the input parameters. The comparisons revealed that an isotropic model using a combination of progressive S-shaped criterion and mobilized dilation angle model is sufficient in capturing the behavior of rock pillars. Subsequently, the pillar model was used to assess the effect of L/W ratio on peak strength. The effect of increasing L/W was minimal for slender pillars and at L/W > 4.

3.2 Introduction

In the last decade, significant advances have been made in numerical modeling tools, both in terms of their ability to simulate physical phenomena at a wide range of scales as well as the associated computational capabilities. In the field of rock mechanics, large-scale design problems are often geometrically complex and are further exacerbated by the heterogeneous nature of most rockmasses. Although empirical relationships can provide rough design parameters, they are generally constrained by limitations of the database which was used for their development. Furthermore, field-scale testing programs developed to minimize the need for abstract analyses may be infeasible due the associated costs. Numerical modeling tools represent a convenient alternative for the purposes of analysis and design. While DEM and FDEM are better suited for investigating small-scale rock damage processes (Munjiza, 2004; Jing and Stephansson, 2007; Ghazvinian et al., 2014; Lisjak and Grasselli, 2014; Farahmand and Diederichs, 2015; Yan et al., 2016; Mayer and Stead, 2017), continuum approaches remain the primary practical tool for mine-scale simulations. The outputs obtained from these models are heavily dependent on the choice of a yield criterion (Hajiabdolmajid et al., 2002; Edelbro, 2010; Walton et al., 2015b).
Recently, the authors have developed a rock yield criterion that accounts for the differing failure modes of sparsely fractured rockmasses over a wide range of expected confining stresses. In order to highlight the relative advantages of using this criterion, a comparative analysis has been performed in this study with four other yield criteria being considered. An associated goal is also to investigate the effect of meso-scale heterogeneity on the overall behavior of these models, as it has not been widely studied in the literature. Because mine pillars are well-studied structures in the field of rock mechanics, they have been chosen as the primary context for this study.

A pillar typically fails through tensile fracturing in the outer peripheral region and through shear in the inner confined portions. Previous studies have found conventional shear yield criteria to be inadequate in capturing the brittle spalling process (Pelli et al., 1991; Martin, 1997); as a result, the Cohesion Weakening Frictional Strengthening (CWFS) strength model was formulated. Over the years, this strength model has been successfully used in modeling brittle damage near underground excavations (Hajiabdolmajid et al., 2002; Edelbro, 2009; Walton et al., 2016; Renani and Martin, 2018a, b). However, for modeling pillars, an integration of CWFS and shear yield is necessary because neither of the two criteria is capable of capturing the macroscopic pillar behavior on its own. To that end, the progressive S-shaped criterion was developed to account for both failure mechanisms.

A number of other yield criteria have been utilized in the past for modeling rock pillars. Some of the major yield criteria used for this purposes include the ultimate S-shaped criterion (Kaiser et al., 2011; Kaiser and Kim, 2015), Strain-softening Mohr-Coulomb (Iannacchione, 1999; Mortazavi et al., 2009), Brittle Hoek-Brown (Martin and Maybee, 2000) and CWFS strength model (Hajiabdolmajid et al., 2002; Walton et al., 2015b). In order to avoid any
confusion with the progressive S-shaped criterion, the non-progressive S-shaped criterion 
developed by Kaiser et al. (2011) will be termed as the ultimate S-shaped criterion in this paper. 
None of these studies demonstrated the ability of a continuum model to reproduce the 
empirically observed trend of pillar strength as a function of width to height ratio (W/H). This is 
unsurprising given these modeling approaches do not properly account for the fundamental 
aspects of brittle rock behavior. The ability of the progressive S-shaped criterion to capture this 
trend is a direct consequence of its development based on the fundamental progressive damage 
mechanisms of intact rock (Sinha and Walton, 2017a; Sinha and Walton, 2018a). In this study, a 
numerical comparison in FLAC$^3$D is used to identify the key differences between these various 
yield criteria.

Selection of a dilation model is equally important as is the selection of an appropriate 
yield criterion. In the past, rock dilatancy has often been neglected by selecting zero dilation 
angles in numerical models (Edelbro, 2009; Chugh and Abbasi, 2011). Although this may be 
acceptable under certain geologic conditions, it is erroneous to extend it to hard rock pillars 
where excavation boundary dilatancy controls the failure dynamics (Walton et al., 2015b). 
Laboratory tests have found dilation angle to vary as a function of plastic shear strain and 
confining stress, that can be adequately captured by a mathematical model (Alejano et al., 2005; 
Zhao and Cai, 2010; Walton and Diederichs, 2015a). This study uses one such dilation model, 
called the Walton-Diederichs (WD) model (Walton and Diederichs, 2015a), in conjunction with 
the progressive S-shaped criterion to simulate rock pillar behavior. Since dilation angle interacts 
with the yield criterion to ultimately control the overall model behavior, the models were re-run 
with a zero and constant non-zero dilation angles and the responses were compared to that 
obtained using the WD model. This approach not only helped in isolating their influences but
also highlighted the impact of using simplified representations of rock dilatancy on rock pillar behavior.

With respect to the issue of accounting for rockmass variability in numerical models, the overall rockmass is typically approximated by an equivalent homogeneous media in continuum models. While this approach has been successfully used in replicating in-situ behavior (Walton et al., 2015b; Sinha and Chugh, 2015), it is of interest to evaluate to what extent stochastic consideration of variability in strength properties influences overall model results. The effect of spatial variability in material has been previously investigated for slopes (Esterhuizen, 1990; Hsu and Nelson, 2006) and room and pillar mine panels (Reinsalu, 2000). In this study, based on the findings of Langford and Diederichs (2015), the input parameters of the progressive S-shaped criterion were randomly derived from a normal distribution with 5% and 10% coefficient of variation (CV) and assigned to individual elements in the model. The variability in the element-level properties was then correlated with the variability in the macro-behavior of slender and squat pillars.

The previous comparative analyses led to the conclusion that a homogenized continuum model using progressive S-shaped yield criterion and a mobilized dilation model is sufficient in modeling rock pillars. Since the validity of the pillar model has been established through a comparison against an empirical pillar database (Sinha and Walton, 2018a), it was utilized here to investigate the effect of length to width (L/W) ratio on pillars strengths.
3.3 Progressive S-shaped criterion: Background and pillar modeling

The development of the progressive S-shaped criterion was prompted by the inability of previous yield criteria to capture the expected behavior of rock pillars. The new criterion is based on the precursory works of Diederichs (2007), Kaiser et al. (2011) and Kaiser and Kim (2015) with modifications to account for the progressive nature of the micro-damage processes in rock. The cumulative plastic shear strain parameter ($\varepsilon^{\psi}$) is well established as a proxy for brittle damage in continuum models and was utilized in the formulation of the progressive S-shaped criterion (Hajiabdolmajid et al. 2002; Zhao et al., 2010a; Walton et al., 2015b; Itasca, 2016a). The criterion combines the CWFS strength model at low confinement and a shear yield model at higher confinement with an ultimate upper envelope conforming to S-shape, as proposed by Diederichs (2007). Figure 3.1 shows a schematic illustration of the progressive S-shaped criterion. The approach in its current form is restricted to nearly-intact rockmasses where the behavior is not significantly influenced by pre-existing fractures.

![Figure 3.1 Conceptual layout of the Progressive S-shaped criterion (after Sinha and Walton, 2018a).](image)

Figure 3.1 Conceptual layout of the Progressive S-shaped criterion (after Sinha and Walton, 2018a).
The criterion consists of three envelopes: (a) Yield, (b) Peak, and, (c) Residual; these three envelopes evolve into each other as a function of $e^{\epsilon}$. The blue line in Figure 3.1 separates the low confinement (corresponding to CWFS model) and high confinement (corresponding to the shear yield) portions of the progressive S-shaped criterion while the red lines correspond to intermediate envelopes. Mechanistically, the three envelopes are related to damage thresholds in rocks and are briefly described as follows: (a) Yield envelope: The left portion corresponds to Crack Initiation (CI) threshold while the right portion is a modified form of the Mogi’s Line. This envelope marks the first point of yield in-situ. (b) Peak envelope: The Yield envelope evolves to the Peak envelope over a user-defined value of cumulative plastic shear strain. The left segment is related to the Spalling Limit while the right side corresponds to the Crack Damage (CD) threshold. (c) Residual envelope: The Peak envelope is degraded to the Residual envelope over the entire range of expected confining stresses. The CI and CD thresholds can be estimated from laboratory tests on intact rocks (Diederichs, 2007; Diederichs and Martin, 2010) while the spalling limit can be determined from the empirical relationship proposed by Diederichs (2007). Although the degradation of the peak to the residual envelope was based on the findings of Martin and Chandler (1994), this portion of the progressive S-shaped criterion is associated with some uncertainty and may require calibration when applied to specific case studies.

Justification for the selection of these damage thresholds as part of the progressive S-shaped criterion and guidelines for determining the input parameters are provided by Sinha and Walton (2018a). For the purposes of this study, the parameters are chosen to conform to Creighton Granite (UCS~220 MPa), with values from Sinha and Walton (2018a). All simulations ran used the Walton-Diederichs (WD) mobilized dilation angle model (Walton and
Diederichs, 2015a). Table 3.1 and Table 3.2 list the progressive S-shaped yield criterion and the WD model input parameters, respectively. When incorporated in a pillar model in FLAC\textsuperscript{3D} (Sinha and Walton, 2018a), this type of model has been shown to replicate the hourglassing phenomenon (Krauland and Soder, 1987) and progressive localization of stress along the mid-height of the pillar (Wagner, 1974) observed in the field. To further test the reliability of the model results, the strength for different W/H ratios were overlaid on an empirical pillar strength database compiled from the literature (shown later in Figure 3.4). The database only considered cases studies with a rock UCS exceeding 200 MPa (comparable to the UCS of Creighton Granite), and this resulted in inclusion of the datasets presented by Hedley and Grant (1972), Hudyma (1988) and Sjoberg (1992). It was found that the model results could precisely demarcate between the failed and stable case histories and exhibited a convex shape that is consistent with what has been typically associated with this relationship. The reader is referred to Sinha and Walton (2018a) for additional details on this study.

Table 3.1 Thresholds and rockmass parameters relevant to the pillar model (from Sinha and Walton, 2018a).

<table>
<thead>
<tr>
<th>Segments of the S-shaped envelope</th>
<th>Threshold in $\sigma_1$-$\sigma_3$ space</th>
<th>Model Input Parameters</th>
<th>Plastic shear strain ($10^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield (Left portion)</td>
<td>$\sigma_1$-$3=81$</td>
<td>40.5</td>
<td>0</td>
</tr>
<tr>
<td>Yield (Right portion)</td>
<td>$\sigma_1$-$3.8\sigma_3=-31.2$</td>
<td>-8.0</td>
<td>35.7</td>
</tr>
<tr>
<td>Peak (Left portion)</td>
<td>$\sigma_1$-$5.6\sigma_3=24.7$</td>
<td>5.3</td>
<td>44.0</td>
</tr>
<tr>
<td>Peak (Right portion)</td>
<td>$\sigma_1$-$2.7\sigma_3=140.8$</td>
<td>43.2</td>
<td>26.9</td>
</tr>
<tr>
<td>Residual (Left portion)</td>
<td>$\sigma_1$-$3.9\sigma_3=0.4$</td>
<td>0.1</td>
<td>36.3</td>
</tr>
<tr>
<td>Residual (Right portion)</td>
<td>$\sigma_1$-$3.5\sigma_3=17.5$</td>
<td>4.7</td>
<td>33.6</td>
</tr>
</tbody>
</table>
3.4 Comparison with traditional continuum modeling approaches

With the ability of the progressive S-shaped criterion to replicate rock pillar behavior established by Sinha and Walton (2018a), four other yield criteria - ultimate S-shaped (non-progressive), strain-softening Mohr Coulomb (MC), brittle Hoek-Brown (HB) and CWFS were chosen for the purposes of a comparative analysis. To ensure that the models are comparable, the yield envelope in each case was developed using the most directly related segments of the progressive S-shaped criterion. The geometry of the pillar models are similar to that in Sinha and Walton (2018a) with a mesh size of 0.166 m. The mesh size was selected based on a sensitivity analysis of its effect on the pre and post-peak portion of the stress-strain responses.

Table 3.2 Dilation parameters used in model (from Walton et al., 2015b).

<table>
<thead>
<tr>
<th>Dilation Parameters</th>
<th>Value assigned</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-mobilization Parameter ($\alpha_0$)</td>
<td>0.001</td>
</tr>
<tr>
<td>Pre-mobilization Confinement Dependence ($\alpha'$)</td>
<td>0.0038</td>
</tr>
<tr>
<td>Dilation Mobilization Plastic Shear strain ($\epsilon_{psm}$)</td>
<td>0.0015</td>
</tr>
<tr>
<td>Low Confinement Peak Dilation Parameter ($\beta_0$)</td>
<td>1.1</td>
</tr>
<tr>
<td>High Confinement Peak Dilation Parameter ($\beta'$)</td>
<td>0.14</td>
</tr>
<tr>
<td>Dilation Decay Parameter ($\epsilon_{ps*}$)</td>
<td>0.01</td>
</tr>
</tbody>
</table>

For strain-softening MC criterion, the peak and residual envelope was chosen to correspond to the left side of the peak and residual envelope of the progressive S-shaped criterion. The selection was based on the fact that the MC criterion was originally developed considering shear-based failures. In case of the brittle Hoek-Brown criterion, only one envelope was defined, following the suggestions of Martin and Maybee (2000). For CWFS, the peak and residual envelopes corresponded to the left side of the yield and peak envelope of the progressive
S-shaped criterion. In case of the ultimate S-shaped criterion, the peak envelope in region i and ii (refer Figure 3.2b) was chosen to correspond to the yield and peak envelope of the progressive S-shaped criterion. The residual envelope was derived by degrading the peak envelope and is described in further details in Section 3.4.1.

Figure 3.2a shows the brittle HB, strain-softening MC and CWFS envelopes while Figure 3.2b illustrates the ultimate S-shaped envelope. All simulations employed the WD dilation angle model with parameters listed in Table 3.2 to isolate the influence of the strength model used on the results obtained.

**3.4.1 Ultimate S-shaped failure criterion**

The need for a tri-linear or S-shaped failure criterion to reproduce hard rock behavior was first proposed by Diederichs (2007). Following his theoretical model, Kaiser et al. (2011) suggested the concept of confinement-dependent GSI to capture the S-shape. The development of this approach was fueled by the flawed applicability of the generalized Hoek-Brown criterion.
in estimation of confined rockmass strength (Valley et al., 2012; Bahrani et al., 2014). The S-shaped envelope considers tensile fracturing/spalling at low confinement and extensile-driven shear at high confinement (Diederichs, 2003), thereby providing realistic strength estimates for confined rockmass. The two segments of the criterion are connected by a transition zone where
\[
\frac{\sigma_3}{\bar{\sigma}_{CS}} = \frac{1}{10}.
\]
In the context of pillars, this means that elements having a confinement higher than \( \frac{\sigma_3}{\bar{\sigma}_{CS}} = \frac{1}{10} \) (termed as outer shell) will fail through shear while those below the threshold (termed as inner shell) will be subjected to spalling (Valley et al., 2012). The point of transition chosen, however, is speculative in nature. If this criterion is indeed used for modeling pillars, the threshold of transition should be modified preferably through a process of back-analysis.

Using Rocscience software Phase\(^{2D}\), Kaiser et al. (2011) assessed the effect of W/H on the strength of pillars. While it may not be possible to explicitly simulate pseudo-shear associated with extensile crack localization in homogeneous continuum models, the overall damage evolution and change in mechanisms as a function of confinement at the meso-scale is represented by the S-shape of the envelope. The model predicted an exponential increase in strength with W/H.

Although this approach has a strong theoretical basis, by only defining the ultimate strength envelope, it neglects the gradual localization of damage that occurs during loading. Failure to account for the manner in which damage initiates and propagates can lead to unrealistic results. Consider Figure 3.2b, where the upper final envelope of the progressive S-shaped criterion was selected as the peak envelope for the ultimate S-shaped criterion and a residual envelope was computed by lowering the higher and lower confinement peak strengths by 50% and 75%. Beyond the low confinement region (\( \sigma_3 > 12.6 \text{ MPa} \)), yield occurs at the peak
envelope. This is in direct contradiction with the findings of Walton et al. (2017) where yield in Indiana Limestone samples initiated well below the peak strength under high confinement conditions. The use of only an ultimate envelope for modeling hard rocks essentially neglects the damage associated with CI and CD stress thresholds. Furthermore, the non-simultaneous mobilization of friction and cohesion for spall-type failure is not accounted for by the manner in which the residual envelope has been defined.

Six models corresponding to W/H of 0.5, 1, 1.5, 2, 3 and 4 were run with peak to residual strength reduction over 0.04 plastic shear strain. The selection of an elastic-brittle-plastic constitutive model in Kaiser et al. (2011) may have been constrained by the available choices in Phase\textsuperscript{2D} - such is not the case in FLAC\textsuperscript{3D}. Also, modeling a strain-dependent degradation is necessary for comparison against the progressive S-shaped criterion. The ultimate S-shaped criterion was incorporated in FLAC\textsuperscript{3D} using the strain-softening MC constitutive model. A piecewise FISH function was called every 1000 solution steps which modified the friction and cohesion of model elements as a function of confinement and plastic shear strain. The stress-strain curves for W/H of 1, 2, 3 and 4 have been shown in Figure 3.3.

Figure 3.4 shows the strength curves for the progressive S-shaped criterion and the four modeling techniques tested. The peak strength for strain-hardening stress-strain curves (related to higher W/H models) was chosen as the point where the curve achieved its maximum. Here, only the strength curve corresponding to the ultimate S-shaped criterion will be discussed while the others will be discussed in their respective sub-sections. For lower (≤1) and higher (≥3) W/H ratios, the trend is similar to the progressive S-shaped criterion with a slightly higher overall strength. The major disparity in the shape of the curves occur in the W/H range of 1-3. This is likely due to the inability of the ultimate S-shaped criterion to account for the initial damage.
process that is associated with CI stress threshold. It has been shown in Sinha and Walton (2018a) that the peak strength of squatter pillars is controlled by the right side of the Peak envelope. Given the identical envelopes in this region (see Fig 2b), the two strength curves are expected to come close/coincide for larger values of W/H. Clearly, the shape of the strength curve for the ultimate S-shaped criterion does not agree with the empirical database. The mechanistic flaw in the envelope and the deviating strength suggests that the ultimate S-shaped envelope may not be suitable for modeling rock pillars in general.

Figure 3.3 Average stress-strain curves for W/H= 1, 2, 3 and 4 for the ultimate S-shaped criterion models.

3.4.2 Mohr-Coulomb (MC) with strain-softening

Mohr-Coulomb is one of the earliest and widely recognized shear strength criteria in the field of rock mechanics. It relates the shear stress and normal stress at failure using two parameters: cohesion and friction angle (Jaegar and Cook, 1979). The simple linear relationship enables easy implementation in modeling software packages. FLAC$^{3D}$ has a built-in strain
softening Mohr-Coulomb constitutive model that allows users to define residual cohesion and friction angle as a function of cumulative plastic shear strain (Itasca, 2016a).

Field testing has shown pillars to exhibit an overall-strain softening behavior. The similarity in shape with the FLAC\textsuperscript{3D} strain-softening constitutive model has often led to its use in the past. It must, however, be recognized that the local (zone-scale) and the global (pillar-scale) responses are not always interchangeable – rather, an appropriate mechanistically accurate model applied at the zone-scale should lead to an emergent global behavior similar to what is observed in the field. The greatest disadvantage of using a strain-softening MC criterion is its inability to account for the tensile spalling process. It is of interest, therefore, to test the outcomes of using

Figure 3.4 Strength curves for different modeling techniques are overlaid on empirical pillar database. The empirical database was segregated for UCS of 200-300 MPa; the stable, unstable and failed case histories are denoted by green, yellow and red color (modified from Sinha and Walton, 2018a).
such a criterion in modeling rock pillars. To that end, 6 models corresponding to W/H ratios of 0.5, 1, 1.5, 2, 2.5 and 3 were simulated using the geometrical setup discussed in Section 3.3.

Figure 3.2a illustrates the peak and residual MC envelopes used in the models. The two envelopes are coincident with the right portions of the Peak and Residual envelopes of the progressive S-shaped criterion. The peak envelope was degraded to the residual envelope over a plastic shear strain of 0.04 (the lag in the plastic shear strain between the Peak and Residual envelope in the progressive S-shaped criterion was 0.05-0.01=0.04). Figures 3.4 and 3.5 show the locus of the peak strength normalized to the UCS for different W/H ratios and the stress-strain curves for W/H ratios of 0.5, 1, 2 and 3, respectively. It seems that the peak strength increases in an exponential fashion, similar to what Iannacchione (1999) and Mortazavi et al. (2009) had observed from FLAC simulations. The reason for such an unrealistic trend is simply due to the fact that the low-confinement tensile spalling process was neglected. Even though this constitutive model captures the brittle to ductile transitional behavior in the stress-strain curves, it severely overestimates the peak strength for squatter pillars.

3.4.3 Brittle Hoek-Brown (HB) strength parameters

Like MC failure criterion, Hoek-Brown failure criterion is one of the most commonly used strength envelopes in the field of rock mechanics. The major difference between the two lies in their mathematical form in the principal stress space - MC is linear while Hoek-Brown is non-linear (Hoek et al., 2002). The convex shape of the HB envelope can be conceptually related to Patton’s joint model (Patton, 1966). At lower confinement, there is a rapid increase in frictional strength due to interlocking of the asperities while at higher confinement, the cohesion increases rapidly due to shearing through intact material. With this conceptual framework and
the knowledge that the surficial tensile fracturing is suppressed due to the generation of hoop stresses under uniaxial loading conditions, it can be understood that HB criterion only captures the shear failure mode of rocks. From this, it can be inferred that such a criterion, without any modification, would not accurately represent the progressive damage process in rock pillars.

Figure 3.5 Average stress-strain curves for W/H = 0.5, 1, 2 and 3 for the MC strain-softening models.

Martin and Maybee (2000) proposed that initial (pre-peak) yield is governed by stress-induced spalling and that a shear failure plane develops only after the peak strength has been achieved. To capture this behavior, Martin et al. (1999) suggested the use of a set of brittle HB parameters, given by $m_0 = 0$, $s = 0.11$ and $a = 0.5$. Plugging these values in the HB criterion gives the following equation:

$$\sigma_1 - \sigma_3 = 0.33\text{UCS}$$

(3.1)
where, \( UCS \) is the Uniaxial Compressive Strength of intact rock. The constant deviatoric strength equation assumes yield to be dominated by cohesion. Although this may be true for slender pillars, it is generally not applicable to wider pillars where the strength is dominated by formation of confined cores where shearing mechanisms occur.

Six pillar models with W/H ratios of 0.5, 1, 1.5, 2, 3 and 4 were run with brittle HB parameters. The average stress-strain curves for W/H ratios of 1, 2, 3 and 4 are shown in Figure 3.6. The flattened shape of the curves at higher axial strain is typical in models where the residual strength is equal to the peak strength; clearly, this is not a realistic interpretation of how pillars behave when they are loaded in the field. When the strength locus is compared to the progressive S-shaped criterion in Figure 3.4, the brittle HB predicts a slightly higher strength for W/H = 0.5-2 and significantly lower strength for squatter pillars. The squat pillar behavior is logical and can be reasoned on grounds of a lower initial and peak strength. The higher strength for slender pillars, on the other hand, is due to the absence of a residual envelope which underestimates the progressive damage localization process. Here the constant yield envelope (which is close to the CI threshold of the progressive S-shaped criterion) enables the stress-states of individual model zones to increase their minimum and maximum principal stresses, culminating in a slightly higher overall pillar strength.

### 3.4.4 Cohesion-Weakening-Frictional-Strengthening (CWFS) model

The Cohesion-Weakening-Frictional-Strengthening model was originally conceived by Schmertermann and Osterberg (1960) based on studies in clay. More recently, Hajiabdolmajid et al. (2002) formally introduced the CWFS strength model for capturing brittle failure in rocks. The driving force behind the development and extension of this model to rocks was the inability
to replicate tensile-driven failure processes using conventional shear yield criteria (Pelli et al., 1991; Castro et al., 1995; Martin, 1997).

Figure 3.6 Average stress-strain curves for W/H = 1, 2, 3 and 4 for the brittle HB models.

The CWFS strength model has been utilized by numerous authors to model the extent and shape of notches that typically form along the boundary of excavations in brittle rock (Hajiabdolmajid et al., 2002; Edelbro, 2009; Walton and Diederichs, 2015b). Walton et al. (2016) performed a back analysis of pillar deformation in a hard rock mine in Sudbury, Canada using a combination of the CWFS strength model and the WD dilation angle model. The model was able to replicate the increasing confinement around the pillar core, pre-peak hardening in the average stress-strain curve, and formation of damage at low confinement regions within the pillar. The results of the study would seem to suggest that the CWFS strength model is capable of capturing the behavior of rock pillars. The key caveat which applies to that study, however, is the low width to height ratio (~1.5) of the modeled mine pillar. The low confinement damage
processes captured by the CWFS strength model are restricted to lower W/H ratios. For higher W/H ratios, the strength is governed by the formation of a confined core and this is expected to cause a deviation between the progressive S-shaped criterion and the classical CWFS model results.

To allow for a direct comparison, six pillar models were run with W/H ratios of 0.5, 1, 1.5, 2, 2.5 and 3. The peak and residual envelopes (Figure 3.2a) were introduced in the FLAC$^{3D}$ model using the strain-softening MC constitutive relationship. The left portions of the Yield and Peak envelopes of the progressive S-shaped criterion correspond to the Peak and the Residual envelopes of the CWFS model in Figure 3.2a.

The strength envelope was transitioned from peak to residual envelope over a cumulative plastic strain of 0.01. Figure 3.7a shows the average stress-strain plot for W/H ratios of 0.5, 1, 2 and 3. Similar to MC, CWFS captures the brittle to ductile transition behavior of pillars. The average strength curve is very similar to the progressive S-shaped criterion models up to W/H ratios of 1.5, beyond which it falls (refer to Figure 3.4). Intuitively, one would expect the strength would be higher because the ultimate strength envelope of the CWFS models is much higher than that of progressive S-shaped criterion for high confinement conditions.

To physically understand the reason behind this anomaly, the stress states for all elements in the W/H = 3 model were plotted and are shown in Figure 3.7b. The near-horizontal alignment of the highly stressed elements was at first surprising; on coloring the elemental stress states on the basis of plastic shear strain, it was found that such an alignment was an artifact of the evolving nature of the envelope and the WD dilation model. The key difference between CWFS and progressive S-shaped criterion is the absence of the modified Mogi’s line. The modified
Mogi’s line enables elements to be stressed at higher confinements without actually undergoing any yield. Since such is not the case with CWFS, yield and associated reduction in dilation angle occurs earlier during the loading phase, causing the strength to fall below the progressive S-shaped model strength curve for squatter pillars. Additional research is necessary to determine the reason behind the similar trend in strength between CWFS and the progressive S-shaped criterion.

Figure 3.7 (a) Average stress-strain curves for W/H = 1, 2, 3 and 4 for the CWFS strength models, (b) Elemental stress states colored by plastic shear strain (millistrain) for the CWFS W/H = 3 model.

3.5 Effect of dilation angle model

The success of numerical models in replicating the behavior of mine-scale structures is dependent on the choice of a yield criterion as well as a dilation angle model. The importance of capturing the mobilized nature of dilation angle has been the focus of limited study (e.g. Zhao et al., 2010a; Zhao et al., 2010b; Walton et al., 2015b). Studies like Chugh and Abbasi (2011), Edelbro (2009) considered a constant zero dilation angle while Martin and Maybee (2000), Mortazavi et al. (2009), Kaiser et al. (2011), and Renani and Martin (2018b) did not provide any
information on the dilation angle used in their study. It appears that the effect of dilation angle on the meso-scale behavior and its emergent effect on the overall model response has often been overlooked. Accordingly, in this study, a comparison of model responses obtained for different choices of dilation angle was performed, with a focus on pillar strength and damage mechanisms.

3.5.1 Comparison of mobilized and constant dilation angle results

Dilation angle plays an important role in the damage localization process as pillars are subjected to increasing load. With onset of inelastic deformation, inelastic lateral strains develop which in turn increase the confinement and strength of neighboring elements. Although localized, this phenomenon can significantly affect the global behavior of pillars (Walton, 2014). The magnitude of lateral plastic strain is controlled by the parameter dilation angle that mathematically relates the plastic axial and lateral strain increments, given by (Vermeer and De Borst, 1984):

\[
\frac{\varepsilon_3^p}{\varepsilon_1^p} = \frac{\sin \psi + 1}{\sin \psi - 1}
\]  

(3.2)

where, \(\varepsilon_1^p\) and \(\varepsilon_3^p\) are the plastic strain rates in the major and minor principal directions, respectively, and, \(\psi\) is the dilation angle. Several studies have found that the dilation angle is a function of confining stress and plastic shear strain and can be better represented by a mathematical model (Alejano et al., 2005; Zhao and Cai, 2010; Walton and Diederichs, 2015a).

The most relevant study in literature which has considered the influence of dilation angle on pillar behavior is that by Walton et al. (2015b), who looked into the differences in damage localization, confining and the vertical stresses on a horizontal cross-section along the mid-
height of granite pillars. Results indicated that the use of a 0° dilation angle underestimates the confining and vertical stresses through the cores of pillars. This observation is intuitive since dilation angle of zero minimizes the right-hand side of equation 3.2, producing the least increase in plastic confining strain. Another set of models in that study utilized a constant 30° dilation angle, as a simplifying approximation to the mobilized WD dilation angle model (Walton et al., 2015b). This too generated contours of plastic yield, vertical and confining stress, highly dissimilar to those obtained using the mobilized WD dilation model. Clearly, the use of a constant dilation angle is not sufficient in describing the complex interdependence of dilatancy and damage on overall model response.

In this study, the W/H ratio of the pillars were varied from 0.5 – 4 with four choices of dilation angle -- (a) WD model, (b) ψ = 0°, (c) ψ = 15°, and (d) ψ = 30°. The model geometry and loading conditions are similar to those described in Section 3.3. Figure 3.8 (a, b) shows the pillar strength as a function of W/H and the stress-strain curves for W/H=2 and 3 with WD, ψ = 0° and ψ = 15° models, respectively.

The trend in pillar strength for all the models are consistent with the sigmoid shape, previously introduced in Figure 3.4. It can therefore be concluded that the distinct form of the curve is a manifestation of the progressive S-shaped criterion, rather than the WD dilation model. For slender pillars, the peak strengths are comparable regardless of the dilation model used; the discrepancies between the different cases widen with increased W/H ratio. Mechanistically, failure in slender pillars is governed by the formation of a cross-shear plane, where dilation angle does not play any significant role. With increase in the W/H ratio, a confined core develops as a response to the dilation of failed peripheral elements. It is under these confinement conditions that the choice of dilation angle has the greatest impact. In terms of the stress-strain curves, the
W/H=3 model with zero dilation angle showed a distinctly different trend. The higher strength for the $\psi = 0^\circ$ model (also refer Figure 3.8a) seems to contradict the general notion that reducing the dilation angle reduces the plastic strain-generated confinement in the core, leading to an overall lower strength. This also contradicts the lower confining stresses observed in pillar simulations of Walton et al. (2015b). This leads one to question whether the pillar strength decreases or increases with reduction in dilation angle as well as what causes the discrepancy in the observed pillar behavior.

Figure 3.8 (a) Variation in strength with W/H ratio for different dilation angles, (b) Stress-strain curve for W/H=2 and 3 with WD model, $\psi = 0^\circ$ and $\psi = 15^\circ$.

3.5.2 Damage processes for different dilation angles

The stress-strain curves in Figure 3.8b provide a reasonable explanation to the questions raised above. In addition to higher overall strength, both the W/H=2 and W/H=3 ($\psi = 0^\circ$) models exhibited a delayed strength mobilization i.e. the models underwent a much larger amount of strain before peak strength was reached. At an intermediate axial strain level (0-0.005 for W/H=2 and 0.005-0.01 for W/H=3), the average stress in the models with WD and $\psi = 15^\circ$
degrees was actually greater. It is, therefore, likely that the confining and vertical stress plots in Walton et al. (2015b) were generated at a stage when the average stress of the $\psi = 0^\circ$ model was below that of the WD model. This reasoning can be applied to the Creighton pillar ($W/H=1.5$) since a delayed strength mobilization was also observed in the $W/H=1.5$ model (similar to $W/H=2$, as shown in Figure 3.8b). If the loading in Walton et al. (2015b) was continued, the stresses in the zero dilation model may have eventually exceeded the WD pillar stresses.

To better understand this phenomenon, the plastic shear strain and confining stresses for the $W/H=3$ model was plotted and is shown in Figure 3.9. The plastic shear strain contours in Figure 3.9 a and b are for WD and $\psi = 0^\circ$ models respectively at a solution step when the WD model reached its peak strength. The confining stress contours in Fig 9 c and d are located at the same solution step. Figure 3.9e shows confining stresses at the solution step when the $\psi = 0^\circ$ model reached its peak strength. Surprisingly, when the WD model reached its peak strength, the volume of plastic yield in the core was lower than in the $\psi = 0^\circ$ model. Laboratory tests on granitic and sedimentary rocks have shown the dilation angle to decrease with increase in confinement and plastic shear strain (Walton and Diederichs, 2015a). Ideally in a pillar, the peripheral elements are supposed to dilate first, followed by a load transfer to the central portion/core and ultimate dilation of the pillar core. The shedding of load due to yielding of the pillar core should be concurrent with the first major fall in the stress-strain curve from peak.

The sequence of compression and dilation that occurs in the pillar core with progressive loading is severely delayed when a dilation angle of zero is chosen in a numerical model. For example, a dilation angle of $15^\circ$, $30^\circ$ and $45^\circ$ yields 1.7, 3 and 5.8 as a plastic strain increment ratio (right side of Eq. 3.2). A dilation of zero assumes the multiplier to be 1, severely underestimating the amount of lateral plastic strain (and confinement) generated for a unit change in
vertical plastic strain. Physically, this means that as the core of the $\psi = 0^\circ$ model yields, it dilates marginally, preventing the core from shedding its confinement. With further damage accumulation and confinement build-up, the stress in the core increases to a much higher value (see Figure 3.9e) in comparison to WD/constant non-zero dilation models.

![Figure 3.9](image)

Figure 3.9 Plastic shear strain contours for $W/H=3$ with (a) WD model, (b) $\psi = 0^\circ$; Confining stress contour for $W/H=3$ with (c) WD model, (d) $\psi = 0^\circ$ at a solution step when the WD model reached its peak strength, (e) Confining stress contour for the $\psi = 0^\circ$ model when it reached its peak strength.

Given the similarity in the stress-strain curves for WD and $\psi = 15^\circ$ model, it seems that a single value can replicate the response of a mobilized dilation model. This could be partly due to the rapid mobilization ($\gamma_m$) of dilation angle and its relative insensitivity to confining stress (for crystalline rocks), enabling a non-zero dilation angle to reasonably approximate the mobilized model. Since this approach is only approximate, the stress-strain curves as well as the variation of strength with $W/H$ ratio do not match perfectly with the WD model results. Although such a
difference may be negligible from a practical perspective, the authors would like to point out that the emergent macro-behavior could be significantly different in softer materials. Wherever possible, a mobilized dilation model should be used for simulating excavation-scale structures.

3.6 **Random assignment of strength parameters**

The previously presented studies have all assumed model elements to have a uniform constitutive behavior. In reality, most rockmasses are heterogeneous on a scale that depends on its origin and constituent minerals. When such a rockmass is assigned uniform strength in a continuum model, it is implicitly assumed that the meso-scale variability in material property can be adequately represented by an equivalent homogeneous media. While such a simplistic approach has been proven to be effective (Edelbro, 2009; Sinha and Chugh, 2015; Walton et al., 2016), it does not consider the potential effects of areas with heterogeneous strength characteristics to control post-yield localization, and ultimately, pillar strength and brittleness.

Unlike grain-based modeling (Lan et al., 2010; Nicksiar and Martin, 2012; Ghazvinian et al., 2014), FLAC$^{3D}$ does not possess the capability of simulating discrete grains. Thus, the primary option for simulating heterogeneity is to assign probability distributions to the principal input parameters of a yield criterion and randomly allocate the properties in a model. This approach is in no way comparable to grain-based modeling because the variability in strength is only modeled at the element scale (0.166 m in this case). Due to computational constraints, the element size used for modeling mine scale structures are much larger than that of typical grains. Therefore, this randomized approach only serves as an assessment of the model’s sensitivity to meso-scale heterogeneity in mechanical properties.
The authors would like to note here that although some studies (e.g. Tang et al., 1997; Guo et al., 2017) have been performed on methodologies for stochastic modeling of rock properties, they are not directly relevant to this current study due to two primary reasons: (1) The objectives of the referenced studies were to indirectly model the progressive failure behavior of rocks by incorporating heterogeneity in rock strength properties. In contrast, the failure process is explicitly represented here by the progressive S-shaped yield criterion and WD dilation angle model. This study’s goal with respect to stochastic modeling is to assess the model’s sensitivity to meso-scale variations in mechanical properties (related to the stochastic nature of the damage thresholds measured in laboratory). (2) In the referenced studies, a Weibull or Gaussian distribution was employed for representing rock strength heterogeneity. With an increase in heterogeneity, a lower global strength was observed and this could be related to the larger number of weaker elements in the model. In this study, the friction and cohesion of different segments of the progressive S-shaped criterion were assigned independently (following a Gaussian distribution). What this means is that a higher coefficient of variation (CV) does not necessarily imply an increase in the number of weaker elements in the model (for example, one element might have a lower $\phi$ for CD but a higher $\phi$ for CI) – the behavior is far more complex. Consequently, a direct comparison between the results presented in this study and those in the referenced studies is not appropriate.

Following the finding of Langford and Diederichs (2015), a Gaussian distribution was assigned to the different parameters of the progressive S-shaped criterion. The mean cohesion and friction angle values were chosen from Table 3.1 while the CV was selected to be 5% and 10%. Fifteen realizations of $W/H = 1$, 2 and 3 pillars were run with each set of parameters. Figure 3.10 shows the model distribution of initial cohesion with 5% and 10% CV.
The progressive S-shaped criterion has 14 input parameters: 6 cohesion values, 6 friction angle values and 2 plastic shear strain values. When developing these models considering heterogeneity, two decisions were made: (a) the intersection point of the low and high confinement portions would be kept consistent for all three envelopes, and, (b) the three envelopes would coincide for some constant value of confinement. The two constraints reduce the degrees of freedom such that the entire criterion can be defined by only 12 principal parameters. Outlined below are the steps that were followed for developing the models:

For every model element, first a random friction angle and cohesion was selected from the Gaussian distributions for the left and right sides of the Peak envelope. The confinement level separating the left and right segment of the progressive S-shaped criterion was then computed.
Next, the cohesion of the left side and the friction angle for both sides of the Yield envelope was selected. The cohesion for the right side of the Yield envelope could then be calculated to ensure the same intersection point of the low and high confinement portions of the Yield envelope as for the Peak envelope. This then constrained the confining stress point at which all three envelopes would coincide.

Finally, the cohesion and friction angle of the left side of the residual envelope was extracted from the Gaussian distribution. The right segment of the residual envelope was then calculated based on the two constraints stated above.

The stress-strain curves for \( W/H = 1 \) and 3 with 5\% and 10\% CV are shown in Figure 3.11. The variability in the peak strength and the post-peak behavior increases with increase in CV. This is an intuitive result, given the wider range of plausible input parameters. The degree of ductility is also affected by the variability in input properties. For example, in Figure 3.11d, there are clear variations in ductility, and there is no consistent relationship between changes in strength and ductility/brittleness. For \( CV=5\% \), the shape of the post peak-portion is very similar in all cases.

In order to investigate how the differences in spatial variability in properties could affect the micro-damage process, plots of plastic shear strain for two solution steps (separated by 1000 steps) were generated and are shown in Figure 3.12 (c, d). The plastic shear strain distribution in the deterministic model for the same solution steps was also obtained (Figure 3.12 a, b). A narrow color range for the plastic shear strain was chosen to readily identify the onset of yield for different model elements. The differences between the stochastic and deterministic models for Step = \( X \) and Step = \( X + 1000 \) is apparent from Figure 3.12 (a-d). The plastic shear strain
distribution (Figure 3.12 a and b) is almost unchanged over the range of steps considered for the deterministic model, implying no additional yielding over 1000 steps. Given the same applied strain path, the dissimilar pattern between the stochastic and deterministic model at Step X could only be justified on grounds of variation in strength properties. Over the 1000 steps, the number of yielded elements increased by six in the plot of the stochastic model yield. This is in stark contrast to what was observed for the deterministic model.

Figure 3.11 Stress-strain curves for (a) CV=5%, W/H = 1; (b) CV=10%, W/H = 1; (c) CV=5%, W/H = 3; (d) CV=10%, W/H = 3.

Figure 3.12 (a-d) only depicts the differences between the stochastic and deterministic model at early stages of loading. To observe the relative differences at a late loading phase (i.e.
closer to the pillar peak strength), plastic shear strain contours were generated for the two models (Figure 3.12 e and f). The color scale was modified such that blue represents ‘no yield’, red represent ‘at/beyond Yield envelope but not close to Peak envelope’, green represents ‘beyond Yield envelope and close to Peak envelope’, cyan represents ‘beyond Peak envelope but not close to Residual envelope’ and magenta represents ‘close to or at Residual envelope’. The larger proportion of yielded elements in the core indicates that damage propagated faster in the stochastic model. Furthermore, the proportion of yielded elements closer to Peak envelope (i.e. green) also seems to be much larger in the stochastic model. Clearly, the mesoscopic yielding process has a significant effect on the overall model response.

Figure 3.12 Contours of plastic shear strain for deterministic W/H = 3 model at (a) step X, (b) step X + 1000; for the stochastic 10% SD model at (c) step X, (d) step X + 1000; (e) \( e^{PS} \) distribution for deterministic model at a solution step near model peak strength; (f) \( e^{PS} \) distribution for stochastic model at the same solution step as in (e).

Figure 3.13 (a, b) shows the mean, standard deviation (SD) and the coefficient of variation (CV) of model strengths. The means correspond fairly well with those of the
deterministic models (W/H = 1: 0.31; W/H = 2: 0.40; W/H = 3: 0.74). With an increase in the CV of the principal input parameters, the SD of the pillar strength increased. However, since the means are different, it is more appropriate to compare CV values instead of SD values. The W/H=1 model exhibited a brittle behavior with a consistent mean and very low CV. Such slender pillars do not develop any confined core and typically fail through formation of through-going shear planes. The W/H = 2 models, on the other hand, exhibited a significantly higher CV in comparison to W/H = 3 for both sets of models. This was in contrast to the authors’ expectation that the variation in the macro-strength should increase as the pillar becomes squatter. With 15 realizations run for each set of model, the trend obtained is not a consequence of insufficient data points and a mechanistic explanation must exist that can justify this trend.

Upon inspection of the post-peak portion of the stress-strain curves, it was observed that some of the W/H = 2 models failed in a brittle fashion while others exhibited a more ductile behavior, including pre-peak hardening to a higher peak strength value (refer Figure 3.13c). However, none of the W/H = 1 or W/H = 3 models showed such variations in macro-failure mode. The only logical explanation then is that W/H = 2 pillar is near the brittle to ductile transition point (for this particular rock type) and slight variations in the distribution of material properties led to either a brittle, a perfectly plastic or a strain-hardening behavior. This explains why the CV value for W/H = 2 models are significantly larger than those for W/H = 3.

For the granite pillar studied, a deterministic approach seems acceptable given the similar average peak strength and post-peak behavior to those observed from the stochastic models considering heterogeneity. The fluctuation in the failure mode for W/H=2 does suggest the potential for stochastic modeling of pillar stability to have some value, especially because many mine pillars are designed with width to height ratios around 2. Rockmasses are heterogeneous by
nature, and capturing the spatial variability in material strength often becomes important for design purposes. The advantage of simulating different realizations with variable material strengths is that a range of Factor of Safety or other indicators of instability could be obtained. This could help in the design of underground structures considering the worst case scenario. Although this study used the findings of Langford and Diederichs (2015) to assign a coefficient of variation to the principal input parameters, other researchers should strive to estimate site specific values from laboratory testing.

3.7 **Effect of length on pillar strength**

Rectangular pillars are used in mines because of three associated advantages: (a) It can alleviate roof problems under high horizontal stress conditions by minimizing the area of exposed roof (Dolinar and Esterhuizen, 2007), (b) It improves mine ventilation by minimizing leakages (Grau et al., 2006) and head losses at corners, and, (c) Smaller numbers of cross-cuts reduce the number of required ventilation terminations. From both production and safety points of view, longer pillars can aid in rapid completion of the development phase of mining and provide greater stability to openings that must remain functional throughout the life of the mine. Bearing in mind these numerous benefits, it is important to quantify the improvement in stability that can be achieved by varying the pillar length.

To date, the number of studies conducted on width effect far supersedes the studies undertaken to investigate the effect of pillar length on pillar behavior. It has often been hypothesized in the past that pillar strength is governed by its shorter dimension; consequently, most of the empirical equations do not account for the ‘length’ term. While this may be true for slender pillars, it is clearly erroneous for squatter pillars where the strength is controlled by the
volume of the confined pillar core. With the validity of the progressive S-shaped criterion as well as the modeling approach established, it was possible to extend the models tested in this study to evaluate this effect.

### 3.7.1 Background

There are five prominent studies that have been conducted on the effect of pillar length on pillar strength over the years. The most recent one by Dolinar and Esterhuizen (2007) discusses the Bauschinger-Johnson (Babcock, 1994), Mark-Bieniawski (Mark, 1990; Mark and Chase, 1997), Grobbellar and Wagner equations (Warner, 1980) and the potential issues that can arise in applying them to rock pillars. Recognizing the strengthening effect of pillar length, Wagner (1980) introduced the concept of effective width given by:

\[ w_{eff} = \frac{4A_p}{C} \]  

(3.3)

where, Ap is the area and C is the circumference of the pillar. For a square pillar, \( w_{eff} \) reduces to the width of pillar while with increasing length, \( w_{eff} \) approaches a constant value of 2*width. Physically, this implies that the rate of increase in strength tapers for longer pillars and beyond a point, the improvement is negligible. Such a trend is demonstrated by the models presented by Dolinar and Esterhuizen (2007). The Mark-Bieniawski equation is a modified version of the Bieniawski equation which analytically computes the ultimate strength by integrating the vertical stress gradient given by Wilson (1983). Since the parameters were developed specifically for squat coal pillars, it is unsuitable for estimating strength of rock pillars in general.

Dolinar and Esterhuizen (2007) used FLAC\(^3\)D models to investigate the effect of length using two different yield criterions - strain-softening Mohr Coulomb and a brittle strength
criterion. An array of models was simulated for L/W ratio of 1-6 and W/H ratio of 0.5-1.5. Results showed that increasing the length substantially increased the strength of squatter pillars but had negligible impact on slender pillars.

To justify the results on mechanistic grounds, Dolinar and Esterhuizen (2007) computed a confining stress factor for each model. The hypothesis was that the overall strength of a rectangular pillar was simply an addition of the strength of the ‘pillar ends’ and the ‘central core’ (measured as confining stress factor). It was found that for slender pillars, the core factor was very low which led to the conclusion that height was a dominant factor in those cases. For the squatter pillars, there was an initial increase in the core factor followed by tapering beyond L/W=5.

This study was the first systematic investigation designed to critically analyze the effect of pillar length. However, since the manner in which the brittle strength criterion was defined is incomplete, the results may not be entirely representative of the true physical process.

3.7.2 Numerical investigation of L/W effect

A set of 30 pillar models corresponding to W/H of 0.5, 1, 1.5, 2, 3, 4 and L/W of 1, 2, 3, 4, 5 was used for this particular study. With all material parameters and boundary conditions held constant (per the Creighton Granite case), modifications were made only to the length of the models discussed in Section 3.3. Figure 3.14 shows the average-stress strain curves for L/W of 2 and 4. Apart from the similar brittle to ductile transitive behavior in both sets of models, the moderate strength increase associated with length is clearly perceptible. The W/H ratio has a greater influence on the pillar strength than the L/W ratio.
Figure 3.13 Error bar plot (1 SD on either side of mean) of normalized peak strengths for (a) CV = 5% with 15 data points, (b) CV = 10% with 15 data points, (c) Average stress-strain curves for CV = 10%, W/H = 2 models.
Because the strength for the L/W models exhibited an S-trend, a sigmoidal function was chosen to fit the data points (Table 3.3) and is given by Equation 3.4 & 3.5. A 3D surface plot of the derived equation is also presented in Figure 3.15. Although the fitted equation may be of little practical usage, it manages to capture the relatively uniform strength for W/H=0.5-1.5 and the non-negligible effect of L/W on strength for squatter pillars. A multiregression analysis with interaction terms, on the other hand, generated a simpler mathematical relationship but failed to capture the S-shape.

\[
\frac{\sigma_p}{UCS} = 0.3 + \frac{0.725}{1 + e^{(6.78 - 2.41 W/H)}} + m \ln(L/W) \tag{3.4}
\]

\[
m = -0.01 + \frac{0.14}{1 + e^{(11.35 - 6.07 W/H)}} \tag{3.5}
\]

where, \( \sigma_p \) is the pillar strength in MPa, UCS is the uniaxial compressive strength of the pillar material, L is length of pillar in meters, H is height of pillar in meters, and, W is width of pillar in meters.
Table 3.3 Normalized peak strength for the 30 models ran as a part of this study.

<table>
<thead>
<tr>
<th>L/W</th>
<th>W/H</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.31</td>
<td>0.31</td>
<td>0.31</td>
<td>0.4</td>
<td>0.74</td>
<td>0.99</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.30</td>
<td>0.31</td>
<td>0.31</td>
<td>0.48</td>
<td>0.83</td>
<td>1.08</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.29</td>
<td>0.30</td>
<td>0.32</td>
<td>0.51</td>
<td>0.9</td>
<td>1.13</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.29</td>
<td>0.29</td>
<td>0.33</td>
<td>0.53</td>
<td>0.93</td>
<td>1.17</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.30</td>
<td>0.29</td>
<td>0.33</td>
<td>0.53</td>
<td>0.94</td>
<td>1.20</td>
<td></td>
</tr>
</tbody>
</table>

When applying Eqs. 3.4 and 3.5, it must be noted that this study only considered pillars where the W/H and L/W ranged from 0.5 - 4 and 1 - 5 respectively. The logarithmic relationship of L/W ratio to the strength is consistent with the findings of Ryder and Ozbay (1990), Dolinar and Esterhuizen (2007) and the effective width concept of Wagner (1980). A similar diminished length effect for L/W>4 on strength was also predicted by Ryder and Ozbay (1990) and Dolinar and Esterhuizen (2007).

From Table 3.3, it can be seen that with increase in length, the strength reduces for W/H of 0.5 and 1 while it increases for the other considered ratios. To comprehend the reason for such a behavior, the volumetric core proportion for each of the models was computed. This was done on grounds similar to Dolinar and Esterhuizen (2007) and following the suggestion of Valley et al. (2012) that the confinement level corresponding to the intersection of the CI and Spalling limit could be used as a demarcation between the outer and inner shell. In our case, the point of intersection was 12.4 MPa, meaning elements in the pillar which exceeded this value were considered as part of the pillar core. The core proportions were determined at the solution step where the respective models achieved their peak strength.
Figure 3.15 Fitted surface shown in 3D with model predicted strength as black dots.

Figure 3.16 and Table 3.4 show horizontal pillar sections and the volumetric core proportion for each of the models, respectively. The red contour lines in the sections characterize the boundary between the outer and inner shell. In case of W/H of 1, no element along the mid-section had a confinement level higher than 12.4 MPa.

In slender pillars, the core proportions are low (<25%) and reduce further with an increase in length. The number of elements constituting the core increases with length but when normalized to the total volume, it shows a decreasing trend. To illustrate why this occurs, confining stress contours along two vertical sections were prepared at distances of 4 m and 8 m from the pillar end for the W/H = 1, L/W = 4 model (see Figure 3.17). It can be seen that elements which contribute towards the core reduce along the length, consequently causing the normalized core proportion to decline.
In case of squatter pillars, the spread of the shear plane across the entire width is restricted by the pillar height, enabling a confined core to form around the center. With increase in length, the longitudinal extent of the core is extended and this corresponds to an increase in the volumetric core proportion. Beyond a L/W of ~3-4, the increase in the number of core elements is balanced by the rise in the total number of pillar elements. As a mathematical explanation, consider the following equation:

\[
Volumetric\ Core\ Proportion\ (\%) = \frac{a + b[(L - 8) WH]}{LWH} \times 100
\]  

(3.6)

where, L, W, H are the length, width and height of a pillar respectively, a is the volume of core for \(L = 8\) m, and, b is the fraction of the extra volume that contributes towards the core for lengths higher than \(L = 8\) m (recall that \(W = 8\) m is fixed in this case). The previous equation can be re-written as:

\[
Volumetric\ Core\ Proportion\ (\%) = \frac{a}{LWH} + b \left(1 - \frac{8}{L}\right) \times 100
\]  

(3.7)

Differentiating both sides with respect to L, we get:

\[
\frac{d}{dL} \left(\frac{Volumetric\ Core\ Proportion}{100}\right) = \frac{100}{L^2} \left(8b - \frac{1}{WH}\right)
\]  

(3.8)

It becomes evident from the inverse square relation that as the length increases, the rate of increase in the volumetric core proportion reduces drastically. This simple analysis was conducted assuming that the proportion of extra elements contributing towards the core is constant. This is a crude approximation made on the basis of vertical cross-sectional contour plots of confinement; nevertheless, it serves to justify the trend in Table 3.4.
Figure 3.16 Contour of $\sigma_3$ along horizontal cross-section of the models. The black line marks the boundary of the core.
The results obtained in this section substantiates the effect of length in improving the overall strength of pillars. The volumetric core proportion trend is similar to that obtained in Dolinar and Esterhuizen (2007). However, the sectional plot of confinement indicates that it may be erroneous to consider only the longitudinal ends of the pillars as inner shell (refer Figure 3.16), as was considered in Dolinar and Esterhuizen (2007). The overall strength and failure behavior is two phased: tensile driven shear in slender pillars and surficial spalling-confined core.
in squat pillars. In the slender pillar case, core formation is obscured by the effect of height causing the strength to remain invariant with changes in length.

Table 3.4 Volumetric Core Proportions (%) for the 30 models.

<table>
<thead>
<tr>
<th>L/W</th>
<th>W/H 0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13.2</td>
<td>24.5</td>
<td>37.9</td>
<td>44.5</td>
<td>55.9</td>
<td>63.3</td>
</tr>
<tr>
<td>2</td>
<td>11.2</td>
<td>16.1</td>
<td>39.5</td>
<td>54.2</td>
<td>58.8</td>
<td>68.6</td>
</tr>
<tr>
<td>3</td>
<td>9.3</td>
<td>11.1</td>
<td>42.7</td>
<td>565</td>
<td>60.2</td>
<td>71.4</td>
</tr>
<tr>
<td>4</td>
<td>7.8</td>
<td>8.8</td>
<td>45.4</td>
<td>58.3</td>
<td>62.7</td>
<td>71.7</td>
</tr>
<tr>
<td>5</td>
<td>6.9</td>
<td>7.4</td>
<td>47.1</td>
<td>59.9</td>
<td>63.4</td>
<td>71.9</td>
</tr>
</tbody>
</table>

3.8 Conclusions

This study has presented a comparative analysis between the progressive S-shaped criterion and four traditional yield criteria (namely brittle Hoek-Brown, Cohesion Weakening–Frictional Strengthening (CWFS), strain softening Mohr-Coulomb and ultimate S-shaped criterion) in modeling rock pillars. While the progressive S-shaped yield criterion is more complicated than the others considered, it also more faithfully captures the range of failure mechanisms that occur in rocks (Sinha and Walton, 2018a), and accordingly is a more useful tool for research. It was found that apart from CWFS, all the other criteria tended to overpredict the strength of hard rock pillars. The resemblance between the results of CWFS and progressive S-shaped criterion may be a consequence of the high brittle to ductile transition for the modeled granitic rock that prevented the majority of the elemental stress-states from transitioning to the right side of the yield criterion. As a result, the emergent model behavior using the two yield
criteria were similar. Further study is required to ascertain how similar the results obtained using these two strength models might be under different conditions.

The choice of representation of dilation angle was shown to have a major effect on the overall model results. A zero dilation angle delayed the progression of the yield process, eventually over-predicting the pillar strengths. It was also found that an appropriate constant non-zero dilation angle can fairly approximate the complexity of a mobilized model. However, such may not be the case in other rock types, where use of a mobilized dilation model may be necessary.

Continuum modeling techniques generally employ an equivalent homogeneous material to simulate heterogeneous rockmasses. Although this approach has been proven to be useful, this study explored how spatial heterogeneity in micro-strength affects pillar behavior. It was found that for both 5% and 10% CV in input parameters, the W/H = 1 models exhibited a brittle behavior while W/H = 3 showed a ductile behavior. The most interesting observation was the variable failure mode for W/H = 2 models indicating that W/H = 2 acted as a brittle to ductile transition point for the rock pillar under study. From a mechanistic perspective, the failure localization at zone-scale was significantly different in comparison to the deterministic model. This led to a wide scatter in the peak pillar strength values. Nonetheless, the average peak strengths were similar between the deterministic and stochastic model sets, and with the exception of W/H = 2, the post-peak behaviors were relatively consistent with those of the deterministic models as well. Given that the modeling approach used was found to be effective compared to the others considered, it was used to study the L/W effect on pillar strength. A modest increase in strength was noted, associated with an increase in volume of the confined
core. The effect of length was minimal for slender pillars and was minimal beyond L/W>4 for all cases.

3.9 Acknowledgements

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CHAPTER 4

MODELING THE BEHAVIOR OF A COAL PILLAR RIB USING THE PROGRESSIVE S-SHAPED YIELD CRITERION

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4.1 Abstract

Spalling of pillar ribs has been a major hazard in the mining industry for decades and in the absence of any rib support guidelines, accidents have continued to occur in recent years. Developing effective support guidelines requires a complete understanding of complex pillar damage mechanisms. Continuum models represent a convenient tool for analyzing this problem, but the behavior of such models are governed heavily by the choice of the constitutive model. In this study, a recently proposed constitutive model was used to simulate the rib fracturing process in a longwall chain pillar at West Cliff Mine. After calibration, the model was able to capture the rib displacement profiles for different locations of the longwall face and the stress evolution 4 m into the pillar.

The rib bolts in the model were found to be yielding over 60% of their length under the headgate loading condition. The model also predicted a steady damage accumulation in the rib for certain face locations, which is consistent with the description of the rib at the site. Damage localized along the upper part of the pillar and underscored the role that the dirt band played in controlling rib deterioration at the site. The ability of the numerical model in replicating the field measurements provides confidence in the capabilities of the new constitutive model. Finally, the
need to use multiple points of calibration is highlighted by comparing the results of the calibrated model to an alternative model calibrated to a smaller amount of data.

4.2 Introduction

Coal pillars are natural supports that are left underground to maintain the structural and functional integrity of mine openings. Most modern mines employ squat pillars with W/H (width to height) greater than 8 for higher productivity (faster development) and efficient ventilation (Grau et al., 2006). Such squat pillars have high peak strengths and a complete collapse under typical mining induced-stress levels is almost impossible (Mortazavi et al., 2009; Esterhuizen et al., 2010a; Kaiser et al., 2011). The major ground control hazard under such a scenario is the slabbing of pillar ribs due to stress-induced spalling. Buckling and failure of ribs has been found to be the leading cause for fatalities in underground coal mines in the United States, and accounted for 35% of all groundfall-related fatalities for the period of 2010-2014 (MSHA, 2015).

The behavior of pillar ribs is governed by numerous variables: coal seam height, mining depth, in-situ coal strength, cleat orientation with respect to entry direction, mining rate, support type and density, mechanical behavior of the interface between the host rock and the coal seam, etc. (Mohamed et al., 2016a). Given the multitude of controlling factors that interact with each other, it can be difficult to analyze coal rib failure using analytical and empirical approaches. This is especially true where the amount of available field data is limited. With the advancement of numerical modeling methods and computational power, complex rock mechanics problems are increasingly being studied using numerical approaches. Unlike empirical methods, numerical methods allow for physics-based modeling of rock engineering structures while avoiding some
of limiting assumptions associated with analytical solutions (e.g. isotropic stress conditions, circular excavation geometry, etc.).

In this study, a recently developed rock yield criterion has been employed in FLAC\textsuperscript{3D} to model the failure behavior of a coal rib under plane-strain conditions. The particular pillar under consideration is located at a depth of 480 m below surface and is a part of the two-entry longwall chain pillar system in the West Cliff Mine in New South Wales, Australia (Colwell, 2006). A calibrated set of model parameters was obtained by matching the model behavior to the extensometer and stress measurements made at the site through an iterative manual back-analysis process.

Continuum models that are calibrated against in-situ data serve as useful tools for studying complex physical phenomena. For example, Walton et al. (2016) used a FLAC\textsuperscript{3D} model to simulate the changes in the internal damage mechanics of a granite pillar due to the presence of dilating stress-induced cracks. Other authors have used continuum models to identify variations in the stress state ahead of an advancing tunnel face (Eberhardt, 2001), analyzing stability of large caverns (Pelizza et al., 2000), and back-analyzing regional in-situ stress fields (Obara et al., 2000). The knowledge gained from such models can be applied to designing underground structures/openings.

Over the years, continuum models have been successfully used for simulating different aspects of coal pillar behavior. Esterhuizen et al. (2010b) used FLAC\textsuperscript{3D} to reproduce the abutment stresses in longwall chain pillars for three different case studies. Zhang et al. (2015) utilized FLAC\textsuperscript{3D} to study the stress changes in longwall chain pillars with various sizes. The principle of yield pillar damage in a Chinese coal mine was studied by Li et al. (2015) using a
plane-strain FLAC\textsuperscript{3D} model. The model was calibrated against field-measured roof-to-floor and rib-to-rib convergences. Shabanimashcool and Li (2012, 2013) used a novel caving algorithm in FLAC\textsuperscript{3D} to replicate bolt loads (in roof) and pillar loads due to longwall face advance at Svea Nord coal mine in Svalbard, Norway. Jiang et al. (2012) used FLAC\textsuperscript{3D} to assess and mitigate coal bump risk using borehole stress-relief technology for a Chinese coal mine.

In all the aforementioned studies, the coal pillars were represented using either the Mohr-Coulomb strain-softening model or the Hoek-Brown model with residual strength parameters. In contrast, this study uses a new constitutive model that is based on the fundamental fracturing process of brittle rocks. This is believed to be more representative of coal mass behavior, given that coal is known to be highly brittle in nature.

A note of caution when employing continuum models with a complex constitutive relationship is to understand the uncertainties that are associated with the modeling method (Bahrani, 2015; Walton and Sinha, 2019). Like most other modeling techniques, continuum models suffer from the issue of non-uniqueness, primarily because of the typically small number of macro-properties that are used to constrain a larger number of input-parameters (Jing, 2003; Bahrani and Hadjigeorgiou, 2018). As a result, multiple parameter sets may have the capability of reproducing various in-situ observations. Therefore, it is extremely important to utilize as much data as possible as part of the model calibration process.

### 4.3 The progressive S-shaped yield criterion

The progressive S-shaped criterion is built upon the precursory works of Kaiser et al. (2000), Diederichs (2003) and Kaiser and Kim (2008), and combines the Cohesion-Weakening-
Frictional-Strengthening (CWFS) strength model (Martin and Chandler, 1994; Martin, 1997; Hajiabdolmajid et al., 2002) at low confinement and a shear yield model at high confinement (Hudson and Harrison, 1997). The criterion is named in recognition of the previously proposed S-shaped criterion for the ultimate strength envelope of rock (Kaiser et al., 2000; Diederichs, 2003; Kaiser and Kim, 2008); note the tri-linear shape (or approximate “S-shape”) of the ultimate strength envelope as shown in Figure 4.1 via the red line (the initial low-confinement linear portion of the envelope is relatively short in this case). The criterion is for massive to sparsely fractured rockmasses and is different from the S-shaped criterion of Kaiser and Kim (2015) that was developed based on triaxial test data for intact rock specimens. The prefix “progressive” refers to the fact that the yield criterion evolves as a function of plastic shear strain in an attempt to simulate the “progressive damage” process that occurs in rocks (Tang, 1997; Hajiabdolmajid et al., 2002; Amitrano and Helmstetter, 2006; Li and Tang, 2015).

The criterion has three major thresholds: (a) Yield threshold: The low confinement portion corresponds to the Crack Initiation threshold (Martin, 1997; Diederichs, 2007), while the high confinement portion is an approximation of Mogi’s Line (Mogi, 1966); (b) Peak threshold: The low confinement portion corresponds to the Spalling Limit (Diederichs, 2007), while the high confinement portion is the Crack Damage threshold (Martin and Chandler, 1994; Diederichs, 2007); (c) Residual threshold: This is a degraded variant of the peak threshold and corresponds to a 30-50% reduction in friction angle (Martin and Chandler, 1994). Figure 4.1 shows the different components of the progressive S-shaped criterion. Note that these three thresholds were termed as the “Yield envelope”, “Peak envelope” and “Residual envelope” in the previous publications by the authors (Sinha and Walton, 2018a; Sinha and Walton, 2019a). The
The yield threshold evolves to the peak threshold over a specific range of plastic shear strain values ($\varepsilon^{ps}$), and then ultimately decays to the residual threshold. The yield threshold can evolve to the peak threshold in two different ways: (a) the friction mobilizes and the cohesion degrades at the same value of plastic shear strain ($\varepsilon_{c,\phi}^{ps}$), or, (b) the cohesion decays first ($\varepsilon_c^{ps}$) followed by the mobilization of friction angle ($\varepsilon_\phi^{ps}$). These values and their relationship to one another ($\varepsilon_\phi^{ps} > \varepsilon_c^{ps}$ or $\varepsilon_\phi^{ps} = \varepsilon_c^{ps}$) are dependent on the rock type; generally speaking, $\varepsilon_c^{ps}$ has been found to be negatively correlated with the in-situ unconfined strength (Walton, 2019). In the
context of the progressive S-shaped criterion, an additional threshold has to be defined for the second case at $e^{ps} = \varepsilon_c^{ps}$ (shown in Figure 4.1 and lies between the yield and the peak threshold) for which the cohesion has degraded but the friction has only mobilized partially. The corresponding friction angle ($\phi_{int.}$) for this threshold can be computed as:

$$\phi_{int.} = \phi_{yield} + \frac{e_p^{ps}}{\varepsilon_\phi} (\phi_{peak} - \phi_{yield})$$

(4.1)

where, $\phi_{yield}$ and $\phi_{peak}$ are the friction angles corresponding to the yield and peak thresholds.

For the degradation of the peak threshold to the residual threshold, a plastic shear strain equal to 4 times $\varepsilon_c^{ps}$ or $\varepsilon_\phi^{ps}$ was chosen. This is based on the experimental findings of Martin and Chandler (1994).

The progressive S-shaped criterion was implemented in FLAC$^{3D}$ using the bi-linear strain-softening constitutive model (Itasca, 2016a). Each segment of the low and high confinement region has a corresponding cohesion and a friction angle. Two primary rules are followed when developing a set of progressive S-shaped criterion parameters: (a) The minor principal stress level ($\sigma_3$) corresponding to the intersection of the low and high confinement portion is kept unchanged, and, (b) The high confinement portion of all the thresholds intersect at a single point. The consideration of the two primary rules constrain the cohesion of the right side of the yield threshold and the cohesion and friction angle of the right side of the residual threshold, such that the total number of independent input parameters drops to 11:

a) 4 cohesion values: Two for the peak threshold and one each for the yield and residual thresholds
b) 5 friction angles: Two each for the yield and peak thresholds and one for the residual threshold

c) 2 plastic shear strains: $\epsilon_{c}^{ps}$ and $\epsilon_{\emptyset}^{ps}$.

Further details on the yield criterion are presented by Sinha and Walton (2018a).

The progressive S-shaped criterion only defines the criteria for yield in the FLAC\textsuperscript{3D} zone elements. For a complete constitutive description, a flow rule also has to be defined. To that end, the mobilized Walton-Diederichs (Walton and Diederichs, 2015a) dilation angle model was employed (this model is discussed in the context of model setup in Section 4.4.2).

4.4 Site description and model development

4.4.1 Site description

The West Cliff Mine is a high-volume Australian longwall mine that extracts coal from the Bulli seam. The particular panel under consideration is located at a depth of 480 m from surface and has a width of ~195 m. The chain pillars of the two-entry system at the site are developed on 42 m x 125 m centers with a development height of 3 m and entry width of 4.8 m. Figure 4.2 shows the generalized stratigraphy at the site. In the original study by Colwell (2006), two adjacent chain pillars referred to as Site A and Site B herein (Figure 4.3) were instrumented with extensometers (7 m length) and hydraulic stress cells, but reliable data could only be obtained from Site A due to failure of the stress cell at Site B. The stress cells were positioned approximately 4 m into the rib at mid-height of the pillar. Displacement and stress monitoring initiated when the longwall face was 52 m inby and continued until the longwall face was 981 m
outby of Site A. Here, the instrumentation data presented by Colwell (2006) has been utilized to calibrate a FLAC$^{3D}$ model.

![Diagram of FLAC$^{3D}$ model showing the general stratigraphy at site. The black lines within the mudstone layers are boundaries between fine and coarse zoning of the model.](image)

Figure 4.2 Geometry of FLAC$^{3D}$ model showing the general stratigraphy at site. The black lines within the mudstone layers are boundaries between fine and coarse zoning of the model.

The immediate roof in the study area is primarily composed of mudstones and interbedded sandstones and was separated from the coal seam by a narrow dirt band. This was thought to be responsible for inducing instability in the entry-side ribs (Colwell, 2006). The pillars at the site were subjected to two complete stages of abutment loads: front abutment and side abutment. The ‘front abutment load’ is defined as the load when the longwall face is at a position of 0 m relative to the pillar (i.e. face at the pillar); the ‘side abutment load’, on the other
hand, is the load long after the face has gone past the pillar. The stress cell and extensometer quantitatively captured the progressive damage mechanism and depth of displacement inflection (i.e. the depth at which a notable increase in the horizontal displacement gradient occurs) as the load on the pillar was increased by longwall face movement. Given the completeness of the data for Site A, it was selected for numerical study and model calibration.

At Site A, two 16 mm diameter, 1.2 m long, fully grouted rockbolts were installed on 1 m spacing along the long axis of the pillar. The upper row of bolts secured a 400 mm wide and 2.4 m long steel strap to the rib.

Figure 4.3 Schematic of Site A and B in relation to the longwall face. Not to scale.
4.4.2 Model development

A 2D plane-strain model of a half pillar and half entry with dimensions of 31 m x 21 m x 1 m was developed in FLAC\textsuperscript{3D} (see Figure 4.2). Discrete interfaces were introduced on either side of the coal seam with mechanical properties listed in Table 4.1. These properties were selected from Mohamed et al. (2016b) who studied the same coal pillar in FLAC\textsuperscript{3D} using a different constitutive model for the coal itself (Mohamed et al., 2015). During the course of calibration, it was found that the stiffnesses of the interface elements had minimal impact on the model behavior. The choice of a low friction angle allowed the host rock to slip along the boundary of the coal seam and mimicked the effect of the thin dirt band. A \(~2^\circ\) dip in the coal seam was also reported by Colwell (2006), but was neglected in the numerical model setup. Such a small angle is unlikely to have any significant effect on the model results.

Only 14 m of the roof and 14 m of the floor were simulated in order to reduce the model runtime. The sides, front, back and bottom were constrained by rollers, and a stress boundary condition was imposed on the top surface. The plane-strain assumption is applicable in this case since the instruments are located near the middle of the longer edge of the pillar. The 3D stress arching effect reduces as one moves away from the cross cut, and is almost non-existent by the time the center of the pillar is reached (Sinha and Walton, 2019c).

The simulation in this study was conducted in three distinct stages similar to those used by Mohamed et al. (2016b):

1) In the first stage, the model was run without any excavation until mechanical equilibrium was achieved. Pre-mining horizontal stresses of 16.3 MPa (along Y) and 3.6 MPa (along
X; Mohamed et al., 2016b) and a vertical stress equivalent to the depth of mining (11.6 MPa) were applied to the model.

2) The next stage consisted of developing the entry using the traction reduction method (Mohamed et al., 2016b). In this method, elements inside the excavation are removed while applying forces equivalent to the pre-mining load on the boundary gridpoints. The forces are progressively relaxed until they become negligible. In this study, the boundary forces were relaxed in 100 steps while achieving mechanical equilibrium at each step. This is important in order to avoid unrealistic coal yielding that is associated with a sudden increase of unbalanced forces in the model. The second stage replicates the development loading condition in the field (i.e. prior to any additional loading from the adjacent longwall advance).

3) In the final stage, bolts were installed in the rib, and the vertical stress along the top of the model was increased by 0.2 MPa/step to simulate the retreat of the longwall face, using the same loading procedure as Mohamed et al. (2016b). The stress increment is small enough to allow gradual damage development along the rib. The model was brought to equilibrium after each increment of vertical stress. A total of 36 steps were implemented to replicate the complete stress data from Colwell (2006).

The authors acknowledge that gateroad loading is complex and asymmetric in nature. However, in absence of any pertinent information regarding loading at West Cliff Mine in Colwell (2006), a constant stress boundary was assumed. This approach was previously used by Mohamed et al. (2016b).
The built-in pile structural element (with rockbolt logic activated) in FLAC\textsuperscript{3D} was employed for simulating rockbolts. Since no pull test data was available for West Cliff Mine, the rockbolt input parameters from Mohamed et al. (2016b) were used (see Table 4.2). In terms of rockmass representation, the coal seam was modeled using the progressive S-shaped criterion, while the host rock was modeled as an elastic material. This assumption appears justified, since no damage or significant deformation was reported in the immediate roof or the floor at the instrumented site by Colwell (2006). The two parameters required for modeling elastic materials in FLAC\textsuperscript{3D} are Young’s modulus and Poisson’s ratio (Itasca, 2016a). Both these rockmass parameters were unavailable for the roof and floor strata at West Cliff Mine, and were therefore estimated based on representative values suggested by Zipf (2006) (see Table 4.3).

Although the progressive S-shaped criterion was originally developed for hard brittle rocks that exhibit spalling failures, it has been shown to be applicable to coal as well given its documented brittle behavior (Mishra and Nie, 2013; Kim et al., 2018). At the site considered in this study, face cleats were oriented along the long axis of the pillar, meaning that the entry-side rib had greater geometrical freedom to undergo buckling under elevated loads once the face cleats connect via intact coal matrix damage (Gao et al., 2014c). With that in mind and recalling that the ribs at the site were heavily supported, the authors believe that the displacements were primarily due to the creation and opening of stress-induced fractures, as buckling would have been largely suppressed by the installed support.

In hard rock pillars, extensile spalling fractures also form parallel to the rib surface. The similarity in the direction of anisotropy in the coal pillar and typical spall fractures in hard rock pillars implies that an isotropic rock yield criterion (like the progressive S-shaped criterion) should be capable of approximating the compound damage process (spalling through intact coal
and minor buckling along cleats) at this site. The authors acknowledge that the calibrated parameter set obtained in this study might not be capable of reproducing rib damage where the cleats are oriented along some other direction.

In continuum models, the macroscopic behavior is controlled by both the yield criterion and the dilation angle model used. Numerous studies have found the dilation angle to vary with damage (quantified using plastic shear strain) and confining stress (Alejano and Alonso, 2005; Zhao and Cai, 2010; Walton and Diederichs, 2015a). In this study, the mobilized Walton-Diederichs dilation angle model was chosen, with initial parameter estimates for coal taken from Walton and Diederichs (2015a). The Walton-Diederichs model utilizes five parameters ($\alpha, \beta_0, \beta', \epsilon_{m}^{ps}$ and $\epsilon_{ps}^{*}$) that define a function of the form $\psi = \psi(\epsilon_{ps}, \sigma_3)$. It has an initial pre-peak dilation angle increase (defined by $\alpha$), a peak dilation angle point (defined by $\beta_0, \beta'$, and $\epsilon_{m}^{ps}$), and a subsequent dilation angle decay (defined by $\epsilon_{ps}^{*}$).

The strength and dilation parameters were calibrated by first varying them individually to understand their effect on the model response, followed by simultaneous changes to multiple parameters (considering the ones that had the greatest impact) until the field measured displacement and stress profiles could be reasonably reproduced. The calibrated coal parameters are listed in Table 4.4, and these parameters correspond to the yield envelopes illustrated in Figure 4.1. The confining stress demarcating the high and low confinement regime was determined to be 5.4 MPa from the model calibration process. A tension cutoff of 3 MPa was selected for the yield threshold, which degraded to 0.1 MPa for the peak and residual thresholds.

In total, 17 parameters were constrained by comparing the model responses to measured stresses and displacement profiles for different locations of the longwall face. Some of the
dilation parameters were not varied as a part of the calibration process ($\beta'$, $e^{ns}$, $e^{ns\_m}$), ultimately reducing the degree of non-uniqueness in the models. Given that we consider 8 pairs of displacement-stress data points at the stress cell location in this study (i.e., 16 independent data points) plus additional data points corresponding to displacements along the extensometer under development and headgate loading conditions, the calibrated parameters obtained from this study can be considered to be well-constrained.

Table 4.1 Input parameters for interfaces located between coal seam and the host rock (from Mohamed et al., 2016b).

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal stiffness (GPa/m)</td>
<td>150</td>
</tr>
<tr>
<td>Shear stiffness (GPa/m)</td>
<td>75</td>
</tr>
<tr>
<td>Cohesion (MPa)</td>
<td>0.5</td>
</tr>
<tr>
<td>Friction angle (degrees)</td>
<td>15</td>
</tr>
</tbody>
</table>

4.5 Results and discussion

Figure 4.4a shows the deformation within the rib under both development and headgate loading conditions in the FLAC$^{3D}$ model. A substantial increase in the lateral displacement as well as the depth of displacement inflection was noted from the extensometer measurements. The calibrated model captures the overall trends in the data, although it slightly overestimates the depth of displacement inflection for the development loading condition. When analyzing the extensometer data from Colwell (2006), care was taken to ensure that all displacements were presented with respect to the deepest anchor (i.e. last data point). Figure 4.4b shows the rib displacement as a function of the induced stress measured 4 m into the rib. The data points
represent the vertical stress induced by the retreat of the longwall face. The total stress can be determined by adding the development load to the measured stress values.

Figure 4.4 (a) Rib displacements from extensometer in FLAC\textsuperscript{3D} model, (b) Displacements as a function of stress change from corrected field data and the FLAC\textsuperscript{3D} model.

Table 4.2 Pile structural element input parameters (from Mohamed et al., 2016b).

<table>
<thead>
<tr>
<th>Property</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s Modulus (GPa)</td>
<td>210</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.3</td>
</tr>
<tr>
<td>Yield load (kN)</td>
<td>105</td>
</tr>
<tr>
<td>Shear bond cohesion (kN/m)</td>
<td>118</td>
</tr>
<tr>
<td>Normal bond cohesion (kN/m)</td>
<td>118</td>
</tr>
<tr>
<td>Shear bond friction angle (deg)</td>
<td>60</td>
</tr>
<tr>
<td>Shear bond stiffness (MN/m/m)</td>
<td>500</td>
</tr>
<tr>
<td>Normal bond stiffness (MN/m/m)</td>
<td>1000</td>
</tr>
</tbody>
</table>

The original data presented by Colwell (2006) only represents the stress change measured by the pressure cell and not the actual stress in the surrounding coal. Mohamed et al. (2016b) computed a multiplying factor of 3 based on Wilson’s equation (1981) to convert the measured
stress to actual rock stress. The corrected data from Mohamed et al. (2016b) were utilized for the purposes of calibration in this study (Figure 4.5). In Figure 4.4b, different magnitudes of rib displacements are reported for the same level of stress change. This means that the rib displacement increased, but the stresses in the rockmass remained constant for certain ranges of the longwall face location. Mechanistically, this can occur if the displacements are concentrated along fractures that formed previously without further fractures forming through the coal. Alternatively, this could represent a time-dependent creep phenomenon. In either case, at a local level, the damage process is discontinuous in nature while the overall behavior at a meso-scale can be approximated by an equivalent continuum.

Table 4.3 Rockmass elastic parameters for different layers in the model (from Zipf, 2006).

<table>
<thead>
<tr>
<th>Layer</th>
<th>Young’s Modulus (GPa)</th>
<th>Poisson’s ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interbedded sandstone (roof)</td>
<td>12</td>
<td>0.26</td>
</tr>
<tr>
<td>Mudstone (roof)</td>
<td>10</td>
<td>0.26</td>
</tr>
<tr>
<td>Coal</td>
<td>3</td>
<td>0.25</td>
</tr>
<tr>
<td>Mudstone (floor)</td>
<td>12</td>
<td>0.26</td>
</tr>
<tr>
<td>Interbedded sandstone (floor)</td>
<td>12</td>
<td>0.26</td>
</tr>
<tr>
<td>Sandstone (floor)</td>
<td>15</td>
<td>0.26</td>
</tr>
</tbody>
</table>

An interesting trend observed in the data is the large increase in rib displacement between stress change values of 4 and 6 MPa. The two sets of data points were separated by about 67 m of longwall face advance. Such sudden changes in displacement tend to be related to discontinuum damage processes, as previously observed by the authors at a longwall mine in the Western US. At the aforementioned Western US mine, the pillar edge collapsed completely within 5-10 m advance of the longwall face (Figure 4.6). Given that the jump in displacement at West Cliff Mine was recorded over 67 m and the rib was not reported to be ‘extensively
damaged’ (Colwell, 2006), it can be inferred that the damage was more progressive than catastrophic in nature (as shown by the model), and that a continuum representation of the rockmass is appropriate. This difference in behavior may have resulted because the rib was supported at the West Cliff Mine, whereas the case shown in Figure 4.6 had no pillar support.

Table 4.4 Calibrated input parameters for coal mass.

<table>
<thead>
<tr>
<th>Progressive S-shaped parameters</th>
<th>Dilation model parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield threshold; left side (cohesion, MPa)</td>
<td>7.1</td>
</tr>
<tr>
<td>Yield threshold; left side (friction angle, degrees)</td>
<td>8</td>
</tr>
<tr>
<td>Peak threshold; left side (cohesion, MPa)</td>
<td>1.9</td>
</tr>
<tr>
<td>Peak threshold; left side (friction angle, degrees)</td>
<td>55</td>
</tr>
<tr>
<td>Residual threshold; left side (cohesion, MPa)</td>
<td>0.57</td>
</tr>
<tr>
<td>Residual threshold; left side (friction angle, degrees)</td>
<td>35</td>
</tr>
<tr>
<td>Yield threshold; right side (friction angle, degrees)</td>
<td>33.7</td>
</tr>
<tr>
<td>Peak threshold; right side (cohesion, MPa)</td>
<td>22</td>
</tr>
<tr>
<td>Peak threshold; right side (friction angle, degrees)</td>
<td>15</td>
</tr>
<tr>
<td>$\epsilon_c^{ps}$ from yield to peak (millistrain)</td>
<td>8.1</td>
</tr>
<tr>
<td>$\epsilon_\delta^{ps}$ from yield to peak (millistrain)</td>
<td>8.5</td>
</tr>
</tbody>
</table>

The axial load along the bottom rockbolt (location shown in Figure 4.2) for Step 8 and Step 36 is shown in Figure 4.7. Step 8 corresponds to an induced stress level of 1.6 MPa at the top of the model, while Step 36 corresponds to 7.2 MPa applied at the top of the model (the rightmost data point in Figure 4.4b). At Step 8, the bolt load is below the yield strength and there is no failure in the bolt steel or at the bolt-rockmass interface. At Step 36, however, the first 0.8 m of the rockbolt has failed, transferring the entire load to the bolt segments between 0.8-1.2 m.
Interestingly, the load level in the last segment also reaches the yield strength assigned to the bolt steel. This implies that with further increase in load (e.g., tailgate loading), the reinforcement capability of the rockbolts might be completely lost. While there is no field data to corroborate these results, it is reasonable to expect complete bolt failures for rib displacements on the order of 0.1 m (Mark et al., 2002; Hadjigeorgiou and Tomasone, 2018). The behavior of the upper bolt is similar to the bottom one and is not shown here to avoid redundancy.

Figure 4.5 Field measured (after Colwell, 2006) and corrected (after Mohamed et al., 2016b) stress change data as a function of longwall face location.

The capability of the FLAC\textsuperscript{3D} model to replicate observed in-situ displacements was further tested by comparing the displacement profiles measured in the field to those in the model for different locations of the longwall face. In a two-dimensional model, it is not possible to explicitly represent a face location. However, with the relationship between the stress levels 4 m into the rib and face location already having been established by Colwell (2006) (Figure 4.5), it was possible to physically relate a model state to a certain longwall face position. A comparison of the rib displacement profiles at 18 m outby, 130 m outby, 217 m outby and 416 m outby from the model and the extensometer is presented in Figure 4.8. Apart from the depth of displacement
inflection for the 18 m outby location (see Figure 4.8a), the model closely matched the observed displacement profiles for all other face positions. The cause for this discrepancy is not fully understood, and might be a limitation of the calibration process, the model loading approach, or the continuum constitutive model that was used. The excellent agreement between the model-derived outputs and field measurements further suggests the model is well-calibrated and represents a reasonable approximation of reality.

Figure 4.6 Extensive rib damage in a Western US longwall mine under front abutment load.

Figure 4.7 Axial load distribution for the lower bolt at Step 8 and Step 36.
4.5.1 Non-uniqueness in continuum models

During model calibration, the authors determined another set of input parameters that could reproduce the measured displacements but not the stresses (see Figure 4.9). The primary difference between the two sets of parameters is the plastic shear strain over which the yield threshold evolved into the peak threshold (Table 4.5). For the displacement and stress calibrated model as previously discussed, a plastic shear strain lag was required between the degradation of cohesion and mobilization of friction. For the displacement-calibrated model discussed here, the
cohesion degraded and the friction mobilized simultaneously over a plastic shear strain of 0.005. The other difference was $\beta_0$ (Table 4.5) in the Walton-Diederichs dilation angle model. This parameter controls the peak dilation angle under low confinement conditions; a higher $\beta_0$ value corresponds to more dilatancy under confined conditions.

Figure 4.9 (a) Rib displacements from extensometer in displacement-calibrated FLAC$^{3D}$ model, (b) Displacements as a function of stress change from corrected field data in the displacement-calibrated FLAC$^{3D}$ model.

The displacement-calibrated model better replicates the displacements observed under development loading conditions in comparison to the displacement and stress calibrated model. However, this model shows a significant mismatch between the rib displacements for stress changes less than 6 MPa (Figure 4.9b). It seems that the yielding induced by prolonging the evolution of the yield to the peak threshold combined with the greater ability to dilate was critical in minimizing the total displacement for lower stress levels and delaying the sudden jump in displacement to occur at a stress change between 4 and 6 MPa. A second difference was the location of strain localization along the coal rib. For the displacement-calibrated model, damage was concentrated more towards the mid-height of the pillar. In comparison, pronounced damage
was noted ~0.6 m below the roof in the displacement and stress calibrated model (Figure 4.10); this result is more consistent with the damage localization presented by Mohamed et al. (2016b) and captures the effect of the soft dirt band at the interface between the coal seam and the roof.

Table 4.5 Input parameters that are dissimilar in the displacement and stress calibrated model and the displacement calibrated model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_c$ (millistrain)</td>
<td>8.1 → 5</td>
</tr>
<tr>
<td>$\varepsilon_0$ (millistrain)</td>
<td>8.5 → 5</td>
</tr>
<tr>
<td>$\beta_0$</td>
<td>0.7 → 0.625</td>
</tr>
</tbody>
</table>

This model comparison illustrates the non-uniqueness issue in numerical models when dealing with problems that are data-limited relative to the model complexity used (Starfield and Cundall, 1988). Consider the case where either the stress cell at Site A was not installed or was damaged. Both sets of parameters discussed above would then have been considered equally representative of the coal seam. This is concerning, since many calibrated models used for forward prediction of ground behavior rely on limited field data. In situations where the number of input parameters are much larger than the number of known outputs, it is possible that many different sets of input parameters can capture the target attributes and are thus considered acceptable (Bahrani and Hadjigeorgiou, 2018). However, there is a possibility that some of those parameter sets are invalidated if additional data became available. It is, therefore, critical to calibrate complex models against numerous known attributes before utilizing them for forward analysis.

The progressive S-shaped criterion has over 10 input parameters, and this can be problematic from a practical standpoint. However, since the criterion is based on the
fundamental damage mechanism of intact rock, many of the different input parameters can be easily determined or estimated (e.g. the CI threshold can be determined from laboratory testing (Martin, 1997; Diederichs, 2007), the spalling limit parameters can be estimated based on commonly applied values (Diederichs, 2007), the high confinement peak threshold can be defined based on CD data from laboratory testing (Martin and Chandler, 1994; Diederichs, 2007), etc.). This helps in reducing the uncertainty in the model input parameters and helps constrain the parameter space for model calibration purposes.

![Image](image.png)

Figure 4.10 Plastic shear strain distribution in (a) the displacement and stress calibrated model, (b) the displacement calibrated model.

### 4.6 Conclusions

This study has presented a calibrated model of a coal pillar rib using a recently developed rock yield criterion (the progressive S-shaped yield criterion) in FLAC$^3$D. The damage evolution from the development loading phase up to the headgate loading phase was simulated by
monotonically loading the model along its top edge. Following calibration, the model was capable of reproducing the field-measured rib displacements and stresses 4 m into the pillar. The model could also replicate the displacement profiles observed for different locations of the longwall face. Overall, this study has found a continuum representation of coalmass using the progressive S-shaped criterion to be promising in capturing coal pillar behavior.

A rapid increase in rib displacements was noted in the field data when the face advanced from 66 m outby to 133 m outby. Such changes can manifest as rib-bursts in the field but in this case, the rib was supported and was described to be in a good condition by Colwell (2006). The model accordingly predicted a gradual increase in rib displacements. The rib bolts in the model were found to have yielded over 60% along their length under the headgate loading condition. A stronger or yieldable bolt might have been beneficial in controlling the rib displacements at the site. Future studies can use this modeling method to assess the efficiency of different rib bolting patterns. Lastly, results from a semi-calibrated model were presented to highlight the need of considering multiple independent types of field measurements when replicating complex physical phenomena (like coal rib damage). Future research will focus on how to incorporate the effect of cleats oriented at an acute angle to the rib surface.

4.7 Acknowledgements

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CHAPTER 5

UNDERSTANDING CONTINUUM AND DISCONTINUUM MODELS OF ROCK-SUPPORT INTERACTION FOR EXCAVATIONS UNDERGOING STRESS-INDUCED SPALLING

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5.1 Abstract

Although the design of underground structures is becoming increasingly dependent on modeling-based analysis approaches, ground control systems continue to be designed primarily using a site-specific trial and error approach. This discrepancy is related to the difficulty of reproducing rock-support interaction behavior in its entirety using commonly applied modeling methods. To help bridge this gap, this study has conducted a comparison of continuum and discontinuum models with a focus on their ability to realistically capture the support effect generated by rockbolts with or without wire mesh. It was found that continuum models can adequately replicate the damage processes and deformation of reinforced rockmasses, but show a negligible change in ground behavior when support elements are explicitly considered within the models. We propose a ‘strain-continuity’ hypothesis where the inability of the modeling method to allow discrete block separation after failure is thought to be responsible for this unrealistic behavior. Two discontinuum bonded-block modeling approaches (Voronoi Tessellation and Trigon) were subsequently tested; realistic behavioral differences between the supported and unsupported model conditions that are consistent with empirical data were exhibited by the Voronoi model. Finally, a conceptual framework has been presented that combines field data
from the literature with the modeling results to provide an understanding of how support responses in the continuum and discontinuum models compare to reality.

5.2 Introduction

The design of support for underground structures has been a topic of extensive research in the field of rock mechanics. Support is an integral part of underground mines, as it helps to maintain the structural integrity of excavations. In particular, mesh and bolts are commonly used to support the roof and walls of underground excavations in rock. Despite its importance, support design is commonly based on empirical approaches or site-specific experience (Larson and Dunford, 1996; Bigby and Cassie, 2003; Colwell, 2006; Mohamed et al., 2016a). The primary reason is the complexity of the interaction between reinforcement elements and the rockmass that makes it difficult to characterize analytically. While numerous studies have focused on developing analytical models for bolt/cable loading mechanics (Hyett et al., 1996; Li and Stillborg, 1999; Cai et al., 2004; Ma et al., 2013), these were limited to rather simplified geological conditions (for example, assuming only a single rock fracture, an elastic rock material, etc.). Other authors like Serbousek and Signer (1987), Indraratna and Kaiser (1990), Ito et al. (2001), and Grasselli (2005) have conducted laboratory testing, but these studies were again constrained by a simplistic consideration of the host material and loading conditions.

With the advent of advanced numerical modeling methods, it is possible to study complex rock mechanics problems that are otherwise difficult to investigate using experimental or analytical approaches. Broadly speaking, there are two high-level classifications for numerical modeling methods used in rock mechanics: continuum and discontinuum. Continuum methods consider the ground as an equivalent continuous material governed by constitutive laws that
account for the evolution of elastic and inelastic strains in response to loading (Fairhurst and Pei, 1990; Jing and Hudson, 2002). Discontinuum methods, on the other hand, explicitly model geologic features that range in size from mm-scale grain-boundary contacts to large m-scale joints, including their ability to fully separate (Jing and Hudson, 2002; Jing and Stephansson, 2007). In terms of scales of applicability, continuum models are better suited for analyzing large structures while discontinuum approaches are typically limited to smaller-scale simulations. The question that remains largely unexplored is how the topic of rock-reinforcement can be studied using continuum and/or discontinuum methods.

While this question is quite broad, this study will focus specifically on the appropriateness of the two modeling approaches in simulating pillar reinforcement. More specifically, this study is concerned with such geological conditions where failure is governed by damage to intact rock material rather than by deformation along discrete, pre-existing structural features. The three-dimensional finite difference method (FDM) software FLAC\textsuperscript{3D} was used as the continuum software of choice for this study, while the two-dimensional discrete element method (DEM) software UDEC was the discontinuum software used. The ability of both modeling approaches to capture rock-support interaction effects is investigated by comparing model results with and without supports (rockbolts and mesh) to observations made in-situ. The ultimate goal is to identify which modeling technique more accurately reflects reality, and is a better tool for assessing support needs.

The function of support in rocks can be broadly classified as (Kaiser et al., 1996): (a) Reinforcement: strengthen the rockmass thereby enabling it to support itself, (b) Retain: provide areal coverage of broken rocks to prevent unraveling, and, (c) Holding: tie retaining elements of the support system to stable rock. In this study, all three mechanisms are considered by using
rockbolts and a combination of rockbolts and mesh. Consequently, the most general term ‘support’ is used throughout the text. Note that the overall goal of this study is not to compare two specific sets of models (corresponding to a particular geologic condition); rather, it is a comparison of the inherent capabilities of the continuum and discontinuum modeling approaches to realistically capture the rock-support interaction behavior. While some previous studies have attempted to model rock-support interaction using numerical techniques in the context of specific case studies (e.g. Vardakos et al., 2007; Gao et al., 2014a; Bai et al., 2016; Mohamed et al., 2016b), none of them have examined the cause and extent of the discrepancy between modeling methods more generally.

Perhaps the most relevant study is by Bahrani and Hadjigeorgiou (2018), who simulated a drift in the George Fisher mine using continuum and discontinuum modeling approaches. Support (cable bolts and a shotcrete liner) was found to be significantly more effective in reducing drift convergence in a discontinuum model than in a continuum model. Almost no difference was noted in drift convergence with and without support elements in the continuum model. A similar observation was made by Sinha and Walton (2017b) when modeling rockbolts in FLAC$^{3D}$ granite pillar models. The authors hypothesized that continuum models might be implicitly accounting for the support effect due to its inability to allow discrete block separation during the stress-induced damage formation process. In other words, when an element on the surface of a pillar yields, the model’s inherent formulation does not allow it to be dislodged from the surface and thus maintains a continuous strain-distribution among its elements (termed as ‘strain-continuity’ in this study). A direct consequence is the generation of confinement in elements further from the excavation wall due to the yielded surficial elements acting as a boundary to the interior zones. While the function of bolts and mesh in a real scenario with
stress-induced fracturing is to hold the damaged rock ‘baggage’ (‘baggage’ indicates the damaged peripheral rock zone that is devoid of any self-supporting capacity; Kaiser et al., 1996) and generate some confinement within the pillar, continuum models tend to simulate this phenomenon implicitly by sustaining the original geometry of the models.

The ‘strain-continuity’ hypothesis is supported by the results of Walton et al. (2016), who successfully replicated the brittle deformation and progressive damage (calibrated to data from two multi-point extensometers) in a hard rock pillar using FLAC$^3$D. The modeled mine used a combination of bolt and mesh to provide support to the pillars, but the numerical model itself did not require any support/structural element to capture the overall trend in the extensometer measurements. Additionally, none of the continuum models presented by Edelbro (2009), Walton et al. (2014) or Renani et al. (2016) considered surficial supports, but all were able to replicate the rock damage process as observed in the field in supported cases. As stated by Renani et al. (2016): “Sensitivity analyses showed that the shotcrete support and final concrete lining [in the model] did not influence the radial displacements and the extent of plastic zone. Hence, the support was excluded from the back analyses to increase the computational efficiency”.

Lorig and Varona (2013), while analyzing the effect of supports in continuum models, note that “Numerical models, especially continuum models, are not usually capable of realistically simulating the disaggregation of the rockmass as it deforms. The primary use of numerical models, therefore, is to demonstrate strain compatibility between the selected support and the deformations resulting after the support is installed”. Implicit in this statement is an acknowledgement that continuum models are effectively modeling the behavior of reinforced ground and can therefore be used to study strain compatibility with support. This compatibility
also enables continuum models to correctly predict bolt loads as measured in the field for reinforced rockmasses, even if the bolt elements used in the models do not influence the modeled ground behavior (Zhang et al., 2015; Raju et al., 2015).

The proposed ‘strain-continuity’ hypothesis is also consistent with the physical testing results of Sakurai (2010). Sakurai (2010) conducted compression tests on synthetic blocks to investigate the reinforcement effect of rockbolts on jointed rockmasses. Three different samples were assembled to represent a hard rockmass, a soft rockmass and a continuum block. Testing revealed a minimal reinforcement effect for the continuum block but a substantial strength gain for the jointed hard rock specimen. It was concluded that in cases where deformation is governed by separation and slippage along joints than through the rock matrix (i.e. hard rockmass case), the effect of bolts is substantial. With respect to numerical modeling approaches, this could be considered analogous to how continuum (unjointed specimen) and discontinuum (jointed specimen) models simulate the rock-support interaction in ground undergoing spalling.

The hypothesized ‘strain-continuity’ concept is, however, not directly applicable to minescale simulations that consider limited discontinuum interfaces between and within roof layers. In such cases, the bolts generate a clamping effect and minimize the bed separation between adjacent strata (Gale et al., 2004; Sahebi et al., 2010; Esterhuizen et al., 2013; Shen, 2014; Sinha and Chugh, 2015; Esterhuizen et al., 2019).

DEM models allow large block displacements as the interfacial contacts are progressively damaged (some examples in context of underground excavation studies are Vardakos et al., 2007, Karampinos et al., 2015; Gao et al., 2015; Walton et al., 2018a) and as such do not suffer from the ‘strain-continuity’ limitation of continuum models. Other authors have successfully
used this approach in simulating large reductions in displacement due to support installation for mining case studies (Gao et al., 2014a; Kang et al., 2015). However, it remains uncertain whether the ground-support interaction behavior exhibited by this modeling technique is realistic. To bridge this gap in the existing literature, this study compares some indicators of support efficiency (in context of excavation wall damage) with those observed in the field. Given that the current norms for designing pillar supports are based mostly on experience, this study intends to establish a modeling method that can be utilized by the mining industry for forward-predicting the efficacy of support layouts in spall-prone ground.

5.3 Methodology

The study of rock-support interaction as applied to pillar walls in numerical models requires appropriate consideration of two different modeling aspects: (a) rockmass representation and (b) support representation.

With respect to rockmass representation in continuum models for spalling ground, we use a recently developed yield criterion based on fundamental damage mechanisms of intact rock (Sinha and Walton, 2018a). It is referred to as the “progressive S-shaped criterion”, as it is based on the concept of an ultimate strength envelope that is S-shaped (Kaiser et al., 2000; Diederichs, 2003; Kaiser and Kim, 2015), and the yield criterion evolves as a function of plastic shear strain (to approximate the “progressive damage” process that occurs in rocks; Tang, 1997; Hajiabdolmajid et al., 2002; Amitrano and Helmutetter, 2006; Li and Tang, 2015). This yield criterion has been successfully used in replicating empirically validated behaviors for strong granitic pillars and moderate strength conglomerate pillars (Sinha and Walton, 2018a).
Two separate analyses were conducted to examine the impacts of support in continuum models: a hypothetical granite pillar study and a longwall coal pillar study. With respect to support representation, built-in cable structural elements (Itasca, 2016c) and explicit bolt models (explicitly representing grout and metal elements as model zones) were considered.

With respect to rockmass representation in discontinuum models, the bonded block modeling approach (BBM) was utilized. BBM represents a material domain as an aggregate of polygonal or triangular blocks that can interact through contacting interfaces. Blocks are modeled as either rigid or elastic; if elastic, they are discretized by constant–strain triangular elements (Itasca, 2014a). The Voronoi Tessellation (polygonal blocks) and Trigon (triangular blocks) approaches have been found to be effective in capturing large-scale and small-scale damage processes in rocks (Christianson et al., 2006; Lan et al., 2010; Ghazvinian et al., 2014; Gao and Stead, 2014; Farahmand and Diederichs, 2015; Azocar, 2016; Sinha and Walton, 2018b). In contrast to the continuum approach, discontinuum models allow block separation as the interfacial contacts are progressively damaged. From a physical standpoint, such models should produce true unconfined surfaces as the blocks detach upon failure, permitting the support to ‘pin’ them against the pillar surface. This ‘pinning’ mechanism was tested in the BBMs through a comparison of axial stress-strain curves and lateral displacements in supported and unsupported models. The increase in volume of rockmass due to stress-induced fracturing, otherwise termed as ‘bulking’ (Kaiser et al., 1996), was also compared with the field data presented by Kaiser et al. (1996) to ensure that the models produced reasonable results.

Ultimately, the results of the continuum and discontinuum analyses were synthesized with the field data from Kaiser et al. (1996) and Colwell (2006) to develop a conceptual model
for how the support responses in the two modeling approaches relate to ground-support interaction in reality.

5.4 Assessing rock reinforcement in continuum models

Spalling is a major hazard for deep underground excavations that has resulted in numerous injuries (fatal and non-fatal) over the years through the formation and collapse of fracture-bounded slabs (Iannacchione and Prosser, 1998; Mohamed et al., 2016b). A common mitigation measure is to install support, especially bolts, normal to the excavation surface. In their absence, extensile-induced fractures that initiate in the peripheral portions can propagate inward and induce global instability in pillars. Pillar wall support is designed to uphold the structural integrity of spalled skin and prevent geometric bulking of the rockmass (Kaiser et al., 1996). The amount of confinement generated by such bolts, however, is negligible in comparison to existing ground stresses. For example, patterns of 20 mm grouted rebar spaced at 1 m and 1.5 m can produce maximum support pressures of only 170 kPa and 75.5 kPa, respectively (Hoek, 1999). In comparison, some deep mines located at ~3 km depth can have in-situ principal stresses as high as 100 MPa (Walton et al., 2018b).

The capability of a continuum model to realistically represent the ground-support interaction effect can be tested by comparing model responses with varying degrees of support to similar scenarios in reality. In this section, two model cases are presented that employ different types and densities of skin support. The first considers a granite pillar model with W/H (width to height ratio) values of 2 and 3. The second study considers the behavior of a longwall chain pillar from the West Cliff Mine in Australia.
5.4.1 Progressive S-shaped yield criterion

The progressive S-shaped criterion was designed to capture the rock damage process for a wide range of confinement levels (Sinha and Walton, 2018a). One clear application is rock pillars where the peripheral portions fail by extensile spalling (at low confinement) and the inner confined core fails via a shear mechanism. Numerous authors have found the Cohesion-Weakening-Frictional-Strengthening (CWFS) model to be effective in modeling the tensile-driven failure process along the boundary of underground excavations (Hajiabdolmajid et al., 2002; Edelbro, 2009; Walton, 2014; Walton et al., 2016). However, as the confining stresses increase (analogous to moving deeper within a pillar), the extensile crack formation process is suppressed and the overall failure can be more appropriately represented by a shear yield criterion (Hudson and Harrison, 1997). It therefore follows that a comprehensive rock yield criterion must combine the characteristics of the CWFS and a shear yield criteria.

The idea of segmented yield criterion is implicitly represented by the S-shaped failure envelopes theorized by Kaiser et al. (2000), Diederichs (2007) and Kaiser and Kim (2015). These precursory works, however, only defined the shape of the ultimate strength envelope and not its evolutionary nature (i.e. how the yield envelope changes with continued rock damage). This is thought to be an important attribute in order to be able to capture the progressive failure process of rocks (Sinha and Walton, 2019a). To that end, Sinha and Walton (2018a) proposed a criterion that combines the CWFS yield model at low confinement and the Mohr-Coulomb yield model at higher confinement with consideration of how these yield envelopes evolve as a function of material damage (represented by plastic shear strain).
Specifically, the criterion consists of three major envelopes (Figure 5.1): (a) Yield Envelope: The low confinement portion corresponds to the Crack Initiation threshold (Martin, 1997; Diederichs, 2007), while the high confinement portion is an approximation of Mogi’s Line (Mogi, 1966). (b) Peak Envelope: The low confinement portion corresponds to the Spalling Limit (Diederichs, 2007), while the high confinement portion is the Crack Damage threshold (Martin and Chandler, 1994; Diederichs, 2007). (c) Residual Envelope: This is a degraded variant of the peak envelope. The progressive damage process of rocks is captured by the evolving nature of the criterion, whereby the yield envelope evolves to the peak envelope and then degrades to the residual envelope over finite values of plastic shear strain. The low and high confinement regimes are demarcated by the confining stress ($\sigma_3$) corresponding to the point of intersection of the Spalling Limit and the Crack Damage threshold (Sinha and Walton, 2018a).

The progressive S-shaped criterion is numerically implemented in FLAC$^{3D}$ via the bilinear strain-softening constitutive model (Itasca, 2016a). Sinha and Walton (2018a) used the criterion to model granite pillars in FLAC$^{3D}$. The confining stress and plastic shear strain dependency of the dilation angle was considered by employing the Walton-Diederichs (WD) dilation model (Walton and Diederichs, 2015a). The model strengths obtained for pillars with W/H ratio values of 1-4 were validated against a database of failed and stable pillar case histories from the literature; the model was also shown to reproduce well documented aspects of pillar behavior such as hourglassing of the core (Krauland and Soder, 1987; Esterhuizen, 2006).

5.4.2 Hypothetical granite pillar models

Given the well-calibrated nature of the pillar models presented by Sinha and Walton (2018a), the W/H=1 model was used by Sinha and Walton (2017b) to assess the effect of skin
supports (Appendix A). In particular, the goal was to ascertain whether the ‘pinning’ mechanism could be captured using explicit and structural element representations of bolts. The current study extends the findings of Sinha and Walton (2018a) to W/H=2 and 3 pillars. The material properties used are identical to the calibrated parameters obtained by Sinha and Walton (2018a).

![Figure 5.1 Schematic diagram of the progressive S-shaped yield criterion (after Sinha and Walton, 2018a).](image)

Two different representations of rockbolts are studied here: the cable structural element built-in to FLAC$^{3D}$ (Itasca, 2016c) and an explicit bolt (Sinha and Walton, 2017b). The explicit bolt simulates the different components of a bolt system (i.e. rock, bolt, grout, grout-rock interface and bolt-grout interface; Windsor, 1997) using FLAC$^{3D}$ zones and interface elements. The rationale behind developing a material model was to examine whether the cable element, which is a simplification of the bolt system (Crockford, 2012), neglects certain aspects of the ground-support interaction mechanism. The bolt steel (24 mm dia, 2.4 m long) and the grout (10 mm thick) were simulated using isotropic elastic and perfectly-plastic Mohr-Coulomb materials,
respectively, while the interfaces were characterized by the Coulomb-sliding model. Figure 5.2 illustrates the different components of the explicit bolt model. In order to ensure a smooth transfer of stress and displacements, the mesh was graded radially outward from the bolt centers.

Figure 5.2 Different components of the explicit bolt model: (a) Rock, grout and bolt steel, (b) Rock-grout and Bolt-grout interface.

Pull tests were conducted on a 4 m x 2 m x 2 m block to determine the input parameters for the bolts. The target load-displacement curve for calibration was based on data from Luke (2016). The entire procedure has been documented by Sinha and Walton (2017b) and the relevant parameters are listed in Tables 5.1 and 5.2 (see Appendix A). The explicit bolt and the cable bolt were subsequently incorporated into the hypothetical granite pillar models with W/H of 2 and 3.

Table 5.1 Properties of different components of the explicit bolt model.

<table>
<thead>
<tr>
<th>Input parameter</th>
<th>Model assigned value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Grout (after Crockford, 2012)</strong></td>
<td></td>
</tr>
<tr>
<td>Young’s Modulus</td>
<td>4.8 GPa</td>
</tr>
<tr>
<td>Strength (UCS)</td>
<td>22 MPa</td>
</tr>
<tr>
<td>Cohesion</td>
<td>6.215 MPa</td>
</tr>
</tbody>
</table>
Table 5.1 Continued

<table>
<thead>
<tr>
<th>Friction</th>
<th>30°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensile strength</td>
<td>2.92 MPa</td>
</tr>
</tbody>
</table>

**Bolt**

<table>
<thead>
<tr>
<th>Young’s Modulus</th>
<th>200 GPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poisson’s ratio</td>
<td>0.28</td>
</tr>
</tbody>
</table>

**Grout-Bolt Interface (after Crockford, 2012)**

<table>
<thead>
<tr>
<th>Shear stiffness</th>
<th>8 GPa/m/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cohesion (Peak/Residual)</td>
<td>2 MPa/0 MPa</td>
</tr>
<tr>
<td>Friction (Peak/Residual)</td>
<td>45°/25°</td>
</tr>
<tr>
<td>Tensile Strength</td>
<td>4 MPa</td>
</tr>
</tbody>
</table>

**Grout-Rock Interface (after Crockford, 2012)**

<table>
<thead>
<tr>
<th>Shear stiffness</th>
<th>9 GPa/m/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cohesion (Peak/Residual)</td>
<td>4 MPa/0 MPa</td>
</tr>
<tr>
<td>Friction (Peak/Residual)</td>
<td>53°/32°</td>
</tr>
<tr>
<td>Tensile Strength</td>
<td>4.5 MPa</td>
</tr>
</tbody>
</table>

Table 5.2 Calibrated properties of the cable element.

<table>
<thead>
<tr>
<th>Input parameter</th>
<th>Model assigned value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s Modulus</td>
<td>200 GPa</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>0.28</td>
</tr>
<tr>
<td>Perimeter</td>
<td>0.13823 m</td>
</tr>
<tr>
<td>Shear stiffness</td>
<td>$5 \times 10^3$ MPa/m/m</td>
</tr>
<tr>
<td>Shear Cohesive Strength</td>
<td>$10^6$ Pa/m</td>
</tr>
<tr>
<td>Tensile Yield Strength</td>
<td>0.3 MN</td>
</tr>
</tbody>
</table>

Three different models were run for each W/H ratio: (a) No Bolt model, (b) Explicit Bolt model, and, (c) Cable Bolt model. To avoid any mesh effects that could influence the comparison, the No Bolt and the Cable Bolt models used the same mesh layout as in the Explicit
Bolt model, with rock properties assigned to the zones representing the bolt and grout in the Explicit Bolt model. The original models by Sinha and Walton (2018a) were developed for 8 m x 8 m pillars. In this case, however, only a 1.6 m thick model section was considered (pseudo plane-strain condition) to ensure that the results are comparable to the discontinuum results (presented later) obtained using the 2D software UDEC. It was not possible to further reduce the model thickness owing to the three-dimensional geometry of the explicit bolts. Effectively, this means that the model represents an infinitely long pillar with an out-of-plane bolt spacing of 1.6 m. Figure 5.3 shows the W/H=2 model geometry with three bolts spaced 1 m apart. Roller boundaries were assigned to the vertical sides of the model to enforce a plane-strain condition and to take advantage of the pillar symmetry by simulating only one-half of the pillar (see Figure 5.3). For W/H=3, only two bolts were installed because of its shorter height. All three sets of models were monotonically loaded (via a velocity boundary condition) through elastic beams on either side of the pillars.

The average stress and strain in the pillars were tracked during the entire loading period. A user-defined routine automatically recorded the relative displacement of either edge of the pillar and averaged the vertical stress over all the pillar elements. The displacements were ultimately converted into strains by dividing by pillar height. The overall behavior was found to be similar with minimal strengthening effect using either representation (structural element or material) of the rockbolts (see Figure 5.4). The slight increase in strength for the Explicit Bolt cases could be either due to the differing constitutive models assigned to the components of the bolt system or it could be the result of actual reinforcement generated by the explicit nature of the bolt.
Excavation wall displacements are a good indicator of the potential location and extent of spalling, and one could argue that rockbolts would be expected to have a more significant impact on these displacements than on the overall strength of a pillar under load (Alejano et al., 2017). Accordingly, displacement contours along a longitudinal section of the W/H=3 model were retrieved for the No Bolt, Cable Bolt and Explicit Bolt models and are shown for comparison in Figure 5.5. These plots were generated when the models reached their respective peak strengths. The range of values in the color scale have been made consistent to facilitate a direct comparison. As can be observed, the explicit bolt model had a slight reinforcing effect and
effectively reduced the thickness of the high displacement region (disp. >40 mm) by ~4-5 mm. However, the differences in the results are minor from a practical standpoint, and do not significantly affect the global stability of the pillars. In reality, the effect of support in minimizing surficial deformations is far greater than what is predicted in these models (Kaiser et al., 1996; Colwell, 2006).

Figure 5.4 Stress-strain curves for the three models with W/H of 2 and 3.

Figure 5.5 Contour plots of normal displacement along longitudinal sections of the three models (W/H=3).
5.4.3 West Cliff Mine case study model

The West Cliff Mine is a high-volume Australian longwall coal mine that extracts from the Bulli seam. The particular panel under consideration is located at a depth of 480 m from surface and has a face length of ~195 m. The chain pillars of the two-entry system at the site are developed on 42 m x 125 m centers with a development height of 3 m and entry width of 4.8 m. In the original study on these pillars by Colwell (2006), two adjacent chain pillars were instrumented with extensometers (7 m length sonic type) and hydraulic stress cells, but reliable data could be obtained from only one of them (Site A) due to failure of the stress cell in the other location (Site B). The stress cells were positioned approximately 4 m into the pillar along the mid-height of the pillar. Displacement and stress monitoring initiated when the longwall face was 52 m inby and continued until the longwall face was 981 m outby of Site A.

Sinha and Walton (2020b) simulated the damage processes in the instrumented pillar using a 2D plane-strain FLAC$^{3D}$ model of half pillar and half entry. Figure 5.6 shows the overall geometry of the model and the stratigraphy at the site. A combination of the progressive S-shaped failure criterion and the WD dilation angle model was assigned to the coal mass, while the host rock was modeled as an elastic isotropic material. The model was shown to accurately reproduce the evolution of stresses and pillar wall displacements as measured in the field for different locations of the longwall face. Given the well-calibrated nature of the model, it was further utilized in this study to investigate the effect of rockbolts.

5.4.3.1 Model results with varied support

Here, the effect of support is investigated by modifying the support in the calibrated model from the true case where two bolts (spaced at 1 m) were installed in the pillar.
Specifically, four models were simulated: (a) No Bolt, (b) 3 Bolts – spaced at 0.75 m, (c) 4 bolts – spaced at 0.6 m, and (d) 4 Bolts and mesh. The rockbolts were originally modeled using the pile structural elements in Sinha and Walton (2020b) but have been simulated here using the cable structural elements to ensure consistency among the structural element types used throughout this study. The displacement and stress measurements at the site were reasonably replicated using the cable elements.

To model the mesh, the built-in liner structural element was utilized. Liner properties that can simulate the behavior of a wire mesh are difficult to estimate. Some rockfall studies (Dhakal et al., 2011; Mentani et al., 2018) have used continuum structural elements with Young’s modulus values in the range of 0.01-0.9 GPa and Poisson’s ratio of 0.30 for modeling steel wire
meshes. Since the goal of this section is to identify whether continuum models can capture the
effect of wire mesh rather than to model a specific type of mesh, a higher Young’s modulus of 1
GPa and a Poisson’s ratio of 0.30 were considered. The liner and its contact interface were
assigned very high strength properties in order to prevent any failure. From a practical
standpoint, this could be considered as a conservative support combination for use in
underground coal mines.

Figure 5.7 shows the modeled pillar wall displacement profiles for the headgate loading
condition. In comparison to the ‘No Bolt’ model, the ‘4 bolts’ model showed an 11 mm decrease
in displacement at the surface of the pillar. No significant effect on displacement was perceivable
beyond 1 m into the pillar. The pillar displacement profiles for the ‘4 Bolts’ model and ‘4 Bolts +
Mesh’ model appear to be almost coincident, which implies that the liner element is not
providing any meaningful resistance to surface rock movements. While not presented in Figure
5.7, only a 50% decrease in the wall displacement could be obtained when the liner modulus was
increased by six orders of magnitude. These results suggest that the support is not interacting
with the coal pillar in the model in a meaningful way.

An interesting trend observed was the reduction in the effect of support as its density was
increased in the model. This is evident from the displacement magnitudes at the surface of the
pillar for the five models presented in Figure 5.7. Specifically, the displacements were: (a) No
Bolt: 81.8 mm, (b) Calibrated model with 2 bolts: 74.7 mm, (c) 3 Bolts: 72 mm, (d) 4 Bolts: 71.1
mm, and, (e) 4 Bolts + Mesh: 70.9 mm. The overall trend can be represented by a negative
exponential function, implying that an exceptionally high support density is required to bring
about any significant change to the overall model behavior. This is consistent with the
aforementioned 50% reduction in model displacements with the extremely stiff liner.
The true interaction between unsupported ground and reinforcement elements is difficult to quantify. Fortunately, in this case, extensometer measurements from two adjacent pillars that had different support layouts were available for comparison. Test site A had the standard pillar wall support (two 16 mm diameter, 1.2 m long, fully grouted bolts were used and were spaced at 1 m along the longer axis of the pillar) as used in other parts of the mine, while Site B had additional (secondary) support in the form of 1.8 m grouted bolts and steel mesh. This is unique in the sense that the two sites are located within 150 m of each other with nearly identical geological and loading conditions, meaning any difference in the measured displacements can be primarily attributed to the presence of secondary supports. Figure 5.7 shows the pillar wall displacement profiles at Site A and B in black solid and broken lines, respectively (Colwell, 2006). These displacement profiles correspond to the headgate loading condition (i.e. when the longwall face was 981 m outby from Site A).

![Comparison of different support combinations with field data](image)

**Figure 5.7** Comparison of different support combinations with field data.
A 67% reduction in displacement was observed along the pillar wall at Site B. This difference is attributed to only two additional bolts every 2 m along the entry and steel mesh. With 4 bolts and a mesh in the coal model, the total displacement predicted was 3.1 times of the displacement measured at Site B. This suggests that the continuum model is ineffective in simulating the effect of secondary support, and more broadly, the rock-support interaction behavior. From a design perspective, this analysis should be considered as a caution to using continuum models for developing ground control systems in underground mines.

5.4.4 Discussion

The fact that unsupported models can replicate the behavior of fully supported pillars implies that the reinforcement effect is implicitly accounted for by the continuum formulation of the models. As a result of this limitation, continuum models calibrated to in-situ data can only be used for analyzing macro-level damage processes post-excavation, but not for forward prediction of ground-support interaction. A possible alternative is the discontinuum modeling approach that allows for explicit separation of failed/fractured material. The ability of this modeling approach to reproduce progressive damage processes of intact rock behavior has been previously established (Lan et al., 2010; Ghazvinian et al., 2014; Gao and Stead, 2014; Farahmand et al., 2015; Sinha and Walton, 2018b). If the discontinuum approach can be shown to realistically capture the rock-support interaction behavior, then the limitations associated with the continuum modeling approach can be overcome. In fact, a combination of the two modeling approaches could be used for design and forward prediction, where the macroscopic-stress distribution can be assessed using the continuum method and the deformation behavior near excavation.
boundaries (with and without support) under the obtained stress levels can be determined using the discontinuum model.

When complex numerical models are used for design or for forecasting future excavation response, one should be aware of the fact that such analyses are inherently associated with a high degree of uncertainty. It is known that continuum and discontinuum models suffer from the potential for non-uniqueness in calibrated inputs, which stems from the fact that a large number of input parameters are constrained by few macroscopic attributes (e.g. field data; Jing, 2003; Bahrani and Hadjigeorgiou, 2018; Sinha and Walton, 2020b). It is therefore extremely important to utilize as much data as possible as part of the model calibration process.

The use of numerical models as a predictive tool is often questioned in the field of rock mechanics. Oreskes et al. (1994) notes that even if a numerical model is capable of replicating present and past observational data, there is no guarantee that the model will perform accurately in predicting the future, primarily because of the dynamic nature of geological systems. In spite of the criticism, we believe that well-calibrated numerical models do have the capability of forecasting failure in a semi-quantitative sense in some cases. This is because numerical models are fundamentally “physics-based”, in that they utilize fundamental physical laws as a part of their solution process.

At present, forward modeling using BBMs should be avoided without significant field-scale calibration first, and even then, such predictive modeling may not always be reliable. In the future however, as these techniques advance, it may be possible to establish a degree of understanding of these systems that will allow forward modeling of previously unencountered field-scale systems based solely on calibration to laboratory data.
5.5 Assessing rock reinforcement in discontinuum models

It is evident from the previous section that a different modeling approach is required to realistically capture the interaction between support elements and a rockmass undergoing stress-induced progressive damage. The two options available to the authors were the DEM (Jing and Stephansson, 2007) and the combined finite-discrete element methods (FDEM; Munjiza, 2004; Lisjak and Grasselli, 2014). While some limited research has been performed on the implementation of support elements in FDEM software (Roberts et al., 1999; Tatone et al., 2015a; Tatone et al., 2015b), it is not yet commercially available in software packages. Moreover, its capabilities have not been fully explored and established in literature. Support elements in DEM, on the other hand, have been utilized by numerous authors for modeling underground excavations (e.g. Deng et al., 2009; Gao et al., 2014a; Gao et al., 2015; Kang et al., 2015; Bai et al., 2016; Li et al., 2016; Oliveira and Diederichs, 2017; Yang et al., 2018). Given its wide application, the DEM approach was selected for further analysis in this study.

In DEM, a material space is represented by an assemblage of blocks that can separate along their contacting interfaces. Based on its usage, DEM can be categorized further into Bonded Block Modeling (BBM), Bonded Particle Modeling (BPM) and rockmass simulation. In BBM, blocks are represented by polygons (Voronoi Tessellation) or triangles (Trigons) while in BPM, circles/spheres are used (Potyondy and Cundall, 2004). The third category of DEM encompasses all models that simulate rockmass behavior, including pre-existing joints and/or fracture networks (Singh and Singh, 2008; Saeidi et al., 2013; Walton et al., 2018a). All simulations in UDEC (DEM software) operate on an explicit time-march algorithm where the differential equations are discretized and solved over very small timesteps (Jing and Stephansson, 2007).
5.5.1 Model development

When modeling intact rock as an assemblage of bonded blocks in UDEC, block geometries can be broadly divided into two categories: (a) Voronoi Tessellation – polygonal blocks, and, (b) Trigon–triangular blocks. In this study, the relative capability of both block types to capture the ground-support interaction is assessed by comparing the behavior of supported and unsupported models. In both cases, a single W/H (i.e. W/H=2) pillar model was developed using elastic, isotropic blocks. While the reinforcing effect of support on overall pillar behavior is expected to vary as a function of pillar W/H, and this relationship could be an interesting topic of research in its own, this study is instead focused on analyzing the capabilities and drawbacks of bonded block models for ground-support interaction modeling in general.

The overall behavior of bonded block models depends on the properties assigned to the blocks and their contacting interfaces. The interfaces are represented by the Coulomb-sliding model in UDEC such that the contacts can fail in shear/tension when the shear strength/tensile strength is exceeded. For the elastic blocks, the input parameters are bulk modulus (K) and shear modulus (G); interfaces are characterized by normal stiffness (k_n) and shear stiffness (k_s), peak cohesion (c_{peak}) and residual cohesion (c_{res}), peak tensile strength (σ_{t,peak}) and residual tensile strength (σ_{t,res}), and, peak (φ_{peak}) and residual (φ_{res}) friction angles. The behavior of the interfaces is similar to a perfectly brittle material where the strength drops to its residual value instantaneously upon failure. The pillar model developed here is 8 m wide and 4 m high, with two elastic rock beams for load transfer on either side (Figure 5.8). These beams were assigned the same elastic properties as the blocks and represent the roof and floor material in an underground mine. For the supported models, rockbolts in the form of cable structural elements and faceplates in the form of liner elements were incorporated. The current study attempts to
model a massive hard rock pillar, similar to the pillars in Creighton mine (Walton et al., 2016). Consequently, no pre-existing fractures were considered in the model, meaning all inelastic deformation is associated with development of new fractures through intact rock.

An appropriate assessment of the capabilities of the two modeling methods must be preceded by calibration of the micro-properties against some established pillar behavior under the unsupported condition. The authors have previously presented modeling results outlining the behavior of supported (Sinha and Walton, 2018a) and unsupported (Sinha and Walton, 2018b) granite pillars in FLAC$^{3D}$. To mimic the pillar degradation process associated with the loss of spalled material along an unsupported pillar edge in a continuum model, Sinha and Walton (2018b) used a ‘deletion’ method (Appendix B). In this method, all boundary elements in the FLAC$^{3D}$ models under tension and exceeding a plastic shear strain limit (associated with the spalling limit) were incrementally deleted as the model was loaded via a constant downward velocity. The same procedure was followed here, but with a slight modification. Previously (Sinha and Walton, 2018b), the ‘deletion’ code was run on pillar models with a square cross-section in plan view. However, since UDEC is a 2D software and operates in a plane-strain mode, these models would be more directly comparable to 3D pillar models with large L/W (length to width) ratios. In this study, a stress-strain curve served as the target for calibration of the bonded block models. That stress-strain curve, termed as the ‘deletion stress-strain curve’ henceforth, was extracted from a FLAC$^{3D}$ ‘deletion’ model having a W/H of 2 and a L/W (length-to-width ratio) of 4.

The UDEC pillars were loaded through a constant velocity of 0.005 m/sec along the top and bottom edges of the rock platens. The average vertical stress over all blocks in the pillar was recorded every 1000 solution steps, as were the relative vertical displacements between the pillar
top and bottom and the relative horizontal displacement at nine points along the left and right edges of the pillar. Typical runtime ranged from 18-24 hours, with the supported pillar models requiring more time to reach their peak strengths. In this study (and the other large-scale studies in this thesis), the ‘Combined’ damping mode was employed. Detailed consideration of damping mode is documented in Appendix C.

5.5.2 Model calibration and results under unsupported conditions

The calibration of the block and contact micro-properties was performed in a sequential fashion. First, the elastic properties of the blocks were varied to match the linear portion of the stress-strain curve. In this stage, the contacts were assigned high stiffness so that their effect on the model behavior was negligible. The next stage consisted of a sensitivity analysis where the cohesion, friction angle and the tensile strength of the contacts were varied individually. This helped to constrain the parameter space that could match the peak strength as well as the general trend of the FLAC3D ‘deletion’ stress-strain curve. The final stage consisted of numerous trial and error runs within the chosen parameter space until a best-fit parameter set was obtained. During the course of model calibration, it was found that a CWFS type contact behavior (i.e. the cohesion decayed and the friction mobilized post failure of the contact) performed better in capturing the post-yield, pre-peak hardening exhibited by the FLAC3D models and expected in reality (Krauland and Soder, 1987; Preston et al., 2013). Such a choice of contact model has previously been employed by other authors to simulate brittle rocks (Ghazvinian et al., 2014) and rockmasses (Oliveira and Diederichs, 2017; Sinha and Walton, 2018b).

Figure 5.9 shows the stress-strain response from the calibrated bonded block models. Both block geometries allowed for the expected pre-peak hardening and the peak strength to be
modeled. Post-peak, the Voronoi model exhibited continued hardening while the Trigon model behaved in a more brittle fashion. The reason could be the greater degree of interlocking in the Voronoi model due to its polygonal block structure (Mayer and Stead, 2017). The Trigon model, on the other hand, has a predisposition towards shear fracturing due to the availability of potential linear failure pathways (Ghazvinian et al., 2014; Mayer and Stead, 2017). Such a modeling approach might be suitable for softer rocks, where the failure mechanism is dominantly shear fracturing.

Figure 5.8 Geometry of a ‘supported’ bonded block pillar model with the faceplates and rockbolts shown in red and black, respectively. The spacing of the bolts varies depending on the number of bolts installed along the height of the pillar.

Table 5.3 lists the block edge lengths and the calibrated micro-parameters for both bonded block models. Voronoi blocks are typically larger in area than Trigon blocks. As a result,
a larger edge length was required for the Trigons to ensure that the total number of elements in each of the models was approximately the same. The calibrated parameters obtained are somewhat sensitive to block size, and this size should generally be related to the scale of displacements intended to be replicated in the models.

Figure 5.9 Average stress-strain curve for FLAC$^{3D}$ ‘deletion’ model, UDEC Voronoi model and UDEC Trigon model.

The higher tensile and lower shear strength for the Voronoi models are a counterbalance to the geometric tendency for damage in such models to localize in an extensile mode (Mayer and Stead, 2017) which is, in fact, consistent with how spalling occurs around underground excavations (Martin, 1997; Diederichs 2007). As the pillars were quasi-statically loaded to peak and beyond, bulking of the pillar walls and hourglassing of the core (Krauland and Soder, 1987) could be clearly observed.

With the block and contact micro-parameters constrained, the next task was to obtain realistic input properties for the bolts and faceplates. Pull-tests with different parameter
combinations were conducted on a cable element embedded in a 2 m x 2 m block until a satisfactory match was obtained between the modeled load-displacement profile and the load-displacement profile presented in Luke (2016). No difference in the calibrated properties was noted when the simulated rock specimen was defined by either Voronoi or Trigon blocks. Table 5.4 lists the calibrated cable element properties. The properties are similar to what have been used in other bonded block studies (Gao et al., 2014a; Kang et al., 2015). The tensile limit also corresponds well with the strain level reported by Li (2013) for grouted rebar bolts. To model faceplates, 5 mm thick, 15 cm long liner structural elements with 200 GPa modulus were utilized, and the interface between the liners and the pillar was assumed to be indestructible.

Table 5.3 Input parameters for the bonded block models.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Voronoi</th>
<th>Trigon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Edge Length</td>
<td>10 cm</td>
<td>12.5 cm</td>
</tr>
<tr>
<td>K</td>
<td>28.6 GPa</td>
<td>28.6 GPa</td>
</tr>
<tr>
<td>G</td>
<td>26.1 GPa</td>
<td>26.1 GPa</td>
</tr>
<tr>
<td>$c_{\text{peak}}$</td>
<td>62.5 MPa</td>
<td>90 MPa</td>
</tr>
<tr>
<td>$c_{\text{res}}$</td>
<td>0.01 MPa</td>
<td>0.01 MPa</td>
</tr>
<tr>
<td>$\phi_{\text{peak}}$</td>
<td>0°</td>
<td>0°</td>
</tr>
<tr>
<td>$\phi_{\text{res}}$</td>
<td>19.1°</td>
<td>34°</td>
</tr>
<tr>
<td>$\sigma_{t,\text{peak}}$</td>
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<td>9 MPa</td>
</tr>
<tr>
<td>$\sigma_{t,\text{res}}$</td>
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<td>0.01 MPa</td>
</tr>
<tr>
<td>$k_n$</td>
<td>12000 GPa/m/m</td>
<td>12000 GPa/m/m</td>
</tr>
<tr>
<td>$k_n / k_s$</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>
Table 5.4 Input parameters for cable structural element.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter</td>
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</tr>
<tr>
<td>Yield strength</td>
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</tr>
<tr>
<td>Shear stiffness</td>
<td>$2 \times 10^3$ MN/m/m</td>
</tr>
<tr>
<td>Elastic Modulus</td>
<td>210 GPa</td>
</tr>
<tr>
<td>Cohesive capacity of grout</td>
<td>0.30 MN/m</td>
</tr>
<tr>
<td>Tensile strain limit</td>
<td>15%</td>
</tr>
</tbody>
</table>

5.5.3 Model results with varied supports and comparison to in-situ data from Colwell (2006)

The performance of 4 different support layouts was assessed using the Voronoi and Trigon pillar models: 3 Bolts, 4 Bolts, 5 Bolts and 5 Bolts + Mesh. In all these supported models, the first bolt was located 0.5 m from the host rock, and the remaining bolts within the pattern were then equally spaced across the pillar surface. The rationale behind this selection was to prevent installing bolts too close to the roof and floor, based on the operational constraints faced by a bolter in underground mines. In the 5 Bolts + Mesh layout, faceplates were omitted and the entire pillar surface was covered by a 5 mm liner structural element in order to mimic a bolt + steel mesh support combination. Again, an indestructible interface was assumed between the mesh and the pillar and the liner was assigned a modulus of 0.1 GPa and a Poisson’s ratio of 0.30.

Figure 5.10 illustrates the vertical stress – axial strain and vertical stress – lateral strain curves for the supported and unsupported Voronoi pillar models. For the Voronoi models, an increase in peak pillar strength was observed with an increase in support density. The pillar peak
strengths are: (a) No Bolt: 90 MPa, (b) 3 Bolts: 95 MPa, (c) 4 Bolts: 100 MPa, (d) 5 Bolts: 111 MPa, and, (e) 5 Bolts + mesh: 112 MPa; note that according to an empirical database of supported pillar cases in-situ, the strength of a supported W/H = 2 pillar in Creighton Granite would be expected to be approximately 95 MPa (Sinha and Walton, 2018a). Although the increase in peak strength is not dramatic, a substantial reduction in lateral strain was observed at fixed stress levels. For example, at $\sigma_v = 80$ MPa, the average lateral displacement for each side of the pillar was: (a) No Bolt: 46 mm, (b) 3 Bolts: 35 mm, (c) 4 Bolts: 32 mm, (d) 5 Bolts: 22 mm, and, (e) 5 Bolts + Liner: 21 mm. This effect of the different layouts is only notable at load levels greater than $\sigma_v = 50$ MPa. This is physically intuitive as a passive support can provide resistance only after being loaded by ground movement along its length. In this case, the movement is related to the initial failure of block contacts and subsequent detachment, manifesting as ‘spalling’ at the excavation scale.

Considering the 5 Bolt cases with and without mesh, it can be seen in the vertical stress-lateral strain plot that the two models show similar behavior up to approximately $\sigma_v = 80$ MPa, beyond which the 5 Bolts + Mesh curve goes above the 5 Bolts curve. The authors believe that the similarity in the initial portion of the curves is due to the low stiffness of the liner element (simulating mesh), which allows it to deform to the shape of the detached/spalled blocks prior to providing any appreciable resistance. As the volume of the ‘baggage’ zone held by the mesh increases at higher load levels, some confinement is generated, which pushes the vertical stress-lateral strain curve over the 5 Bolts curve. This is pictorially depicted by the black and brown boxes in Figure 5.10a.

Colwell (2006), on the basis of extensometer measurements from multiple coal mines, concluded: “the steel mesh … does not have a significant impact on the total rib displacement.
However, the extensometry … suggests that the use of steel mesh significantly reduces the extent of softening …”. In order to evaluate the models with this statement in mind, the horizontal stress contour was extracted from the 5 Bolts and 5 Bolts + Mesh models at a $\sigma_v$ of 90 MPa (see Figure 5.11); at this stress level, the difference in average lateral strain was less than 0.005 (20 mm) between the models (refer Figure 5.10a). The edge of the low confinement ($\sigma_3 \leq 4$ MPa) damaged zone in the pillar, marked by white solid lines, was found to be farther away from the center of the pillar for the 5 Bolts + Mesh model in comparison to the 5 Bolt model. While this confirms what Colwell (2006) had stated with respect to the “extent of softening”, the model does predict some (minor) reduction in lateral displacements as a result of mesh installation. The difference could be attributed to either the pillars in Colwell (2006) not being loaded to large strain levels or the inability of coal pillar walls to stabilize for modest changes in confinement due to the presence of closely spaced weakness planes (i.e. cleats). The ‘holding’ mechanism of the mesh can be seen in Figure 5.11 (refer to the black boxes) where the bulking at the top right corner of the pillar is markedly suppressed.

Figure 5.10 (a) Vertical stress versus average lateral strain, (b) Vertical stress versus average axial stress for the Voronoi pillar models.
The Trigon models exhibited a slightly different trend with increasing support density (Figure 5.12). The peak strengths for the Trigon models are: (a) No Bolt: 91.5 MPa, (b) 3 Bolts: 97 MPa, (c) 4 Bolts: 106 MPa, (d) 5 Bolts: 108 MPa, and, (e) 5 Bolts + Liner: 105 MPa. These values show that the strength does not increase uniformly as the support density was increased in the model. In terms of both the axial and lateral strains, the 5 Bolt layout was stronger and caused lesser bulking in comparison to the 5 Bolt + Mesh layout. The reason for this unexpected behavior is the lower bulking associated with the shear fracture patterns that develop in the Trigon models. This can be seen by comparing the ranges of the lateral strain axes in Figures 5.10a (Voronoi) and 5.12a (Trigon). Under such low strains, the faceplates provided greater reinforcement than the mesh – the mesh requires greater displacements to create any significant effect in the model. No reduction in the depth of softening was noted when the horizontal stress contours of the 5 Bolt and the 5 Bolt+Mesh Trigon models were compared.
The question as to whether faceplates are more efficient in comparison to wire mesh is a debatable topic. Colwell (2006) attempted to combine different support elements and patterns into one rating scale, termed as Rib Support Rating (RIBSUP). While describing this rating scheme, Colwell (2006) goes on to mention that when a liner (described as straps and mesh in Colwell (2006)) is utilized and securely fitted to a pillar using bolts and faceplates, then it is sufficient to ignore the impact of faceplates. On the other hand, if no liner (e.g. mesh) is present, then the rating should account for the faceplate. In both the Trigon and Voronoi models, the mesh was securely attached to the pillar by the first node of the cable elements. This means that the Trigon pillar models would be expected to show a trend similar to the Voronoi models. A close examination of the fracturing process revealed the formation of shear triangles at the periphery of the pillars. In contrast, the Voronoi pillar models formed localized ‘slab’-like extensile fractures along its periphery, thereby quickly mobilizing mesh resistance (see Figure 5.11). Based on this observation, it seems that the Trigon model is not capable of appropriately
capturing the bulking process, and therefore may not be the best tool for modeling spalling
damage in intact brittle rocks.

5.5.4 **Analysis of bulking factor and comparison with empirical data**

The bulking of the rockmass was found to occur in a non-uniform fashion along the
width of the pillar in both the Voronoi and Trigon models. In practice, the bulking factor is often
defined as a percentage of volume increase (area increase, in this case) within the yield zone
from the undamaged state (Walton, 2014; Oliveira and Diederichs, 2017). To evaluate the
bulking factor, the areas between separated blocks were calculated using MATLAB’s image
analysis tool; note that any volume change associated with elastic deformation of the unbroken
model blocks was not considered in this calculation, as it was insignificant in comparison to the
amount of volume change associated with the discontinuum separation of blocks.

The entire analysis of bulking factor was conducted at the peak stress level of the
unsupported Voronoi and Trigon models. From a support comparison perspective, this is
meaningful, as one can only compare the effect of different support layouts for a particular stage
of mining operation (i.e. under a constant load level). In reality, the load will be locally
redistributed to adjacent pillars as damage initiates along its periphery. Since in this case only
one pillar is being simulated, the system can be thought to be load-driven such that at a constant
stress value, the average load in the different models is the same.

The analysis was conducted in three steps: (1) First, snapshots of the model geometry
were taken for the Voronoi and the Trigon models at $\sigma_v = 90$ MPa and $\sigma_v = 91.5$ MPa,
respectively. These pictures were imported into MATLAB using its image processing tools.
Calculating a bulking factor requires two different quantities: the extent of yield zone and the proportion of volume increase within this zone. To calculate the damage extent, a scale factor was assigned to the pixels and the entire pillar (starting from the center) was segmented into 0.25 m wide bins. Within each bin, the proportion of white spaces was found by converting the picture from RGB to grayscale and counting the number of dark and light pixels. A bulking factor for each bin could then be calculated, and these values were represented as a function of position within the pillar (see Figure 5.13 for these results from the Voronoi model). To ensure that the counting process did not extend beyond the pillar edge, a region of interest bounding the pillar was also defined. The point at which the segmented bulking factor started to escalate marked the edge of the yield zone. In the final step, the total area of cracks within the bins defining the yield zone was determined by simple addition and used in calculating the overall bulking factor. Note that the calculation assumed the yield zone to extend across the entire height of the pillar. This means that the final bulking factor values presented are slightly lower than the bulking factors for the true yield zone.

Approximately 0.75 m reduction in the total width of the yield zone was observed in the Voronoi models when support was added. No such reduction was noted in the Trigon models. Additionally, the extent of the yield zone was found to be much smaller (~0.25 m on either side) in the Trigon models. The most prominent difference between the Voronoi and Trigon behavior is the contrast in bulking within the first 0.5 m of the pillar wall between the unsupported and supported case. While the Voronoi models exhibited segmented bulking factor differences (both sides of the pillar) of 15% and 6% between the No Bolt and 3 Bolt cases, the same values were only 3% and 4% in the Trigon models.
To establish which discontinuum method is more realistic, the model results were compared to the empirical data of Kaiser et al. (1996). It is difficult, however, to obtain a uniform support pressure from numerical models for a particular support layout. The support pressure will depend largely on the degree to which each support element is mobilized under a given stress condition. To overcome this difficulty, the empirical guidelines of Hoek (1999) were used to estimate approximate equivalent uniform pressures. Hoek (1999) provided four equations for mechanical anchored bolts (17 mm, 19 mm, 25 mm and 34 mm diameter) while only one equation is provided for grouted rebar (20 mm diameter). Since the cable elements in this study correspond to grouted rebars, the maximum support pressures computed from the mechanical bolt equations had to be corrected. Specifically, the ratio of the support pressure for a 20 mm diameter grouted rebar to the support pressure of a 20 mm diameter mechanical anchored bolt was used to obtain the corrected support pressure for the different bolt layouts.
The guidelines for shotcrete from Hoek (1999) could not be directly applied to the 5 Bolts + Mesh model since the liner element is considered to be analogous to a wire mesh here. An alternative was to convert the peak strength of mesh from Dolinar (2006) into support pressure and add it to the 5 Bolt layout. Figure 5.14 compares the model-derived overall bulking factor values with empirical data from Kaiser et al. (1996) as a function of support pressure. It is useful to note here that the empirical dataset put forward by Kaiser et al. (1996) implicitly assumes the support elements to be strong enough to prevent any failure of the support system. While this might be true in some cases, there is a possibility of structural failure (of the supports) if the support density considered is not adequate. Under such a scenario, the behavior of the “supported” ground will be similar to that of the unsupported rockmass.

Figure 5.14 Comparison of model predicted bulking factor with empirical data as a function of support pressure.

The influence of reinforcement can be readily discerned for the Voronoi models in Figure 5.14. The effect is far less pronounced in the Trigon models, largely due to the failure localizing
as non-dilatant shear rather than tensile fracture development along the pillar periphery. This support effect trend is clearly not in accordance with the trend proposed by Kaiser et al. (1996). The trend for Voronoi, on the other hand, appears to generally match the empirically expected trend.

5.6 A conceptual framework of rock-support interaction in excavations experiencing stress-induced damage

This paper has individually detailed the effect of supports in continuum and discontinuum models of excavations. Here, the results are synthesized into a conceptual framework to obtain a clearer understanding of how the support responses in the two modeling approaches relate to ground-support interaction in reality. Figure 5.15 is a graphical representation of the proposed conceptual model. The actual ground-support interaction is represented by a region rather than a single curve due to variability in the rockmass mechanical attributes and support properties that affect their mutual interaction.

Within this framework, the ground support interaction curve (hatched region in Figure 5.15) is divided into three sections: (1) The initial (leftmost) region signifies the condition where the support system is unable/barely able to reinforce the target rockmass. This is related to the failure of individual support elements or the support system as a whole under unanticipated levels of ground deformation. (2) Beyond this region, the effect of support increases drastically with a corresponding increase in support density. This is analogous to the drastic drop in bulking factor for support pressures less than 200 kPa in the empirical data of Kaiser et al. (1996). The maximum marginal “value added” for increases in support density can be obtained in this range. (3) The final section of the curve corresponds to potentially over-designed support layouts for
which the gains in performance are significantly diminished (analogous to the portion of the empirical dataset beyond support pressures of 200 kPa).

Figure 5.15 Conceptual model outlining the relationship between support density and its effect on underground excavations in rock.

The second and third sections of the ground-support interaction curve can also be observed when the pillar wall displacements reported by Colwell (2006) are plotted as a function of support pressure. Extensometer data was available for 7 longwall mines, but only 5 were considered where a direct comparison between support layouts was possible. To ensure comparable load levels, displacements corresponding to a headgate side loading condition (i.e.
when the face has long passed the instrumented pillar) were selected, and the corresponding values were scaled with the depth of mining.

The support layouts were converted into support pressure following the recommendations of Hoek (1999) and then plotted against the total displacement/depth of mining (Figure 5.16). As Hoek (1999) did not provide any details on how to convert mesh support into an equivalent pressure, the support pressure determined in Section 5.5.4 was used. Any mesh that covered half the height of the pillar was assigned half this support pressure value.

The magnitude of lateral wall displacement at the mid height of the pillar is an indicator of the amount of stress-induced bulking of the rockmass. As can be seen in Figure 5.16, the slopes of the lines decrease as one moves along the support pressure axis, meaning that the effect of support decreases as its density is monotonically increased. The trend is consistent with the empirical data of Kaiser et al. (1996) and the support effect framework proposed in this study. It is not surprising that the first section of the conceptual curve is missing, given that the support in mines has evolved over years to manage expected displacement levels.

In terms of modeling approaches, continuum models implicitly account for the first two sections of the ground support interaction behavior. This is because as the support density is increased beyond a critical value in the ‘maximum gain’ region, the bulking factor becomes small enough such that the overall behavior of the ground can be represented by an equivalent continuum. Any additional support installed within the model has a marginal effect and is analogous to the third section of the ground support interaction curve. Note that the slight upward slope given to the last section is consistent with the negative exponential trend seen in Section 3.3.1 for the support effect in continuum models.
Discontinuum models, on the other hand, were found to be capable of reproducing the entire range of rock-support interaction behavior. Here, the Voronoi and Trigon BBM modeling approaches are both encapsulated within the “discontinuum” term, but it appears that the Voronoi results are more consistent with actual rock-support interaction than the Trigon results. Overall, the discontinuum modeling approach is a far superior tool relative to the continuum modeling approach for assessing and designing pillar reinforcements, and this is primarily because of the presence of local discontinuum elements (i.e. the interfaces between neighboring blocks) allowing blocks to separate upon failure.

The existence of the first section of the ground-support interaction curve was tested by running 2 additional Voronoi pillar models that had one bolt located along its mid-height. The details of the 2 models are as follows: (a) 1 Bolt: No changes were made to the cable parameters. (b) 1 Bolt (Variant 1): The tensile yield strength was reduced to 0.15 MPa. Since Hoek (1999)
only provides the maximum support pressure as a function of bolt diameter and spacing, these support layouts could not be converted into equivalent pressures. However, it is understandable that the two models correspond to increasing levels of support capacity, meaning that they should be spaced horizontally in terms of “support density” as shown in Figure 5.15.

The reductions in bulking factor and lateral strain with respect to the No Bolt case were computed for all the Voronoi models and are plotted in Figure 5.17. As expected, the lower support capacity cases led to support system failure and showed almost no change in pillar behavior from the unsupported cases.

The divergence between bulking factors and lateral strain reductions at higher support densities can be attributed to the fact that bulking factor calculation only considers the peripheral portion of the pillar. Lateral strain, on the other hand, is determined over the pillar width and consequently dampens the total support effect. The qualitative similarity in the overall trend to Figure 5.15 provides confidence in the proposed framework. In the ‘inadequate support’ region (i.e. Variant 1), failure of the bolts was observed at early stages of loading. This juncture between the ‘inadequate support’ section and the ‘highest gain’ section is site-dependent and can be related to the rock type, existing stresses, yield depth and magnitude of bulking.

The most important attribute of the ground-support interaction conceptual model is the existence of an optimum point (see Figure 5.15). From a design perspective, a support combination should ideally be placed at the optimum point or in the maximum gain region. Whether a support layout within the maximum gain-region is capable of reinforcing the target rockmass is dependent on specific site conditions. In this study, the 4 bolt layout could be considered as the optimum point for the Voronoi models (refer to Figure 5.17).
develop such a curve for a given excavation, then the chances of over-design could be largely alleviated without compromising safety; as demonstrated in this study, such a curve can be developed through the use of discontinuum bonded block models.

![Diagram](image)

Figure 5.17 Bulking factor and lateral strain reduction in Voronoi pillar models.

### 5.7 Conclusions

This study has presented a systematic comparison of the capabilities of continuum and discontinuum models in capturing ground-support interaction behavior for managing stress-driven damage in sparsely fractured rockmasses. While continuum models can replicate the stresses and deformations observed in supported rockmasses, they significantly underestimate the effect of support when support is explicitly considered in the models. The primary reason is their inability to allow block detachment post-failure, thereby imposing a boundary condition along the exterior of the model. This proposition is supported by studies conducted by other authors and was verified here by comparing model behaviors with and without support elements.
Bonded-block discontinuum models, on the other hand, permit explicit block separation as the block contacts are damaged due to loading. The two primary block representations that are used in practice (Voronoi Tessellation and Trigons) were tested with different combinations of skin support. The Voronoi approach was found to exhibit a behavior that is more consistent with previously reported observations of actual in-situ behavior. The difference was primarily due to fractures localizing as non-dilatant shear in Trigon models instead of extensile fractures as observed in the Voronoi models. This is thought to be caused by the shape of the constituent blocks in the two types of models.

Finally, field-data from literature and results from the numerical models were synthesized to develop a conceptual model for rock-support interaction. Based on support effectiveness, the curve was divided into three regions: an initial inadequate support region, followed by a maximum gain region, and ultimately an overdesigned region. Within this framework, the discontinuum modeling approach has been identified to capture the entire range of rock-support interaction behavior and is therefore a superior tool for the design of support systems for underground excavations. Future research will focus on using the Voronoi approach to model the effect of different support configurations in the context of specific mining case studies.

5.8 Acknowledgements

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CHAPTER 6

MODELING THE BEHAVIOR OF A COAL PILLAR RIB USING BONDED BLOCK MODELS (BBMS) WITH EMPHASIS ON GROUND-SUPPORT INTERACTION

This paper has been submitted to the journal *Tunnelling and Underground Space Technology* (Sinha and Walton, 2020d).

6.1 Abstract

Rib spalling is a major hazard in the mining industry and in absence of coal rib support guidelines, accidents have continued to occur in recent years. Developing effective support guidelines requires a complete understanding of pillar damage mechanisms as well as the rock-support interaction mechanism. Bonded Block Models (BBMs) represent a convenient tool for this purpose, as they can reproduce the rock fracturing process reasonably well, but it is not known whether this modeling technique can quantitatively replicate the impact of reinforcement (bolts) on otherwise unsupported ground. To bridge this gap in research, we employed the BBM approach to simulate the behavior of a supported coal pillar rib located in a longwall mine in Australia. This case study presents a unique opportunity in that two otherwise identical chain pillars with different support densities adjacent to one another were instrumented. After calibrating a model against displacement and stress measurements made over the course of mining in one pillar, when the support in the calibrated BBM was modified to match that of the adjacent chain pillar, it could predict displacements to within 6 mm of what was measured in-situ. Given the ability of the BBM to match field-measured displacements and stresses and also ground behaviors for varying support densities, it seems that such a model has the potential to be used as a support design tool. Lastly, the effect of block shape was investigated by replacing the
elongated blocks with isotropic polygonal blocks. This model could not reproduce the ground-support interaction very well, likely due to the inaccurate geometric representation of an anisotropic rock like coal.

6.2 Introduction

Recent years have seen a rapid increase in the use of discontinuum modeling tools for studying the rock fracturing process, both at the laboratory-scale (Lan et al., 2010; Bahrani et al., 2014; Bewick et al., 2014; Abdelaziz et al., 2018; Sinha et al., 2020) and at the field-scale (Coggan et al., 2012; Lan et al., 2013; Gao and Stead, 2014; Shen, 2014; Bai et al., 2016). In comparison to the continuum approach, where material damage is approximated through inelastic yield, discontinuum models attempt to explicitly simulate the rock fracturing process by allowing the elements to detach and separate. The major advantage of using such a modeling approach is that it does not require defining complex constitutive relationships and operates based on simple physical laws (e.g., Newton’s laws of motion). Additionally, discontinuum models have been shown to better replicate the ground-support interaction mechanism in comparison to continuum models and therefore have the potential to be used as a support design tool (Bahrani and Hadjigeorgiou, 2018; Sinha and Walton, 2019b). The necessity for developing such tools is exemplified by the continued confidence placed on experience-based support design approaches (Larson and Dunford, 1996; Colwell, 2006; Mohamed et al., 2016a).

While a number of discontinuum modeling techniques exist (PFC$^{2D}$: Wanne et al., 2004; Cai et al., 2007; Kias and Ozbay, 2013; FDEM: Elmo and Stead, 2010; Li et al., 2019b; Vazaios et al., 2019, etc.), this study is focused on the Bonded Block Modeling (BBM) method as implemented in Itasca’s Universal Distinct Element Code (UDEC). In BBMs, a material space is
represented by bonded polygonal (Voronoi Tessellation) or triangular (Trigon) blocks that can detach along the contacts when the tensile and/or shear strength of the contact is exceeded. Although the vast majority of the previous studies have focused on laboratory-scale rock fracturing processes, there has been some success in reproducing field-scale behaviors as well. Coggan et al. (2012) and Gao and Stead (2014) modeled the shear fracture formation above coal mine entries, while Gao et al. (2014b) simulated the longwall caving process. Christianson et al. (2006) conducted numerical triaxial tests on lithophysical tuff specimens using Voronoi blocks to aid in the design of the Yucca mountain nuclear waste repository. Preston et al. (2013) investigated the effect of aspect ratio (i.e. width to height ratio) on the strength of a limestone pillar. Muaka et al. (2017) used an integrated discrete fracture network (DFN) – Voronoi approach to understand the destabilizing effect of clay-filled shear structure on the stability of rock pillars. While both Voronoi and Trigons have been used in field-scale applications, Sinha and Walton (2019b) have recently shown that unlike Voronoi, Trigon models tend to show less of a reduction in bulking when supports are added than would be expected in reality (at least for two-dimensional models). Accordingly, we only considered the polygonal block geometry in this study.

Coal is a brittle, anisotropic material, and its mechanical response is largely controlled by its cleat structure (Kim et al., 2018; Song et al., 2018). As cleats are natural planes of weakness, their orientation with respect to the roadway influences the ground control issues observed at a site (Smith, 1992; Jones et al., 2014). Gao et al. (2014c) modeled the anisotropic behavior of coal in PFC^3D by representing the coal matrix using bonded spheres and the cleats and bedding using a DFN. A similar approach was applied by Vardar et al. (2019) where the coal matrix was simulated using Trigons in 2D and the cleats were simulated using a DFN. In terms of actual coal
mine case studies, Bai et al. (2016) used Voronoi to simulate the behavior of an entry housed in a water-rich environment. Other notable works include those by Kang et al. (2015), Chen et al. (2016b), and Yang et al. (2017) using the Trigon modeling approach. However, none of these coal-mine case studies considered cleats and/or their effect on roadway deformations. To allow fractures to form preferentially along the direction of cleats (as reported by Colwell, 2006 for the site under consideration), we use elongated Voronoi blocks to model coal, similar to the approach taken by Ghazvinian et al. (2017). The representation of a coal pillar using elongated Voronoi blocks is based on three major assumptions: (1) Small-scale heterogeneities in the coal do not affect the macroscopic behavior of the pillar; (2) Cleats primarily act as weakness planes rather than pre-existing discontinuities; and (3) There are no significant large-scale joints that affect the coal pillar behavior. For the site under consideration, no significant jointing or micro-scale heterogeneity in the coal was reported (Colwell, 2006).

This study is a continuation of the authors’ efforts to better understand the capabilities of BBM, extend its application to large structure analysis, and establish it as a tool for designing skin supports in underground mines (Sinha and Walton, 2019b; Walton and Sinha, 2020). Specifically, this study is focused on modeling the West Cliff Mine case study (Colwell, 2006), which is unique in the sense that two adjacent pillars were instrumented, but the pillars had different rib support patterns. Given the proximity of the two pillars and the fact that both were given a single geological description by Colwell (2006), the two pillars can be considered similar from a geological perspective, and any differences in the observed behavior can be directly linked to the differing support patterns.

The West Cliff Mine case study was previously modeled by Mohamed et al. (2016b) and Sinha and Walton (2020b) in FLAC3D. Mohamed et al. (2016b) used a user-defined coal rib
constitutive model while Sinha and Walton (2020b) employed the progressive S-shaped yield criterion (Sinha and Walton, 2018a). Subsequently, Sinha and Walton (2019b) tested the calibrated continuum FLAC\textsuperscript{3D} model with the addition of extra bolts (above and beyond those installed in the field) and showed that the incorporation of extra bolts suppressed the rib displacements by no more than 7%. In contrast, the rib displacement measured at the second pillar (with 2 extra bolts) was only \~30\% (70\% reduction) of that at the pillar to which the model was calibrated. This previous finding highlights the inability of continuum models to directly reproduce the effect of reinforcement on ground behavior. In this study, we demonstrate that BBMs can replicate this local reinforcement influence on ground behavior. Specifically, we have calibrated the behavior of a coal pillar in the West Cliff Mine against field-measured displacements and stresses and used this calibrated model to evaluate the influence of support in comparison to what was observed in-situ.

6.3 Site description and model setup

6.3.1 Description of the site and instrumentation

The West Cliff Mine is a two-entry longwall coal mine located along the south-east coast of Australia. The particular panel under consideration (Panel 515) is 480 m below ground surface, with the chain pillars spaced at 42 m and 125 m center-to-center across and along the long axis of the panel, respectively. At the instrumented sites, the entry was 4.8 m wide and 3 m high. Colwell (2006) installed a 7 m long multi-point extensometer and a stress cell, each, in two adjacent pillars, referred to as Site A and B herein. Both instruments were installed horizontally at the mid-height, with the stress cell located 4 m into the pillar to monitor the stress changes.
associated with longwall face advance and progressive rib fracturing. Although the stress cells were installed when the longwall face was ~450 m inby of Site A, the extensometers were not until the face was about 72 m inby of Site A. Monitoring was continued until the longwall face was ~981 m outby of Site A.

The key difference between Sites A and B is that at Site A, the rib section was supported by two 1.2 m long, 16 mm diameter resin grouted rebars while at Site B, two additional 1.8 m long rebars were installed (4 bolts in total). The 1.2 m and 1.8 m bolts were spaced at 1 m and 2 m, respectively along the entry. At both sites, some mesh was also installed – Site A had a 400 mm tall strip of mesh along the upper row of bolts while Site B had an additional 500 mm mesh along the pillar bottom. Mesh was not explicitly modeled in this study, as its effect is negligible in comparison to bolts, and also because it only extended partially along the seam height. Therefore, the only type of support considered in this study is rockbolts as reinforcing elements, which serve to strengthen the rockmass and improve its self-supporting capacity (Kaiser et al., 1996).

Unfortunately, the stress cell at Site B did not function properly, and model calibration was therefore conducted using the displacements and stresses measured at Site A. Once the calibration was complete, two additional 1.8 m long bolts were installed in the model and the peak rib displacement measured at Site B was compared to that in the model.

In the continuum models of Mohamed et al. (2016b) and Sinha and Walton (2020b), it was assumed that the extensometers and stress cell measurements corresponded to two stages of loading: (1) Development – this corresponds to a state when the entries have relaxed completely after initial excavation; and (2) Headgate – this is related to the stress redistribution caused by
the approach and passage of the adjacent longwall face. Accordingly, in these previous models, the first set of measurements by the extensometer at Site A was considered to be associated with entry relaxation, and all subsequent measurements were considered to be related to headgate loading.

While this might not be an issue for continuum models where the support elements only demonstrate strain-compatibility with the deformations of the rock (Lorig and Varona, 2013) rather than significantly influence it, the timing of support installation is very important in discontinuum models (Bouzeran et al., 2017). Colwell (2006) reported that the rib bolts at West Cliff Mine were installed within 4 m from the face. For brittle materials like coal, entry relaxation occurs very close to the face (Mohamed et al., 2016b), meaning that the entries were probably fully (or almost fully) relaxed when the bolts were installed. Accordingly, the bolts were installed in the BBM after full relaxation of the entries.

In the field, the extensometers were installed after bolt installation, and the first set of measurements at -52 m face location (~12 mm rib displacement) was interpreted to be associated with the “development” condition in Mohamed et al. (2016b) and Sinha and Walton (2020b). Given that the extensometers were installed well behind the entry face, it is likely that full entry relaxation associated with face advance had already occurred by the time the extensometers were installed. Upon recognizing this, the stress cell data at Site A were examined to evaluate whether or not this initial displacement could be associated with headgate loading. Figure 6.1 shows the stress measurements made at Site A as a function of the longwall face location. It can be seen that the measured stresses do not increase until the face is about 25 m inby of the instrumented pillar, indicating that significant headgate loading had not initiated up to this point. There was also no change in rib displacement between -52 m face location and +2 m face location. Based
on all this, we believe that the displacements measured in the time period between its installation at -72 m longwall face location and -52 m longwall face location were not related to development loading or headgate loading, and may correspond to time-dependent deformation mechanisms or other unknown phenomena. Since the exact cause of the displacements (~12 mm) measured at -52 m face location is not known, we considered these measurements as the baseline against which to compare all displacements associated with headgate loading and zeroed all subsequent extensometer measurements (and model results) with respect to this stage.

Figure 6.1 Stress change as measured in the field with advance of the longwall face (after Mohamed et al., 2016b).

6.3.2 Description of the BBM setup

Figure 6.2 shows the plane strain BBM setup of a half pillar and half entry with dimensions of 21 m x 31 m that was used in this study. Only the first 4 m of the coal pillar was modeled using elongated Voronoi blocks to allow for explicit fracture formation and separation. This value was selected based on the 1-2 m depth of significant fracturing as identified from the
Site A extensometer data. The rest of the coal pillar and the roof and floor layers were modeled using continuum zones. Each Voronoi block in the pillar was discretized by multiple constant strain-triangular zones; these zones can deform elastically or inelastically depending on the constitutive model assigned to them (Itasca, 2014a).

Figure 6.2 Overall geometry of the BBM pillar model. The zoomed in view shows the blocks and zones.
Sinha and Walton (2020c) demonstrated that it is not possible to reproduce pillar displacements at deeper locations with elastic blocks while attaining realistic displacements at the rib surface. In particular, they showed that when the blocks are purely elastic, the surficial displacements could be matched but if an attempt is made to match displacements at locations deeper within the pillar (~0.5 m - 1 m), then it is not possible to achieve such a match without the surficial displacements being notably overestimated. This is because extremely small blocks cannot be used due to computational limitations, and cm-scale blocks lead to large geometric mismatch which cannot be overcome for cases with blocks that are not especially deformable (such as fully elastic blocks). As a viable alternative, one can use an inelastic constitutive model in the zones such that damage near the pillar periphery is explicitly represented by contact failure while finer-scale damage occurring deeper within the pillar is approximated by a combination of contact failure and zone yield. A similar methodology was followed in this study, where a Cohesion-Weakening-Frictional-Strengthening model (CWFS; Hajiabdolmajid et al., 2002) was assigned to all zones in the coal layer (both within the Voronoi blocks and in the fully continuum portion of the pillar). The roof and floor layers were simulated as elastic, with properties listed in Table 6.1 (from Mohamed et al., 2016b).

The CWFS strength model was initially developed for simulating brittle fracturing in rocks (Martin and Chandler, 1994; Hajiabdolmajid et al., 2002) and it is known that coal is a highly brittle material (Kim et al., 2020). Although the CWFS strength model has not been directly employed for simulating coal pillars in the past, it was in part used by Sinha and Walton (2020b) through the application of the progressive S-shaped criterion, which essentially combines the CWFS strength model at low confinement and a shear yield model at higher confinement (Sinha and Walton, 2018a). Since the focus of this study is on the local fracturing
behavior along the pillar periphery (low-confinement conditions), the use of only a CWFS strength model was thought to suffice. If the focus were on the global strength of the pillar, then the consideration of both the low as well as the high confinement section of the progressive S-shaped criterion would have been required.

Lastly, the contacts between the coal layer and the host rock were simulated using a low strength joint element. The joint elements were assigned zero tensile strength, 0.5 MPa cohesive strength and 15° friction angle. Such low values were selected to allow the host rock to slip along these boundaries and mimic the weakening effect of the dirt bands between the pillar and the surrounding rock as reported by Colwell (2006). All other joints within the roof and floor layers, as well as the joint between the elongated Voronoi section and the continuum zone section of the coal layer, were made indestructible (construction joints).

Table 6.1 Rockmass elastic parameters for different layers in the model (from Mohamed et al., 2016b).

<table>
<thead>
<tr>
<th>Layer</th>
<th>Young’s Modulus (GPa)</th>
<th>Poisson’s ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interbedded sandstone (roof)</td>
<td>12</td>
<td>0.26</td>
</tr>
<tr>
<td>Mudstone (roof)</td>
<td>10</td>
<td>0.26</td>
</tr>
<tr>
<td>Coal</td>
<td>3</td>
<td>0.25</td>
</tr>
<tr>
<td>Mudstone (floor)</td>
<td>12</td>
<td>0.26</td>
</tr>
<tr>
<td>Interbedded sandstone (floor)</td>
<td>12</td>
<td>0.26</td>
</tr>
<tr>
<td>Sandstone (floor)</td>
<td>15</td>
<td>0.26</td>
</tr>
</tbody>
</table>

The most recent version of UDEC (Version 7) has the capability of generating elongated polygonal blocks using the built-in Voronoi generator, but the current study was performed in Version 6 that cannot automatically create these blocks. For that reason, a 3 m high and 12.8 m
wide block had to be first built in RS2 and populated with Voronoi blocks of 0.1 m edge length, then imported into MATLAB and compressed into a 3 m x 6.4 m block (elongation factor of 2; 2.4 m entry + 4 m pillar). Finally, the block edges were entered in UDEC as crack elements.

In these models, we chose to use an elongation factor of 2, based on some laboratory-scale unconfined compression tests (UCS) with loading at 0° and 45° to the elongation direction. We calibrated the model peak strengths to those observed in laboratory tests by Kim et al. (2018) (Figure 6.3), and achieved a $\frac{UCS_{0^0}}{UCS_{45^0}}$ ratio of 1.38, similar to the ratio of 1.4 observed in the test data. For these models, the bottom edges were constrained via rollers and a very slow velocity was applied to the top boundaries to load the specimens. The calibrated contact parameters are listed in Table 6.2. Since the extent of anisotropy is controlled by the elongation factor (Ghazvinian et al., 2014), the ability to reproduce a ratio of ~1.4 provides confidence in the chosen value. An elongation factor of 2 was also employed by Ghazvinian et al. (2017) and Zhu et al. (2020) to simulate laminated rocks.

The field-scale simulations were conducted according to the following scheme:

(a) In the first step, the model was run without any excavation until mechanical equilibrium was attained. In this step, pre-mining horizontal stresses of 3.6 MPa (in-plane) and 16.3 MPa (out-of-plane) and a vertical stress of 11.6 MPa (Mohamed et al., 2016b) were applied to the model.

(b) In the next step, the first 2.4 m of the entry was extracted and the unbalanced forces were relaxed in 10 stages using the built-in ZONK function. This progressive relaxation is necessary in order to avoid unrealistic yielding/fracturing along the entry due to sudden
increase of unbalanced forces in the model. Once the entry was completely relaxed, rockbolts and faceplates were installed in the model and all displacements were initialized to zero (i.e. with respect to the start of headgate loading).

Table 6.2 Contact parameters for the laboratory-scale BBM.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Peak cohesion; $c_{\text{peak}}$ (MPa)</th>
<th>Residual cohesion; $c_{\text{res}}$ (MPa)</th>
<th>Peak friction angle; $\phi_{\text{peak}}$ (°)</th>
<th>Residual friction angle; $\phi_{\text{res}}$ (°)</th>
<th>Tensile strength*; $\sigma_{t,\text{peak}}$ (MPa)</th>
<th>Normal stiffness (GPa/m/m)</th>
<th>Shear stiffness (GPa/m/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>14</td>
<td>0.5</td>
<td>20</td>
<td>7.5</td>
<td>1.5</td>
<td>500,000</td>
<td>250,000</td>
</tr>
</tbody>
</table>

* Residual tensile strength ($\sigma_{t,\text{peak}}$) was set to 0.

Figure 6.3 Stress-strain curves for unconfined compression tests with loading along 0° and 45° to the block elongation direction. Fracture patterns post-simulation is shown in the inset.
(c) In the last step, the vertical stress along the model top boundary was increased at 0.2 MPa/stage while bringing the model to mechanical equilibrium after each stage to simulate the retreat of the longwall face. This is the same loading procedure followed by Mohamed et al. (2016b) and Sinha and Walton (2020b). While gateroad loading is undoubtedly more complex than that considered in this study, in the absence of other pertinent information about the site, a constant stress approach was used as a simplification. A total of 35 such stages were implemented to replicate the complete stress data from Colwell (2006).

6.4 Model calibration and results

6.4.1 Model parameters and calibration

The BBM model inputs were calibrated using an iterative manual back-analysis approach. Before delving into the results, it is necessary to review the different parameters that govern the behavior of such models. The parameters can be broadly sub-divided into two categories: coal mass parameters and support parameters. The coal mass parameters can be further sub-divided into zone parameters and contact parameters. As noted previously, the CWFS strength model was applied in the model zones within the coal layer. This model requires defining Young’s modulus (E), Poisson’s ratio (ν), peak and residual cohesions, tensile strengths and friction angles, and a critical plastic shear strain (ε<sub>ps</sub>) to control the rate of softening. As the CWFS strength model mimics the degradation of cohesion and mobilization of friction with damage (ε<sub>ps</sub>), the peak cohesion is larger than the residual value, while the peak friction angle is smaller than its residual counterpart (Hajiabdolmajid et al., 2002; Walton et al., 2016). The same
CWFS parameters were used in the Voronoi zones as well as the continuum portion of the coal seam. However, no yield was observed in the continuum portion of the coal seam after the headgate loading was complete, meaning the behavior of this portion of the pillar was effectively elastic.

Corrections in the zone modulus values are often required when separate continuum and Voronoi sections are modeled due to the presence of relatively low stiffness discontinuum contact elements in the latter (Garza-Cruz et al., 2014; Ghazvinian et al., 2017). Such corrections were not necessary in this case as the actual material being modeled is soft enough that the relative values of the contact stiffnesses used were sufficiently high to ensure that the effective modulus of the Voronoi section was the same as that of the continuum section. For the contacts, the strength parameters are similar to those for zones ($c_{\text{peak}}$, $c_{\text{res}}$, $\sigma_{t,\text{peak}}$, $\sigma_{t,\text{res}}$, $\varphi_{\text{peak}}$, $\varphi_{\text{res}}$); the normal and shear stiffness define the contact elastic behavior. The drop from peak to residual is instantaneous for the contacts.

The rockbolts were modeled using the rockbolt element available in UDEC, and the faceplates were modeled using the beam structural element. All faceplates were made elastic with $E$ of 200 GPa, $\nu$ of 0.3, and a rock-to-faceplate friction angle of 25°. The remainder of the rock-to-faceplate interface strength properties were set to zero. Estimation of the rockbolt properties is much more difficult, as pull test data from the site was not available. There are two key parameters that govern the anchor characteristics of rockbolt elements: stiffness ($K_{\text{bond}}$) and cohesive strength ($S_{\text{bond}}$) of the grout. UDEC manual (Itasca, 2014b) provides the following equation for estimating $K_{\text{bond}}$:

$$K_{\text{bond}} \approx \frac{2 \pi G}{10 \ln(1+2\ell/D)}$$

(6.1)
where, $G$ is the grout shear modulus, $D$ is the bolt diameter and $t$ is the thickness of the annulus (i.e. borehole radius minus bolt radius). If the annulus of the 16 mm rockbolt was around 3-3.5 mm and the resin grout modulus is 2.25 GPa (Farmer, 1975), then the range of $K_{\text{bond}}$ to be tested should be approximately 1.6-1.8 GN/m/m. Zipf (2006) provided practical values of $S_{\text{bond}}$ for simulating grouted rockbolts in different coal measure rocks. A range 120 kN/m to 150 kN/m, corresponding to grip factors of 0.3 to 0.4 ton/inch, was ultimately tested during the calibration process. $E$, $\nu$, tensile strength for the bolts were set to 210 GPa, 0.3, and 105 kN, respectively, per Mohamed et al. (2016b).

It is acknowledged that spacing of the rockbolt nodes may have some effect on its interaction with the blocks, particularly in cases where there are multiple blocks between nodes. In this case, the nodes were spaced at 0.05 m such that there was at least one node in each of the bolted blocks (a detailed analysis of the influence of node density on model results is presented in Appendix D). Lastly, as UDEC scales various support parameters depending on the spacing in the out-of-plane direction (Itasca, 2014b), this was set to 1 m and 2 m for the 1.2 m and 1.8 m bolts, respectively.

The Voronoi block contact parameters had the greatest degree of uncertainty associated with them and they were modified over a much wider range than the others. The greatest confidence was placed in the support and zone input parameters (meaning these were modified least from their initial values). Some erratic trends, similar to those reported in Sinha and Walton (2020c), were also observed in the current study. In particular, it was found difficult to control the displacements and stresses by small systematic changes to the different input parameters; the behaviors, however, were consistent with expectation when large changes were introduced to the parameters. This issue will be discussed in context of actual model results. Secondly, when the
zone strength parameters were made too strong, the rib displacements were found to decrease drastically. This occurred due to block movements contributing more towards the rib displacements and further highlights the need to introduce inelasticity in the Voronoi blocks, especially when the block size cannot be made very small.

Table 6.3 lists the calibrated set of model parameters. A high contact tensile strength was required for the contacts to prevent the blocks from buckling into the entry at low stress levels. Damage initiated first at the pillar corners through zone yield that eventually led to explicit fracturing along the block boundaries. If the damage was forced to initiate along block boundaries first, then the final model displacements were too large. This is illustrated in Figure 6.4 in form of rib displacement contours after the development relaxation stage (i.e. before “zeroing” for comparison of the headgate loading displacements to the extensometer data) with inelastic (Figure 6.4a) and elastic (Figure 6.4b) blocks. As can be seen, when the blocks were elastic, the displacements with the same contact properties were more than 200% of those with inelastic blocks. It seems that it is important to allow finer-scale damage to initiate first via zone yield at the corners in order to prevent extensive fracturing along the rib. Although the initial zone strength is lower than the contact strength in the model with inelastic zones, due to point loading and wedging of blocks, damage ultimately progressed via explicit cracking rather than through zone failure along the rib (i.e. under low confinement conditions).

With respect to the CWFS parameters, the critical plastic shear strain and the peak cohesion had to be changed from 0.0081 and 7.1 MPa in Sinha and Walton (2020b) to 0.035 and 8.4 MPa. The critical plastic shear strain is a zone-size dependent parameter and must be increased with reduction in zone size to obtain similar behaviors (Itasca, 2016b). The zones in
Sinha and Walton (2020b) were cubic and 0.1 m long; the zones are much smaller in this study, and an increase in critical plastic shear strain is therefore justifiable. With respect to the change in peak cohesion, the increase can be explained on grounds that the overall strength of a rockmass, composed of intact rock blocks and bounded by explicit fracture pathways, is lower than the strength of the intact rock components. In other words, to achieve an equivalent rockmass strength to that represented by the continuum model of Sinha and Walton (2020b) when accounting for explicit fractures, the material strength within the Voronoi blocks needed to be increased.

6.4.2 Model results

Figure 6.5 compares the rib displacement profile and stress profile as measured in the field with those from the calibrated BBM. The overall shape of the displacement profile was well replicated by the BBM (Figure 6.5a). From both the field data and the model results, it can be seen that the depth of significant fracturing lies in the range of 1-2 m from the rib surface, beyond which the behavior is nearly elastic. This explains why only the outer 4 m of the coal pillar was modeled using the elongated Voronoi blocks.

The measured change in vertical stress 4 m into the pillar (stress cell location in the field) was also well reproduced by the BBM (Figure 6.5b). The continued increase in stress is likely related to both the advance of the longwall face (see Figure 6.1) as well as the fracturing of the outer skin pushing the stresses deeper into the pillar. Additionally, the sudden jump in displacement between the 4.5 MPa and 6 MPa stress data points could be replicated by the model, but the jump may occur slightly sooner in the model than in reality (it is difficult to assess definitively given the lack of multiple data points in this range and the variability in the data).
is at this stage when the depth of fractured contacts suddenly increased in the BBM from \( \sim 0.3 \) m to \( \sim 1.1 \) m.

Table 6.3 Calibrated set of parameters.

<table>
<thead>
<tr>
<th>Zones - CWFS</th>
<th>Contacts</th>
<th>Rockbolt</th>
</tr>
</thead>
<tbody>
<tr>
<td>E (GPa)</td>
<td>( c_{\text{peak}} ) (MPa)</td>
<td>13.9</td>
</tr>
<tr>
<td>Peak cohesion (MPa)</td>
<td>( c_{\text{res}} ) (MPa)</td>
<td>0</td>
</tr>
<tr>
<td>Residual cohesion (MPa)</td>
<td>( \varphi_{\text{peak}} ) (°)</td>
<td>37</td>
</tr>
<tr>
<td>Peak friction angle (°)</td>
<td>( \varphi_{\text{res}} ) (°)</td>
<td>27.5</td>
</tr>
<tr>
<td>Residual friction angle (°)</td>
<td>( \sigma_t ) (MPa)</td>
<td>12</td>
</tr>
<tr>
<td>Tensile strength (kN)</td>
<td>Normal stiffness (GPa/m/m)</td>
<td>80,000</td>
</tr>
<tr>
<td>Critical plastic shear strain from peak to residual</td>
<td>Shear stiffness (GPa/m/m)</td>
<td>40,000</td>
</tr>
</tbody>
</table>

During model calibration, the authors identified multiple parameter sets that exhibited slightly better agreement with the stress data but showed mismatch with respect to the rib displacement profile. Two such model results, termed as Alternate 1 and 2, are shown in Figure 6.6. The only difference between the parameters of Alternate 1 and 2 and the calibrated model is that Alternate 1 and 2 had \( S_{\text{bond}} \) values of 125647.46 N/m and 125647.47 N/m while the calibrated model had a \( S_{\text{bond}} \) of 125620 N/m. As can be seen, the model behaviors were extremely sensitive to small parameter changes. Moreover, the increase in the \( S_{\text{bond}} \) necessarily did not lead to a delayed increase in displacements when the change was very small (the displacement jump in Alternate 1 occurred slightly later than in Alternate 2), but it did when the
change was larger (Alternate 1 and 2 had both had delayed displacement increases in comparison
to the calibrated model). Since the calibrated model showed reasonable fit to both the
displacement profile and the stress data, the authors decided to use this parameter set for the rest
of the study.

![Figure 6.4 Rib displacement contours after development relaxation with (a) Inelastic, (b) Elastic
blocks. Note that these displacements are presented relative to the initial unexcavated condition
rather than the post-development-relaxation datum used to compare model results to the
extensometer data.]

The evolution of fractured contacts and yielded zones in the calibrated model as a
function of headgate loading is shown in Figure 6.7. The depth of fracturing increased from
Stage 10 to Stage 15 and then remained almost constant up to Stage 33. The rapid displacement
increase shown in Figure 6.5b occurred at Stage 15 in the model. Based on a comparison of
Stage 10 and Stage 15 in Figure 6.7, one can recognize how the depth of fractured contacts has
almost tripled. This also occurred in the field when the face crossed the 58 m outby location and
then remained constant until the end of the monitoring period (Figure 6.8). Interestingly, this
increase in depth of fracturing could not be replicated by both Sinha and Walton (2020b) and Mohamed et al. (2016b) using FLAC$^{3D}$ models. We believe that it was possible to reproduce this behavior in BBM because of its ability to model discontinuous and localized damage processes. Continuum models maintain strain-continuity within their domain and as a result cannot simulate such localized high-strain phenomena.

Figure 6.5 (a) Rib displacement profile, (b) Stress change versus rib displacement as measured in field and those in the calibrated model. The displacements are presented with respect to the displacements measured when the longwall face was 52 m inby in the field (initiation of headgate loading at a longwall face position of -52 m).

The size of the blue regions, signifying yielded zones with $\varepsilon_{ps}$>0.005, increased with increasing vertical stresses. At an early stage (Stage 10), there was some yield at the corners. As loading continued, the extent of the yielded regions increased and formed a V-shaped region just at the edge of the fractured contacts. Such a V-shaped shear zone appears to be characteristic of stress-driven brittle failure processes, and has previously been observed in model results by Carter et al. (2008), Edelbro (2009), Sinha and Walton (2018a, 2020c) and Renani and Martin (2018b). Within the region of explicit fracturing, the number of yielded zones was minimal. This means that failure near the pillar boundary indeed occurred via contact failure (highly dilatant).
while it occurred via zone yield for regions deeper within the pillar (minimal dilation). Lastly, it can be noted how the failure of the joint elements bounding the coal seam also propagated deeper, allowing the coal layer to slip and accommodate the deformations due to fracturing and yielding of the coal pillar.

Figure 6.6 (a) Rib displacement profiles, (b) Stress change versus rib displacement as measured in field and those in two alternate models. The displacements are presented with respect to the displacements measured when the longwall face was 52 m inby in the field (initiation of headgate loading at a longwall face position of -52 m).

Figure 6.9 shows the axial load in the upper and lower rib bolt for the calibrated model. At early stages of loading, some local peaks can be identified in the upper rib bolt that are associated with fracture development in the model (Figure 6.9a). As soon as the depth of fracturing increased at Stage 15, the bolt attained yield strength for ~50% of its length, and there was also some failure of the bolt-grout interface. With continued loading, the bolt elements closest to the rib also attained yield strength and the length of the yielded section increased slightly. The decay in the bolt axial load to the right of the yielded region implies that it is still transferring some amount of the load into the coal and providing reinforcement to the fractured rib.
Figure 6.7 Fracture pattern and yielded zones for 5 different stages of headgate loading. Stage “1” corresponds to the model state prior to headgate loading (i.e. after development entry relaxation), and Stage “33” corresponds to the model state after 33 stages of headgate load increase.

Figure 6.8 Raw displacement profiles for 18 m, 58 m, 130 m, 217 m and 416 m outby face positions (Colwell, 2006).
In the lower bolt, a slightly different trend was observed (Figure 6.9b). It began to yield at an early stage of loading (Stage 10) at a specific location that corresponded to a local fracture in the model. As the headgate loading continued, a steady decline was noted in the peak load level. A closer look at the model results revealed a complete failure of the bolt-grout interface at Stage 15. Consequently, as the ribs continued to deform laterally, the entire rib bolt slipped and this resulted in a loss of axial load. To further understand this trend, it is useful to revisit the structural representation of bolts in UDEC. The interaction between the rockbolt nodes and zone vertices (also called gridpoints) is simulated by a spring/slider system in UDEC (Itasca, 2014b). When the local differential movement between a node and its neighboring gridpoints increases, the load in the grout increases as well as a linear function of the grout stiffness, until the peak strength is attained. Upon attaining the peak grout strength, the rockbolt slides with additional deformation in the rock, providing only a constant resistive force (equivalent to the grout strength) to the zone gridpoints. With that in mind, the axial load in the bolts is calculated with respect to the strain between neighboring nodes of the rockbolt. It seems that at later stages of loading, when the rib started to buckle, the entire rockbolt moved with the ground, reducing the differential movements between neighboring nodes and hence the associated axial loads. At all stages of simulation, the displacement of the outermost node in the lower bolt was found to be greater than in the upper bolt (referring to Figure 6.10 for one such stage; the rock displacement provides an approximation of the nodal displacement) and this explains why such a behavior was not observed in the upper rib bolt.

The bolt load profile shown in Figure 6.9a is rather different from Sinha and Walton (2020b) due to the fact that Sinha and Walton (2020b) did not consider any faceplates. Accordingly, for all rock-grout interfaces that failed, the axial loads were very low and, in some
cases, almost zero. In this study, faceplates were simulated using elastic beam elements and the last nodes of the rockbolts were merged with the central nodes of the faceplates. Because of this indestructible connection, failed sections of the rockbolts still continued to carry load up to their yield strength (Hyett et al., 2012). Although the model shows that the bolts are providing some reinforcement, it seems that they are approaching their ultimate capacities and with further loading (e.g. second abutment loading), it is possible that their reinforcement capability will be completely lost. This is somewhat supported by the description of the tailgate intersection (location where the longwall face intersects the tailgate entries) in Colwell (2006) for the adjacent panel: “The effects of second front abutment loading associated with the current longwall were evident to a distance of approximately 30 m outby of the face. Rib spall within this zone of 0.5 m to 1 m was observed on the block side rib…”. When 0.5 – 1 m of the rib was reported to have visibly spalled, it is likely that rockbolts, which were only 1.2 m long, were ineffective by this point.

Figure 6.9 Axial forces in the (a) top, (b) bottom rib bolt as a function of stage number.
6.5 Effect of rib supports

With the reliability of the BBM established, the next task was to investigate the effect of alternate support patterns on the model response. To that end, the calibrated model was re-run with no support and with 4 bolts. Figure 6.10 shows the horizontal displacement contour for the unsupported, 2 bolt (calibrated to Site A) and 4 bolt (Site B) models. The displacement at the mid-height of the pillar at the periphery is also shown in Figure 6.10. A remarkable increase in displacement from 59 mm to 116 mm (or 97% increase) was observed when the two rockbolts were removed from the model. Comparing with the continuum FLAC$^{3D}$ model of Sinha and Walton (2020b), only a nominal 5-6 mm increase was noted when the supports were omitted.

For the 4 bolt model, the rib displacement at the periphery was 29 mm (Figure 6.10). This corresponds to a 50% drop in displacements with respect to the 2 bolt model and is only 6 mm less than what was measured in the field (Colwell, 2006; Sinha and Walton, 2019b). It is interesting to note here that the match to Site B displacement was not a focus of model calibration, but an emergent behavior of the BBM when the two extra bolts were added (same properties as those in Table 6.3). The similarity between the model-predicted displacements and field measurements confirms that BBMs can reproduce ground behaviors under varying support conditions.

While there are many possible explanations for the 6 mm discrepancy between the 23 mm rib displacement measured at Site B and the 29 mm rib displacement in the 4 bolt model, we believe it may relate to the different degrees of rib damage at Site A and B at the start of the headgate loading stage (Stage 1). In the field, almost zero rib displacement was measured at Site B while ~12 mm was measured at Site A when the longwall face was 52 m inby of Site A.
What this means is that after the extensometers were installed at both sites, there was some movement at Site A before the first set of readings was taken (~52 m longwall face position). Due to the presence of additional support, any such movement that would have occurred at Site B was suppressed, and hence no deformation was recorded by the extensometer. As the BBM was initially calibrated to Site A and then used to simulate Site B, a ~12 mm displacement was present in both models following the development relaxation stage (before zeroing the displacements in the model; see Figure 6.4a). The difference in the rib conditions at the start of the headgate loading stage was therefore ignored in the BBM, and this might have led to the 6 mm mismatch between the model and the field data.

To better understand how the progressive damage development was affected by the incorporation of 2 additional bolts, vertical stress changes at a point 4 m into the pillar (stress measurement location) were plotted as a function of the rib displacement and are shown in Figure 6.11. An additional model was run with an out-of-plane spacing of 1 m for the 1.8 m bolts and is also shown in Figure 6.11; this alternative case corresponds to a higher support density than what was installed at Site B. For purposes of comparison, the plot for the calibrated 2 bolt model is also shown (same as in Figure 6.5b). As can be seen, the point of the sudden displacement increase was delayed by ~4 MPa in the 4 bolt (2 m) model. It should also be noted that the magnitude of increase is significantly lower in the 4 bolt (2 m) model, implying that the bolts not only reduced the depth of significant fracturing (Figure 6.10) but also suppressed the dilatancy within the stress-fractured region.

The sudden jump in the 4 bolt (2 m) model occurred at loading Stage 26, which corresponds to a ~270 m outby longwall face location for Site A. The equivalent longwall face location was determined by identifying the stress magnitude at loading Stage 26 in the calibrated
2 bolt model and then relating it to Figure 6.1 (recall that the stress data in Figure 6.1 is for Site A). According to the extensometer data of Colwell (2006), the sudden increase in displacement at Site B occurred when the face was about 440 m outby of Site A (Stage 29). The slightly premature displacement increase at Stage 26 in the model (Figure 6.11) is also consistent with the potential effect of the different rib conditions that existed at the start of headgate loading. If less contacts had failed after development relaxation (Stage 1) for the 4 bolt model, then it might have been possible to further delay the jump to occur closer to a stress corresponding to 440 m outby of site A. Lastly, it can be seen in Figure 6.11 that when the out-of-plane spacing for the 1.8 m bolts was reduced to 1 m, the jump did not occur at all and the displacements were further suppressed. Indeed, the higher support density layout is effective in preventing the generation and separation of stress-induced spalling fractures.

Further analysis of the support effect was conducted via calculation of bulking factors for the unsupported, 2 bolt and 4 bolt (2 m out-of-plane spacing) models. Bulking factor is defined as the percentage of volume increase within the yielded zone from an undamaged state (Walton, 2014; Oliveira and Diederichs, 2017). Since UDEC is a 2D software, it is the area generated by the stress-fractures rather than the volume which was considered during the calculation. For determination of bulking factor (BF), the following equation from Kaiser et al. (1996) was used:

\[
BF = \frac{u_w - u_{df}}{d_f}
\]  

(6.2)

where, \(u_w\) is the displacement at the rib, \(u_{df}\) is the displacement at the depth of failure, and \(d_f\) is the thickness of the fractured rock. To determine the three listed parameters, lateral displacements of all gridpoints along 5 horizontal lines were extracted from the model. These lines were extended to 1.5 m into the rib and were spaced at 0.5 m vertically (Figure 6.12a inset).
Figure 6.12a shows the lateral displacements for the 2 bolt model. The numbers in the legend represent the location of the horizontal lines from the base of the coal seam. Although it would have been simpler to select just one section along the mid-height of the pillar and compute the BF, it would not be representative of the entire pillar because of the non-uniform shape of the damaged zone (see Figure 6.10). In particular, fracturing was greatest at the center and diminished along the edges. It was therefore decided to compute the bulking factor separately along the 5 horizontal lines.

\( d_f \) was manually identified as the point where the perturbations in the lateral displacements diminished and the curve became smooth. These points are marked by the black solid circles in Figure 6.12a. Once the \( d_f \) was identified, it was rather straightforward to determine \( u_w \) and \( u_{df} \) from Figure 6.12a as the displacements at the rib surface and at \( d_f \), respectively. As the horizontal axis in Figure 6.12a corresponds to the undeformed location of the gridpoints, the comparison to an undamaged state is implicitly accounted for in the BF equation. Additionally, when displacements across the fractured region are subtracted (numerator of Eq. 6.2), the explicit contact damage and the inelastic yield in the block zones are accounted for (although the latter is minimal at the pillar periphery).

The computed BF for all three models and their mean values are shown in Figure 6.12b. As expected, the BF dropped in an exponential fashion with increase in support density, which is consistent with the empirical findings of Kaiser et al. (1996). A direct (quantitative) comparison against the empirical data of Kaiser et al. (1996) is not appropriate in this case because that data is based on observations in hard rocks (like granite). However, since coal is brittle in nature and also undergoes spalling, the trend is at least expected to be similar.
Figure 6.10 Rib displacement contours for unsupported condition, 2 bolt condition (calibrated) and 4 bolt condition. The displacements are presented with respect to the displacements measured when the longwall face was 52 m inby in the field (initiation of headgate loading at a longwall face position of -52 m).

The decline in the marginal benefit of added support with an increase in support density can be explained using the conceptual framework for ground-support interaction presented by Sinha and Walton (2019b). Specifically, one can split the ground-support interaction curve, plotted in support effect-support density space, into three segments: (1) Inadequate support segment – Support density is not adequate and it breaks leading to minimal effect on ground
behavior; (2) Maximum gain segment – Increase in support density has the maximum marginal "value added" in this region; and (3) Overdesigned segment – an excessive amount of support has been added to the system, and the effect of any further support on the ground is limited. In this final segment, the ground has already been sufficiently reinforced such that it behaves as a continuum. With this framework in mind, it seems that the 2 bolt layout is in the Maximum gain segment and it manages to suppress the dilation of fractures efficiently, while the 4 bolt layout is near the boundary between the Overdesigned and Maximum gain segments. Note this classification only holds for the specific loading condition tested in this study. With additional mining-induced stresses (e.g. second abutment loads) coming onto the pillar, the rib can fail even with the 4 bolt layout (as previously noted), and at that point, the layout could be considered as ‘inadequate’ or ‘under-designed’.

Figure 6.11 Rib displacement versus stress change for the 2 bolt model (calibrated) and the 4 bolt models. The displacements are presented with respect to the displacements measured when the longwall face was 52 m inby in the field (initiation of headgate loading at a longwall face position of -52 m).
Figure 6.12 (a) Methodology followed for determination of the edge of the fractured region, (b) Bulking factors along horizontal lines located at different heights in the coal pillar for the unsupported, 2 bolt and 4 bolt BBM.

### 6.6 Effect of block shape

The modeling of fracturing in anisotropic rock depends heavily on the shape anisotropy of the constituent blocks. For coal, the block aspect ratio was established through simulated small-scale compression tests in this study. To further understand how block shape might be controlling the fracturing and yield in the models, as well as its mechanical interaction with support, a separate back-analysis was conducted using isotropic Voronoi blocks. For this purpose, the model geometry described in Section 2.1 was employed and the elongated blocks were replaced with regular isotropic blocks of the same height.

The rationale behind selecting the same height was twofold: the number of blocks along the seam height should be kept consistent with that in the elongated block model, and the same zone size should be used as in the elongated block model; this eliminates the need to re-calibrate the zone inelastic parameters. Additionally, the support parameters listed in Table 6.3 could also
be used, meaning that the only modifications required were in the contact parameters (this is natural, since the contact geometry was the only other change made in the model).

A manual back-analysis was subsequently conducted using the isotropic Voronoi block geometry, and it was possible to determine a set of contact properties that resulted in a reasonable match with the field extensometer and stress measurements. Table 6.4 lists the calibrated contact parameters for this model, and Figure 6.13 compares the model results to the field measurements at Site A. A lower contact tensile strength was required, as the propensity of the blocks to separate laterally was lower in this case. As can be seen, the model was able to match the field extensometer measurements very well (Figure 6.13a) and this agreement is marginally superior to that obtained in the elongated block model case. The trend of the stress data (Figure 6.13b) was also well reproduced for the entire range of headgate loading considered.

Table 6.4 Calibrated set of contact parameters for regular Voronoi model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$c_{\text{peak}}$ (MPa)</th>
<th>$c_{\text{res}}$ (MPa)</th>
<th>$\varphi_{\text{peak}}$ ($^\circ$)</th>
<th>$\varphi_{\text{res}}$ ($^\circ$)</th>
<th>$\sigma_t$ (MPa)</th>
<th>Normal stiffness (GPa/m/m)</th>
<th>Shear stiffness (GPa/m/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>16.5</td>
<td>0</td>
<td>41</td>
<td>27.5</td>
<td>8</td>
<td>80,000</td>
<td>40,000</td>
</tr>
</tbody>
</table>

Figure 6.14 and 6.15b shows the horizontal displacement contours after the entry loading stage and the headgate loading stage, respectively. The displacement at the pillar mid-height after the development relaxation stage is similar to that in Figure 6.4a; the fracturing, however, is much more localized in the latter case. For the 2 bolt model, the depth of fracturing and displacement at pillar mid-height are also similar (compare Figure 6.10, and Figure 6.13a and 6.5a).
Figure 6.13 (a) Rib displacement profiles, (b) Stress change versus rib displacement as measured in field and those in the different models. The displacements are presented with respect to the displacements measured when the longwall face was 52 m in by in the field (initiation of headgate loading at a longwall face position of -52 m).

Figure 6.14 Rib displacement contours after development relaxation stage. Note that these displacements are presented relative to the initial unexcavated condition rather than the post-development-relaxation datum used to compare model results to the extensometer data.

The calibrated model was subsequently re-run without support (unsupported) and with 4 bolts, considering out-of-plane spacings of 1 m and 2 m for the longer bolts. The displacement versus vertical stress results for these models are shown in Figure 6.13b. As expected, the unsupported model underwent a rapid increase in displacement at early stages of headgate
loading but stabilized at around a stress of 2 MPa. Following this point, the rate of displacement increase was similar to that of the 2 bolt model. In comparison to the elongated model, the displacement at the pillar mid-height is much lower following the headgate loading phase. This is explained by the breakage and separation of a large portion of the rib in the upper half of the pillar (Figure 6.15a); the lateral displacement of the separated part is 162 mm while at the mid-height, it is only 72 mm. Such a behavior was not observed in the elongated block model, as the blocks buckled along the vertical failure planes in that case.

Figure 6.15 Rib displacement contours after (a) unsupported condition, (b) 2 bolt condition, (c) 4 bolt condition. The displacements are presented with respect to the displacements measured when the longwall face was 52 m inby in the field (initiation of headgate loading at a longwall face position of -52 m).
Both the 4 bolt models exhibited delayed displacement increases in comparison to the 2 bolt model. At late stages of headgate loading, the displacements started to rise, first in the out-of-plane 2 m model followed by the 1 m model (out-of-plane 1 m layout corresponds to greater support density than 2 m). This increase is attributed to a combination of both grout failure and bolt breakage. Overall, the addition of two bolts seems to restrict rib cracking and dilation (Figure 6.15c), but eventually the movements exceeded the reinforcement capacity of the rock support.

The difference in the behaviors observed in the elongated and isotropic block models can be simplistically explained as follows: in the elongated block models, there is a buckling tendency in the lateral direction, and incorporation of supports tend to bind/tie these layers together. From Euler’s buckling theory (Timoshenko and Gere, 2009), it is known that the critical bulking load is related to the area moment of inertia, which for a beam is proportional to \((\text{width})^3\). If rockbolts are capable of effectively binding the layers together, then it would raise the critical bulking load dramatically. This explains why drastic changes in rib behavior (Figure 6.12b) were obtained in the elongated block models with inclusion of additional bolts. This is of course a simplification of the actual ground behavior, which involves fracturing between the layers and differential buckling, but the explanation serves as a useful conceptual model as a first order approximation. Note that buckling is a well-documented mode of failure associated with coal ribs (Smith, 1992; Seedsman, 2006; Jones et al., 2014).

In the regular Voronoi models, the blocks do not exhibit a pure buckling tendency, but rather also incorporate a notable degree of shearing in their movement. This is because these blocks are isotropic in shape and thus have equal pathways for failure in the vertical and horizontal direction. When the models were examined more closely, it was found that the
fracture openings in the elongated block model were more uniformly distributed along multiple contacts but were concentrated along a limited number of block edges in the regular model; this is logical, as there are a smaller number of sub-vertical fracture elements in the isotropic Voronoi model being used to create the same displacement profile as in the anisotropic model. Due to the aforementioned shearing mechanism and the high strain concentrations, some of the rockbolts were also found to rupture under the headgate loading condition in the isotropic Voronoi models. On the contrary, loading in the rockbolts in the elongated block models was mostly in the axial direction, and consequently almost no bolt elements ruptured. From this discussion, it is understandable that the isotropic blocks do not reproduce the same rock damage mechanisms as are reproduced by the elongated blocks. An additional finding is that just because a BBM model is well calibrated (e.g. the 2 bolt case), it cannot be expected to produce reasonable forward predictions (e.g. the 4 bolt case) if the representation of ground behavior (in this case, the anisotropic buckling) is not correct.

6.7 Conclusions

In this study, the Bonded Block Modeling (BBM) approach was used to simulate the rib damage process in a longwall chain pillar located in West Cliff Mine (Australia). The anisotropy of coal mass was represented using elongated Voronoi blocks. At the West Cliff Mine, Colwell (2006) installed extensometers and stress cells in two adjacent chain pillars that had different rib support densities and collected data as the longwall face approached and passed the instrumented pillars. The field data corresponding to the chain pillar with lower support density was specifically utilized for constraining the BBM input parameters. The model had the same rockbolt layout as was present at the site. After calibration, the model was able to replicate the
rib displacement profile, stress changes as a function of longwall face location, and the depth of fracturing.

The model was subsequently re-run without any support and with extra support to mimic the support condition at the adjacent chain pillar in the West Cliff Mine. In absence of any support, the model predicted very high displacements (a ~97% increase) - much larger than what was obtained in previous continuum models of the same site. By incorporating the appropriate amount of additional support corresponding to the Site B pillar, model displacements within 6 mm of those recorded at this pillar were obtained. These results indicate that the elongated Voronoi block approach is not only capable of reproducing the rib damage phenomena but also the ground-support interaction mechanism, and therefore has the potential to be used as a support design tool.

Finally, to understand the influence of block shape on damage development in the pillar models, a model with isotropic Voronoi blocks instead of elongated blocks was calibrated. Ultimately, it was found that this representation cannot reproduce the ground-support interaction mechanism accurately, which is ultimately related to the inability of isotropic blocks to properly simulate the behavior of anisotropic ground. In particular, the elongated block models showed a buckling tendency, and rockbolt installation increased the width of material undergoing buckling, thereby causing significant changes in the model results. In the isotropic Voronoi model, the buckling propensity was lower, and a greater degree of shearing was observed along the block contacts.
6.8 Acknowledgements

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CHAPTER 7

A STUDY ON BONDED BLOCK MODEL (BBM) COMPLEXITY FOR SIMULATION OF LABORATORY-SCALE STRESS-STRAIN BEHAVIOR IN GRANITIC ROCKS

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7.1 Abstract

The emergent macroscopic behaviors of Bonded Block Models (BBM) are governed largely by the properties assigned to their three components - blocks, contacts and zones (blocks are discretized using finite-difference zones). Over the years, different representations of these components, including elastic/inelastic zones and heterogeneous/homogeneous blocks and contacts (corresponding to different mineral grains and associated mechanical properties), have been employed to simulate various aspects of rock behavior. However, there is a lack of understanding of the capabilities of these model representations with respect to their ability to replicate specific rock mechanical attributes. The goal of this study was to test a variety of model representations and evaluate their capabilities in terms of reproducing mechanical behaviors observed for a granitic rock type.

It was found that inelastic zones were necessary to capture high confinement peak strengths while incorporation of heterogeneity was necessary to replicate the microcracking process. The heterogeneous, inelastic BBMs could match all the calibration targets and is identified as the best representation for modeling the full range of granitic rock behaviors. To help researchers select an appropriate BBM representation for modeling different aspects of rock
behavior, a summary table outlining the capabilities of the different model representations is also provided.

7.2 Introduction

Grain-scale microstructures in crystalline rocks are known to control their emergent macroscopic mechanical response to loading (Wong et al., 2006; Lan et al, 2010; Farahmand et al., 2015). When such rocks are loaded under compression in a laboratory setting, a heterogeneous stress field is created within the specimen due to the elastic mismatch among constituent grains and the presence of micro-flaws (pores, cleavages, strings of grain boundary cavities; Sprunt and Brace, 1974; Tapponnier and Brace, 1976; Dey and Wang, 1981; Kranz, 1983; Lan et al., 2010). As the local tensile strength in the vicinity of a flaw or at a grain boundary is eventually exceeded, new microcracks are created (Gallagher et al., 1974; Dey and Wang, 1981; Kranz, 1983; Blair and Cook, 1998a; Diederichs, 2003). The microcracks continue to grow and coalesce with loading, ultimately leading to macroscopic failure via axial splitting (under unconfined conditions) or shear band development (under high confinement; Bieniawski, 1967; Martin and Chandler, 1994; Eberhardt et al., 1999a). The overall damage evolution therefore hinges on the complex interaction between microcracks and the stress field.

The microstructure of a rock is generally characterized by grain size, grain shape, grain types and their distributions, contact properties, elastic properties and crystallographic orientations (Fritzen, 2011). Lan et al. (2010) describes three sources of microstructural heterogeneity: (a) Geometric: due to variability in shape and size of the grains, (b) Elastic: due to stiffness contrast of constituent minerals, and, (c) Contact: due to variable length, orientation and mechanical properties of grain contacts. Although the influence of geometric heterogeneity has
been studied using experimental techniques in the past (Olsson, 1974; Hugman and Friedman, 1979; Wong et al., 1996; Eberhardt et al., 1999b), the number of polygonal block-based and grain-based modeling studies that investigate additional aspects of heterogeneity has increased rapidly in recent years (Lan et al., 2010; Chen et al., 2016a; Park et al., 2017; Abdelaziz et al., 2018; Liu et al., 2018; Zhou et al., 2019).

The grain-scale modeling approach is a subset of the Discrete Element Method (DEM; Cundall, 1971), and essentially represents a material space using aggregates of detachable blocks. Two important attributes that have led to its widespread acceptance are the general resemblance in shape of the modeled blocks to actual mineral grains and the ability of the modeling methods to allow for realistic fracture formation and opening. Over the years, this modeling approach has been successfully used in reproducing various features of the rock damage process (Kazerani and Zhou, 2010; Lan et al., 2010; Ghazvinian et al., 2014; Bewick et al., 2014; Bahrani et al., 2014; Fabjan et al., 2015; Farahmand and Diederichs, 2015; Chen et al., 2016a; Mayer and Stead, 2017; Park et al., 2017; Zhou et al., 2019; Li et al., 2019b). The polygonal block shape has also been recently incorporated in the Finite-Discrete Element Method (FDEM) framework to improve the grain representations (Abdelaziz et al., 2018). Details on FDEM and its application in small and large-scale rock engineering problems can be found in Munjiza (2004), Mahabadi (2012), Mahabadi et al. (2012), Mahabadi et al. (2014), Lisjak et al. (2016), etc.

In DEM models, different approaches exist for representing the grains/blocks and their respective contacts. Particle Flow Code Grain-based models (PFC-GBMs) use a combination of bonded circular/spherical particles (Potyondy and Cundall, 2004) in the grains and smooth-joints (Mas et al., 2008) along the grain contacts. Alternatively, polygonal or triangular GBMs,
popularly known as Bonded-Block Models (BBMs), employ a continuum mesh within the blocks and a Coulomb-slip model for the block contacts (Itasca, 2014a). As pointed out by Gao et al. (2016), PFC-GBMs have a high intrinsic porosity due to the spherical shape of the particles and it is difficult to model low porosity rocks with this approach. BBMs, on the other hand, have a highly interlocked block structure and as such do not suffer from the porosity issue. While both these approaches are capable of realistically replicating the rock fracturing process (e.g. Lan et al., 2010; Farahmand and Diederichs, 2015; Wang and Cai, 2018; Zhou et al., 2019 etc.), this study will focus only on the BBM approach.

A BBM developed to mimic a multi-minerallic rock should ideally incorporate all sources of micro-structural heterogeneity discussed above. Yet previous BBM studies have adopted some major simplifications that neglect one or more sources of heterogeneity. For example, Ghazvinian et al. (2014) employed a homogeneous block model (uniform block and contact properties) to reproduce the Unconfined Compressive Strength (UCS), Young’s modulus (E) and Poisson’s ratio (ν) of Lac du Bonnet granite. The same rock was simulated using a mineralogically heterogeneous BBM (4 mineral block types and 10 contact types) by Farahmand and Diederichs (2015), and that model was able to reproduce UCS, E, ν, triaxial strengths, and pre-peak cracking thresholds (Crack Initiation and Crack Damage thresholds). It is apparent that the BBM in Farahmand and Diederichs (2015) was more complex and accordingly could reproduce more macroscopic rock attributes/properties (as recorded in the laboratory) than those in Ghazvinian et al. (2014). Since each of the previous BBM studies have employed a single model setup and demonstrated match to certain rock attributes, the relationship between model complexity and its ability or inability to capture other aspects of rock mechanical behavior is not well understood.
In this study, we attempt to identify the degrees of model complexity that are necessary to replicate a variety of rock behaviors. To this end, a large suite of laboratory-derived mechanical properties for a granitic rock were identified, and attempts were made to replicate as many of these properties (referred to as calibration targets herein) as possible using different model representations. The model results presented were calculated using input parameters that were obtained through an iterative manual back-analysis process.

7.3 Review of pertinent literature

The grain-based modeling approaches only started to receive attention in the last decade, as a mean to overcome the drawbacks of the Bonded Particle Method (BPM). BPM was developed by Cundall and Strack (1979), Potyondy et al. (1996), Diederichs (1999) and Potyondy and Cundall (2004) and is the precursor to most of the micromechanical modeling techniques available today. In BPM, a material is represented by an assemblage of circular discs or spheres that interact via parallel bonds and linear contact laws. Since its introduction, a number of limitations were identified in BPM (Cho et al., 2007) and modifications were proposed to overcome these limitations (Potyondy and Cundall, 2004; Cho et al., 2007; Potyondy, 2012; Scholtés and Donzé, 2013; Ding and Zhang, 2014). Most of these modifications identified the lack of particle-particle interlocking in BPM and tried to overcome them by fusing neighboring spheres/discs.

The polygonal grain-based modeling approach, typically applied using Voronoi Tessellations, attempts to better represent the geometric characteristics of polycrystalline rocks, and as such does not suffer from the lack of particle interlocking (Ghazvinian et al., 2014; Mayer and Stead, 2017). As described in Section 7.2, there are two popular ways of defining the
polygonal grains: PFC-GBM and BBM. PFC-GBM, as the name suggests, is typically built using the Itasca software PFC\textsuperscript{2D} or PFC\textsuperscript{3D}. BBM, on the other hand, operates using the UDEC or 3DEC software.

The block zones in BBM can be assigned either an elastic or an inelastic constitutive model; if inelastic, then appropriate strength parameters like cohesion, friction angle, tensile strength have to be defined. The ‘contacts’ between neighboring blocks operate per the Coulomb slip model and are characterized by deformational (stiffness) and strength (cohesion, friction angle and tensile strength) parameters. Since its emergence, the 2-dimensional UDEC-BBM has been widely used to study microfracturing in rocks (Christianson et al., 2006; Lan et al., 2010; Gao and Stead, 2014; Fabjan et al., 2015; Farahmand and Diederichs, 2015; Noorani and Cai, 2015; Chen and Konietzky, 2014; Tan et al., 2016; Gao et al., 2016; Park et al., 2017; Li et al., 2017a; Bahaaddini and Rahimi, 2018; Sinha and Walton, 2018b; Li et al., 2019b), scale effects in rocks (Norouzi et al., 2013; Stavrou and Murphy, 2018), damage around tunnels and underground excavations (Coggan et al., 2012; Gao and Stead, 2014; Shen, 2014; Bai et al., 2016), spalling in rock pillars (Preston et al., 2013; Sinha and Walton, 2019b), etc. Similar studies have been conducted using its 3-dimensional counterpart, i.e. 3DEC-BBM (e.g., Ghazvinian et al., 2014; Garza-Cruz et al., 2014; Garza-Cruz and Pierce, 2014; Wang and Cai, 2018, 2019).

In BBM, the individual blocks typically are either polygons with four or more sides (Voronoi) or triangles (Trigon). The Voronoi blocks are more representative of the petrographic characteristics of crystalline rocks, but there has been some success in modeling granites using Trigons as well (Gao and Stead, 2014; Gao et al., 2016). A key issue with the use of Trigons is the resulting predisposition towards shear fracturing due to the availability of linear failure
pathways (Ghazvinian et al., 2014; Mayer and Stead, 2017; Sinha and Walton, 2019b). As we move towards broader application of BBM, there are concerns that Trigon BBM may not have the capability to properly represent volumetric deformation even when strength is accurately captured (Sinha and Walton, 2019b). It follows that Voronoi BBM, because of its interlocking capabilities, might be better suited for studying the deformation and damage in brittle geomaterials.

Although Voronoi BBM has shown promise in various domains of rock engineering, the focus of this study is on laboratory-scale modeling efforts. As indicated above, there have been a large number of attempts to model the progressive fracturing process in rocks under compression. When considering such prior works (see summary Table 7.1), it is interesting to note the various representations of zones, blocks and contacts that have been employed historically to model the same rock type. For example, Lac du Bonnet granite was modeled using homogeneous, elastic BBMs by Ghazvinian et al. (2014), homogeneous, inelastic BBMs by Noorani and Cai (2015) and heterogeneous (blocks and contacts) elastic BBMs by Lan et al. (2010), Nicksiar and Martin (2014), Chen and Konietzky (2014), Farahmand and Diederichs (2015) and Park et al. (2017). Within this latter category (homogeneous, elastic BBM), two different contact representations were used: Lan et al. (2010), Nicksiar and Martin (2014) and Park et al. (2017) assigned different stiffness values to contacts based on the pairs of bounding mineral blocks, but assigned the same strength parameters to all the contacts. Farahmand and Diederichs (2015) and Chen and Konietzky (2014), on the other hand, assigned different stiffnesses as well as strength properties to each group of contacts. Clearly, the BBMs in Farahmand and Diederichs (2015) and Chen and Konietzky (2014) are more complex than those in Lan et al. (2010), Nicksiar and Martin (2014) and Park et al. (2017), in the sense that there are
more input parameters in their models. It is also evident from Table 7.1 that as the models became more complex, a larger number of calibration targets could be matched. To better understand the relationship between BBM representation and its capabilities, this study selects granitic rock as a starting point, since granite is the most widely modeled rock type using BBM.

Laboratory-scale BBMs are, like all models, an approximation of reality – the question is what level of approximation (or conversely, model complexity) is acceptable for a particular problem in hand? To answer this question and lay the groundwork for BBMs developed as a part of this study, results from previous BBM studies have been examined. Before delving deeper into the BBM literature, a quick overview of the progressive rock-damage process is presented (Martin and Chandler, 1994):

1) Under compression, rocks initially behave in a non-linear fashion due to closure of pre-existing cracks.

2) This is followed by a linear response where the rock behaves as a fully elastic material.

3) At around 30-50% of the peak strength, microcracks orientated sub-parallel or parallel to the direction of major principal stress starts forming randomly within the volume of the specimen. This stress level is termed as the Crack Initiation (CI) threshold.

4) As loading continues, microcracks start to interact and coalesce at around 70-90% of the peak strength. This is called the Crack Damage (CD) threshold and marks the yield point of the rock.

5) Microcracks continue to coalesce and grow until a macroscopic shear plane is formed across the specimen. This is followed by a rapid drop in axial load.
Table 7.1 Summary of previous BBM studies. Boxes are left blank where relevant information was not provided.

<table>
<thead>
<tr>
<th>Rock Type</th>
<th>Blocks &amp; Contacts</th>
<th>Zones</th>
<th>2D/3D</th>
<th>E, ν</th>
<th>UCS</th>
<th>Triaxial Strengths</th>
</tr>
</thead>
<tbody>
<tr>
<td>Granite</td>
<td>Homogeneous</td>
<td>Rigid</td>
<td>2D</td>
<td>✓</td>
<td>✓</td>
<td>σ₃ ≤ 10% UCS</td>
</tr>
<tr>
<td>Granite</td>
<td>Homogeneous</td>
<td>Rigid</td>
<td>3D</td>
<td>✓</td>
<td>✓</td>
<td>σ₃ ≤ 10% UCS</td>
</tr>
<tr>
<td>Sandstone</td>
<td>Homogeneous</td>
<td>Elastic</td>
<td>2D</td>
<td>✓</td>
<td>✓</td>
<td>σ₃ ≤ 35% UCS</td>
</tr>
<tr>
<td>Marble</td>
<td>Homogeneous</td>
<td>Elastic</td>
<td>2D</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Granite</td>
<td>Homogeneous</td>
<td>Elastic</td>
<td>3D</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Strong Rock ²</td>
<td>Homogeneous</td>
<td>Elastic</td>
<td>2D</td>
<td>✓</td>
<td>✓</td>
<td>σ₃ ≤ 10% UCS</td>
</tr>
<tr>
<td>Granite</td>
<td>Homogeneous</td>
<td>Elastic</td>
<td>3D</td>
<td>✓</td>
<td>✓</td>
<td>σ₃ ≤ 10% UCS</td>
</tr>
<tr>
<td>Granite</td>
<td>Homogeneous</td>
<td>Inelastic</td>
<td>2D</td>
<td>✓</td>
<td>✓</td>
<td>σ₃ ≤ 30% UCS</td>
</tr>
<tr>
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<td>Homogeneous</td>
<td>Inelastic</td>
<td>3D</td>
<td>✓</td>
<td>✓</td>
<td>σ₃ ≤ 10% UCS</td>
</tr>
<tr>
<td>Sandstone</td>
<td>Heterogeneous</td>
<td>Elastic</td>
<td>2D</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Granite</td>
<td>Heterogeneous ³</td>
<td>Elastic</td>
<td>2D</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Granite</td>
<td>Heterogeneous ³</td>
<td>Elastic</td>
<td>2D</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Granite</td>
<td>Heterogeneous</td>
<td>Elastic</td>
<td>2D</td>
<td>✓</td>
<td>✓</td>
<td>σ₃ ≤ 10% UCS</td>
</tr>
<tr>
<td>Granite</td>
<td>Heterogeneous</td>
<td>Elastic</td>
<td>2D</td>
<td>✓</td>
<td>✓</td>
<td>σ₃ = 30 MPa</td>
</tr>
<tr>
<td>Granite</td>
<td>Heterogeneous</td>
<td>Elastic</td>
<td>2D</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Granite</td>
<td>Heterogeneous ³</td>
<td>Elastic</td>
<td>2D</td>
<td>✓</td>
<td>✓</td>
<td>σ₃ ≤ 16% UCS</td>
</tr>
</tbody>
</table>

✓ - matched; ² for unconfined conditions; ³ for σ₃ ≤ 10% UCS; ⁴ underestimated; ⁵ properties similar to granite; ⁶ contacts have homogeneous strength & heterogeneous stiffness.
Table 7.1 Continued

<table>
<thead>
<tr>
<th>Tensile strength</th>
<th>Crack Initiation</th>
<th>Crack Damage</th>
<th>Stress-strain (unconfined)</th>
<th>Stress-strain (confined)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>✓</td>
<td>✓ a</td>
<td>✓ a</td>
<td>Brittle</td>
<td>Hardening</td>
<td>Kazerani and Zhou (2010)</td>
</tr>
<tr>
<td>✓</td>
<td>✓ a</td>
<td>✓ a</td>
<td>Brittle</td>
<td>Hardening</td>
<td>Wang and Cai (2019)</td>
</tr>
<tr>
<td>✓</td>
<td></td>
<td></td>
<td>Brittle</td>
<td>Hardening</td>
<td>Bahaaddini and Rahimi (2018)</td>
</tr>
<tr>
<td>✓</td>
<td>✓ a</td>
<td></td>
<td>Brittle</td>
<td></td>
<td>Norouzi et al. (2013)</td>
</tr>
<tr>
<td>✓</td>
<td></td>
<td></td>
<td>Brittle</td>
<td></td>
<td>Ghazvinian et al. (2014)</td>
</tr>
<tr>
<td>✓</td>
<td></td>
<td></td>
<td>Brittle</td>
<td>Hardening</td>
<td>Stavrou and Murphy (2018)</td>
</tr>
<tr>
<td>✓</td>
<td>✓ a</td>
<td>✓ a</td>
<td>Brittle</td>
<td>Hardening</td>
<td>Wang and Cai (2019)</td>
</tr>
<tr>
<td>✓</td>
<td>✓ a</td>
<td></td>
<td>Brittle</td>
<td></td>
<td>Noorani and Cai (2015)</td>
</tr>
<tr>
<td>✓</td>
<td></td>
<td></td>
<td>Brittle</td>
<td></td>
<td>Li et al. (2017a)</td>
</tr>
<tr>
<td>✓ a</td>
<td>✓ a</td>
<td></td>
<td></td>
<td></td>
<td>Lan et al. (2010)</td>
</tr>
<tr>
<td>✓ a</td>
<td>✓ a</td>
<td></td>
<td></td>
<td></td>
<td>Nicksiar and Martin (2014)</td>
</tr>
<tr>
<td>✓ b</td>
<td>✓ b</td>
<td></td>
<td>Brittle</td>
<td>Hardening</td>
<td>Farahmand and Diederichs (2015)</td>
</tr>
<tr>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td>Hardening</td>
<td>Chen and Konietzky (2014)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Hardening</td>
<td>Chen et al. (2016a)</td>
</tr>
<tr>
<td>✓</td>
<td>✓ a</td>
<td>✓ a</td>
<td>Brittle</td>
<td></td>
<td>Park et al. (2017)</td>
</tr>
<tr>
<td>✓</td>
<td>✓ c</td>
<td>✓ c</td>
<td>Brittle</td>
<td>Hardening</td>
<td>Li et al. (2019b)</td>
</tr>
</tbody>
</table>
A homogeneous BBM with rigid blocks is the simplest model representation with the least number of input parameters. Such models neglect grain deformation and have been used under the pretext that mineral blocks are stiffer than the block contacts and so their deformability can be neglected (Kazerani and Zhao, 2010). Wang and Cai (2019), in a recent study, showed that BBM with rigid blocks perform in a similar manner to elastic block models. With that in mind and noting the phenomenological similarity between rigid blocks and elastic blocks with high modulus, the rigid block representation was not considered explicitly in this study.

The homogeneous, elastic BBM has been successfully used to replicate the elastic constants (E and ν), UCS, triaxial strengths and tensile strengths (σ_t) of various granites (UDEC-BBM: Stavrou and Murphy, 2018; 3D-BBM: Ghazvinian et al., 2014; Wang and Cai, 2019), sandstones (Bahaaddini and Rahimi, 2018) and marbles (Norouzi et al., 2013). Interestingly, the only studies that report matching CI and CD are Ghazvinian et al. (2014) and Wang and Cai (2019), and both these studies were conducted in 3D.

From a mechanistic perspective, elastic blocks imply unbreakable mineral grains, meaning that these models should be capable of simulating conditions where damage occurs predominantly via grain boundary fracturing (i.e. inter-granular) rather than via intact grain damage (i.e. intra-granular or trans-granular). Previous fundamental studies on rocks have found microfracturing to initiate at grain-boundaries under unconfined and low confinement conditions, with the density of intra-granular fractures increasing with confining stress (Sprunt and Brace, 1974; Tapponnier and Brace, 1976; Wong, 1982; Kranz, 1983; Eberhardt et al., 1999c; Haimson and Chang, 2000; Lee et al., 2006). The change in mode of fracture formation with confinement is also generally consistent with recent GBM studies (Hofmann et al., 2015; Peng et al., 2018a; Abdelaziz et al., 2018). Note that for the purposes of this study, the terms ‘intra-granular’
(existing within a grain; Kranz, 1983) and ‘transgranular’ (crossing multiple grains; Kranz, 1983) are not differentiated because of the difficulty of identifying them in BBM; instead, the umbrella term ‘intra-granular cracks’ is used throughout.

It follows that elastic homogeneous BBMs should be capable of representing the peak strengths under unconfined and low-confinement conditions only. This is confirmed by the fact that former BBM studies considering granites have either not reported triaxial simulation results (e.g. Ghazvinian et al., 2014) or performed calibration for a limited range of confinement (e.g. 0-10% UCS). Additionally, the strength envelopes were almost perfectly linear, implying that models run at higher confinements would likely lead to peak strengths larger than expected in reality.

Noorani and Cai (2015) modified the zone constitutive model to strain-softening type, with the goal of capturing the combined effect of grain boundary and intra-granular fracturing in BBM. The use of a softening constitutive model allows the zones to deform inelastically upon yield, which is phenomenologically equivalent to grain damage. This approach was shown to be capable of capturing the UCS, tensile strength, triaxial strengths up to $\sigma_3 = 30\%$ UCS, CD under unconfined conditions, and the general phenomenon of reduced dilatancy at higher confinement. However, due to the homogeneous nature of the specimen, it is likely that the unconfined and confined CIs (these values were not reported in the study) and the confined CDs were not replicated in these models.

With respect to the representation of the intra-granular fracturing process, a sub-tessellation approach was demonstrated by Gao et al. (2016) and Wang and Cai (2018). In that approach, each mineral block was further split into sub-blocks, and grain damage was
represented by cracking along the sub-block contacts. Such an approach is appealing, since it allows for explicit simulation of intra-granular fracturing. This approach does have two major limitations, however: (1) Creation of sub-blocks increases the computational demands and model runtime drastically; (2) A small number of sub-blocks (5-6) enforces kinematic constraints on where fractures can develop within a particular block. Accordingly, where it is not computationally feasible to generate a large number of sub-blocks (see Abdelaziz et al., 2018), a potential alternative is to utilize continuum zones with an inelastic constitutive model to approximate intra-granular damage. Although the focus of this study is on the inelastic zone approach, some results obtained using the sub-tessellation approach of Gao et al. (2016) are presented in Appendix E.

The introduction of different mineral blocks and contacts are means of incorporating stiffness heterogeneity within BBMs. Dey and Wang (1981) have shown that the stiffer mineral at a welded contact experiences additional boundary traction when an external stress is applied to the system. If these stresses exceed the local tensile strength at the contact, new grain boundary cracks form. In BBM, since it is not possible to include microscopic flaws (cavities, pores) at grain-boundaries, initiation of microcracking is governed mainly by grain movements (translation and rotation; Ghazvinian et al., 2014) and elastic property mismatch. While the contribution of elastic property mismatch towards generation of local tensile stress is negligible in homogeneous BBMs, the effect is pronounced in heterogeneous BBMs. Accordingly, most of the heterogeneous BBM studies in the literature have been able to match the unconfined CI threshold (Lan et al., 2010; Nicksiar and Martin, 2014; Park et al., 2017). Farahmand and Diederichs (2015) went a step further by demonstrating that heterogeneous, elastic BBMs can capture CI under confined conditions as well.
With respect to peak strength in heterogeneous models, Lan et al. (2010), Nicksiar and Martin (2014), Chen and Konietzky (2014) and Park et al. (2017) only report calibration under unconfined conditions, whereas, Farahmand and Diederichs (2015) and Chen et al. (2016a) show triaxial results up to $\sigma_3 = 10\%$ UCS. The strength envelopes were linear because of the elastic nature of the blocks. From the literature survey, it was not possible to identify the relative benefits of assigning different strength properties to only the block contacts, as the studies employing the homogeneous contact strength simplification (e.g., Lan et al., 2010; Park et al., 2017) did not report triaxial simulation results.

A peculiar behavioral aspect of elastic BBMs is the notable pre-peak hardening after yield under confined loading (Farahmand and Diederichs, 2015; Chen et al., 2016a; Bahaaddini and Rahimi, 2018; Li et al., 2019b; Wang and Cai, 2019). In a real rock, grain boundary fractures connect via intra-granular fractures, resulting in shorter pre-peak hardening phases. In absence of such grain fracturing capabilities, a longer fracture path along the block contacts has to be followed in elastic BBMs that ultimately result in an extended hardening phase. This drawback can be overcome if the zones within the blocks are allowed to yield and soften (Wang and Cai, 2019; Sinha and Walton, 2019d).

A review of literature has provided some insight into the strengths and limitations of the different model representations in BBM and has also helped identify research questions that remain unanswered to date. A key problem with drawing conclusions on the basis of the previous studies is the lack of similitude in their methodologies (e.g. loading mechanism, boundary conditions, 2D versus 3D, different rocks and calibration targets), and the absence of any discussions pertaining to the inabilities of the respective BBMs. To allow for a more quantitative comparison and to accomplish the ultimate goal of determining the optimum level of model
complexity for various applications, a granite and its associated geomechanical parameters were selected, and attempts were made to reproduce as many attributes as possible with the various BBM representations.

7.4 Laboratory data

The rock selected for this study is from the 7910 ft (2.4 km) level of Creighton mine located in Sudbury, Canada. The mineralogical composition of the footwall rocks collectively referred to as “Creighton Granite” varies, but they are all treated as a single geomechanical unit (Walton, 2014). The average petrographic composition is approximately 30% Quartz, 55% Na-Feldspar and 15% Biotite (Sinha and Walton, 2019d). The mineral grains are highly irregular in shape with an average size of 2.25 mm (Sinha and Walton, 2019d).

Uniaxial and triaxial (0-60 MPa confinement) compression tests as well as Brazilian tensile tests were conducted on Creighton Granite samples in the CANMET Natural Resource Laboratory in Ottawa, Canada and the results were previously analyzed by Walton (2014). The average UCS is 203 MPa and the average direct tensile strength (estimated by applying a correction factor to indirect Brazilian tensile strength data per the findings of Perras and Diederichs (2014)) is 9 MPa. The high $\frac{UCS}{\sigma_t}$ ratio (23) is an indication of the brittle nature of the rock, and reflects the high spalling potential for the rock (Diederichs, 2007). The Hoek-Brown (HB) failure envelope was fitted to the dataset following the guidelines of Hoek and Brown (1980) to obtain a $m_i$ of 20.9 (Walton, 2014). The peak strengths and some representative laboratory stress-strain curves are shown in Figure 7.1a and b. For each test, the axial and the lateral strains were recorded in real time to allow for the characterization of CI, CD and volume
changes in the specimens; although Walton (2014) did not previously report CI and CD values for Creighton Granite, they were estimated for this study as described below.

Figure 7.1 (a) Peak strengths (Walton, 2014), (b) Two laboratory stress-strain curves for each level of confinement (one solid and one dashed), (c) Crack Damage thresholds.

CD is commonly accepted as the long-term strength of the rock and is defined as the point of non-linearity in the axial stress-axial strain curve (Martin and Chandler, 1994; Diederichs, 2007; Diederichs and Martin, 2010). CD can also be identified as the point of volumetric strain curve reversal (only for unconfined conditions; Diederichs, 2003) or by
monitoring acoustic emissions from the rock specimen (Eberhardt et al., 1998). Since no acoustic data was available for Creighton Granite, CD was determined from the axial stress-strain curve based on the point of onset of tangent modulus decrease (Diederichs, 2003; Walton et al., 2015a; Walton et al., 2017) and is shown in Figure 7.1c. The mean unconfined CD is 128.7 MPa, which is only 63% of the mean UCS. This value is closer to the lower limit of CD thresholds typically observed in granitic rocks (Bieniawski, 1967; Zhao et al., 2013; Xue et al., 2014; Peng et al., 2018b).

The CI threshold marks the onset of extensile microcracking in the rock specimen and is identified as the point of crack-volumetric strain reversal (Martin and Chandler, 1994; Walton et al., 2017; Peng et al., 2018b) or non-linearity in the axial strain-lateral strain curve (Ghazvinian, 2010). The calculation of crack volumetric strain is dependent on the elastic constants (i.e. Young’s Modulus and Poisson’s ratio), and is particularly sensitive to changes in Poisson’s ratio (Eberhardt et al., 1998). Most of the axial stress – lateral strain curves for Creighton Granite exhibited marked non-linearity from the beginning of each test (likely due to crack-closure). The behavior was more pronounced in the confined test curves, which made it difficult to reliably estimate a Poisson’s ratio. This is documented in detail in Appendix F.

As an alternative, the Inverse Tangent Lateral Stiffness (ITLS) method (Ghazvinian, 2010) was used to determine the CI thresholds. The advantage of this approach is that it relies solely on the shape of the axial stress-lateral strain curve for identifying the CI. The inverse tangent lateral stiffness can be calculated using the following equation (Ghazvinian, 2010):

\[
\varepsilon_I \Delta = \frac{\Delta \varepsilon_I}{\Delta \sigma_I}
\] (7.1)
where, $\Delta \sigma_1 = \sigma_{i+8} - \sigma_{i-8}$ \( (i = 1, 2, 3 \ldots) \) and $\sigma_i$ is the $i^{th}$ axial stress data point, $\Delta \varepsilon_V = \varepsilon_{l_{i+8}} - \varepsilon_{l_{i-8}}$ \( (i = 1, 2, 3 \ldots) \) and $\varepsilon_{li}$ is the $i^{th}$ lateral strain data point.

The recommended bin size of 16 can be modified depending on the resolution of test data. In this study, the bin size of 16 was employed following a moving average analysis to reduce the noise in the ITLS values. Figure 7.2a shows one such graph from a UCS test plotted against $\sigma_1$. The point where the ITLS starts to deviate from linearity (marked by a green circle in Figure 7.2a) is CI. The CI thresholds determined using the ITLS approach are shown in Figure 7.2b, along with the best-fit line.

![Figure 7.2](image)

Figure 7.2 (a) Inverse Tangent Lateral Stiffness (ITLS) method for identifying CI demonstrated for unconfined test data, (b) Crack Initiation threshold determined using the ITLS method.

Interestingly, the coefficient of the confinement term in the CI equation is 2.77, which is nearly double what Diederichs (2003) obtained \( (\sigma_1 = 90 + 1.4 \sigma_3) \) through back-analysis of in-situ damage observations at the Creighton mine. Walton et al. (2016) also found a constant deviatoric CI threshold to reasonably replicate the damage evolution in a pillar at Creighton mine. On the contrary, this study as well as those by Chang and Lee (2004) and Peng et al.
(2018b) have obtained a much higher confinement dependency from laboratory tests on granites. The exact cause for this disconnect between laboratory data and in-situ observations is not well understood and is a topic for future research. One possible reason could be the generation of Hoop stresses in the specimen that delays the onset of stable cracking (Diederichs, 2007). This rationale, however, does not conform with the assertion that CI is a material-specific stress threshold and as such should be unaffected by boundary conditions and testing geometries (Diederichs, 2007; Diederichs and Martin, 2010).

With respect to elastic properties, Walton (2014) noted a modest confinement dependency of the Young’s modulus, with average values ranging from 52.7 GPa (for UCS) to 79.7 GPa (for $\sigma_3$= 60 MPa). This variability was explained as an effect of pre-damage caused by stress relaxation as the core samples were being extracted. The upper limit of the modulus is therefore most likely to be indicative of the intact in-situ rock behavior. However, since the specimens tested in laboratory were already pre-damaged, and recalling that BBMs are zero porosity systems with no simple method to incorporate open cracks, for the purposes of comparison with BBM models, we will use the mean Young’s modulus across the range of confining stresses considered. Arzua et al. (2014) and Walton et al. (2015a) suggested that in brittle rocks, the confinement dependency of elastic modulus can be mathematically represented by a logarithmic function that transitions to a linear function below a certain confinement (see Eq. 7.2).

$$E(\sigma_3) = E_0 + \begin{cases} \frac{\omega' \sigma_3}{e^{\left(\frac{\omega' + \omega_0}{\omega'}\right)}} & \text{when } \sigma_3 < e^{\left(\frac{\omega' + \omega_0}{\omega'}\right)} \\ \omega' \ln(\sigma_3) - \omega_0 & \text{when } \sigma_3 > e^{\left(\frac{\omega' + \omega_0}{\omega'}\right)} \end{cases} \quad (7.2)$$
where, $E_o$ is the mean elastic modulus under unconfined condition, $\omega'$ and $\omega_o$ are fit parameters. Walton (2014) reported the fit parameters for Creighton Granite to be 13.91 and 29.31, respectively. In order to obtain a representative modulus value, Eq. 7.2 was integrated from $\sigma_3 = 0 - 60$ MPa and then divided by 60 MPa, which gave a value of 69 GPa. This value was considered as the calibration target for the BBMs.

The average Poisson’s ratio for Creighton Granite was calculated to be 0.26 (with a standard deviation of ~0.03). Walton et al. (2016) had previously reported a Poisson’s ratio of 0.1 for Creighton Granite. The discrepancy in the values is attributed to the methodologies opted in the two studies - Walton et al. (2016) selected the stress range for calculating the Poisson’s ratio from the initial linear section of the axial stress - lateral strain curve while here we chose the stress range from the linear section of the axial stress - axial strain curve per ISRM guidelines (Bieniawski and Bernede, 1979; Fairhurst and Hudson, 1999). It is not clear which of the two techniques provides a better estimate of Poisson’s ratio, given that the initial linear section of axial stress - lateral strain curve includes some degree of crack closure in axial strain while the linear section of the axial stress - axial strain curve often exceeds the CI threshold of the rock. In other words, the stress range in which the rock behaves in an elastic manner sometimes does not concur in the axial strain and lateral strain space.

7.5 Model setup and description

The applicability of BBMs for simulation of brittle damage in Creighton Granite was tested through a series of simulated Brazilian, UCS and triaxial tests in the explicit distinct element software UDEC. Figure 7.3 shows the geometry of the Brazilian and UCS / triaxial model setup. The Brazilian sample is 55 mm in diameter and was loaded through two steel
platens on either side. The UCS / triaxial sample is 120 mm in height and 55 mm in width, and was loaded via a constant velocity boundary along the model top. In both sets of models, the bottom was restrained by roller boundaries. The laboratory stress-strain curves had controlled post-peak behavior that implies that the loading system was very stiff; as a result, no platens were included in the UCS / triaxial models, which effectively simulates an infinitely stiff loading system. The lack of any lateral constraints or platen friction on the specimen ends simulates a frictionless platen condition; with that said, previous studies have found the effect of platen friction on test results for slender specimen geometries (length/width ≥ 2) to be negligible (Mogi, 2006; Hemami and Fakhimi, 2014; Gao et al., 2018). Further discussion on this topic and associated model results are presented in Appendix G.

Figure 7.3 Brazilian and Uniaxial / triaxial model setup (after Sinha and Walton, 2019d).
In the triaxial simulations, the hydrostatic stress was first applied to the specimen, followed by the application of deviatoric loading. The loading velocity was 0.005 m/sec for the Brazilian test and 0.01 m/sec for the UCS / triaxial models. Although these values are large in comparison to the typical loading rates in laboratory tests, the timestep itself is extremely small in UDEC (Itasca, 2014a). This means that a large number of solution steps are required for displacing the boundaries by unit distance. The velocities chosen are small enough to mimic a pseudo-static loading condition and are consistent with those used by Kazerani and Zhao (2010), Fabjan et al. (2015) and Starvou and Murphy (2018). Also, the default ‘Local’ damping mode in UDEC was employed in all the models in this study (Itasca, 2014a).

Many previous studies (e.g., Tan et al., 2016; Li et al., 2017a; Park et al., 2017) have incorporated true grain morphologies derived from SEM or digital image processing techniques (Li et al., 2019b) in grain-based models. As pointed out by Wang and Cai (2018), BBMs with block structures that are statistically similar to the actual grain size distribution of the target rock unit are often computationally intensive and impractical for calibration. At a minimum, however, the average block size should be close to the mean grain size of the rock. Given that this study encompasses a large number of model calibrations and that the geomechanical attributes correspond to rocks with slightly different petrographic characteristics, no attempt was made to incorporate the ‘true’ grain size distribution in the BBMs. Instead, a block edge length of 2 mm was selected that gave an equivalent average block diameter of 2.25 mm (Sinha and Walton, 2019d). The blocks were then segregated into three mineral classes using a randomization scheme. This resulted in areal mineral proportions of 15.0%, 29.9% and 55.1% for Biotite, Quartz and Plagioclase, respectively. Note that the built-in Voronoi generator in UDEC tends to create a near-uniform grain size distribution (Li et al., 2006). Even with a very low value of
‘iteration’, elongated blocks for Biotite minerals could not be created; therefore, the blocks in the model do not truly represent mineral grains.

In UDEC, a block can be defined as rigid or deformable; if deformable, the blocks are discretized using constant strain triangular zones (Itasca, 2014a). The zones can be set to behave elastically or inelastically, depending on the constitutive model assigned to them. For this study, the ratio of block to zone edge length was chosen to be 2, based on the recommendation of Fabjan et al. (2015). In addition to the zone strength properties, a complete BBM description includes zone elastic properties and contact properties. The input properties could either be uniform across the entire model (termed as homogeneous) or different for different mineral blocks and their respective contacts (termed as heterogeneous). Incorporation of block/contact heterogeneity and/or zone inelasticity increases the number of input parameters and thereby amplifies the non-uniqueness issue in BBMs. From a mechanistic standpoint, however, the added complexities may be necessary to capture some well documented aspects of the rock failure behavior. When model complexity is increased, it is of course necessary to increase the number of macroscopic calibration targets as the number of input parameters increases.

To comprehend the relative benefits of increasing model complexity, the following combinations were tested: (a) Elastic zone with homogeneous blocks and contacts, (b) Inelastic zones with homogeneous blocks and contacts, (c) Elastic zones with heterogeneous blocks and contacts, and, (d) Inelastic zones with heterogeneous blocks and contacts. Results from one supplementary case are also presented in addition to those listed above: elastic zones with heterogeneous blocks but homogeneous contacts. The rationale behind opting for a slightly simpler representation with respect to those in Lan et al. (2010), Nicksiar and Martin (2014) and Park et al. (2017) was to isolate the effect of block heterogeneity.
For each level of model complexity, the BBM input parameters were adjusted to match as many laboratory-derived attributes as possible. The calibration methodology is similar to those in Ghazvinian et al. (2014), Farahmand and Diederichs (2015) and Wang and Cai (2019), and is briefly described below:

(a) For the homogeneous models, elastic constants (E and v) for the zones were set to 75 GPa and 0.25, respectively. The emergent specimen-scale Young’s modulus is known to depend on the contact normal ($j_{kn}$) and shear ($j_{ks}$) stiffness, while the Poisson’s ratio is related to their ratio (Kazerani and Zhou, 2010; Ghazvinian et al., 2014). In order to simplify the calibration process, the ratio was set to 1.5 (within the range prescribed by Fabjan et al. (2015) and identical to that used by Farahmand and Diederichs (2015)) and the $j_{kn}$ was varied until the Young’s modulus and Poisson’s ratio matched those reported in Section 7.3. For the heterogeneous models, a similar approach was taken, with the exception that the blocks were assigned values of E and v from Bass (1995) based on their mineral composition (see Table 7.2). Each mineral block type or mineral-mineral contact type had consistent properties across the entire model (i.e. no randomized distribution was considered within a mineral or mineral-mineral contact type).

(b) The CI threshold and the Brazilian tensile strength are largely dependent on the tensile strength of the block contacts (Kazerani and Zhou, 2010; Fabjan et al., 2015; Wang and Cai, 2018). The contact strength was varied until a reasonable match with CI and tensile strength values was attained.

(c) The contact cohesion values and friction angles affect the CD, macroscopic cohesion and friction angle, and, consequently the unconfined and confined peak strengths. Based on
parametric studies conducted as a part of the calibration process, it was found that the contact cohesion had a greater impact on the sample cohesion, while the contact friction significantly affected the sample friction angle. These findings are consistent with those of Wang and Cai (2018). A best-fit combination of friction angle and cohesion was ultimately chosen from the parametric study.

Table 7.2 Elastic modulus, Poisson’s ratio and Density of common minerals in granite (from Bass, 1995).

<table>
<thead>
<tr>
<th>Mineral</th>
<th>Young’s Modulus (GPa)</th>
<th>Poisson’s Ratio (ν)</th>
<th>Density (g/cc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quartz</td>
<td>94.5</td>
<td>0.08</td>
<td>2.65</td>
</tr>
<tr>
<td>Plagioclase</td>
<td>88.1</td>
<td>0.26</td>
<td>2.63(^a)</td>
</tr>
<tr>
<td>Biotite</td>
<td>33.8</td>
<td>0.36</td>
<td>3.05</td>
</tr>
</tbody>
</table>

\(^a\)From Mavko et al. (2009).

(d) To simplify the calibration of the contact properties in the heterogeneous models, the properties from Farahmand and Diederichs (2015) were selected as a baseline, followed by systematic modifications. Some rules were followed during the manual back-analysis process: (i) Quartz-Quartz (QQ) contacts were assigned higher strength properties than Plagioclase-Plagioclase (PP) contacts, followed by Biotite-Biotite (BB). All bi-minerallic contacts were assigned lower strengths, with QB and PB lower than QP due to the softer biotite association. (ii) The residual contact friction angle was kept constant at 5\(^o\), as an increase in residual contact friction angle over 10\(^o\) resulted in very high triaxial test peak strengths. (iii) The residual cohesion and tensile strength for all contacts were zero.

(e) In the inelastic models, the calibrated contact properties from their elastic counterparts were used as an initial set of parameters. Iterative modifications to the contact and zone
strength parameters were ultimately required in order to match the peak strengths for all confining stresses. Typically, the inelastic properties had a minimal effect on the model behavior at low confinement, but their influence increased at higher confinements.

The best parameter set reported for each model type is based on two assumptions. First, if during model calibration, a particular type of model (or complexity level) was found incapable of reproducing all the laboratory-derived attributes, then a logical subset was chosen based on an understanding of the rock damage process. For example, the homogeneous, elastic model could not simultaneously capture the peak strengths under low and high confinement. Since elastic blocks neglect any grain damage and given that intra-granular fracturing is an important failure mechanism under high confinements, we chose to replicate the unconfined and low-confinement attributes while allowing disagreement with the high confinement attributes. Secondly, the entire calibration was performed using a single block geometry. The effect of block arrangement on the model results is beyond the scope of this study.

CI and CD in BBMs are commonly identified as the points of tensile crack and shear crack acceleration, respectively (Ghazvinian et al., 2014; Nicksiar and Martin, 2014; Farahmand and Diederichs, 2015). A review of existing literature revealed that there is no standardized method for selecting such acceleration points. To develop a method that can be used to identify the CI and CD from the crack curves consistently, the % of tensile and shear cracks normalized to the total number of tensile and shear cracks at the onset of residual strength was compared to the CI and CD determined from the ITLS and non-linearity of the axial stress-strain curves in the models, respectively. The purpose of normalizing to the total number of cracks at residual stress level rather than to the total number of contacts was to amplify any change in slope in the crack
curves. An example of the normalized tensile and shear crack curves for the heterogeneous, inelastic model is shown in Figure 7.4.

Interestingly, the CI from the ITLS and the CD from the non-linearity of the BBM axial stress-strain curves coincided with the point of intersection of the two initial linear segments of the tensile and shear crack curves. The linear segments and points of intersection are shown with black broken lines and red circles in Figure 7.4. Since the same trend was observed in all the model types, this methodology was selected for determining CI and CD in this study.

Figure 7.4 Methodology for identifying (a) CI, (b) CD, from the normalized number of tensile and shear cracks in the model. These graphs correspond to the heterogeneous, inelastic BBMs (Section 7.6.4).

In the UCS and triaxial models, the stresses were calculated as a sum of reaction forces along the top edge of the model, divided by the specimen width. To ensure that the models were at a pseudo-static condition, average vertical zone stresses in three domains that are 40 mm high and 55 mm wide and spanning from the model bottom to the top were also tracked during the simulations and care was taken to ensure that these three curves coincided with the stress determined from the reaction forces. Vertical and lateral strains were computed based on the
locations of all gridpoints and 11 pairs of gridpoints along the shorter and longer edges of the model, respectively. In the BTS model, the sum of reaction forces ($P$) along the top of the platen was converted into tensile stress per the equation $\sigma_t = \frac{2P}{\pi D t}$, where $D$ is the diameter and $t$ is the thickness of the model (1 unit here). All monitoring was done using user-defined FISH functions executed once every 1000 solution steps.

Rocks loaded under confined conditions show reduced dilatancy due to suppression of brittle extensile fractures. This phenomenon has been qualitatively explored in previous BBM studies using volumetric strain plots (Noorani and Cai, 2015; Farahmand and Diederichs, 2015; Li et al., 2019b). An alternative approach, based on the determination of peak dilation angles, is employed here which allows for a more quantitative comparison with the laboratory data. A brief description of the approach is presented below. More details can be found in Walton and Diederichs (2015a).

The macroscopic dilatancy of a specimen can be expressed in terms of the dilation angle ($\psi$) and is related to the maximum ($\dot{\varepsilon}_1^p$) and minimum ($\dot{\varepsilon}_3^p$) strain increments as (Vermeer and de Borst, 1984):

$$\sin(\psi) = \frac{\dot{\varepsilon}_1^p + 2\dot{\varepsilon}_3^p}{2\dot{\varepsilon}_3^p - \dot{\varepsilon}_1^p}$$

Several studies (Alejano and Alonso, 2005; Zhao and Cai, 2010; Walton and Diederichs, 2015a) have found the dilation angle to be a function of plastic shear strain ($\gamma^p$) and confining stress ($\sigma_3$). The plastic shear strain parameter quantifies damage to a rock specimen and can be determined from internal variables as:
\[ \gamma^p = \epsilon_1^p - \epsilon_3^p \]  

(7.4)

where, \( \epsilon_1^p \) and \( \epsilon_3^p \) are maximum and minimum inelastic strains, respectively. Determination of the instantaneous dilation angle requires estimation of the axial and lateral (or maximum and minimum) plastic strain increments. Walton and Diederichs (2015a) developed a new approach (Eqs. 7.5 and 7.6) that uses the concepts of elasticity theory and the definition of CI and CD in Diederichs and Martin (2010) for calculating these quantities. The equations essentially subtract the elastic component of the total strain, along with axial plastic and lateral plastic strains at CD and CI, to obtain \( \epsilon_1^p \) and \( \epsilon_3^p \) respectively.

\[ \epsilon_1^p = \epsilon_1 - \epsilon_{1(CD)} - \frac{\sigma_{1-CD}}{E} \]  

(7.5)

\[ \epsilon_3^p = \epsilon_3 - \epsilon_{3(CI)} + \nu \frac{\sigma_{1-CI}}{E} \]  

(7.6)

Lastly, to satisfy the condition of yield at CD, i.e. \( \gamma^p = 0 \), the plastic shear strain obtained from Eq. 7.4 needs to be shifted by \( \gamma_{CD}^p \). The maximum value of instantaneous dilation angle is the peak dilation angle for each specimen. The peak dilation angles for 10 MPa, 20 MPa, 40 MPa and 60 MPa BBMs, normalized to the peak dilation angle for UCS, were ultimately determined and compared to the corresponding values from laboratory testing. To provide for a better understanding of how the volumetric characteristics of the models evolve with loading, volumetric strain – axial strain curves have also been presented for all the model cases.

### 7.6 Results

In this section, the results corresponding to best-fit parameter sets are presented in order of increasing model complexity. Some alternative parameter combinations and their results are
also discussed. The parameters presented in any given section represent one of the many similar parameter sets that fit the laboratory data equally well. During the model calibration stage, we found that typically a range for each parameter rather than a unique parameter set is obtained from back-analysis. While the values reported here are chosen from the middle of this range, selection of other similar parameter combinations would not have affected the conclusions of this study.

7.6.1 Homogeneous, elastic BBM

Figure 7.5 (a-d) shows the model-predicted stress-strain curves for UCS and triaxial simulations, peak strengths, CI and CD, respectively. The corresponding input parameters are listed in Table 7.3. Elastic BBMs are known to exhibit prolonged pre-peak hardening under confinement (Stavrou and Murphy, 2018; Bahaaddini and Rahimi, 2018; Wang and Cai, 2019), and such a behavior was observed in the models. Following the onset of yield (axial strain non-linearity), the model behaved in a brittle fashion only for the UCS condition. The actual stress-strain curves for Creighton Granite were brittle even under a confining stress of 60 MPa. Clearly, the macroscopic behavior of the BBM is inconsistent with that of the real rock.

In spite of the unrealistic hardening in the stress-strain curve, the peak strengths up to $\sigma_3=20$ MPa matched the laboratory data. Interestingly enough, this range of confinement, i.e. 0 - ~10% UCS, lines up perfectly with the previous attempts to capture triaxial strengths using homogeneous, elastic BBMs (Stavrou and Murphy, 2018). For $\sigma_3=40$ MPa and $\sigma_3=60$ MPa, the peak strengths are significantly overestimated and this can be explained by the lack of grain-fracturing capabilities in these models. For the Brazilian test, the corrected tensile strength was 8.6 MPa, which is only 0.4 MPa less than the average tensile strength of Creighton Granite.
Fracture initiation in homogeneous BBMs is governed largely by point loads (wedging) generated during grain translation and rotation. Since the block structure is compact, a considerable amount of axial straining is required to generate adequate tensile stress within the model. Due to this, the onset of microfracturing is somewhat delayed. As shown in Figure 7.5c, the discrepancy between the model-predicted CI and the laboratory data increases with an increase in confinement. While the CI for the unconfined condition is within the range of laboratory data, further reduction was not possible without increasing the data-model misfit in tensile strength. This is because CI is controlled entirely by the contact tensile strength (Kazerani and Zhou, 2010; Fabjan et al., 2015; Wang and Cai, 2018), which also happens to affect the Brazilian tensile strength.

The CD values predicted by the BBM are close to the upper bound of the laboratory data for $\sigma_3=0$-20 MPa confinement and are clearly too high in the 40 and 60 MPa triaxial models. The lack of pre-peak hardening under unconfined conditions, manifesting in a larger CD value, can also be observed in other UDEC-BBM studies (Fabjan et al., 2015; Stavrou and Murphy, 2018). In an attempt to lower the CD for the UCS model, a separate sensitivity analysis was conducted. It was found that instead of a softening-type contact (i.e. simultaneous degradation of friction and cohesion upon failure), a CWFS-type contact (cohesion degraded but friction mobilized post failure) performed better in allowing greater pre-peak hardening prior to failure. The softening-type and CWFS-type contacts are illustrated using schematic diagrams in Figure 7.6. In the “CWFS contact” type model, the contact cohesion degraded from 100 MPa to 0 MPa and the friction mobilized from $0^\circ$ to $52^\circ$. While this model type could also capture the tensile strength fairly well (8.1 MPa), the triaxial strengths were far too high (Figure 7.5b). A similar contact representation was used by Ghazvinian et al. (2014). In light of the current results and the fact
that Ghazvinian et al. (2014) only performed calibration for unconfined conditions, it can be concluded that a CWFS-type contact is not suitable for simulating confined rock behavior.

Figure 7.5 (a) Stress-strain curves for 0-60 MPa confinement, (b) Model strengths for Base and ‘CWFS contact’ model in $\sigma_1$-$\sigma_3$ space, (c) CI Thresholds, (d) CD Thresholds, (e) Volumetric strain versus axial strain.
In the softening contact-type model, the CI threshold crossed the CD threshold around a confining stress of 20 MPa. When confining stresses are high in BBMs, local tensile stresses are naturally suppressed, which forces the microfractures to initiate in a shear mode. Only when the blocks slide past one another and wedging is enhanced do the contacts start to fail in tension. This sequence of microfracturing does not occur in reality and highlights the role that stiffness heterogeneity plays in inducing damage in rocks.

Table 7.3 Input parameters for homogeneous, elastic BBM.

<table>
<thead>
<tr>
<th>Contact normal stiffness (jkn; GPa/m)</th>
<th>Contact cohesion (jcoh; MPa)</th>
<th>Contact friction (jfric; deg)</th>
<th>Contact residual cohesion (jrcoh; MPa)</th>
<th>Contact residual friction (jrfic; deg)</th>
<th>Contact tensile strength (jtens; MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>230,000</td>
<td>66</td>
<td>43</td>
<td>0</td>
<td>35</td>
<td>17</td>
</tr>
</tbody>
</table>

Figure 7.6 Graphical description of the softening-type and CWFS-type contact behavior.

A CI threshold larger than CD threshold is physically not meaningful and has not been reported in the literature for any crystalline rocks. Since the definitions of CI and CD do not hold for these higher confinement BBMs, the peak dilation angle comparison is not presented. Figure 7.5e shows the volumetric strain-axial strain plot for the homogeneous elastic BBMs. The
volumetric expansion as well as the rate of dilation decreased noticeably with confinement. This is in keeping with how rocks respond under compression (Yuan and Harrizon, 2004; Zhao and Cai, 2010; Arzua and Alejano, 2013) and is consistent with other BBM studies as well (Noorani and Cai, 2015; Farahmand and Diederichs, 2015; Li et al., 2019b).

### 7.6.2 Homogeneous, inelastic BBM

Intra-granular fracturing can be implicitly modeled in UDEC-BBM by allowing the zones to yield inelastically according to a strain-softening constitutive model. The abundance of such fractures in rocks tested under confinement has been confirmed in the past using image analysis techniques (Moore and Locker, 1995) and SEM (Sprunt and Brace, 1974; Tapponnier and Brace, 1976; Haimson and Chang, 2000). Despite its importance, the majority of previous BBM studies have only considered elastic blocks in their simulations.

The incorporation of an inelastic constitutive model in the zones undoubtedly improves the phenomenological capability of the model but it also increases the number of input parameters by a factor of two. The additional (zone) parameters are the peak and residual cohesion, peak and residual friction angle, peak and residual tensile strength, dilation angle and the plastic strain over which the peak values are degraded to their residual counterparts (assumed here to be identical for all parameters). The zone strength parameters should lie within a reasonable range as expected for actual mineral grains, although these values are not directly measurable in laboratory and are neither well constrained in literature. Hence, the input parameters reported are based solely on the ability of the models to match the laboratory attributes, with some intermediate reliability checks against the values used by Noorani and Cai (2015) and Wang and Cai (2019).
Figure 7.7 (a-d) shows the stress-strain curves, peak strengths, CI and CD from the best-fit model. The relevant input parameters are listed in Table 7.4. The most prominent difference with respect to elastic BBMs is the absence of significant pre-peak hardening in the stress-strain curves (Figure 7.7a); a similar observation was made by Wang and Cai (2019). The overall shape is fairly consistent with the laboratory curves for Creighton Granite. Surprisingly, the post-peak drop modulus values in the $\sigma_3=40$ MPa and $\sigma_3=60$ MPa models were larger than the UCS, 10 MPa and 20 MPa models and this trend is not consistent with true rock behavior (Alejano et al., 2009; Arzua and Alejano, 2013; Chen et al., 2014). The disagreement is likely related to the failure being dominated by the inelastic damage of the zones rather than by opening of block contacts.

A gradual increase in the post-peak drop modulus can also be observed in the stress-strain curves of Wang and Cai (2019). Intuitively, if the drop modulus is dependent on the rate of degradation of the zone strength envelope, then a larger plastic shear strain lag should result in a slower decay in stress. Increasing the $\varepsilon_{ps}$ indeed resulted in a lower drop modulus, but it also increased the $\sigma_3=40$ MPa and $\sigma_3=60$ MPa peak strengths significantly. The zone strength parameters could not be lowered to counterbalance this strength rise, because it led to large mismatch in the peak strengths for the UCS, $\sigma_3=10$ MPa and $\sigma_3=20$ MPa models.

The calibrated BBM could capture the peak strengths for the entire range of confining stress tested (Figure 7.7b). For this set of parameters, the tensile strength was 9.2 MPa. As can be seen in Figure 7.7c, CI was overestimated for all the triaxial models. Under unconfined conditions, the value was close to the upper limit of the laboratory data. Recalling that the homogeneous, elastic BBM also behaved in a similar fashion, it could be inferred that homogeneous models are not capable of capturing the true pre-peak micro-damage process in its
entirety. CI values were not reported by Noorani and Cai (2015) and were only reported for UCS conditions by Wang and Cai (2019).

The CD thresholds only approximately matched laboratory values in the UCS, $\sigma_3=10$ MPa and $\sigma_3=20$ MPa models, but are overestimated in the $\sigma_3=40$ MPa and $\sigma_3=60$ MPa models (Figure 7.7d). The lack of pre-peak hardening in the UCS model does not agree with the results from Noorani and Cai (2015), who reported a CD of 71% UCS. The key difference in model parameters that is believed to have caused this effect is that Noorani and Cai (2015) opted for the same zone tensile strength as those of the block contacts (12.1 MPa), on the grounds that intra-granular fracturing is prevalent in UCS and contributes to the large dilation commonly seen in laboratory tests. The plot of yielded zones in Noorani and Cai (2015) further indicates that almost all zones in the model had failed in tension. Although it is true that UCS tests on granitic rocks typically involve large volumetric strains, there is no evidence in literature that the damage is pervasive across the entire volume of the specimen nor that the damage is dominantly intra-granular in nature (Eberhardt et al., 1999c; Lee et al., 2006). It is also counterintuitive for an intact mineral grain to have the same tensile strength as that of a contact, as contacts should be inherent points of weakness within the grain structure.

The distribution of damaged contacts in the triaxial models were markedly localized. Part of the reason is that as the zones started to contribute more towards the failure of the specimen, less contacts were involved and block motions were arrested. The zones, because of their continuum formulation, do not possess the same dilating capacity as the block contacts. This also explains why the volumetric strain magnitudes are lower than in the elastic models (compare Figure 7.7e and Figure 7.5e).
Figure 7.7 (a) Stress-strain curves for 0-60 MPa confinement, (b) Model strengths in $\sigma_1-\sigma_3$ space, (c) CI Thresholds, (d) CD Thresholds, (e) Volumetric strain versus axial strain.
Table 7.4 Input parameters for homogeneous, inelastic BBM; $\varepsilon_{ps}$ is the plastic strain over which the parameters degraded from peak to residual values.

<table>
<thead>
<tr>
<th>Contact properties</th>
<th>Zone properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>$j_{kn}$ (GPa/m)</td>
<td>Cohesion (MPa)</td>
</tr>
<tr>
<td>$j_{coh}$ (MPa)</td>
<td>Peak ($\varepsilon_{peak}$)</td>
</tr>
<tr>
<td>$j_{fric}$ (deg)</td>
<td>Friction angle ($^\circ$)</td>
</tr>
<tr>
<td>230,000</td>
<td>100</td>
</tr>
<tr>
<td>65</td>
<td>55</td>
</tr>
<tr>
<td>50</td>
<td>38</td>
</tr>
<tr>
<td>$j_{fric}$ (deg)</td>
<td>Tens strength (MPa)</td>
</tr>
<tr>
<td>$j_{tens}$ (MPa)</td>
<td>Peak ($\sigma_{t,peak}$)</td>
</tr>
<tr>
<td>32.5</td>
<td>38</td>
</tr>
<tr>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td>$\varepsilon_{ps}$</td>
<td>Peak dilation angle ($\psi_{peak} - ^\circ$)</td>
</tr>
<tr>
<td>0.03</td>
<td>30</td>
</tr>
<tr>
<td>$\psi_{peak}$</td>
<td>Residual dilation angle ($\psi_{res} - ^\circ$)</td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

7.6.3 Heterogeneous, elastic BBM

The contribution of microstructural heterogeneity towards the progressive damage in rocks has already been discussed. Based on the results so far, one can understand that it is not possible to reproduce tensile microcracking under confined conditions with a homogeneous block assumption. Farahmand and Diederichs (2015) showed that CI under unconfined and low confinement conditions can be realistically captured using elastic, heterogeneous models. To ascertain whether heterogeneous BBMs have the capability of predicting damage initiation under even higher confinement, 3 sets of elastic properties, corresponding to Quartz, Plagioclase and Biotite mineral grains, were assigned to the block model. In addition to the elastic parameters (Table 7.2), the models require 6 sets of contact properties. It is apparent that the degree of non-uniqueness is much higher in these models, with over 40 input parameters. Table 7.5 lists the final set of input parameters. This set of models is termed as ‘Base’, and the associated results are presented below.
Figure 7.8 (a-d; Base) shows the stress-strain curves, model strengths in $\sigma_1$-$\sigma_3$ space, CI thresholds and CD thresholds, respectively. Similar to the homogeneous, elastic BBMs, the models exhibited enhanced strain hardening prior to failure (Figure 7.8a). The UCS and triaxial strengths up to $\sigma_3=20$ MPa are well captured, but the high confinement strengths are overestimated. The most prominent difference with respect to the homogeneous models is the ability to match the CI and CD for the entire range of confining stress (Figure 7.8c and d).

Table 7.5 Contact input parameters for heterogeneous, elastic BBM (Base).

<table>
<thead>
<tr>
<th>Contacts</th>
<th>$j_{kn}$ (GPa/m)</th>
<th>$j_{coh}$ (MPa)</th>
<th>$j_{fric}$ (deg.)</th>
<th>$j_{tens}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q-Q</td>
<td>280000</td>
<td>130</td>
<td>65</td>
<td>31</td>
</tr>
<tr>
<td>P-P</td>
<td>250000</td>
<td>112</td>
<td>63</td>
<td>31</td>
</tr>
<tr>
<td>B-B</td>
<td>130000</td>
<td>98</td>
<td>60</td>
<td>22</td>
</tr>
<tr>
<td>Q-P</td>
<td>230000</td>
<td>90</td>
<td>60</td>
<td>24</td>
</tr>
<tr>
<td>Q-B</td>
<td>230000</td>
<td>67</td>
<td>55</td>
<td>21</td>
</tr>
<tr>
<td>P-B</td>
<td>230000</td>
<td>64</td>
<td>55</td>
<td>20</td>
</tr>
</tbody>
</table>

In order to better understand the effect of microstructural heterogeneity on stress localization in the unconfined models, the minor principal stresses ($\sigma_3$) were extracted from the elastic, homogeneous and elastic, heterogeneous BBMs at an average axial stress of 89 MPa, which is the value of CI for the unconfined heterogeneous elastic BBM (see Figure 7.9). The goal is to compare the range of tensile stresses developed within the two models at the same loading stage. As can be seen, much larger tensile stresses localize within the heterogeneous model in comparison to its homogeneous counterpart. Tensile stresses over ~20 MPa are also highly localized in the model. The difference in the extent of stress perturbation explains why the CI could be represented in the unconfined and the triaxial models (similar reasoning), even
though the contact micro-tensile strengths assigned are much larger than those in the homogeneous BBMs.

It appears that the early initiation of microcracks eventually affects its subsequent growth and interaction. A closer look at the crack development modes revealed that even in the $\sigma_3=60$ MPa model, tensile cracks dominated up to slightly beyond the CD threshold. The abundance of tensile cracks at advanced stages of loading indicates the continued role that microstructural heterogeneity plays in controlling the fracture evolution process.

After peak strength, all models exhibited instantaneous drops in the axial stress before completely losing their load carrying capacity. The post-peak drop modulus values are high and are not consistent with the laboratory test data. A similar behavior can be noted in the stress-strain curves from Chen et al. (2016a). It seems that the lack of zone inelasticity is responsible for the over-prediction of the $\sigma_3=40$ MPa and $\sigma_3=60$ MPa strengths as well as the mismatch in the post-peak behavior.

The volumetric strains and the normalized peak dilation angles were determined from the axial stress - strain and axial stress - lateral strain curves (see Figure 7.8e and f). The decreasing trend in the laboratory data in Figure 7.8f signifies a change in the mode of fracture formation from highly dilatant extensile (axial) cracking to medium-low dilation shear (oblique) cracking as noted by Walton and Diederichs (2015a). This confinement-dependent dilatancy phenomenon is well captured by the models. The data-model discrepancy at higher confinements (Figure 7.8f) can be attributed to the lack of intra-grain damage (i.e. elastic zones) and the greater ability of block contacts to dilate in comparison to the zones.
Figure 7.8 (a) Stress-strain curves for 0-60 MPa confinement, (b) Model strengths in $\sigma_1$-$\sigma_3$ space, (c) CI Thresholds, (d) CD Thresholds, (e) Volumetric strain - axial strain, (f) Normalized peak dilation angle versus confinement plots for the ‘Base’ and ‘Homo contact’ models.
All of the aforementioned results correspond to BBMs that had different stiffnesses and strengths assigned to the block contacts. There is another contact representation in literature that is characterized by different contact stiffnesses (i.e. $j_{kn}$, $j_{ks}$) but similar contact strengths (Lan et al., 2010; Nicksiar and Martin, 2014; Park et al., 2017). This is a major simplification relative to the completely heterogeneous model, and it reduces the number of input parameters appreciably. From a mechanistic standpoint, the rationale for the use of different stiffnesses yet similar
strengths is somewhat ambiguous; one would expect the strengths as well as the stiffnesses to be
different for different mineral contact associations. The assumption might be based on the
concept that different stiffnesses would induce different amounts of stress at the contacts. If the
same logic is followed, then one can argue that elastic zone mismatch will generate enough stress
heterogeneity and the use of uniform contact properties (strength and stiffness) in a BBM might
be a sufficient approximation for practical purposes.

Surprisingly enough, when a homogeneous contact BBM with $j_{kn} = 22500 \text{ GPa/m}$, $j_{kn}/j_{ks}$
$= 1.5$, $j_{coh} = 89 \text{ MPa}$, $j_{fric} = 57.5^\circ$ and $j_{tens} = 25 \text{ MPa}$ (termed as ‘Homo contact’) was run, it was
found to be capable of reproducing all the laboratory attributes equally well (Figure 7.8). Only a
slight discrepancy was noted between the CI predicted by the Base and ‘Homo contact’ models
at higher confinements. This could be attributed either to the reduced phenomenological
capability of the ‘Homo contact’ models (i.e. no contact heterogeneity) or it could be a limitation
of the calibration process itself. In any case, it appears that the consideration of elastic block
heterogeneity is far more important than contact heterogeneity towards capturing the micro-
damage process.

7.6.4 Heterogeneous, inelastic BBM

Based on the results so far, two conclusions can be drawn: 1) Microstructural stiffness
heterogeneity is necessary for reproducing the confined CI and CD; 2) Allowing intra-granular
fracturing via zone yield is critical to allow accurate representation of confined peak and post-
peak behavior. A BBM with both these characteristics should be able to match all the laboratory
derived attributes. The authors are unaware of any previous studies that have considered block
and contact heterogeneity and zone inelasticity simultaneously. The reason is likely the difficulty
of calibrating complex models that have over 60 input parameters to constrain. Here we matched 22 different laboratory attributes (5 peak strengths, tensile strength, 2 elastic constants, 5 CIs, 5 CDs and 4 normalized dilation angle values) and qualitatively compared the post-peak behavior with those from the laboratory. Although there are many more model inputs than calibration targets (~3 times), some of the model input parameters (E, v of minerals) were either constrained from literature or were not varied (residual friction angle, cohesion and tensile strength of contacts, etc.). The total number of parameters that were constrained was only 38, which lowers the ratio of input parameters to calibration targets to 38/22=1.73. This value is comparable to prior studies conducted using UDEC-BBM. Additionally, even for parameters that were varied as part of the calibration process, many could be bounded within a limited range of physically reasonable values. For example, the friction angle and tensile strength for the contacts was varied in the range of 40º-65º and 10-35 MPa, respectively (Chen and Konietzky, 2014; Farahmand and Diederichs, 2015).

Given the approach adopted in this study (gradual increase in complexity with intermediate calibration steps), it was possible to integrate both block and contact heterogeneity and zone inelasticity in a BBM and constrain it with modest calibration efforts. Specifically, the contact properties listed in Table 7.5 and the strain softening parameters in Table 7.4 were selected as baselines and then modified systematically until the calibration targets were met. In order to limit the degree of model non-uniqueness, only one set of strain-softening zone parameters was assigned to the entire model. The use of three sets of zone strain-softening parameters for the three mineral types may be more physically accurate, but was not considered as the current set of models was able to match all recorded attributes of Creighton Granite. Table 7.6 and 7.7 lists the best-fit input parameters.
Table 7.6 Contact input parameters for heterogeneous, inelastic BBM.

<table>
<thead>
<tr>
<th>Contacts</th>
<th>( j_{kn} ) (GPa/m)</th>
<th>( j_{coh} ) (MPa)</th>
<th>( j_{fric} ) (deg.)</th>
<th>( j_{tens} ) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q-Q</td>
<td>280000</td>
<td>105</td>
<td>63</td>
<td>28</td>
</tr>
<tr>
<td>P-P</td>
<td>250000</td>
<td>95</td>
<td>61</td>
<td>27</td>
</tr>
<tr>
<td>B-B</td>
<td>130000</td>
<td>80</td>
<td>58</td>
<td>20</td>
</tr>
<tr>
<td>Q-P</td>
<td>230000</td>
<td>90</td>
<td>53</td>
<td>21</td>
</tr>
<tr>
<td>Q-B</td>
<td>230000</td>
<td>60</td>
<td>51</td>
<td>18</td>
</tr>
<tr>
<td>P-B</td>
<td>230000</td>
<td>60</td>
<td>51</td>
<td>18</td>
</tr>
</tbody>
</table>

The stress-strain curves obtained are relatively brittle, with modest pre-peak hardening (Figure 7.10a) and are generally consistent with the laboratory data. During the model calibration phase, it was found that a high \( \varepsilon_p \) in the strain-softening model was required to obtain a consistent reduction in post-peak modulus with confinement. This value is much larger than those used by Noorani and Cai (2015) and Wang and Cai (2019); however, \( \varepsilon_p \) is a zone size-dependent parameter and a direct comparison is therefore not possible. A large \( \varepsilon_p \) implies a slower drop from the peak to the residual stress level and could be interpreted to represent the progressive aggregation of microfractures within a zone-sized region in a mineral grain before its strength is reduced.

Table 7.7 Zone input parameters for heterogeneous, inelastic BBM.

<table>
<thead>
<tr>
<th>Cohesion (MPa)</th>
<th>Friction angle (deg)</th>
<th>Tensile strength (MPa)</th>
<th>Dilation angle (deg)</th>
<th>( \varepsilon_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak ( (\varepsilon_{\text{peak}}) )</td>
<td>Residual ( (\varepsilon_{\text{res}}) )</td>
<td>Peak ( (\phi_{\text{peak}}) )</td>
<td>Residual ( (\phi_{\text{res}}) )</td>
<td>Peak ( (\sigma_{t,\text{peak}}) )</td>
</tr>
<tr>
<td>118.5</td>
<td>48.5</td>
<td>65</td>
<td>47.8</td>
<td>51.5</td>
</tr>
</tbody>
</table>
Figure 7.10 (a) Stress-strain curves for 0-60 MPa confinement, (b) Model strengths in $\sigma_1$-$\sigma_3$ space, (c) CI Thresholds, (d) CD Thresholds, (e) Volumetric strain - axial strain, (f) Normalized peak dilation angle versus confinement.
Figure 7.10b shows the peak strengths in $\sigma_1-\sigma_3$ space. To the best of our knowledge, this is the first time that the non-linear trend of the strength envelope has been clearly replicated using UDEC-BBM. The CI and CD thresholds were also reproduced in these models (Figure 7.10c and d). The tensile strength was 8.8 MPa, which is only 0.2 MPa lower than the average tensile strength of Creighton Granite. Failure in the BTS specimen (also in the previous models) occurred through the formation of a diametrical crack between the two platens. Once the peak tensile strength was attained, the stress-strain curve dropped in a brittle fashion. These results are similar to those reported by Farahmand and Diederichs (2015), Gao et al. (2016) and Park et al. (2017). The trend of normalized peak dilation angle as well as the volume change in the specimen was also consistent with the laboratory data (Figure 7.10e and f), and highlights the well-calibrated nature of the BBM.

### 7.6.5 Crack types and zone yields in different BBMs

The percentages of damaged contacts and yielded zones at peak strength for the different models are presented in Figure 7.11. The contacts have been further sub-divided on the basis of whether they fractured in shear or in tension. While it is possible to compare the number of intra-granular and grain-boundary fractures in PFC-GBMs, a direct comparison in this case is not physically meaningful. In PFC-GBM, the breakage of one parallel bond at the disc contact can be correlated to one fracture in real rock. In UDEC-BBM, the yield of one zone does not necessarily relate to a single fracture; rather it relates to an aggregate of fractures, and the number of fractures within this aggregate at an instant defines the position of the zone strength envelope between the peak and the residual. In other words, an increase in plastic shear strain implies a physical increase in the concentration of intra-granular fractures. Since this
phenomenon is abstract and difficult to quantify, only the % of yielded zones is reported in Figure 7.11. The total number of zones and contacts in the models are 42506 and 19333, respectively.

A general drop in the number of tensile cracks and a rise in the number of shear cracks was noted with confinement. A similar change in grain-boundary failure mode from tensile to shear has been reported from other numerical modeling studies by Hofmann et al. (2015),
Abdelaziz et al. (2018), Peng et al. (2018a) and Li et al. (2019b). Interestingly, all the 10 MPa models showed a larger number of tensile grain-boundary cracks at peak strength in comparison to the UCS models. The exact cause for this behavior is not fully understood, but may be related to the fact that triaxial models are confined and require more damage (e.g. hardening) before the peak strength is attained. When the BBM is unconfined, development of a few extensile fractures is sufficient to lose its load carrying capacity. This proposition is consistent with previous modeling studies (Hofmann et al., 2015; Peng et al. 2018a) and the fact that the cumulative % of shear and tensile cracks in all the confined BBMs are larger than in the unconfined ones.

The role of intra-granular fracturing in controlling the failure of confined BBMs is evident from Figure 7.11b and d. The number of yielded zones increased consistently with increase in confinement, barring only the $\sigma_3=60$ MPa models. It seems that damage is more localized in the $\sigma_3=60$ MPa models and thereby involves a smaller number of zones in the failure process (Figure 7.11b and d). This can be verified from plots of yielded zones in the $\sigma_3=40$ MPa and $\sigma_3=60$ MPa heterogeneous inelastic models, which are shown in Figure 7.12. In addition to the localized nature, the damage intensity ($\epsilon_{pp}$) in the $\sigma_3=60$ MPa model was also proportionately larger than in the $\sigma_3=40$ MPa model.

7.7 Discussion

As previously documented in the literature, the homogeneous, elastic block models could match the unconfined and low confinement peak strengths from the laboratory, but performed poorly in capturing the confined peak strengths and the microfracture evolution process. Their application should therefore be restricted to studying rock behavior under low confining stresses only. The ability to model the microfracturing process was much enhanced when different
mineral blocks were introduced in the BBM. The elastic mismatch played a vital role in inducing initial microcracks as well as subsequent interaction and growth. Like the homogeneous BBMs, these models also overestimated the peak strengths at high confinement owing to the lack of grain fracturing capability. Realistic spalling around a circular tunnel was modeled by Farahmand et al. (2018) using a scaled-up version of this approach.

Figure 7.12 Contour of yielded zones in the 40 MPa and 60 MPa heterogeneous, inelastic BBM.

Some authors have proposed a modification to the completely heterogeneous BBM representation that has different contact stiffnesses but uniform strengths. In order to isolate the effect of contact heterogeneity relative to that of block stiffness heterogeneity, a supplementary set of models was run with heterogeneous blocks (variable properties by mineral type) but
homogeneous contact properties (both stiffnesses and strengths). The data-model fit was found to be as good as those in the completely heterogeneous counterpart. It seems that the effect of elastic block heterogeneity is far more pronounced and overshadowed the effect of contact heterogeneity. Given that these BBMs required only one set of contact parameters and were easier to calibrate, modelers might consider employing this semi-heterogeneous representation in future.

The heterogeneous, elastic BBM (with or without contact heterogeneity) managed to capture all the laboratory-derived attributes commonly considered for calibration. Replicating more advanced attributes like peak strength at high confinements, realistic pre-peak hardening, confinement-dependent dilatancy and post-peak modulus required additional phenomenological capability in the form of inelastic zones. The yield of zones within blocks mimics the intra-granular fracturing process, which is known to be an important damage mechanism both under high confining stress and in the post-peak regime. Note that it was important to consider zone inelasticity in conjunction with block heterogeneity to capture all the aforementioned attributes. Block heterogeneity is strongly tied to the initiation and propagation of cracking in BBMs and omitting its effect led to overestimation of CI and CD. The heterogeneous, inelastic BBM could reproduce all the known geomechanical attributes of Creighton Granite, and is therefore a reasonable approximation of reality.

The above findings are concisely summarized in Table 7.8. The authors believe that this table will assist future researchers in selecting a BBM representation that is suitable to a particular problem. When using this table to decide what constitutes a suitable model representation, one should consider the availability of calibration data as well as the ability of the model to capture the mechanisms relevant to the problem. For example, if BBM is used for
studying the fracture evolution in centrally cracked Brazilian discs (CCBD, Chen and Konietzky, 2014), then a heterogeneous, elastic representation suffices. This is because heterogeneous, elastic BBMs can realistically capture the micro-damage process in rocks. On the other hand, if the goal is to investigate the scale effects on peak strength at low confinements, then a homogeneous, elastic BBM might be sufficient (Stavrou and Murphy, 2018). In any case, to use BBMs for predicting behavioral changes in rocks outside the set of conditions for which it is calibrated requires confidence that the model is a reasonable approximation of reality under the new set of conditions as well based on mechanistic logic. Accordingly, exercising prudence in interpreting and analyzing results and incorporating as much data as possible into the calibration process is paramount before using BBMs for predictive purposes.

7.8 Conclusions

Over the years, a number of different simplifications (or model types) have been used in BBM studies for simulating the progressive damage process in granitic rocks, but the relative advantages and disadvantages of these model types are not well understood. To shed some light on this topic, this study has presented results for models of increasing complexity, and attempts were made to explain any data-model misfit on mechanistic grounds. Note that although the parameter space was explored extensively, we cannot definitively rule out the possibility that some other combination of parameters might produce similar macroscopic behavior, or that some parameter sets exist that could reproduce macroscopic behaviors more effectively than those identified in the paper. In any case, since these micromechanical models are physics-based, in that they try to capture the actual physics of rock damage, it was possible to identify physical reasons whenever some aspect of rock behavior could not be replicated.
Table 7.8 Capabilities of different BBM representations.

<table>
<thead>
<tr>
<th>Macroscopic attributes</th>
<th>Homogeneous, Elastic</th>
<th>Homogeneous, Inelastic</th>
<th>Heterogeneous, Elastic</th>
<th>Heterogeneous, Inelastic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic constants (E, ν)</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>UCS</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Tensile strength</td>
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<td>✓</td>
<td>x</td>
<td>✓</td>
</tr>
<tr>
<td>Low confinement peak strength (σ₃ ≤10% UCS)</td>
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<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>High confinement peak strength (σ₃ &gt;10% UCS)</td>
<td>x</td>
<td>✓</td>
<td>x</td>
<td>✓</td>
</tr>
<tr>
<td>Unconfined CI</td>
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<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Confined CI</td>
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<td>✓ a</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Unconfined CD</td>
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<td>✓ a</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Confined CD</td>
<td>x</td>
<td>x</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Pre-peak behavior (confined)</td>
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<td>✓</td>
<td>x b</td>
<td>✓</td>
</tr>
<tr>
<td>Post-peak behavior (confined)</td>
<td>x c</td>
<td>✓</td>
<td>x c</td>
<td>✓</td>
</tr>
<tr>
<td>Confinement dependent dilatancy</td>
<td>✓ d</td>
<td>✓ d</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

x - cannot match; a slightly overestimates; b excessive strain-hardening; c rapid drop; d captures qualitative trends only.

It was found that block and contact heterogeneity must be incorporated in BBMs to allow for a realistic representation of the microfracturing process to be achieved. It was also established that the relative contribution of elastic block heterogeneity to the generation of tensile damage in the models is far more significant than that of contact heterogeneity. In terms of the representation of peak strength under high confinement conditions, it is important to
assign an inelastic constitutive model to the block zones; this allows the zones to yield and soften and mimics the intra-granular fracturing process. A heterogeneous block and contact BBM with inelastic zones could replicate not only the UCS and triaxial strengths but also the confined CIs, CDs, and the phenomenon of confinement-dependent dilatancy. Because this type of complex model has a large number of input parameters, it is important to utilize many target attributes from laboratory testing in the calibration process.

Lastly, the authors would like to acknowledge that a plane-strain model has been used here to study the behavior of three-dimensional rock specimens. The lack of an extra degree of freedom undoubtedly has some effect on the model results, but a corresponding three-dimensional study of this scope with realistic block size is currently not possible due to computational constraints. With further advances in computational capabilities and parallelization of 3DEC, it might be possible in future to replicate this study using 3D-BBMs.

7.9 Acknowledgements

The research conducted for this study was funded by the National Institute for Occupational Safety and Health (NIOSH) under Grant Number 200-2016-90154. The authors would like to extend their gratitude for the financial support. Special thanks to Dr. Mark Larson and Dr. Bo-Hyun Kim for reviewing this manuscript prior to submission and providing valuable suggestions.
CHAPTER 8

INVESTIGATION OF THE MICROMECHANICAL DAMAGE PROCESS IN A GRANITIC ROCK USING AN INELASTIC BONDED BLOCK MODEL (BBM)

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8.1 Abstract

This study numerically investigates the damage process in a granite using an inelastic multi-minerallic block model in two-dimensional Universal Distinct Element Code (UDEC). In addition to the commonly considered calibration targets like uniaxial and triaxial strengths, tensile strength and Young’s modulus, attention was paid to reproduce additional attributes such as post-peak response, residual strengths and confinement-dependent dilatancy to minimize the non-uniqueness potential of the models. The fracture pattern transitioned from axial cracking to shear banding as the specimen confinement was increased from 0 to 60 MPa. Most notably, the model could exhibit the Cohesion-Weakening-Frictional-Strengthening (CWFS) behavior that is typically associated with brittle rocks. The progressive damage mechanism in the unconfined Bonded Block Model (BBM) was subsequently studied using the 2D Digital Image Correlation (2D-DIC) approach. To date, the application of the 2D-DIC approach has been restricted only to real material testing; this study, therefore, is an attempt to extend its applicability to numerical models. It was found that 2D-DIC is capable of imaging the simulated microcracking process very well and the results were similar to those observed from real testing on a different granitic rock. Lastly, the numerical-based DIC results were analyzed to clarify that even if the point of
axial stress-axial strain non-linearity does not coincide with the point of volumetric strain reversal in unconfined BBMs, the axial stress-axial strain non-linearity approach should always be used for determining the Crack Damage (CD) threshold in BBMs.

8.2 Introduction

Grain-scale heterogeneity in brittle rocks plays an important role in controlling its emergent macroscopic response under compression (Farahmand and Diederichs, 2015; Lan et al., 2010; Wong et al., 2006). Micromechanical observations during testing (Brace et al., 1966; Fairhurst and Cook, 1966; Martin and Chandler, 1994; Sprunt and Brace, 1974; Tapponnier and Brace, 1976) and numerical studies (Farahmand and Diederichs, 2015; Peng et al., 2018a) have been used to identify that the initial mode of crack formation is in tension, which later transitions to a shear mode as cracks begin to interact. The stress levels at which the tensile and shear cracks initiate are referred to as the Crack Initiation (CI) and Crack Damage (CD) thresholds, respectively (Diederichs, 2003; Diederichs and Martin, 2010; Martin and Chandler, 1994). It is generally accepted that heterogeneity (geometric and stiffness) is responsible for the generation of the local tensile stress concentrations that cause damage initiation at CI (Blair and Cook, 1998a, 1998b; Dey and Wang, 1981; Gallagher et al., 1974). Beyond CD, previously isolated microcracks begin to interact and propagate, and this process eventually leads to the failure of the specimen via axial cracking (unconfined/low confinement conditions) or shear band formation (under confined conditions) (Bieniawski, 1967; Eberhardt et al., 1999c; Martin and Chandler, 1994).

At the grain-scale, the microcracking process in unconfined and confined compression tests is somewhat different. When the specimen is unconfined, cracking primarily occurs along
grain boundaries, as these are inherent points of weakness in the rock microstructure. Under confined conditions, however, due to geometrical interlocking and mobilization of frictional forces along the grain boundaries, damage occurs both through grain-boundary cracking and crushing and shearing within the grains (intra-granular fracturing) (Abdelaziz et al., 2018; Eberhardt et al., 1999c; Haimson and Chang, 2000; Hofmann et al., 2015; Lee et al., 2006; Peng et al., 2018a; Sprunt and Brace, 1974; Tapponnier and Brace, 1976; Wong, 1982).

From a macroscopic perspective, as damage evolves in brittle rocks, the cohesional and frictional components of strength do not degrade simultaneously. Martin and Chandler (1994) illustrated through laboratory testing on Lac du Bonnet Granite that it is the cohesion component of strength that degrades first, followed by friction mobilization. Hajiabdolmajid et al. (2002) later developed the Cohesion-Weakening-Frictional-Strengthening (CWFS) model for this behavior, and this model has since then been used to characterize various rock types (Walton, 2014, 2018; Walton and Diederichs, 2015b).

While laboratory testing has historically been the primary approach for studying micromechanical damage processes in brittle rock, recent years have seen a rapid rise in the use of advanced numerical modeling tools (Abdelaziz et al., 2018; Chen et al., 2016a; Lan et al., 2010; Liu et al., 2018; Villeneuve et al., 2009; Zhou et al., 2019). Such models are convenient for study of certain aspects of physical processes that are difficult (or impossible) to study through laboratory testing. A model calibrated to limited laboratory data also provides the opportunity to test and evaluate potential system behaviors under a wide variety of different conditions. For example, a reliable micromechanical model of a granite calibrated to laboratory data could be potentially used to evaluate the effects of grain shape and size on damage processes, strength, and deformation characteristics. Since the accuracy of such analyses is
highly dependent on the extent to which various physical phenomena are reproduced by the model used, it is important to continue to improve modeling approaches. This study is such an attempt to advance the laboratory-scale numerical representation of damage and deformation processes in granitic rocks.

In terms of simulating the rock fracturing process, numerical modeling techniques can be broadly sub-divided into two groups - continuum and discontinuum. Continuum models implicitly represent the strain-weakening process by allowing the elements to yield in an inelastic manner (Chen et al., 2007; Li et al., 2003; Villeneuve et al., 2009; Zhu and Tang, 2004). The evolution of damage in such cases is controlled primarily by the constitutive relationships assigned to the elements. Alternatively, explicit simulation of the microcrack growth and coalescence process can be achieved either through the use of the Discrete Element Method (DEM) or the Finite-Discrete Element Method (FDEM), both of which are physics-based models that allow the formation of true discontinuum fractures (Cundall, 1971; Mahabadi, 2012; Munjiza, 2004). This study is focused on the Bonded Block Modeling (BBM) approach for the study of intact rock damage, which is implemented using DEM (Ghazvinian et al., 2014; Itasca, 2014a; Kazerani and Zhou, 2010; Lan et al., 2010).

Bonded Block Models (BBMs) represent a material space as an aggregate of polygonal or triangular blocks that interact with each other through their contacting interfaces. The contacts fail when the shear stress or the normal stress acting on them exceeds the shear strength or the tensile strength, respectively (Itasca, 2014a). In laboratory-scale models, the blocks can be considered to represent the mineral grains in a rock, and the interfaces between the blocks correspond to grain-boundaries. While both polygonal and triangular BBM have been used for simulating small (Farahmand and Diederichs, 2015; Gao and Stead, 2014; Gao et al., 2016; Lan
et al., 2010) and large (excavation) scale (Bai et al., 2016; Christianson et al., 2006; Farahmand et al., 2018; Gao and Stead, 2014) damage processes in rocks, polygonal blocks (generated as a Voronoi Tessellation) are preferred for modeling granites due to the closer resemblance in shape of the unit blocks to actual mineral grains (Gao et al., 2016; Ghazvinian et al., 2014; Li et al., 2019b).

Voronoi BBMs are often developed in the Itasca software UDEC (Itasca, 2014a). Each of the polygonal blocks in UDEC can be considered to be either rigid or deformable; if deformable, the blocks are discretized by a continuum mesh or ‘zones’. Additionally, a deformable block can be assigned an elastic or inelastic constitutive model. Since UDEC does not allow block division once the model has been stepped, the use of inelastic zones within the blocks allows for the intra-granular damage process to be approximated. Accordingly, the use of an inelastic constitutive model for the zones is important when modeling rocks under high confinement or in the post-peak portion of the stress-strain curve, where intra-granular fracturing is prevalent (Abdelaziz et al., 2018; Peng et al., 2018a). To represent the full range of rock behavior, a micromechanical model should therefore incorporate both elastic heterogeneity and zone inelasticity. To the authors’ knowledge, only two studies applying this approach using Voronoi BBMs (Sinha and Walton, 2019d, 2020a) have been made to date. All other previous studies have used one of the three following representations: (a) Homogeneous Elastic (e.g. Ghazvinian et al., 2014; Stavrou and Murphy, 2018) – Only one set of properties was assigned to all blocks and contacts and the zones were elastic, (b) Homogeneous Inelastic (e.g. Noorani and Cai, 2015; Wang and Cai, 2019) – Only one set of properties was assigned to all blocks and contacts, but the zones were inelastic, and, (c) Heterogeneous Elastic (e.g. Chen and Konietzky, 2014; Lan et al., 2010; Park et al., 2017) – The properties were based on the mineral type, contact properties were assigned
individually based on the mineral-mineral associations, and the zones were elastic. Accordingly, none of these studies were able to produce reasonable post-peak stress-strain behavior across a wide range of confining stresses; a detailed discussion on the advantages and disadvantages of each of the three model representations can be found in Sinha and Walton (2020a). Even in studies that have employed zone inelasticity and block heterogeneity (Sinha and Walton, 2019d, 2020a), a close match to the post-peak shapes and the residual strengths for all levels of confinement considered could not be attained. These models were also not able to reproduce the CWFS strength behavior that is typically associated with brittle rocks.

Additionally, for the models that have been previously developed, no simple approach exists within UDEC to evaluate how grain-boundary crack growth and intra-granular yield interact to influence strain field within the simulated specimens. UDEC has built-in functions for extracting strains, but these values correspond only to the block zones; consequently, the component of strain induced by block separation cannot easily be considered at the grain-scale within the specimens. Although it is possible to extract the block separation magnitudes (i.e. displacements) using built-in UDEC functions, translating these into strain values is problematic and there is no well-established methodology for this purpose. The primary issue is that strain calculation requires a reference length, and this reference length should ideally be zero for two nodes on separate blocks that were previously in contact but have separated.

Based on the existing state-of-the-art as described above, the following research gaps were identified: (1) No previous BBM has been able to closely match the post-peak strength drops and residual strengths of brittle rocks for multiple levels of confinement, (2) No previous BBM study has replicated macroscopic CWFS strength behavior, and, (3) There is no existing methodology for quantifying the strains created due to block separation in UDEC. The
motivation for this study is that if it can be established that BBMs are capable of reproducing the post-peak behavior (addressing research gaps 1 & 2), then it will be possible to gain more insight into the underlying damage mechanisms; this is critical given that it is very difficult to control this part of the stress-strain curve in actual compression tests (Diederichs, 1999, 2003). The use of such BBMs to study post-peak rock behavior will of course rely on the existence of analysis approaches (research gap 3) that allow for consideration of the interaction of block separation and zone yield, both of which are prevalent in the post-peak portion of the stress-strain curve (Abdelaziz et al. 2018; Sinha and Walton, 2020a).

This study adopted a mobilized dilation angle model in the zones to address research gaps 1 and 2. This is in contrast to the models in Sinha and Walton (2020a), which used a dilation model that linearly degraded the dilation angle from a peak to a residual value, independent of confining stress. Those BBMs were not able to fully reproduce the observed post-peak response or the expected CWFS behavior of Creighton Granite. In terms of analyzing the grain-scale strain field perturbations (research gap 3), a convenient methodology is to use 2D Digital Image Correlation (2D-DIC), but its applicability to BBM (and more broadly to numerical models) is yet to be established. To that end, this study employs the 2D-DIC approach in context of the UCS BBM to accomplish two primary goals: (1) Quantify strains within the specimens, considering both crack and separation and zone strains (which is currently not possible with the built-in functions in UDEC), and, (2) Extend the 2D-DIC approach from real rock testing to numerical models.


8.3 Rock description and model setup

8.3.1 Description of Creighton Granite

The rock selected for this study is from the footwall of the Creighton mine in Sudbury, Canada (Walton, 2014). There is some variability in the mineralogical composition of the rocks across the footwall, but they are treated as a single geomechanical unit (termed as “Creighton Granite”; Walton, 2014). The rock has highly irregular mineral grains with an average size of 2.25 mm and is composed (on average) of approximately 55% Na-Feldspar, 30% Quartz and 15% Biotite (Sinha and Walton, 2019d). The average UCS and direct tensile strength for Creighton Granite are 203 MPa and 9.0 MPa, respectively. The tensile strength was estimated from BTS data by applying a conversion factor of 0.86 (Perras and Diederichs, 2014). For the purposes of constraining the BBMs, triaxial data for 0-60 MPa confining stress (Walton, 2014), CI, CD, Young’s Modulus (E), Poisson’s Ratio (ν) and peak dilation angles were utilized (Table 8.1) (Sinha and Walton, 2020a; Walton, 2014).

The peak strengths and the CD thresholds were characterized using the Hoek-Brown formulation for intact rocks (Hoek et al., 2002), given by:

\[
\sigma_1 = \sigma_3 + [\sigma_{ci} \text{ or } CD(\sigma_3=0)] \left( m_i \frac{\sigma_3}{[\sigma_{ci} \text{ or } CD(\sigma_3=0)]} + 1 \right)^{0.5}
\]  

(8.1)

where, \( \sigma_1 \) and \( \sigma_3 \) are the major and minor principal stresses, \( \sigma_{ci} \) is the uniaxial compressive strength obtained by fitting Eq. 8.1 to uniaxial, triaxial and tensile strength data, \( CD(\sigma_3=0) \) is the CD threshold under unconfined conditions, and, \( m_i \) is a fit parameter. Note that \( \sigma_{ci} \) and \( CD(\sigma_3=0) \) might not be equal to the average UCS and average CD since they are obtained from a statistical
fitting process that considers data other than those obtained under uniaxial loading conditions. Contrary to the non-linear trend of peak strengths and CD thresholds, CI generally varies linearly as a function of confining stress.

**8.3.2 Description of the mobilized dilation model**

Volumetric changes in rock specimens under compression can be studied using the dilation angle \( (\Psi) \) (Vermeer and De Borst, 1984). It has been found that the dilation angle in rocks varies both as a function of confining stress and plastic shear strain - plastic shear strain being a proxy for specimen damage (Alejano and Alonso, 2005; Walton and Diederichs, 2015a). A recently proposed mobilized dilation angle model, termed as the Walton-Diederichs (WD) model (Walton and Diederichs, 2015a) adjusts the dilation angle as a function of both confinement \( (\sigma_3) \) and plastic shear strain \( (\varepsilon_{ps}) \). The model formulation is presented in Equations 2 & 3.

\[
\Psi(\sigma_3, \varepsilon_{ps}) = \begin{cases} 
\frac{\alpha \varepsilon_{ps} \Psi_{peak}}{\varepsilon_{ps}^{m} e^{(\alpha-1)/\alpha}} \quad \text{when } \varepsilon_{ps} < \varepsilon_{ps}^{m} e^{(\alpha-1)/\alpha} \\
\Psi_{peak} \left( \alpha \ln \left( \frac{\varepsilon_{ps}^{m}}{\varepsilon_{ps}} \right) + 1 \right) \quad \text{when } \varepsilon_{ps}^{m} e^{(\alpha-1)/\alpha} \leq \varepsilon_{ps} < \varepsilon_{ps}^{m} \\
\Psi_{peak} e^{\left( \frac{-\varepsilon_{ps}^{m} \varepsilon_{ps}}{\varepsilon_{ps}^{m}} \right)} \quad \text{when } \varepsilon_{ps} \geq \varepsilon_{ps}^{m}
\end{cases}
\] (8.2)

\[
\Psi_{peak}(\sigma_3) = \begin{cases} 
\varphi \left( 1 - \frac{\beta'}{e^{-\left(1-\beta_0-\beta'/\beta'\right)/\beta'}} \sigma_3 \right) \quad \text{when } \sigma_3 < e^{-\left(1-\beta_0-\beta'/\beta'\right)/\beta'} \\
\varphi (\beta_0 - \beta' \ln \sigma_3) \quad \text{when } \sigma_3 > e^{-\left(1-\beta_0-\beta'/\beta'\right)/\beta'}
\end{cases}
\] (8.3)

The WD model has five parameters \( (\alpha, \beta_0, \beta', \varepsilon_{ps}^{m} \text{ and } \varepsilon_{ps}^{m'}) \) and is characterized by an initial pre-peak segment (defined by \( \alpha \)), which increases up to a peak point (defined as a function...
of confinement by $\beta_0$, $\beta'$, and $\epsilon_m^{ps}$, and then decays following a negative exponential function (defined by $\epsilon^{ps*}$). The parameters $\beta_0$ and $\beta'$ define the sensitivity of peak dilation angle to confinement, $\epsilon_m^{ps}$ is equal to the plastic shear strain at which the peak dilation is achieved, $\alpha$ controls the curvature of the pre-mobilization section of the dilation model, and, $\epsilon^{ps*}$ controls the rate of decay in dilation angle. For a particular rock, these parameters can be determined by fitting the mathematical model to uniaxial and triaxial laboratory compressions test data (Walton, 2014).

Table 8.1 Geomechanical properties of Creighton Granite (Sinha and Walton, 2020a; Walton, 2014).

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>E (GPa)</td>
<td>69</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.26</td>
</tr>
<tr>
<td>Peak Strength (MPa)</td>
<td>$\sigma_3 + 181 \left( \frac{\sigma_3}{8.66} + 1 \right)^{0.5}; m_t =$</td>
</tr>
<tr>
<td>CD (MPa)</td>
<td>$\sigma_3 + 133 \left( \frac{\sigma_3}{16.4} + 1 \right)^{0.5}; m_t = 8.1$</td>
</tr>
<tr>
<td>CI (MPa)</td>
<td>$98 + 2.77 \sigma_3$</td>
</tr>
<tr>
<td>$\beta_0$</td>
<td>1.1</td>
</tr>
<tr>
<td>$\beta'$</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Walton et al. (2016) previously determined the WD parameters for Creighton Granite and some of the relevant ones are listed in Table 8.1. The confinement-dependent dilation parameters that describe the how the peak dilation angle varies as a function of confining stress ($\beta_0$ and $\beta'$) are specifically reported here since they were chosen as calibration targets for the BBM. In other words, the BBM was calibrated to quantitatively exhibit the confinement-dependent dilation phenomenon, which is the relative inability of a specimen to expand/dilate when confined.
8.3.3 Bonded Block Model setup and calibration

Numerical simulation of Creighton Granite was conducted using two separate model setups in UDEC – BTS and UCS / triaxial models (Figure 8.1). The BTS model was circular with a diameter of 55 mm, while the UCS / triaxial model was rectangular with a height and width of 120 mm and 55 mm, respectively. Two steel platens were positioned on either edge of the BTS specimen and loading was conducted by restraining the bottom platen while allowing the top platen to move downward per a constant velocity. In the UCS / triaxial setup, no such platens were considered and the specimens were loaded directly via a constant velocity boundary along the model top. The lack of platens replicates an infinitely stiff loading system, which is justified in this case as the stress-strain curves for Creighton Granite from the laboratory had controlled post-peak behavior. For conducting the triaxial tests, a stress boundary was assigned to the lateral edges of the model. The velocities chosen for the BTS and UCS / triaxial models were 0.005 m/sec and 0.01 m/sec, respectively, which are slow enough to mimic pseudo-static loading conditions (Fabjan et al., 2015; Kazerani and Zhao, 2010; Stavrou and Murphy, 2018).

Grain size distribution is known to have some effect on the macroscopic response of BBM and it is accordingly ideal to map grain shapes and sizes using advanced image processing techniques and incorporate them into the modeled block structure (Li et al., 2017a; Park et al., 2017; Tan et al., 2016). While such an approach has advantages in terms of model realism, in this particular case, we are attempting to model the behavior of rocks with a range of petrographic characteristics rather than a single specimen. Wang and Cai (2018) mention that where it is not possible/feasible to model the true grain size distribution, the average block size should at least be closer to the mean value. With that in mind, a 2 mm block edge length and a small ‘iteration’ value of 5 was input into the UDEC Voronoi generator to create a highly
irregular block structure that had an average block size of 2.25 mm (Sinha and Walton, 2019d). The ‘iteration’ parameter controls the irregularity of the block structure; a higher value corresponds to more regularity. The authors would like to point out here that irrespective of the ‘iteration’ parameter value, elongated blocks conforming to the typical shape of biotite minerals could not be created. Therefore, the block structure in Figure 8.1 is only an approximation of the true grain morphology of Creighton Granite. Any disagreement between the actual grain geometries and the modeled grain geometries is implicitly accounted for in the material parameters obtained from model calibration. Once the model was constructed, the blocks were segregated randomly into three mineral groups, following the areal proportions as listed above. The elastic constants and density values were assigned separately for each of the mineral types based on the values recommended by Bass (1995) and Mavko et al. (2009).

To mimic the intra-granular fracturing process, the Mohr-Coulomb strain-softening constitutive model was assigned to the zones within the different mineral blocks (Noorani and Cai, 2015; Wang and Cai, 2019). It would be most realistic to assign three different sets of strain-softening parameters to the three mineral types; as part of this study, we developed such a model, and it was found to be capable of capturing all the laboratory-derived attributes of Creighton Granite. However, models run with a single set of inelastic parameters (derived by areal averaging the inelastic strength parameters for the three mineral grains) were also found to perform equally well in replicating the calibration targets. Since both models behaved similarly in reproducing the calibration targets, it was decided to employ the less complex homogenized inelastic zone property model for further analysis in this study. Note that it was only the inelastic strength parameters which were homogenized; the elastic constants and densities were assigned separately by mineral type.
The only refinement introduced in the strain-softening framework of UDEC is the representation of the dilation angle. Instead of a plastic shear-strain based dilation model, we considered the mobilized WD model (Walton and Diederichs, 2015a) in the zones. The motivation for the use of a mobilized dilation model is based on logical extrapolation – if a mechanism applies to a rock specimen, then it should apply to its constituents as well. This can also be justified on grounds that the mechanism by which a fracture is formed and its response to loading is not dependent on the scale of the problem. Whether at a micro scale or a macro-scale,
with shearing (expressed via $\varepsilon_{ps}$ in the WD model), the asperities on a newly-developed fracture surface will degrade and dilatancy will reduce. On the other hand, if confinement is increased, then the ability of extensile fractures to open will be limited and this will manifest in a lower peak dilation angle.

When the WD dilation model is used in conjunction with a strain-softening constitutive model, the pre-peak parameters are not required (Walton and Diederichs, 2015a). In other words, the entire dilation model can be defined by three parameters ($\beta_0, \beta', \varepsilon_{ps}^*$). This is beneficial both from a calibration as well as a phenomenological standpoint. Three input parameters are required in the default plastic shear strain-based dilation model in UDEC, and so the choice of the WD mobilized dilation angle model does not increase the number of input parameters. Using only 3 parameters, both the plastic shear strain and confinement dependencies of dilation angle are still represented in the model zones. The WD parameters used in the BBM zones were chosen directly from Walton et al. (2016). While these were also varied as a part of the calibration process, the model results were found to not be especially sensitive to the dilation parameters over the range of values typical for most rocks and minerals; rather, the use of the mobilized model itself appeared to have a large impact in and of itself (relative to a simplified dilation model in UDEC).

The calibration methodology followed for determining the best-fit parameter set has been discussed in the context of simpler models by Sinha and Walton (2020a). In Sinha and Walton (2020a), results corresponding to a heterogeneous, inelastic BBM were also presented, but that model utilized the built-in plastic shear strain-based dilation model instead of the mobilized dilation model. Various parametric combinations were tested, and it was found impossible to capture the post-peak behavior and the residual strengths of Creighton Granite with a basic
plastic shear-strain-dependent dilation model. In particular, those models underpredicted the residual strengths at low confinement and overpredicted them at higher confinement. Such a finding can be explained based on the fact that a confinement-independent dilation model would tend to underestimate the grain dilatancy at low confinement and to overestimate it at higher confinement. Since higher dilation angles generate more local confining stresses, this leads to anomalously low model strengths at a given strain level under low confinements and anomalously high model strengths under high confinements. Of course, the influence of the specific dilation angle model used is only notably when there is a significant amount of zone yield in the BBM (i.e. in the post-peak portion of the stress-strain curve and under triaxial conditions).

With all this in mind, we employed a model considering the complete strain-dependent dilation angle mobilization and decay process and confinement-dependency to replicate the post-peak behavior and the residual strengths of Creighton Granite. The contribution of the dilation model only becomes significant when the zones start to yield within the model, and this was found to occur close to the CD threshold. It therefore follows that the behavior of the current BBMs is similar to those in Sinha and Walton (2020a) up to the CD threshold. The calibrated zone parameters (strength and dilation model) and the block contact parameters are listed in Table 8.2 and 8.3, respectively. The results from an alternative model calibrated only to the peak unconfined and confined strengths can be found in Appendix H.

While other studies have attempted to identify mineral-specific parameters to simplify the calibration process for different rock types (Villeneuve, 2008; Villeneuve et al., 2009, 2012; Villeneuve and Siratovich, 2015), the strength parameters in this study (Table 8.2) were chosen...
purely on the basis of their ability to reproduce the different calibration targets. With that said, the stiffness properties for the minerals were not calibrated and were selected as “mineral-specific” inputs from Bass (1995) and Mavko et al. (2009). It should also be noted that the parameters in Table 8.2 were obtained specifically for Creighton Granite and as such might not be directly applicable to other rock types. To simulate other rock types, either the methodology outlined in Sinha and Walton (2020a) could be followed or the parameters in Table 8.2 could be used as a starting point followed by iterative adjustments.

Table 8.2 Zone input parameters for the calibrated BBM; \( \varepsilon_{ps} \) is the plastic strain over which the parameters degraded from peak to residual values.

<table>
<thead>
<tr>
<th>Cohesion (MPa)</th>
<th>Friction angle (deg)</th>
<th>Tensile strength (MPa)</th>
<th>WD model parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak ( (c_{peak}) )</td>
<td>Residual ( (c_{res}) )</td>
<td>Peak ( (\phi_{peak}) )</td>
<td>Residual ( (\phi_{res}) )</td>
</tr>
<tr>
<td>118.5</td>
<td>48.5</td>
<td>65</td>
<td>47.8</td>
</tr>
</tbody>
</table>

8.3.4 Methodologies for analyzing the BBM results

To track the evolution of stress, axial strain and lateral strain, user-defined FISH functions were implemented in UDEC. In the UCS / triaxial models, axial stress was determined by dividing the sum of the reaction forces along the model top by the width of the specimen. Axial and lateral strains were determined by tracking each of the gridpoints along the shorter edges and 10 pairs of gridpoints along the longer edges of the model, respectively. In addition, the mode of fracture formation (i.e. tensile or shear) was also tracked for all contacts in the model. For the BTS model, the sum of reaction forces \( (P) \) along the top platen was converted to tensile stress using the equation: \( \sigma_t = \frac{2P}{\piDt} \), where \( D \) is the diameter and \( t \) is the thickness of the model (unity for a 2D model).
Table 8.3 Contact input parameters for the calibrated BBM (Q=Quartz; P=Plagioclase; B=Biotite).

<table>
<thead>
<tr>
<th>Contacts</th>
<th>$j_{kn}$ (GPa/m) x10^4</th>
<th>$j_{coh}$ (MPa)</th>
<th>$j_{fric}$ (deg.)</th>
<th>$j_{tens}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q-Q</td>
<td>28</td>
<td>105</td>
<td>63</td>
<td>28</td>
</tr>
<tr>
<td>P-P</td>
<td>25</td>
<td>95</td>
<td>61</td>
<td>27</td>
</tr>
<tr>
<td>B-B</td>
<td>13</td>
<td>80</td>
<td>58</td>
<td>20</td>
</tr>
<tr>
<td>Q-P</td>
<td>23</td>
<td>90</td>
<td>53</td>
<td>21</td>
</tr>
<tr>
<td>Q-B</td>
<td>23</td>
<td>60</td>
<td>51</td>
<td>18</td>
</tr>
<tr>
<td>P-B</td>
<td>23</td>
<td>60</td>
<td>51</td>
<td>18</td>
</tr>
</tbody>
</table>

It is important to replicate the laboratory-obtained CI and CD thresholds in BBMs alongside peak strengths to ensure that the models are replicating the micro-damage process accurately. The CI threshold is generally determined from laboratory data using the reversal point of crack volumetric strain or non-linearity in the axial stress – lateral strain curve, while the CD threshold can be identified from the reversal of volumetric strain or the point of non-linearity in the axial stress – axial strain curve (Diederichs, 2003; Diederichs and Martin, 2010; Martin and Chandler, 1994; Nicksiar and Martin, 2012). Crack volumetric strain ($\varepsilon_{\nu,\text{crack}}$) represents the inelastic portion of the total volumetric strain ($\varepsilon_{\nu}$) and is calculated using the following equation (Martin and Chandler, 1994):

$$\varepsilon_{\nu,\text{crack}} = \varepsilon_{\nu} - \left(\sigma_1 - \sigma_3\right)\frac{(1-2\nu)}{E}$$

(8.4)

The volumetric strain reversal approach has been found to overestimate the confined CD thresholds in crystalline rocks. Diederichs (2003) explained the cause of this discrepancy based on the simultaneous increase in the axial and lateral strain rate following the initial interaction of
microcracks, which creates a lag in the onset of dilatancy. As a result, the point of axial stress-strain non-linearity is the universal definition of the CD threshold (Diederichs and Martin, 2010).

In micromechanical models, it is more common to identify the CI and CD thresholds from the point of acceleration in the tensile and shear crack curves, respectively (Diederichs, 1999; Farahmand and Diederichs, 2015; Ghazvinian et al., 2014). The authors in a recent study developed an approach to determine these ‘acceleration points’ from tensile and shear crack curves normalized to the total number of tensile and shear cracks in the models at the residual stress levels (Sinha and Walton, 2020a). The points of first significant change in slope in the two normalized curves were chosen as the CI and CD, respectively (see Figure 8.2a). These points also correspond well with the points of non-linearity in the axial stress – lateral strain curve (CI) and the axial stress – axial strain curve (CD) from the BBMs. The match between the shear crack acceleration point and the point of non-linearity in the axial stress – axial strain curve is illustrated using a moving average plot of tangent modulus values in Figure 8.2b. Tangent modulus is a measure of the slope of the axial stress-axial strain curve and the deviation from horizontal indicates inelastic deformation in the axial direction (i.e. the CD threshold).

During the course of model calibration, the authors observed a lag in the point of volumetric strain reversal and acceleration of the shear cracks in the UCS model (Figure 8.2a). The lag in the point of volumetric strain reversal and the axial stress – axial strain non-linearity is concerning, as most of the previous Voronoi BBM studies have identified the unconfined CD by choosing a point on the shear crack curve that is coincident with the reversal of volumetric strain (Farahmand and Diederichs, 2015; Noorani and Cai, 2015; Wang and Cai, 2019). To further highlight this discrepancy in the context of the models run as a part of this study, the volumetric strain reversal point is shown on the tangent modulus plot in Figure 8.2b. As can be observed,
the volumetric strain reversal occurs at a much later phase when the tangent modulus has
dropped by almost 20%. More interestingly, in each of the cited studies (Farahmand and
Diederichs, 2015; Noorani and Cai, 2015; Wang and Cai, 2019), this lag can be clearly noted.
Since this study attempts to replicate the CD thresholds for both unconfined and confined
conditions, the shear crack approach was followed throughout. The crack approach is preferred
in BBM studies for determining the CI and CD as it is reliable and easier to implement in
practice. The appropriateness of the axial stress-axial strain approach (coincident with the
acceleration point in the shear crack curve) in the context of BBMs is verified later using the
DIC approach.

Lastly, to compare the volumetric changes in the specimen with those measured in the
laboratory, a normalized peak dilation angle (peak dilation angle normalized against the peak
dilation angle under unconfined conditions) was calculated for each BBM. There are two
advantages of using the normalization approach: (1) It is mathematically more convenient to fit a
curve to such data as it has to pass through (0, 1); (2) It is easier to understand how changes to
the input parameters affect the shape and size of the mathematical model. The readers are
referred to Rahjoo and Eberhardt (2016) for more discussion on this topic. Previous BBM
studies have only qualitatively explored the confinement-dependent dilatancy phenomenon using
volumetric strain - axial strain plots (Farahmand and Diederichs, 2015; Noorani and Cai, 2015).
The use of normalized peak dilation angle instead allows for a simple yet quantitative
comparison with the laboratory data.

The determination of peak dilation angle for a specimen requires calculation of two
separate quantities: the instantaneous dilation angles and the maximum plastic shear strains ($\gamma^p$).
The methodology for calculating these two quantities can be found in Walton and Diederichs (2015a) and is briefly summarized here:

Figure 8.2 (a) Methodology for identifying CI and CD as shown for UCS BBM results, (b) Tangent modulus plot for UCS BBM indicating the discrepancy in the point of volumetric strain reversal and shear crack acceleration, (c) Identifying the peak dilation angle from dilation angle versus maximum plastic shear strain plots.

(1) The instantaneous dilation angle ($\psi$) was determined from the equation (Vermeer and De Borst, 1984):

$$\sin(\psi) = \frac{\dot{\varepsilon}_1^p + 2\dot{\varepsilon}_3^p}{2\dot{\varepsilon}_3^p - \dot{\varepsilon}_1^p}$$  

(8.5)
where, $\varepsilon_1^p$ and $\varepsilon_3^p$ are the maximum and minimum plastic strain increments, respectively.

(2) The maximum plastic shear strain ($\gamma^p$) is also a proxy for specimen damage and is mathematically calculated as:

$$\gamma^p = \varepsilon_1^p - \varepsilon_3^p$$  \hspace{1cm} (8.6)

If the dilation angle is constant, the two plastic parameters $\gamma^p$ and $\varepsilon_{ps}$ can be related via a constant at any point during plastic deformation (Walton, 2014). The relationship is more complex if the dilation angle varies as a function of the plastic parameter, but generally speaking, a constant conversion factor is applied in practice (Alejano and Alonso, 2005).

(3) To satisfy the condition of yield at CD, i.e. $\gamma^p = 0$ (Chandler, 2013; Zhao and Cai, 2010), $\gamma^p$ was shifted by the value of $\gamma_{CD}$ at the CD threshold ($\gamma_{CD}^p$). This is necessary as the onset of non-linearity in the axial and lateral strains are generally not coincident in rocks (Eberhardt et al., 1998; Zhou and Cai, 2010).

(4) There are two ways of computing $\varepsilon_1^p$ and $\varepsilon_3^p$: conducting loading-unloading tests and using the irrecoverable strains at the end of the unloading cycles to approximate the plastic strains or using the theory of elasticity to subtract calculated elastic components from the total strain to give $\varepsilon_1^p$ and $\varepsilon_3^p$, respectively (Walton and Diederichs, 2015a). In this study, the latter approach was used to compute the plastic strain components.
(5) The increments of plastic strains were then obtained by subtracting the plastic strains at an earlier stage of loading to those from a later stage. The axial strain gap between the ‘initial’ and ‘final’ point for each increment was 0.1 millistrain.

An example of instantaneous dilation angle plotted as a function of maximum plastic shear strain for the UCS and $\sigma_3=60$ MPa BBM is shown in Figure 8.2c. The maximum value of dilation angle is the peak dilation angle for the specimen (marked by green circle in Figure 8.2c). Once the peak dilation angles were identified, it was relatively straightforward to determine the normalized peak dilation angles.

### 8.4 Calibrated BBM results and discussion

Figure 8.3a compares the model-predicted peak and residual strengths to those measured in laboratory and Figure 8.3b shows the stress-strain curves for the UCS, $\sigma_3=10$, 20, 40 and 60 MPa BBMs. As can be seen, the non-linear shape of the peak strength envelope as well as the residual strengths are well captured by the models. To ascertain if the shape of the post-peak segments of the stress-strain curves were consistent with those of Creighton Granite, average post-peak segments were calculated from the raw laboratory stress-strain curves.

For this purpose, all stress-strain curves for a single level of confinement were first normalized such that the peaks coincide at (1, 1), the strain axis was divided into bins of 0.01 length and vertical stresses in each bin were averaged; finally, the averaged curve was re-scaled such that the peak point corresponded to the average peak strength and strain for that particular level of confinement. Figure 8.3b compares the laboratory post-peak behavior with those obtained from the BBMs. The reason why the stress-strain curves begin at different $\sigma_1$ values is
because a hydrostatic stress equivalent to the confining stress level was initialized in the BBM before the models were stepped (or, equivalently, before deviatoric stresses were applied). Since the hydrostatic loading phase was not considered in the models, the laboratory curves were truncated to start at the end of the hydrostatic loading phase as well.

A good match was obtained for all confinements barring only the $\sigma_3=60$ MPa model. The cause for the discrepancy can be attributed to the fact that there was only one stress-strain curve for $\sigma_3=60$ MPa that attained the residual stress level (see Figure 8.3a), and that specimen happened to have a quartz vein in it (potentially leading to a more brittle response; Turichshev and Hadjigeorgiou, 2017). The models were also able to replicate the CI thresholds for the entire range of confinement considered (Figure 8.3c). These results are similar to those in Sinha and Walton (2020a), but are provided here for completeness.

Figure 8.3d shows the CD thresholds for Creighton Granite determined using the axial stress – axial strain non-linearity approach and the points of volumetric strain reversal. The two values diverged with increasing confinement as is expected based on the findings of Diederichs (2003) and Diederichs et al. (2004). While the models were able to replicate the CD thresholds (obtained using the shear crack acceleration approach) and the volumetric strain reversal points fairly well, a mismatch was noted at $\sigma_3 = 0$ (unconfined conditions). In particular, the volumetric strain reversal occurred at $\sigma_1=171$ MPa while the axial stress – axial strain non-linearity occurred at $\sigma_1=141$ MPa in the BBM. Such a mismatch is not present at $\sigma_3 = 0$ in the laboratory data.

When rocks are loaded above the CD stress level, significant inelastic damage is induced in both the axial and lateral directions that results in irreversible volume change (Bieniawski, 1968; Lajtai, 1998). To better understand this phenomenon in terms of the numerical models, it is
useful to revisit Figure 8.2c that shows the instantaneous dilation angle as a function of maximum plastic shear strain. A reduction in the peak dilation angle with confinement is perceptible, but the models fail to capture the subsequent decay in dilatancy. One would expect the decay to be minimal under low confining stresses, but increase with an increase in confinement (Arzua and Alejano, 2013; Walton and Diederichs, 2015a; Zhao and Cai, 2010). This is because at low confinement, fractures open laterally and the associated dilation angles are high ($\epsilon_1^p \sim 0$ in Eq. 8.5 $\rightarrow \psi \sim 90^\circ$). When the system is sufficiently confined, tensile Mode I crack opening is impeded and shearing begins along the grain boundaries. With continued loading, the asperities on the grain boundaries and on the newly developed fracture surfaces are progressively damaged, which manifests as a decay in dilation angle. The fact that the BBMs do not register this decay (see Figure 8.2c) is related to the contacts being unable to simulate the destruction of asperities. How to incorporate asperities in a BBM, either implicitly or explicitly, remains an interesting research question for future studies.

Despite the inability of the model to replicate the dilation decay phenomenon, its ability to quantitatively replicate the confinement dependency of dilation angle (Figure 8.3e) represents a notable improvement over previous models that have only qualitatively demonstrated this phenomenon (Farahmand and Diederichs, 2015; Noorani and Cai, 2015).

The simulated Brazilian tensile test stress-strain curve corresponding to the best-fit parameter set is shown in Figure 8.3f. The peak strength is 10.2 MPa, which is only 0.2 MPa lower than the average BTS of Creighton Granite. Such a small mismatch is considered acceptable given that the standard deviation in the laboratory measured BTS values was 3 MPa. The fracture pattern in the BTS BBM just past the peak stress is also shown in Figure 8.3f and is consistent with those observed in Creighton Granite specimens post-testing.
Figure 8.3 (a) Peak and residual strengths in $\sigma_1$-$\sigma_3$ space, (b) Average stress-strain curves for 0-60 MPa confinement, (c) CI Thresholds (after Sinha and Walton, 2020a), (d) CD Thresholds, (e) Normalized peak dilation angle versus confinement, (f) BTS stress-strain curve with fracture pattern just beyond peak. Note that the CD values calculated using the volumetric strain reversal approach in (d) have been shifted to the right by 2 MPa for visualization purposes. The black and blue data points in (a, c-e) are derived from laboratory while the red data points are from the BBMs. Figure (f) is from BBM.
8.4.1 Cohesion-Weakening-Frictional-Strengthening (CWFS) model

In the last decade, Hajiabdolmajid et al. (2002) introduced the CWFS model to simulate the failure behavior of brittle rocks. The development of this strength model was motivated by the inability of the conventional yield criteria (e.g., Mohr-Coulomb, Hoek-Brown) to replicate the damage zones around tunnels in massive to sparsely fractured rockmasses. As shear yield criteria assume cohesive and frictional strength to mobilize simultaneously, it is unsurprising that such models were unable to replicate the brittle rock damage process (recall that cohesion degrades first followed by friction mobilization in brittle rocks).

The CWFS model allows the cohesion to degrade and the friction angle to mobilize as a function of plastic shear strain. Since its introduction, the strength model has been successfully employed by numerous authors in reproducing notch formation around tunnels (Edelbro, 2009; Hajiabdolmajid et al., 2002; Zhao et al., 2010a). More recently, Walton et al. (2016) used the CWFS model to numerically replicate the displacements measured by 2 multi-point extensometers in a pillar at Creighton mine. With the results of Walton et al. (2016) in mind, and noting the high ratio of UCS to tensile strength for Creighton Granite (22.7 in this case), the CWFS model can be expected to be applicable to Creighton Granite as well (Diederichs, 2007).

The cohesion and friction angle variation with maximum plastic shear strain were calculated for the calibrated BBM and is shown in Figure 8.4. To calculate these curves, axial stress versus maximum plastic shear strain curves were first developed for the unconfined and low confinement models ($\sigma_3 \leq 20$ MPa). Then, for different maximum plastic shear strains, straight lines were fitted to axial stress versus confinement plots using linear regression. The fit parameters in the $\sigma_1 - \sigma_3$ space could then be converted into cohesion and friction angle using
equations available in the literature (Hudson et al., 2002). During the course of this calculation, it was necessary to consider only the low confinement models to minimize the curvature in the strength data and to represent processes that are dominantly brittle (Walton, 2018).

Figure 8.4 Cohesion and friction angle evolution with maximum plastic shear strain calculated at the specimen-scale.

As can be seen in Figure 8.4, the BBMs indeed exhibit a CWFS behavior in the initial stages of damage. To the authors’ knowledge, this is the first time that a micromechanical model has been shown to capture the CWFS behavior. Post-mobilization at ~6 millistrains, a continued decay occurred in the friction angle while the cohesion (almost) leveled off and this is consistent with the findings of Martin and Chandler (1994). A similar behavior could not be obtained using the BBMs in Sinha and Walton (2020a), likely due to the improper representation of dilatancy in the zones. Zone dilation angle only influences the model behavior when there is a large amount of zone yield (i.e. under triaxial conditions and in the post-peak). As the CWFS calculation
considers the post-peak model states for unconfined and triaxial simulations, it is understandable why the BBMs in Sinha and Walton (2020a) were unable to produce such a behavior.

In any case, similar CWFS behavior has been observed previously from laboratory testing on conglomerate, Carrara marble, limestone and Stanstead Granite (Walton, 2014, 2018; Walton and Diederichs, 2015b). Overall, the similarity in the trends of cohesion and friction angle to those in Martin and Chandler (1994) provides confidence in the ability of the BBM to replicate the strength characteristics of brittle rock.

8.4.2 Fracture formation with loading

The reliability of the BBM was further confirmed by comparing the fracture pattern in the models to trends reported in literature for similar rocks (Arzua and Alejano, 2013; Tan et al., 2015). The model fracture patterns correspond to a stage when the respective residual stress levels were attained (Figure 8.5). Since these models possess intra-granular fracturing capability via zone yield, all zones that were yielded and those zones which underwent significant inelastic shear during the loading process have also been highlighted in Figure 8.5 in blue and green, respectively.

The overall fracture mechanism transitioned from axial cracking to shear-banding as the confinement was increased from 0 to 60 MPa as is expected based on previous experimental studies on rocks (Arzua and Alejano, 2013; Mair et al., 2002; Tan et al., 2015; Walton et al., 2017; Zhao and Cai, 2010). The inclination of the shear band (with respect to the horizontal direction) also decreased from ~77° in the $\sigma_3 = 20$ MPa BBM to ~63.5° in the $\sigma_3 = 60$ MPa BBM. A decrease in the shear band angle with confinement has been reported by Arzua and Alejano
Vardoulakis (1980) provided a mathematical relationship between the orientation of the shear band ($\theta$) and friction angle ($\varphi$) and dilation angle ($\psi$) for granular materials, given by:

$$\theta = 45^\circ + \frac{1}{4} (\varphi + \psi) \quad (8.7)$$

Figure 8.5 Fracture patterns in the UCS, 20 MPa and 60 MPa models at their respective residual stress level.

To obtain the instantaneous friction angles, a polynomial was first fitted to the BBM peak strengths for $\sigma_3 = 0-60$ MPa and then local derivatives were obtained at the confining stresses of interest. When this instantaneous friction angle and the instantaneous dilation angle at peak for the $\sigma_3 = 20$ MPa and $\sigma_3 = 60$ MPa were plugged into equation 8.7, $\theta$ values of 73$^\circ$ and 64$^\circ$ were
obtained, respectively. The theoretical shear band angles are therefore within 5% of those observed in the BBM specimens.

Brittle UCS specimens often undergo axial splitting and are associated with large volumetric dilation. Some of these elongated fractures can be seen in the center and along the edges of the UCS model. There is only minimal zone damage, indicating that the failure occurs predominantly via grain-boundary fracturing. The extent of zone damage somewhat increases in the $\sigma_3 = 20$ MPa model, but elongated, axial fractures can still be observed (this justifies the use of $\sigma_3 \leq 20$ MPa models in Section 8.4.1). With further increase in confinement, a localized shear band develops across the specimen. Many more zones (with greater damage intensity) are involved in the ultimate failure, indicating the importance of intra-granular fracturing under confined loading conditions.

Figure 8.5 only shows the final fracture patterns at residual stress levels but not the development of fractures with loading. To that end, plots of failed contacts at CD (141 MPa), the peak stress level (195 MPa) and 80% of the peak stress in the post-peak portion of the stress-strain curve (156 MPa) were extracted from the UCS BBM and are shown in Figure 8.6. At CD, some of the failed contacts interacted to form fractures about 2-4 mm long (1-2 block edge lengths). Increases in crack density with loading eventually resulted in the creation of elongated macro-fractures aligned roughly along the direction of the minimum principal stress (“Peak” in Figure 8.6). As the specimen was loaded further post-peak, the macro-fractures started to open, resulting in large volumetric dilation along the top left and bottom right corners of the specimen (blue lines in Figure 8.6). A similar behavior was noted in the confined BBMs with the only difference being the lower bulking in the specimens.
Geometrical attributes of microcracks can provide more insight into the progressive damage development in the BBMs. To that end, orientations of the microcracks in the UCS specimen were determined at $\sigma_1=141$ MPa (CD), $\sigma_1=175$ MPa (volumetric strain reversal) and $\sigma_1=195$ MPa (Peak) (see Figure 8.7a). These orientations are defined with respect to the horizontal direction and range from $0^\circ$ to $180^\circ$ (inset of Figure 8.7a). While interpreting Figure 8.7 (a-c), it should be noted that the tops of the blue bars correspond to the sums of all the cracks.

In the initial phases of loading (up to CD), most of the cracks were oriented parallel or sub-parallel to the major principal stress direction ($60^\circ$ - $120^\circ$). The crack generation process begins when the lateral strain generated by the Poisson effect exceeds a critical value. At a local scale, the lateral strain creates extensile normal stresses on the block contacts and the contacts rupture...
when the stresses exceed the respective tensile strength. Since the strains are generated normal to the longitudinal axis of the specimen, sub-vertical microcracks are naturally expected (also recall that cracking is primarily extensile in nature up to CD; Figure 8.2a).

As the applied load exceeds the CD, shear cracks develop within the specimen (Farahmand and Diederichs, 2015; Park et al., 2017). These cracks are generally inclined to the vertical direction, and this leads to widening of the orientation distribution. A similar behavior
can be observed in the models of Park et al. (2017). It also seems that the orientation data follow a Gaussian distribution (mean at 90°) for all load levels.

The predominance of axially oriented microcracks in the early phases of loading has been previously reported by Paterson and Wong (2005) on the basis of optical observations on granite, with some cracks inclined up to ±35°. These cracks are generally confined to the boundaries of mineral grains and are extensile in nature (Eberhardt et al. 1999c; Lee et al., 2006; Peng and Johnson, 1972; Tapponnier and Brace, 1976). As the loading is continued up to the peak, the proportion of the highly inclined cracks increases in reality (Wawersik and Brace, 1971; Wulff et al., 1999), similar to what is observed in the BBM (Figure 8.7a).

In the confined BBMs, although tensile cracks initiate first (Sinha and Walton, 2020a), the number of shear cracks eventually exceeds the number of tensile grain-boundary cracks. Accordingly, the orientation distribution has its peak near 90° at the CD threshold (inset of Figure 8.7b), but flattens out (or even becomes slightly bi-modal) between 50° and 130° at the peak strength. The crack geometries in Figure 8.7b correspond to the σ₃=60 MPa BBM at average axial stress levels of σ₁=373 MPa (CD), σ₁=495 MPa (volumetric strain reversal) and σ₁=626 MPa (Peak). To further analyze why the distribution did not resemble a Gaussian distribution, the shear and tensile crack orientations were plotted separately for the σ₃=60 MPa case when the model had reached its residual stress level (Figure 8.7c). It can be seen in Figure 8.7c that the flattening of the distribution is a compound effect of the bi-modal nature of the shear crack orientation distribution and Gaussian nature of the tensile crack orientation distribution.
The bi-modal nature of the shear cracks can be explained on grounds that the 90° plane corresponds to a plane of zero shear strain on a Mohr’s Circle (for the overall specimen stress state), and the frictional forces on less inclined contacts are influenced more by the vertical stress rather than the lateral stress. The two peaks located ± 20° away from the vertical are therefore the preferential conjugate orientations for shear crack development. This is supported by the observations of Wawersik and Brace (1971) who found the crack population oriented at ≥ ±20° to the vertical to increase in rocks tested under high confining stresses. These inclined microcracks eventually develop into a macroscopic shear fracture by linking up the en-echelon inclined cracks (Paterson and Wong, 2005). Note here the similarity in the orientation of the macroscopic shear band as predicted by Eq. 8.7 (Vardoulakis, 1980) for \(\sigma_3=60\) MPa (±26° to vertical) and the peaks of the shear crack count in Figure 8.7c. The other confined BBMs exhibited similar behavior and the corresponding results are therefore not presented.

8.5 Investigating the damage localization in UCS BBM using two-dimensional Digital Image Correlation (2D-DIC)

The 2D Digital Image Correlation (2D-DIC) technique has found increasing use in recent years for studying the stress-induced fracturing process in rocks (Li et al., 2017b; Shirole et al., 2019; Xing et al., 2018; Zhou et al., 2018b). It is a robust, non-destructive, non-contact optical technique via which the full-field displacement and strain fields across a specimen surface can be monitored in real time (Pan et al., 2009; Sutton et al., 2009). Although most of the previous applications of 2D-DIC pertain to physical laboratory tests on natural and man-made materials, an attempt has been made here to extend its applicability to numerical models, with the specific objective of capturing the effects of block separation on the effective strains in the simulated
rock material. The UCS BBM is chosen for this purpose since failure in the UCS model is dominated by block separation (refer Figure 8.5) and this is the component of strain that is difficult to estimate using the functions available within UDEC.

8.5.1 Background of 2D-DIC

The theory of 2D-DIC has been reviewed in detail by numerous authors (e.g. Chu et al., 1985; Pan et al., 2009; Sutton et al., 2009), and accordingly only a brief overview of the technique is presented here. 2D-DIC determines in-plane displacement and strain fields by tracking different regions or subsets on a specimen surface (in the form of digital images) as it deforms progressively during the loading process. Typically, a random speckle pattern of gray-scale values serves as the carrier of the deformation information of the specimen surface (Pan et al., 2009; Schwartz et al., 2013). The idea is to identify the location of a subset in the deformed state with respect to its location in the reference (or un-deformed) image. Each subset is composed of a number of pixels with different gray scale values; the stochastic nature of the gray patterns helps in distinctly identifying the location of the subset in the different images (Hedayat et al., 2014; Pan et al., 2009).

The similarity of a subset in the deformed image and in the reference image is evaluated by correlation and cross-correlation criteria (Pan et al., 2009; Sutton et al., 2009). The position of the peak of the correlation coefficient distribution defines the position of a particular subset in the deformed image, and thereby the displacement of the subset. This calculated displacement is assigned to the center of the subset. Similarly, the displacements at the center of other subsets are calculated to obtain the full-field displacement across the surface of the specimen. Once the displacement field is calculated, the strain-field can be determined following the pointwise local
least-square technique of Pan et al. (2007, 2009). Pan et al. (2009) discussed how a numerical differentiation of the raw displacements might yield unreliable strains due to amplification of the noise in the displacement field. Accordingly, a smoothing approach (like the local least-square technique described below) is necessary before the strains can be estimated.

In the local least-square technique, a calculation window is first defined around a subset center for which the strain is to be calculated. A linear plane is then fitted to the local displacement field in this window following the least-square technique. The coefficients of the equations then allow for the determination of the Green-Lagrange strain tensor \( E^* \) and thereby the principal strains across the specimen surface. The out-of-plane components are ignored when calculating the principal strains from the normal \( \varepsilon_{xx}, \varepsilon_{yy} \) and shear strain \( \varepsilon_{xy} \) components (see Eqs. 8.8-8.10) of \( E^* \) (Chu et al., 1985; Tung et al., 2008):

\[
\varepsilon_{xx} = \frac{\partial u}{\partial x} + \frac{1}{2} \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial x} \right)^2 \right] 
\]

(8.8)

\[
\varepsilon_{yy} = \frac{\partial v}{\partial y} + \frac{1}{2} \left[ \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 \right] 
\]

(8.9)

\[
\varepsilon_{xy} = \frac{1}{2} \left[ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right] + \frac{1}{2} \left[ \frac{\partial u}{\partial x} \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \frac{\partial v}{\partial y} \right] 
\]

(8.10)

where, \( u \) and \( v \) are the displacement fields along the x and y direction, respectively.

### 8.5.2 2D-DIC setup in BBM and validation

A successful 2D-DIC analysis requires the fulfilment of four important requirements: (1) A random speckle pattern is needed across the surface of the specimen, (2) The speckle size should be small and the gray-scale intensity values should have a relatively well-distributed
histogram (Lin and Labuz, 2013), (3) The axis of the camera used for capturing the images must be parallel to the surface normal of the specimen, and, (4) The camera should be placed sufficient far away such that the error in displacement measurements due to out-of-plane deformations are minimal (Pan et al., 2009). The last two requirements are automatically met in this study as the images for DIC analysis were extracted from a numerical model (no camera) and there is no out-of-plane deformation owing to the plane-strain formulation of UDEC.

To develop the artificial speckle pattern for the UCS BBM, the following steps were followed:

1. The addresses of each gridpoint (gridpoints are the vertices of the triangular zones) in the un-deformed or un-run model were extracted. Across a block contact, UDEC usually duplicates a gridpoint such that each of the gridpoints are affiliated to one of the blocks in contact (refer to the red circle in Figure 8.8a). This is necessary so as to permit failure and separation along the contacts (Itasca, 2014a).

2. From the list of gridpoint addresses and their coordinate locations, all duplicate gridpoints were identified and removed (Figure 8.8a). This step is vital to be able to incorporate the strains induced by the explicit opening of cracks. When two blocks start to detach in the BBM, additional deformation will be induced in the zones across the separating contact since one of the duplicate gridpoints has been removed. By tracking the vertices of all the zones, the strains due to both inelastic damage and block separation can be accounted for in the analysis. It is important to note that this step is performed as a post-processing step (i.e. the duplicate gridpoints are not removed in the BBM models but from the list of gridpoints that are tracked during the simulation). In context of the
UCS BBM, the contribution of zone damage towards the strain field perturbations can be expected to be minimal (refer Figure 8.5).

(3) The coordinates of all the gridpoints in the final list were extracted from UDEC once every 5000 solution steps and were subsequently imported in MATLAB. Using a combination of Delaunay triangulation and surface plots, the triangular zones were reconstructed and colored randomly per a uniform distribution (Figure 8.8b). For all subsequent deformed images, it was ensured that the same color value was assigned to a particular speckle (analogous to the assumption that a speckle does not change color during the damage process). The use of a uniform distribution for coloring the triangular speckles fulfils the well-distributed histogram requirement of DIC analysis. As for the size of the speckles, zones are the smallest elements in a BBM, and obtaining a speckle smaller than the zone size is not possible. The accuracy of the computed strain and displacement fields could be possibly enhanced by reducing the zone size, but only at the expense of longer simulation time.

(4) It might seem intuitive to conduct the DIC analysis without step ‘2’ (i.e. with explicit fractures in the image) as is done for real rock tests, but there are specific issues with employing this approach. First, if the images are extracted directly from UDEC, the line thickness in block geometry plots tends to lead to an underestimation of the strains due to block separation. Secondly, if the images are reconstructed in MATLAB, as was done in this case, it is not possible to create Delaunay triangles with intermittent white spaces (i.e. explicit fractures).
Figure 8.8 (a, b) Methodology followed for generating an artificial speckle pattern from the BBM, (c) Failed contacts from BBM overlaid on the DIC strain image at peak strength, (d) Comparison of the axial stress - axial strain curves from BBM and DIC analysis.

Once the artificially speckled images for the un-deformed and deformed model geometry were generated, it was subsequently analyzed in VIC-2D software. Subset and step sizes of 15 pixels and 5 pixels, respectively, were chosen for analysis. The step size defines how far apart the center of the neighboring subsets are located and typically it is selected to be smaller than
half the subset size (Hedayat et al., 2014; Patel and Martin, 2018). A step size of 5 ensured that the neighboring subsets overlapped with each other, which leads to added accuracy in the estimated displacement field (Hedayat et al., 2014).

The reliability of the 2D-DIC analysis was evaluated first by comparing the fracture pattern in the UCS BBM at peak strength to the regions of strain localization from the DIC analysis. From Figure 8.8c, it can be seen that the high strain regions (red) coincide well with the failed contact locations in the BBM. The DIC analysis was not extended all the way to the boundary of the specimen due to issues with selection of the strain calculation window; along the edges of an image, there are fewer pixels in the strain calculation windows and this leads to substantial noise in the estimated strain field. As a method validation, the full-field axial strains were averaged at each stress level across the specimen surface and compared to the axial strains measured from the numerical model. The two curves match well up to slightly beyond the peak strength (Figure 8.8d), and this establishes the applicability of 2D-DIC to numerical modeling results.

DIC analyses are not suited for analyzing large deformations, and consequently model states in the post-peak regime were not examined in great detail. In particular, the basic DIC assumption of preserving the speckle pattern during the deformation process is violated (Bourcier et al., 2013) and the correlation/cross-correlation methodology for mapping the subsets across the different images starts to fail.
8.5.3 2D-DIC analysis: Results and discussion

Figure 8.9 shows the minor principal strain ($\varepsilon_{33}$) for the UCS BBM at five different stress levels, normalized to the average specimen-scale lateral strain at peak strength ($-1.31 \times 10^{-3}$; the negative value indicates extensional strain). The minor principal strain values are more sensitive to the microcracking process than the other strain directions and were therefore employed for further analysis (Shirole et al., 2019). As can be seen, at low levels of axial stress (44% UCS), the strain field is somewhat non-uniform, and this is attributed primarily to the elastic mismatch between the mineral blocks, as the amount of microcracking in the specimen prior to the CI threshold should be limited (Eberhardt et al., 1998; Lan et al., 2010). At an axial stress equal to 90% of the UCS, the strain concentration distinctly increases and forms sub-vertical bands. The elongated strain patterns signify the formation of extensile microcracks along the longitudinal axis of the specimen, and this is known to be the dominant mode of damage development in brittle rocks (Ghazvinian, 2015; Lan et al., 2010; Walton, 2014). The predominance of tensile cracks in the BBM at this loading stage also supports this reasoning (Sinha and Walton, 2020a). The extent of strain localization can be further appreciated from the fact that at a load corresponding to 90% of the UCS, the local strains are >300% of the specimen-scale lateral strain at the peak load. As the sample was loaded to its UCS and beyond, damage zones were clearly perceptible, and these regions coincided with the failed contacts in the BBM (see Figure 8.8d). Overall, the results indicate that the DIC analysis was successful in capturing the progressive damage formation in the model.

Diederichs (1999) and Lan et al. (2010) previously demonstrated using micromechanical models how the heterogeneity in the stress fields increases with increasing specimen damage. The study by Diederichs (1999) was performed using an early version of Itasca’s Particle Flow
Code (PFC), while Lan et al. (2010) used an elastic heterogeneous BBM. Lan et al. (2010) further related the variations in the axial stress field with the CI and CD observed in their models. With the full field strains computed for the UCS BBM, it was possible to visualize this heterogeneity in terms of strains rather than stresses at different load levels. Figure 8.10a shows the strain ellipses at \( \sigma_1 = 25\% \) UCS, \( \sigma_1 = 44\% \) UCS (CI), \( \sigma_1 = 73\% \) UCS, \( \sigma_1 = 90\% \) UCS (CD) and \( \sigma_1 = \) UCS load levels as obtained from the 2D-DIC analysis. The length of the major and minor axes of the ellipses correspond to the standard deviation of the minor (\( \varepsilon_{33} \)) and major (\( \varepsilon_{11} \)) principal strains respectively and signifies the heterogeneity in the strain fields.

![Figure 8.9 Strain fields in the UCS BBM for different stress levels obtained from the DIC analysis.](image)

Prior to occurrence of any micro-damage (below 44% UCS; CI), some heterogeneity can be observed in both the major and minor principal strain fields. This is attributed to the multi-minerallic nature of the model, where the different mineral blocks deformed in a dissimilar
fashion depending on the local stresses and elastic modulus. Shirole et al. (2020) also reported a more homogeneous strain field in Lyon sandstone (composed of quartz grains with some clay cement) in comparison to Stanstead and Barre Granites (multi-minerallic). With increasing load levels, the ellipses became larger and indicate an increase in heterogeneity with progressive damage. An interesting thing to note here is that the standard deviation of the minor principal strain field is always larger than that of the major principal strain field, and the ellipses become more elongated at greater load levels (i.e. after more damage has been induced). This implies that the minor principal strain field is more sensitive to the microcracking process, which is somewhat intuitive as the cracking in the UCS model is dominantly along the specimen axis (refer to Figure 8.7a).

Taking advantage of this fact and utilizing some laboratory test results on Stanstead Granite specimens (Shirole et al., 2020), a comparison of the $\varepsilon_{33}$ standard deviation values (as a proxy for damage) were made to ascertain if the strain field perturbations in the BBM were realistic. This comparison is unique in the sense that it relates to local strains rather than stresses, which can be directly measured during laboratory compression tests (using strain gauges or DIC). The geomechanical parameters and petrological characteristics of Stanstead Granite are different than those of Creighton Granite, and therefore a quantitative comparison would not be appropriate. With that in mind but recalling that both rocks are roughly granitic in composition, it can be expected that the overall trends and magnitudes of $\varepsilon_{33}$ standard deviation will at least be similar. The average UCS and modulus of Stanstead Granite is 129 MPa and 44 GPa (Shirole et al., 2020), while the mineralogical composition is approximately 25% Quartz, 65% Feldspar and 9% Biotite (Nasseri and Mohanty, 2008). The mean (volume averaged) grain size is 1.34 mm (Nasseri and Mohanty, 2008), which is smaller than that of Creighton Granite. For more details
on Stanstead Granite testing and associated 2D-DIC analysis, readers are referred to Shirole et al. (2020).

Figure 8.10 (a) Strain ellipses for 5 different load levels, (b) Comparison of the standard deviation of minimum principal strains in the BBM and for Stanstead granite (Shirole et al., 2020) as a function of percentage failure strength.

Figure 8.10b shows representative photos of Stanstead Granite and Creighton Granite specimens. Note the similarity in the mineralogical compositions of the two rock types. Since the UCS values of the two rock types are different, it was necessary to plot the standard deviations as a function of normalized failure strength so as to be able to compare the overall trends in the values. CI and CD for Stanstead Granite were computed using the reversal of crack volumetric strain and axial stress-axial strain non-linearity approach, respectively (Shirole et al., 2020). The axial and lateral strains were determined through averaging the $\varepsilon_{yy}$ (vertical) and $\varepsilon_{xx}$ (horizontal) strain fields across the entire surface of the specimen. For Stanstead Granite, the point of axial stress – axial strain non-linearity (i.e. the CD threshold) was found to coincide with the volumetric strain reversal point, as has been observed for other crystalline rocks.
With regards to the standard deviation curves, the following observations can be readily made:

(1) The overall trend and the magnitudes of the $\varepsilon_{33}$ standard deviation in the Creighton Granite UCS BBM are very close to those of Stanstead Granite.

(2) The CD values for Stanstead Granite and Creighton Granite are located at similar distances for their respective points of acceleration in the standard deviation curves. This serves as a secondary confirmation that the axial stress – axial strain non-linearity or the shear crack approach yields a reliable CD value for the BBM under unconfined conditions.

(3) The point of volumetric strain reversal corresponds to a model state that has experienced more damage than is typically expected in real rocks at CD.

A major point of concern in relation to the last observation is that many previous BBM studies (Farahmand and Diederichs, 2015; Noorani and Cai, 2015; Wang and Cai, 2019) have used the stress at volumetric strain reversal to determine CD under unconfined conditions. It is therefore likely that the CD was overestimated and the micro-properties obtained from model calibration may be slightly off. The exact cause for the mismatch between the volumetric strain reversal point and axial stress – axial strain non-linearity point in UCS-BBM is not well understood and should be studied further. In any case, it is recommended that future studies employing block models should use the axial stress – axial strain non-linearity point or ‘acceleration’ of shear cracking (as discussed in Section 8.3) for identification of the CD threshold.
8.6 Conclusions

Bonded Block Models (BBM) are a useful tool for studying the microfracturing process in rocks. The model input parameters are generally constrained by comparing the macroscopic behavior of the models to those observed from laboratory testing. Conventional calibrations are restricted mostly to pre-peak attributes, like Young’s Modulus, Poisson’s ratio, unconfined CI, unconfined CD, tensile strength and low confinement peak strengths. In this study, a multi-minerallic BBM with inelastic zones has been developed, and it was shown to not only match the commonly-considered calibration targets, but also the confined CD, post-peak behavior (including residual strength), high-confinement strengths and the decrease in peak dilation angle as a function of confinement for a granitic rock (Creighton Granite). To replicate the post-peak stress-strain behavior, residual strength, and dilation angle, it was necessary to use a mobilized (confinement and plastic shear strain dependent) dilation angle model for the model blocks.

As a methodological advancement and to improve our understanding of how strain heterogeneity evolves in block models, a 2D-Digital Image Correlation (2D-DIC) analysis was conducted on the UCS model results. It was found that the minor principal strains ($\varepsilon_{33}$) are more sensitive to the microcracking process than the major principal strains ($\varepsilon_{11}$). Given the greater sensitivity to damage development, standard deviations of $\varepsilon_{33}$ at different load levels were computed and compared to those for a different granitic rock for which laboratory test data including DIC results were available. A close correspondence in the overall trend was observed. With the methodology established for UCS BBM, future studies can apply it to confined BBMs as well to obtain strains due to both block separation and zone yield.
Lastly, the $\varepsilon_{33}$ standard deviation values were utilized to examine the discrepancy in the point of volumetric strain reversal and the axial stress-axial strain non-linearity in the unconfined BBM. Using the 2D-DIC results, it was confirmed that the axial stress-axial strain non-linearity is indeed the more appropriate indicator of CD in the unconfined BBM, and the volumetric strain reversal point corresponds to a slightly advanced damage state. The close correspondence of the 2D-DIC analysis results and the ability of the model to match pre- and post-peak attributes highlights the phenomenological capabilities of the calibrated BBM and more generally the BBM approach.

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CHAPTER 9

INVESTIGATION OF PILLAR DAMAGE MECHANISMS AND ROCK-SUPPORT INTERACTION USING BONDED BLOCK MODELS (BBMS)

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9.1 Abstract

In this study, Bonded Block Models (BBMs) are used to investigate the pillar damage mechanisms and rock-support interaction in massive-to-sparsely-fractured rockmasses. Hypothetical granite pillar models of width-to-height (W/H) ratio of 1, 2 and 3 are developed, and the input parameters are constrained by matching the stress-strain response of the BBMs to the stress-strain curves from FLAC\textsuperscript{3D} models that were previously calibrated to an empirical pillar strength database. Two different block representations are also considered – elastic and inelastic. It was found that inelastic blocks are necessary to capture the behavioral transition from strain-softening to pseudo-ductile with increase in pillar W/H.

Post-calibration, different rockbolt combinations are tested in the BBM and their influence on the pillar strength and lateral deformations are analyzed. To facilitate interpretation of the support effect, bulking factors are also computed for various stages of pillar loading. It was found that as the support density is increased, the peak pillar strengths also increase but the effect is dependent on the W/H. For the W/H=1 pillar, rockbolts only affected the residual strengths, which is consistent with previous experimental findings from literature. Deformation of the outer stress-fractured region and bulking systematically decreased with increasing support density, but the exact trend evolved as the pillars were loaded to various points on their stress-
strain curves. Lastly, a BBM pillar was developed with explicit intra-block fracturing capability (i.e., individual blocks could break) and the support analysis was repeated with this model. These results are compared to those from the inelastic BBM to understand if the continuum representation of damage within the inelastic blocks led to some underestimation of the rock-support interaction mechanism. It was ultimately concluded that the continuum inelastic representation of smaller-scale damage within individual blocks allows for a more appropriate representation of the rock-support interaction than the explicit intra-block representation.

9.2 Introduction

Pillars are an integral load bearing member in underground mines and play an important role in upholding the functional integrity of the mine openings. As mining progresses deeper and deeper, these pillars are subjected to even higher stresses, which increases the incidence and severity of ground control issues, including but not limited to spalling, buckling, pillar bursting, etc. (Kaiser et al., 1996; Diederichs, 2007). From a macroscopic failure standpoint, the design of pillars is well-studied (Hedley and Grant, 1972; Krauland and Soder, 1987; Potvin et al., 1989; Lunder and Pakalnis, 1997; Martin and Maybee, 2000) and typically involves consideration of a ratio of pillar strength to expected stress. On the contrary, design of skin supports for controlling stress-induced failures at the pillar periphery is not so straightforward, primarily because of our lack of understanding of the complex interaction between support elements and unsupported ground undergoing stress-induced damage. Some limited practical support guidelines, however, have been developed for mining and hard rock tunneling applications (Kaiser et al., 1996; Kaiser, 2019).
The focus of this study is on brittle damage mechanics and rock-reinforcement interaction in highly stressed massive rockmasses, examined in the context of mine pillars. The study of these topics is practically important in that it may ultimately lead to improved design of pillars and pillar support which will improve worker safety in underground mines. While field instrumentation (e.g. extensometer, stress cell, instrumented rockbolts, etc.) could be used for investigating these phenomena, such is expensive, time consuming and typically yields point estimates of rockmass response to loading (Forbes et al., 2020). An alternative tool for this purpose is the Bonded Block Modeling (BBM) technique, a subclass of Discrete Element Models (DEMs; Cundall, 1971), which represents a material domain by an aggregate of polygonal (Voronoi) or triangular (Trigon) blocks. These blocks can separate once the ‘contacts’ between the blocks fail in shear or in tension. Other discontinuum modeling tools like Particle Flow Code (Wanne et al., 2004; Cai et al., 2007; Kias and Ozbay, 2013) or Finite-Discrete Element Method (Elmo and Stead, 2010; Lisjak et al., 2014; Lisjak et al., 2017; Li et al., 2019a; Vazaios et al., 2019) are commercially available and might have the potential to reproduce behaviors similar to those presented in this study. The current study, however, focuses specifically on understanding the capabilities of Voronoi BBM, as implemented in Itasca’s Universal Distinct Element Code (UDEC). Two major advantages of using Voronoi BBM in comparison to other available options are its well-established structural element suite (i.e. in UDEC; Gao et al., 2015; Bai et al., 2016; Bahrani and Hadjigeorgiou, 2017) and its ability to realistically simulate the tensile fracturing (Dadashzadeh, 2020) and bulking process associated with brittle rock damage (Sinha and Walton, 2019b).

To date, notable pillar-specific studies conducted using Voronoi BBMs include those by Preston et al. (2013), Bai et al. (2016), Muaka et al. (2017) and Wang et al. (2019). Preston et al.
(2013) evaluated the effect of height on the ultimate strength of jointed limestone pillars in Missouri using Voronoi BBMs. No support was considered in these models, however. Instabilities related to water-rich roof layers in a Chinese coal mine were analyzed using entry-scale Voronoi models by Bai et al. (2016). The effect of support was not explored in detail since supports were found to be inefficient in controlling the large displacements at the site. Muaka et al. (2017) utilized the Voronoi approach to develop a methodology for designing jointed hard rock pillars that were transected by clay-filled shear zones. Wang et al. (2019) studied the failure modes of coal pillars due to excavation of an adjacent underlying coal seam in China using the Voronoi approach. Neither Muaka et al. (2017) nor Wang et al. (2019) considered the influence of support in their models. Based on this, it is apparent that there is a need for research focusing on assessment of rock-support interaction and intact rock bulking using Voronoi BBM.

Broadly speaking, the study of rock-support interaction embodies two key components: rockmass representation and support representation. Rockmass representation deals with the replication of the macro-mechanical properties of the rock structure (in this case, a pillar) as well the underlying micro-mechanical processes. Once an appropriate rockmass representation is achieved, support can be incorporated in a given model and the resulting behaviors compared to those of its unsupported counterpart. For the purposes of this study, the only support considered is rockbolts, simulated using cable structural elements. Other discontinuum modeling studies (not employing Voronoi BBM) like Gao et al. (2015), Bai et al. (2016) and Shreedharan and Kulatilake (2016) have also used cable elements to model rockbolts. Rockbolts typically reinforce/strengthen the rockmass and improve its self-supporting capacity (Kaiser et al., 1996).
9.2.1 Rockmass representation

In massive, highly stressed ground, damage occurs via brittle spalling along the pillar periphery (Hoek et al., 1995; Diederichs, 2007; Diederichs et al., 2004) and via shearing inside the pillar core (Li et al., 2019a). This is because closer to the periphery, the confining stresses are low, which promotes the development and growth of extensile fractures (Diederichs, 2003). As one progresses inward, the propensity for extensile fracture development is suppressed due to the high confining stresses (Diederichs, 2003), forcing the damage to occur in a shear mode. Such a change in failure characteristics as a function of position within the pillar is well documented in the literature (Chen, 1993; Esterhuizen and Ellenberger, 2007; Kang et al., 2015; Bai et al., 2019; Li et al., 2019a). As shearing is associated with sliding rather than separation, the associated bulking and dilation of the damaged rockmass is naturally lower than what is observed at the pillar periphery. It follows that a numerical model developed to simulate a rock pillar must be able to capture this transition in failure behavior.

Aside from the failure mode, another variable that plays an important role in modulating the macroscopic stress-strain response and the peak strength of pillars is its width to height ratio (W/H) (Martin and Maybee, 2000; Mortazavi et al., 2009; Kaiser et al., 2011; Sinha and Walton, 2018a). As the W/H is increased, the overall pillar behavior transitions from brittle to pseudo-ductile and the peak strength increases as well. It will be shown later in this study that the failure mode and W/H are inter-related, in that the relative contribution of extensile fracturing and confined shearing towards ultimate failure is dependent on the W/H of the pillar.

Sinha and Walton (2018b) previously attempted to model granite pillars (specifically Creighton Granite from Sudbury, Canada) using Voronoi BBM, and in that study, the input
parameters were constrained by matching the BBM stress-strain curves to stress-strain curves from a FLAC³D model for W/H=1, 2 and 3 (see Appendix I). A major drawback of that study is that the match to the stress-strain curves for W/H=1, 2 and 3 were obtained with three different sets of BBM parameters, meaning that the parameters were not fundamental to the material being modeled. Sinha and Walton (2018b) attributed this issue to the elastic, unbreakable nature of the blocks in the BBMs. To remedy this problem and identify the optimal BBM representation for modeling granite pillars, we tested two options in this study – elastic blocks (same as in Sinha and Walton, 2018b) and inelastic blocks. The key difference between the two representations is that in the inelastic BBM, the constant strain triangular continuum zones used to discretize each block can yield and deform inelastically, while they can only deform in a linear elastic fashion in the elastic BBM (Itasca, 2014a). As will be shown in this study, this extra complexity is necessary to replicate rock behavior under different loading conditions.

Inelastic BBMs are much more difficult to calibrate due to the large number of input parameters. Sinha and Walton (2020a) showed that although such increase in the number of input parameters is inevitably associated with an increase in the non-uniqueness potential of the model (Jing, 2003; Bahrani and Hadjigeorgiou, 2018), it can also significantly improve model performance in terms of the number of calibration targets that can be replicated. To date, there have been few applications of inelastic BBM at the laboratory-scale (Noorani and Cai, 2015; Wang and Cai, 2019; Sinha and Walton, 2020a) and at the field-scale (Yang et al., 2017, 2018; Xue et al., 2019; Sinha and Walton, 2020c, d). Yang et al. (2017, 2018) and Xue et al. (2019) used entry-scale trigon models to examine the failure mechanisms around deep coal mine openings and refined the support patterns at the site. In Sinha and Walton (2020c, d), observed pillar deformation trends in the Creighton mine and in the West Cliff coal mine were
successfully reproduced, indicating that inelastic BBMs might have broader applicability. Note that the granite pillar BBM in Sinha and Walton (2020c) was subjected to a load path (both vertical and shear loading) from the calibrated mine-scale FLAC\textsuperscript{3D} model of Walton et al. (2016) and was focused on the site-specific damage development in a pillar at Creighton mine. In contrast, the models presented herein are hypothetical, loaded purely in the vertical direction and attempt to simulate the progressive damage process of granite pillars for multiple W/H cases using a single parameter set, perhaps for the first time using polygonal BBM.

9.2.2 Rock-support interaction

The authors are unaware of any available field data that definitively indicates what aspects of the pillar behavior (beside dilatancy/bulking) are affected by the presence of support. One laboratory-based study that is relevant in this regard is that by Alejano et al. (2017), who attempted to quantify the reinforcement effect of pillar strapping by conducting laboratory tests on cabled specimens. They found that cabling reduced the lateral deformations and increased the residual strength, but did not have any notable effect on the peak strength. While this study enhances our understanding of rock-support interaction, it is limited to standard laboratory-scale specimens with only cable straps as means of support. Nevertheless, given that the specimens were slender (W/H=0.5) in Alejano et al. (2017), one might expect similar changes in behavior with the addition of reinforcement to slender pillars. Such propositions cannot be made for W/H=2 and 3 pillars that exhibit inelastic hardening prior to attaining their peak strengths.

In terms of numerical investigations of rock-support interaction, there is only a limited number of studies that have been conducted to date (Gao et al., 2014a; Kang et al., 2015; Yang et al., 2017; Bai et al., 2019). Gao et al. (2014a), Kang et al. (2015) and Yang et al. (2017, 2018)
presented a comparison of roof deflection with and without support in a coal mine entry, while Bai et al. (2019) presented some qualitative support guidelines for yielding coal pillars based on a discontinuum model (supports were not tested in the models). As a first step towards understanding the influence of reinforcement on ground behavior in numerical models, Sinha and Walton (2019b) employed the elastic W/H=2 BBM from Sinha and Walton (2018b) to evaluate the effect of reinforcement on pillar behavior. In that study, different support patterns were tested with both Trigon and Voronoi block geometries. Unlike the Voronoi models, the Trigon models were found to show less of a reduction in bulking when supports were added than would be expected in reality. These and other models were used to also demonstrate how continuum models tend to underestimate the reinforcement effect of supports due to their enforcement of strain-continuity.

With respect to the support-analysis study of Sinha and Walton (2019b), there are two main limitations concerning the pillar BBMs that were presented: (1) the fact that the input parameters used only applied to a specific W/H rather than the material more generally suggests that the model’s predictive performance under differing conditions (i.e. with added support) may be poor; (2) the effect of reinforcement on the behavior of different W/H pillars was not analyzed.

9.2.3 Objectives and methodology

To bridge the aforementioned gaps in the literature, the present study attempts to answer the following questions: (1) Is it possible to calibrate models for W/H=1, 2 and 3 pillar cases with a single set of model parameters? (2) Can BBMs reproduce the pillar damage mechanisms described above? (3) How do rockbolts influence the strength and deformation of pillars? (4) Is
the effect of rockbolts similar for different W/H? (5) Does zone yield in inelastic BBM, which is a continuum representation of damage, lead to some underestimation of support effect? In order to answer the first four research questions, rock damage and support effect were examined in the different W/H pillar BBMs. For the last question, we compared the reinforcement effect in the inelastic block model and an elastic block model, but the blocks in the elastic model were allowed to break along certain pre-defined failure pathways, thus representing the development of explicit damage within Voronoi blocks. This explicit block breakage technique has been previously employed by Gao et al. (2016), Wang and Cai (2018) and Liu et al. (2019) for laboratory-scale simulations.

Input parameters in BBMs are typically constrained by matching the macroscopic behavior of the model with those measured in laboratory or in the field. Here, we used the stress-strain curves from the FLAC$^{3D}$ granite pillar models presented by Sinha and Walton (2018a), which in turn were calibrated to match the predicted peak strengths for multiple W/H pillar cases according to an empirical pillar strength database (Hedley and Grant, 1972; Hudyma, 1988; Sjoberg, 1992). Accordingly, the calibrated BBM in this study also matches the empirical pillar strength predictions. By matching the peak strengths of unsupported BBMs to the empirical pillar strength prediction, it is effectively assumed that the pillars considered in the empirical strength database were unsupported or minimally supported. Although it is not explicitly mentioned in the relevant empirical studies that the pillars in question were unsupported, observations like peeling of the fractured wall material, sloughage up to the center of the pillar and recommendations to use cables and rockbolts to prevent the disintegration of fractured wall material seem to indicate that majority of the pillars were unsupported (Hedley and Grant, 1972; Hudyma, 1988; Sjoberg, 1992). Lastly, as UDEC is a 2D software and simulates a plane-strain
condition, the stress-strain curves for long pillars (length to width ratio of 4) were employed as the calibration targets (Sinha and Walton, 2019a).

9.3 Unsupported pillar models

9.3.1 Model setup and parameter description

Figure 9.1 shows the model setup used in this study. The edge length of each block was selected to be ~10 cm, as this is considered to be small enough so as to not impose any kinematic constraints on fracture development and growth. Field-scale models have historically used block sizes ranging from 6.5 cm to 50 cm (Damjanac et al., 2007; Preston et al., 2013; Bai et al., 2016; Wang et al., 2019; Dadashzadeh, 2020). For all three pillar geometries, the width of the pillar was set to 8 m and the height was varied to achieve the target W/H ratio. Loading was conducted via two elastic continuum beams on either side that are analogous to the roof and floor in an underground mine. Because in a typical mining scenario, pillars are strained mostly by the deflection of the immediate roof, we assigned a roller boundary to the bottom edge of the lower elastic beam and a very slow downward velocity to the top edge of the upper elastic beam. Additionally, to ensure homogeneity of the applied strain, the velocities were scaled with respect to the pillar W/H: 0.015 m/s for W/H=1, 0.0075 m/s for W/H=2 and 0.005 m/sec for W/H=3. Loading was advanced in this pseudo-static manner until the peak strengths were attained in the different W/H pillar models.

UDEC discretizes each deformable block by multiple constant strain triangular ‘zones’ (Figure 9.1; Itasca, 2014a). These zones can be elastic or inelastic depending on the constitutive model assigned to them (referred to as ‘elastic BBM’ and ‘inelastic BBM’ in this study,
respectively). For the inelastic BBMs, we applied the Cohesion-Weakening-Friction-Strengthening (CWFS; Hajiabdolmajid et al., 2002, 2003) model in the zones. This strength model essentially degrades the cohesive strength and mobilizes the frictional strength simultaneously or non-simultaneously as the plastic shear strain ($\varepsilon_{ps}$) increases, and is based on the fundamental extensile damage mechanism in brittle rocks (Martin and Chandler, 1994; Diederichs, 2003). Since its inception, the CWFS strength model has been employed by numerous authors to model the brittle rock damage process (Zhao et al., 2010a; Lee et al., 2012; Walton et al., 2014; Renani and Martin, 2018b; Dadashzadeh, 2020).

Walton et al. (2016) also used the CWFS strength model to simulate granite pillars (uniaxial compressive strength, UCS, of ~200 MPa) in Creighton mine, Canada. For rocks with similar UCS, the empirical pillar strength database, corrected numerically for length (i.e. for long pillars; Sinha and Walton, 2019a), suggests that W/H=1, 2 and 3 pillars in a UCS = 200 MPa granite should have peak strengths of 67 MPa, 116 MPa and 203 MPa, respectively. These values were therefore considered as the calibration targets for the BBMs in this study.

To calibrate the BBMs to the FLAC$^{\text{3D}}$ stress-strain curves, pillar stresses and strains had to be recorded during the course of the model simulations. For that reason, vertical stresses averaged over all zones in the pillar and the displacements across the top and bottom edge of the pillar were tracked. The displacements were ultimately converted into strains by dividing them by the height of the pillar in question.

The input parameters in BBMs can be broadly sub-divided into two groups: zone properties and contact properties. For elastic zones, the only parameters to be defined are Young’s Modulus (E) and Poisson’s ratio ($\nu$), while for inelastic zones, additional strength
parameters like peak and residual cohesion, peak and residual friction angle, peak and residual tensile strength, and critical $\epsilon_{ps}$ values that define the evolution of each of these strength components are needed. The Coulomb-slip behavior of the contacts is defined by normal and shear stiffness ($j_{kn}$, $j_{ks}$), peak and residual cohesion ($c_{\text{peak}}$, $c_{\text{res}}$), peak and residual friction angle ($\phi_{\text{peak}}$, $\phi_{\text{res}}$) and peak and residual tensile strength ($\sigma_{t,\text{peak}}$, $\sigma_{t,\text{res}}$). Unlike the zones, the drop from the peak to residual value for each strength component is instantaneous for the contacts.

Figure 9.1 Geometry and boundary conditions of the pillar model.
In the supported models (Section 9.4), additional parameters corresponding to the cable structural elements were also defined (Table 9.1). These were chosen directly from Sinha and Walton (2019b), who conducted simulated pull tests to match a load-displacement profile from Luke (2016). Cable element node spacing was set to 0.1 m to ensure that there was at least one node in each of the bolted blocks. Rockbolt faceplates (20 mm long, 5 mm thick) were simulated using beam structural elements (Itasca, 2014b). These elements were assigned a modulus of 200 GPa, a Poisson’s ratio of 0.25 and a rock-to-plate interface friction angle of 25°. All other interface strength properties were set to 0. To ensure effective load transfer between the rockbolts and the faceplates, the central node of each faceplate was attached to the last node of the corresponding rockbolt via an indestructible connection.

Table 9.1 Input parameters for cable structural element (Sinha and Walton, 2019b).

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter</td>
<td>24 mm</td>
</tr>
<tr>
<td>Yield strength</td>
<td>0.30 MN</td>
</tr>
<tr>
<td>Shear stiffness</td>
<td>$2 \times 10^3$ MN/m/m</td>
</tr>
<tr>
<td>Elastic Modulus</td>
<td>210 GPa</td>
</tr>
<tr>
<td>Cohesive capacity of grout</td>
<td>0.30 MN/m</td>
</tr>
<tr>
<td>Tensile strain limit</td>
<td>15%</td>
</tr>
</tbody>
</table>

9.3.2 Elastic block models

Elastic BBMs have a small number of input parameters and are relatively easy to calibrate. A manual back-analysis was conducted in this study to reproduce the FLAC$^{3D}$ stress-strain curves from Sinha and Walton (2019a). In this approach, individual parameters are first modified systematically to understand their influence on the stress-strain response, followed by
changing multiple parameters at the same time. Table 9.2 lists the calibrated contact parameters and the respective stress-strain curves are shown in Figure 9.2. Because not all points on the stress-strain curve can be considered to represent equally critical calibration targets, the use of $R^2$ or another quantitative measure of fit is not appropriate; instead, a qualitative comparison was performed. Accordingly, the obtained parameter combination does not necessarily represent the “best-fit” model, but rather one of several possible similar combinations that could all be considered reasonably calibrated.

![Figure 9.2 Pillar stress-strain curves for W/H=1-3 with elastic blocks.](image)

A drop in the stress-strain curves immediately following yield is observed and is related to the brittle extensile fracturing along the periphery (temporary loss of load-carrying capacity) that pushed the stresses further into the pillar. Despite testing a variety of different parameter combinations, a match against the W/H=1, 2 and 3 stress-strain curves with one set of input parameters could not be attained. Further explanation regarding the attempts to identify a single set of calibration parameters for all three elastic BBM W/H cases is presented in Appendix J.
Some interesting inferences can be made from the trends of the parameters in Table 9.2. W/H=2 and 3 both required smaller contact tensile strengths in comparison to W/H=1, likely because the slender geometry (W/H=1) did not generate enough confining stress to suppress the tensile fracturing process. A steady decline in the peak cohesion value as a function of W/H is also consistent with the idea that frictional strength is important at larger W/H while the cohesive strength is more important for slender pillars. Lastly, \( \varphi_{\text{res}} \) had to be set to low values in order to prevent the geometric interlocking of the polygonal blocks as they are forced to rotate past one another (Mayer and Stead, 2017). If \( \varphi_{\text{res}} \) was raised over 12-13°, extremely large peak strengths for W/H=2 and 3 and a large residual strength for W/H=1 were obtained.

Table 9.2 Contact parameters for the elastic BBM.

<table>
<thead>
<tr>
<th>Models</th>
<th>( c_{\text{peak}}^* ) (MPa)</th>
<th>( \varphi_{\text{peak}} ) (°)</th>
<th>( \varphi_{\text{res}} ) (°)</th>
<th>( \sigma_{t,\text{peak}}^* ) (MPa)</th>
<th>( j_{kn} ) (GPa/m/m)</th>
<th>( j_{ks} ) (GPa/m/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W/H=1</td>
<td>120</td>
<td>34</td>
<td>5</td>
<td>40</td>
<td>100,000</td>
<td>50,000</td>
</tr>
<tr>
<td>W/H=2</td>
<td>80</td>
<td>35</td>
<td>10</td>
<td>17.5</td>
<td>100,000</td>
<td>50,000</td>
</tr>
<tr>
<td>W/H=3</td>
<td>50</td>
<td>46</td>
<td>8</td>
<td>17.5</td>
<td>100,000</td>
<td>50,000</td>
</tr>
</tbody>
</table>

* \( \sigma_{t,\text{res}} \) and \( c_{\text{res}} \) were set to 0.

9.3.3 Inelastic block models

The key to the successful implementation of the inelastic block representation is to allow the highly dilatant peripheral fracturing to occur explicitly via contact failure and the finer-scale shearing (limited dilatancy) inside the pillar via zone yield. This means that as the failure mode transitions from tensile to shear and the severity of localization decreases, the element within the BBM simulating the majority of the damage also changes from contact failure to zone failure.
Since in this study we are attempting to model granite pillars similar to those at Creighton mine, it was most logical to select the contact and zone strength properties from Sinha and Walton (2020c) as the starting point for calibration, followed by some minor modifications to account for the differences in loading path between the two studies. Table 9.3 lists the calibrated set of model parameters, and the corresponding stress-strain curves are shown in Figure 9.3. These zone strength properties are very similar to those recommended by Walton (2019) for a brittle rock with UCS of ~200 MPa and m\(_i\) of 20.9 (i.e. Creighton Granite; see Walton, 2014). The only modification introduced in the CWFS strength model is the additional decay in the friction angle from 47.5° to 30° over a large critical plastic shear strain of 0.05. Such a decay was also reported by Martin and Chandler (1994) and Renani and Martin (2018a) and is attributed to the loss of frictional strength due to destruction of asperities along initially rough stress-induced fracture surfaces. The small effective residual cohesive strength corresponds to stress-independent interlocking of rock fragments and/or rock-bridges that remain in the system after the initial fracturing has occurred (Walton, 2019).

Contact parameters are less well-constrained based on prior study and were thus modified over a much wider range during the calibration process. In some cases, the physical meaning of the individual contact parameters is not as straightforward as might be assumed. For example, one might expect \(\varphi_{res}\) to be related to the basic friction angle in some fashion, but BBM behaviors are heavily influenced by the geometric interlocking of the blocks, meaning that values of \(\varphi_{res}\) less than the basic friction angle (Alejano et al., 2012) are typically required (Christianson et al., 2006; Farahmand et al., 2018; Stavrou and Murphy, 2018; Dadashzadeh, 2020).

The calibrated inelastic BBM was able to successfully reproduce the peak strengths for all three W/H geometries and also the brittle response for W/H=1 and strain-hardening behavior
for W/H=2 and 3 (Figure 9.3). It was possible to obtain this behavior, especially for the W/H=3 geometry, due to the gradual reduction of the zone friction angle for large critical plastic shear strains. In other words, deeper within the pillar where confining stress is high, failure occurs in a shear mode. Based on classical shear yield criteria, it is known that rocks lose strength following failure; this strain-softening behavior was implicitly modeled in the pillar ‘core’ by allowing the friction angle to degrade with increasing damage. Such a modification was not necessary in Walton et al. (2016) and Sinha and Walton (2020c) because of the slender geometry of the pillars (W/H=1.5) in those studies.

Table 9.3 Contact and zone parameters for the inelastic BBM.

<table>
<thead>
<tr>
<th>Zones - CWFS</th>
<th>Contacts</th>
</tr>
</thead>
<tbody>
<tr>
<td>E (GPa)</td>
<td>( \sigma_{\text{peak}} ) (MPa)</td>
</tr>
<tr>
<td>Peak cohesion (MPa)</td>
<td>( \sigma_{\text{res}} ) (MPa)</td>
</tr>
<tr>
<td>Residual cohesion (MPa)</td>
<td>( \varphi_{\text{peak}} ) (°)</td>
</tr>
<tr>
<td>Peak friction angle (°)</td>
<td>( \varphi_{\text{res}} ) (°)</td>
</tr>
<tr>
<td>Residual friction angle (°)</td>
<td>( \sigma_t^* ) (MPa)</td>
</tr>
<tr>
<td>Tensile strength (MPa)</td>
<td>( j_{\text{cr}} ) (GPa/m/m)</td>
</tr>
<tr>
<td>Critical plastic shear strain from peak to residual</td>
<td>( j_{\text{ks}} ) (GPa/m/m)</td>
</tr>
</tbody>
</table>

* \( \sigma_t^* \) was set to 0.

**9.3.3.1 Damage mechanics in the model**

With the ability of the inelastic BBM to replicate the macroscopic strengths and behaviors of pillars with different W/H ratios established, further analysis was performed to
ascertain if the underlying damage mechanisms were reproduced as well. Figure 9.4 shows the failed contacts and yielded zones at various points on the stress-strain curves for each case. The following observations can be made based on the model results:

(1) Damage in all the models initiates first by zone yield at the corners, followed by contact failure. Although the initial strength of the contacts is higher than that of the zones (Table 9.3), fracturing progressed via contact failure rather than zone yield along the pillar periphery due to point loading and wedging of blocks (Figures 9.4b, 9.4f and 9.4h).

(2) In the W/H=1 pillar, even in the post-peak, the number of yielded zones is minimal, and pervasive contact damage can be noted throughout the entire pillar width. Indeed, the slender geometry prevents generation of high confining stresses within the pillar and failure consequently progressed via contact damage. The lack of core generation is evinced by the small region of yielded zones at the center of the W/H=1 model (Figure 9.4g).
Figure 9.4 Failed contacts and yielded zones at various stages of loading in the W/H=1-3 pillar model.

(3) For larger W/H cases, confining stresses are generated relatively early in the loading process, forcing the failure to occur through zone yield. In the late stages of loading (Figures 9.4e, 9.4j and 9.4k), the intensity of zone damage starts to increase in the core.
This intensity is greatest in the W/H=3 model, as it is loaded to a larger total axial strain level. The depth of fractured contacts is also smaller in the W/H=3 model in comparison to the W/H=2 model at equivalent levels of axial strain. With continued loading, contact damage ultimately propagated all the way to the center of the pillar (Figures 9.4e and 9.4k).

(4) The transition from extensile cracking to shearing with depth is controlled by the W/H geometry, and this fundamental behavior is well captured in these models. The relative contribution of explicit contact damage and implicit zone damage towards the ultimate failure is also evident in Figure 9.4.

(5) Spalling and separation of blocks can be observed along the entire pillar periphery. The extent and severity of block detachment is greater in the larger W/H geometry models. Discontinuum models explicitly simulate these highly dilatant processes and are therefore better suited for analysis of rock-support interaction (Sinha and Walton, 2019b).

To ensure that the failure of the contacts along the pillar periphery occurred in tension, the normalized number of tensile and shear cracks and yielded zones are plotted against pillar stress for the W/H=2 geometry (see Figure 9.5); corresponding plots for W/H=1 and 3 are not shown, as they are similar to Figure 9.5. Tensile cracks and zone yield are denoted by black solid and broken lines, respectively, while shear cracks and zone yield are indicated by red solid and broken lines, respectively. It can be seen that when the stress-strain curve registers the onset of non-linear pillar behavior (between ‘a’ and ‘b’ in Figure 9.4), the majority of the damage in the model has occurred by tensile cracking (refer to the blue circles in Figure 9.5). For clarity, the plot of failed contacts at this loading state is also shown in the inset. From all this, we can infer
that initial boundary cracking indeed occurred in a tensile mode, as one would expect in a
massive brittle rockmass undergoing spalling (Diederichs, 2003, 2007). It is also due to this
reason that the W/H=2 and 3 models exhibited a loss in load carrying capacity right after
attainment of yield in both the elastic (Figure 9.2) and the inelastic (Figure 9.4) BBMs.

A secondary observation from Figure 9.5 is that the number of contacts failing in shear
starts to escalate immediately after this extensile failure phase in the inner portions of the pillar.
Ultimately, the number of failed contacts levels off and zone shear becomes the dominant
damage mode. Note the negligible role that zone tensile failure plays during the entire loading
process - this is logical, as tensile failure occurs explicitly along the periphery and tensile stresses
are suppressed deeper within the pillar.

9.4 Supported inelastic pillar models

In this section, we investigate how installation of rockbolts might influence the overall
behavior of the three pillar W/H cases, and for that purpose, the three W/H models were re-run
with different support combinations: (1) $W/H=1$: 6 bolt and 10 bolt, (2) $W/H=2$: 3 bolt, 5 bolt
and 6 bolt, and, (3) $W/H=3$: 3 bolt, 4 bolt and 5 bolt. In terms of bolt spacing, these support
combinations correspond to (1) $W/H=1$: 1.4 m and 0.78 m, (2) $W/H=2$: 1.5 m, 0.75 m and 0.60
m, and, (3) $W/H=3$: 0.83 m, 0.55 m and 0.42 m. For installing the rockbolts, a 0.5 m gap was left
at the top and bottom of the pillar and the bolts were then spaced out in the vertical direction
between these bounds. The idea was to prevent the installation of rockbolts very close to the
pillar top and bottom, which is based on the operational constraints typically faced by a rock
bolter in an underground mine.
To allow for easy comparison between the supported and unsupported models, support pressures were computed, following the equations provided by Hoek (1999). The equations in Hoek (1999) correspond to the equivalent maximum pressures for support patterns, which represent the upper bound of what is mobilized in the field. Hoek (1999) lists four equations for mechanically anchored rockbolts corresponding to 17 mm, 19 mm, 25 mm and 34 mm diameters and only one equation for 20 mm grouted rebar. As the resin grouted rockbolt in this study is 24 mm in diameter (chosen from Sinha and Walton, 2019b), the maximum support pressures computed from the mechanically-anchored bolt equations had to be used along with the 20 mm grouted rebar equation. Specifically, the equations for 20 mm and 24 mm diameter mechanical rockbolts were first determined via linear interpolation, and then the ratio of the support pressure for a 20 mm diameter mechanical rockbolt to the support pressure for a 20 mm diameter grouted rebar was applied to obtain the support pressure estimate for the 24 mm grouted rebar.
9.4.1 Effect of support

Figure 9.6a shows the stress-strain curves for the most heavily supported BBM and the unsupported BBM for each W/H case. It is apparent from Figure 9.6a that the overall response does not change very much with incorporation of support; the peak strengths, however, increase with increasing support density for the W/H=2 and W/H=3 pillars (Figure 9.6b). As rockbolts pin the spalled slabs/blocks to the pillar, this generates some confinement, which would otherwise not be present if the failed rockmass is allowed to fracture and collapse into the adjacent excavations. From these results, it seems that the addition of support pressure has increased effect on squatter geometries (Figure 9.6b), and this can be explained by the greater contribution of zone yield towards the ultimate failure of these pillar, combined with the fact that the gap in maximum principal stress between the peak and residual zone strength envelope increases at higher confining pressure (recall that the friction angle of the peak and residual envelopes per Table 9.3 are $\theta^0$ and 47.5°, and thus they diverge in the principal stress space).

Figure 9.6 (a) Stress-strain curves for the unsupported and most heavily supported models for W/H=1, 2 and 3, (b) Peak strength as a function of support pressure for the unsupported and supported models. Zero support pressure corresponds to the unsupported model.
Interestingly, for W/H=1, the peak strength remains almost invariant, but the residual strengths increase with addition of support. This is consistent with the laboratory findings of Alejano et al. (2017) for slender cabled laboratory specimens. A logical question with respect to the lack of a reinforcement influence on the W/H=1 pillar peak strength is if rockbolts generate some amount of confining pressure on the periphery, then why does the strength remain unchanged, even though peak strength is well-known to increase with increasing confining stress (Arzua and Alejano, 2013; Walton et al., 2014; Walton, 2018)? The answer lies in the passive nature of the confining stress generated by rockbolts (and in the cabled samples of Alejano et al., 2017) that is activated only when sufficient deformation (i.e. fracturing) has occurred in the rockmass. In laboratory tests with slender specimen geometries, large lateral deformations do not occur until the specimen is loaded to the post-peak (Lajtai, 1988; Li et al., 2011; Chen et al., 2015), and this is also observed in the current models. Consequently, any support-generated confinement can only influence the pillar behavior in the post-peak. Note how the various W/H = 1 stress-strain responses are different in the post-peak regime in Figure 9.6. Because the rockbolts prevented the disaggregation of the pillar edge and allowed for some friction to mobilize along the failed contacts, a higher post-peak strength is obtained.

Figure 9.7 shows the horizontal displacements in the W/H=2 unsupported and 5 bolt models at different loading stages. The depth and severity of fracturing increase as the unsupported pillar is strained monotonically by the two loading beams. With inclusion of 5 bolts, not only are the lateral displacements significantly suppressed, but so is the depth of fracturing. These results suggest that rockbolts can delay the inward propagation of fractures by generating local confinement (reinforce) and also efficiently pin and hold the spalled blocks, if the support density employed meets the appropriate strain demand. Of course, faceplates play an important
role in controlling the surficial deformations by increasing the effective supported area of the pillar boundary (Gray, 1998).

![Contour of horizontal displacement](image)

Figure 9.7 Contour of horizontal displacement in the W/H=2 pillar model without any bolts and with 5 bolts.

### 9.4.2 Bulking Factor (BF)

Bulking factor is a convenient index for quantifying the effect of reinforcement and is defined as the increase in volume (or area in 2D) within the yielded/fractured region with respect to an undamaged state (Kaiser et al., 1996; Walton, 2014; Oliveira and Diederichs, 2017). In physical terms, BF represents the dilation of the broken rockmass along the excavation
boundary, and this index has found wide application in mining. For instance, one can estimate the approximate displacement demand on supports by multiplying a representative BF (Kaiser et al., 1996) with a semi-empirical estimate of failure depth (Martin et al., 1999; Kaiser, 2014). It is important to note here that our previous study on rock-support interaction (Sinha and Walton, 2019b) treated the BF and the influence of support as independent of geometry or loading conditions, based on the rough empirical guidelines of Kaiser et al. (1996). In this study, we computed the BFs for all the BBMs at various stages of loading.

The calculation was conducted for the supported and unsupported models at four distinct points (or analysis stages) along their strain-strain curves. Each of the four points correspond to four model step numbers; these numbers are the same in the unsupported and supported models with a particular W/H geometry. In other words, as the models are loaded by a constant downward velocity, choosing a fixed step number would mean that the outer surfaces of the loading beams also converged by a fixed amount. In-situ, this corresponds to a pillar that is being compressed/strained by the host rock following nearby production activities. The axial strains corresponding to the four analysis stages are listed in Table 9.4. For a given W/H, the axial strains are not equal in the unsupported and supported models because the stress-strain responses are not exactly identical (Figure 9.6a).

### 9.4.2.1 Calculation methodology

To compute the bulking factors, the following equation from Kaiser et al. (1996) was employed:

\[
BF = \frac{u_w - u_d}{d_f} \quad (9.1)
\]
where, \( u_w \) is the displacement at the pillar edge, \( u_{df} \) is the displacement at the depth of failure, and, \( d_f \) is the thickness of the stress-fractured region. To obtain the three parameters, lateral displacements of all gridpoints (the vertices of the triangular zones) along multiple horizontal lines, spaced at 0.5 m vertically, were extracted from the BBMs. These horizontal lines and the displacements along these lines (left half of the 8 m pillar) for one model are shown in Figure 9.8. Using this figure as an example, \( u_w \) corresponds to the displacement at \( x=0 \) for each line, \( d_f \) corresponds to the points where the displacement perturbations diminished and the curves became smooth (marked by black circles), and \( u_{df} \) corresponds to the lateral displacement at \( x=d_f \). The approach used for determination of \( d_f \) and \( u_{df} \) is described in detail in Appendix K.

Table 9.4 Axial strains (10^{-3}) at which bulking factors are computed.

<table>
<thead>
<tr>
<th>Stage</th>
<th>W/H=1</th>
<th>W/H=2</th>
<th>W/H=3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unsup-</td>
<td>Unsup-</td>
<td>Unsup-</td>
</tr>
<tr>
<td></td>
<td>ported 6 bolt</td>
<td>ported 3 bolt</td>
<td>ported 3 bolt</td>
</tr>
<tr>
<td>1</td>
<td>1.1 1.0 1.0</td>
<td>2.1 1.9 1.9</td>
<td>2.9 2.7 2.6</td>
</tr>
<tr>
<td>2</td>
<td>1.7 1.5 1.5</td>
<td>4.1 3.8 3.8</td>
<td>4.7 4.3 4.1</td>
</tr>
<tr>
<td>3</td>
<td>2.7 2.4 2.7</td>
<td>5.6 5.2 5.3</td>
<td>6.5 6.0 5.8</td>
</tr>
<tr>
<td>4</td>
<td>3.5 3.3 3.3</td>
<td>7.2 6.9 6.7</td>
<td>9.2 9.0 9.2</td>
</tr>
</tbody>
</table>

While a simpler approach would have been to determine the bulking factor along a single horizontal line along the pillar mid-height, such an approach would not be representative of the entire pillar, as the depth of fracturing is non-uniform (Figure 9.7). Accordingly, bulking factor is computed along multiple lines, as shown in Figure 9.8. To generate these displacement profiles, gridpoint addresses along the lines were extracted before running the models, followed by importing them back into the models at the four analysis stages (Table 9.4) to query the respective displacements. Because un-run model coordinates are used in the x-axis of Figure 9.8,
comparison to an undamaged state is implicitly accounted for in this calculation. Additionally, the subtraction in the numerator of Eq. 9.1 accounts for both the explicit separation of blocks as well as the inelastic deformation within the zones, although the majority of the bulking observed in the models corresponds to explicit block separation.

![Figure 9.8 Methodology for determination of bulking factor from horizontal displacement profiles. The numbers in legend represent the location of the horizontal lines from the base of the pillar. The black open circles represent the edges of the fractured region. The displacements correspond to the left half of the W/H=2 unsupported model (Stage 4 in Table 9.4). All data points are spaced at 0.1 m along the x-axis.](image)

This methodology is somewhat different from that used by Sinha and Walton (2019b) to compute the bulking factors. In that study, the image analysis tool in MATLAB was employed to directly obtain the areal increase due to stress-fracturing from images of the BBM. Such an approach undoubtedly yields a more representative estimate of bulking factor, but could not be used in this study due to the large block separations observed at late stages of loading (Figure
The separation of blocks made it challenging to identify a spatial domain over which to conduct the MATLAB image analysis.

### 9.4.2.2 Results and discussion

The results of the bulking factor analysis are summarized in Figure 9.9 using two sets of graphs for each W/H case: BF versus inelastic axial strain and BF versus support pressure. Inelastic axial strain was obtained by simply subtracting the elastic strain, which is the pillar stress divided by the elastic modulus (80 GPa), from the total strain at the four analysis stages. The results from the BF analysis indicates a significant variation in the BF values for each model state analyzed (Figures 9.9a, c, e). Generally, the BFs are larger at the pillar centers and are lower at the top and bottom edges. Some large BFs are, however, obtained at the pillar edges in cases where block separation had occurred (these regions also had lower $d_f$, thereby raising the BF).

BF increases with increasing inelastic axial strain, as expected, since inelastic axial strain serves as a damage index for the pillar as a whole. No consistent trend is observed in the data as a function of support pressure; some are concave, some are linear and some are convex. A divergence between the supported and unsupported bulking factors can be observed with increasing inelastic axial strain. The difference in BF between the unsupported and supported conditions is larger for the W/H=1 and 2 models in comparison to W/H=3. This means that supports are performing more efficiently in suppressing the dilation of the stress-induced fractures in these models.
In absence of any consistent non-linear relationship, a linear equation was fitted to the mean BF values as a function of inelastic axial strain. The $R^2$ values for all the fits are greater than 0.85. These equations were then employed to develop curves for BF versus support pressure at various inelastic axial strains, as shown in the right panels in Figure 9.9. This interpolation based on the linear curve fits was required because the BFs for supported and unsupported models did not correspond to the same inelastic axial strain at the particular analysis stages considered (Figures 9.9a, c, e). At low inelastic axial strain, the effect of rockbolts is negligible in the W/H=1 model, as minimal lateral deformation occurred at this stage. With further increases in inelastic axial strain, the trend transformed into a near-exponential form, consistent with the empirical data of Kaiser et al. (1996).

For W/H=2, the BF – support pressure relationship initially exhibited an exponential trend, but became linear between support pressures of 0 and 145 kPa at later stages of loading. It can be observed from Figure 9.9d that the BF for the 3 bolt model (support pressure of 145 kPa) increases rapidly in comparison to the 5 bolt and 6 bolt cases (290 kPa and 363 kPa). A closer look at the model results indicated that the cable elements in the less supported model underwent grout failure and some of the elements in the central bolt also broke axially at later loading stages. Consequently, the rockbolts are not performing as effectively as the rockbolts in the 5 bolt and 6 bolt layouts. The W/H=3 case exerted the greatest displacement demand on the support. Again, failure of cable elements was observed in the least supported model (3 bolt), which triggered the rapid increase in the BF observed in Figure 9.9f. The 4 bolt and 5 bolt layouts did not exhibit any bolt breakage, but also had some grout failures. For that reason, the bulking factors increased more in these models with continued pillar damage than in the W/H=2, 5 bolt and 6 bolt models.
Figure 9.9 Bulking factor as a function of inelastic axial strain for (a) W/H=1, (c) W/H=2, (e) W/H=3, and bulking factor as a function of support pressure for (b) W/H=1, (d) W/H=2, (f) W/H=3. The solid circles and lines in (a), (c) and (e) represent the mean BF values for each model at the different analysis stages. Greater support pressure corresponds to smaller spacing between the rockbolts.
The increasing demand on the support with increase in W/H is evident from the fact that the drop in BF between the unsupported condition and the intermediate support layout (i.e. 6 bolt for W/H=1, 3 bolt for W/H=2 and 3 bolt for W/H=3) tapers off at large inelastic axial strain, even though the equivalent support pressures are greater. Design of rock supports in grounds undergoing large deformation should therefore be based on balancing the anticipated displacement demand and the deformation capability of the support element (Kaiser, 2019). As shown in Figure 9.9, this displacement demand varies with loading, meaning that for an efficient support design, an estimate of surficial deformation over the functional life of the excavation is required.

Sinha and Walton (2019b) proposed a conceptual framework for the rock-support interaction. In that framework, the rock-support interaction curve, plotted in the support density - support pressure space, was split into three regions (Figure 9.10): (i) Inadequate support region – Support density is not adequate and it breaks, leading to minimal effect on ground behavior; (ii) Maximum gain region – Increase in support density has the maximum marginal “value added” in this region with respect to some performance metric (e.g. BF reduction); and (iii) Overdesigned region – an excessive amount of support has been added to the system, and the effect of any further support on the ground is limited. The rock-support interaction itself is represented by a region rather than a single curve due to variability in the rockmass mechanical attributes and support properties that affect their mutual interaction. With this framework in mind, it is estimated that at an inelastic axial strain of 0.007, the 10 bolt \((W/H=1)\) and 6 bolt \((W/H=2)\) models are either in the Overdesigned region or approaching the boundary between the Overdesigned and Maximum Gain regions; 6 bolt \((W/H=1)\), 5 bolt \((W/H=2)\), 4 bolt \((W/H=3)\) and 5 bolt \((W/H=3)\) models are in the Maximum Gain region; and 3 bolt \((W/H=2\) and 3) models are
either in the Inadequate segment or in the lower portion of the Maximum Gain region (Figure 9.10). These classifications only apply to the specific loading condition considered, i.e. 0.007 strain. At lower inelastic axial strains, the 3 bolt \((W/H=2)\) model could be considered to be higher up in the Maximum gain segment, for example.

![Diagram](image)

Figure 9.10 The rock-support interaction curve and the approximate locations of the different supported BBMs on this curve at an inelastic axial strain of 0.007 as estimated from the simulation results. The arrows indicate the uncertainty in classification for some of the models. Greater support density corresponds to smaller spacing between the rockbolts.

### 9.5 Elastic pillar models with explicit intra-block fracturing capability

In inelastic BBMs, fracturing within the blocks is approximated via zone yield. As zone yield is a continuum representation of damage and given that continuum models tend to
underestimate the support effect (Sinha and Walton, 2019b), there is a possibility that the support influence on pillar behavior reported in Section 9.4 is slightly underestimated relative to reality. To test this proposition, a BBM was developed that had explicit intra-block fracturing capability, and then the rock-support interaction analysis was repeated. The corresponding BBM is shown in Figure 9.11a. This model is slightly different from the laboratory-scale BBMs of Gao et al. (2016) and Liu et al. (2019) that considered intra-block fracturing. In Gao et al. (2016) and Liu et al. (2019), cracks were inserted between the block centroids and their vertices to further split the Voronoi blocks, while in this study, the BBM was first discretized by triangular zones and then the locations of the zone edges within each block were used to insert additional cracks within the BBM. It is for this reason that more contacts or failure pathways could be introduced in the current BBM. The inter and intra-block contacts are shown by green and black lines in Figure 9.11a.

The runtime of UDEC BBMs are dependent on the number of gridpoints and the contacts (Itasca, 2014a). As these intra-block BBMs contain a large number of blocks, to attain an acceptable runtime, a symmetry condition was applied, and the zone size was set such that each triangular block corresponded to exactly one constant strain zone. This limited the number of gridpoints and contacts to be considered by the solution algorithm.

For determining if failure of the intra-block contacts could appropriately model the damage within blocks, the inter-block contacts were assigned the same properties as those listed in Table 9.3 and all blocks were made elastic, meaning that the only parameters considered for modification were those of the intra-block contacts. Despite numerous trials, a single set of parameters that could match the stress-strain curves for all three W/H geometries could not be identified, likely because of the less complex nature of the contact constitutive model in
comparison to the zone constitutive model. Accordingly, the support analysis was conducted using a BBM calibrated only to the W/H=2 case. The intra-block contact properties for the calibrated model and a comparison of the stress-strain curve with the corresponding inelastic BBM are presented in Table 9.5 and Figure 9.11b.

Figure 9.11 (a) Geometry of the model with explicit intra-block fracturing capability, (b) Stress-strain curves of different unsupported and supported BBMs.

The explicit model was able to match the overall shape of the stress-strain curve. Fracturing in this model initiated by inter and intra block tensile cracking along the periphery, which later transitioned to intra-block shear failure at locations deeper within the pillar at late stages of loading (Figure 9.12a). It is interesting to note how the intra-block contact strength properties are much larger than the inter-block ones (Table 9.3) yet internal damage progressed
primarily via intra-block shearing. The reason for this behavior is the predisposition of triangular blocks or trigons towards shear fracturing due to the availability of linear failure pathways (Ghazvinian et al., 2014; Mayer and Stead, 2017). The horizontal deformation pattern in this model for the last recorded stage can be found in Figure 9.12b.

Table 9.5 Contact parameters for the intra-block contacts.

<table>
<thead>
<tr>
<th>c_{peak}^* (MPa)</th>
<th>\phi_{peak} (°)</th>
<th>\phi_{res} (°)</th>
<th>\sigma_{t,peak}^* (MPa)</th>
<th>j_{kn} (GPa/m/m)</th>
<th>j_{ks} (GPa/m/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>145</td>
<td>45</td>
<td>40</td>
<td>30</td>
<td>100,000</td>
<td>50,000</td>
</tr>
</tbody>
</table>

* \sigma_{t,res} and c_{res} were set to 0.

Figure 9.12 (a) Crack count at various points along the stress-strain curve, (b) Horizontal displacement contour for the W/H=2 model at the terminal point of the stress-strain curve in (a). The total number of inter and intra-type contacts in this model are 9840 and 16022, respectively.

For the rock-support interaction analysis, three support patterns were tested – 3 bolt (1.5 m spacing), 5 bolt (0.75 m spacing) and 6 bolt (0.60 m spacing). The stress-strain curves for these models are shown in Figure 9.11b. Subsequently, bulking factors at four different model stages were computed for the unsupported, 3 bolt, 5 bolt and 6 bolt BBMs, followed by
determining the bulking factors for different magnitudes of inelastic axial strain (as was done in Section 9.4.2). The results are illustrated in Figure 9.13.

![Figure 9.13](image.png)

Figure 9.13 (a) Bulking factor as a function of inelastic axial strain, (b) bulking factor as a function of support pressure. The solid lines in (a) represent the mean BF values for each model at the different analysis stages. Greater support pressure corresponds to smaller spacing between the rockbolts.

In comparison to the inelastic W/H=2 BBM, it can be seen how the explicit model predicts higher strength gain with addition of supports: for the 5 bolt model, the strength increased by 19.7% and 16.2% in the explicit and the inelastic model, respectively, from their unsupported counterparts. The discrepancy is much larger in the lateral deformation behavior and bulking factor values. In particular, all explicit models exhibited low peripheral displacements due to the breakage of the Voronoi blocks into smaller triangles at late stages of loading. This is expected, as block breakage inhibits the development of large geometric mismatches in these models that are typically observed in Voronoi BBMs. The tendency of trigons to exhibit less volume changes in comparison to Voronois under the same loading condition has been previously reported by Sinha and Walton (2019b). Perhaps this issue could be
overcome by sub-tessellating each Voronoi block with more Voronois, but this was not attempted for two reasons: (1) If the built-in Voronoi generator is used for this purpose, then very small Voronoi sub-blocks will be created. An alternate block generation technique is therefore required; (2) Since the sub-blocks are polygonal in shape, they will have to be discretized by multiple constant-strain zones, which will increase the model runtime drastically. For context, the elastic, inelastic and explicit W/H=2 models took ~15-18 hours, ~1 day and ~3-3.5 days to run, respectively, on a machine with a 10-core Intel i7-6950X processor and 64 GB of RAM. Depending on the number of sub-Voronois and the number of zones within each sub-Voronoi, we estimate a sub-tessellated model might take anywhere between 7-15 days to complete simulation on a similar machine.

Lastly, we compared the reinforcement effect in the inelastic and explicit BBMs by normalizing the bulking factors for the supported BBMs with respect to the unsupported bulking factors and these are listed in Table 9.6. Clearly, at all four inelastic axial strains, the drop in bulking factor with the addition of support is much larger in the inelastic BBM, suggesting that the bolts are performing more efficiently in suppressing the stress fractures in the inelastic BBM. It also seems that while allowing more explicit damage increases the influence of rockbolts, this increased effect might not be necessarily correct (more effect on strength rather than deformation, which is contrary to the experimental findings of Alejano et al. (2017)). The authors would like to reiterate that the results presented here are not just a manifestation of the BBM being semi-calibrated (i.e. calibrated to only one W/H stress-strain curve). This is because the inter-block properties are identical in both the explicit and inelastic model and the macroscopic axial behaviors are also almost identical, meaning that we have purely isolated the influence of the two different intra-block representations.
Table 9.6 Comparison of normalized bulking factors in the inelastic BBM (Section 9.4.2) and in the BBM with explicit intra-block fracturing capability.

<table>
<thead>
<tr>
<th>Inelastic axial strain ($10^{-3}$)</th>
<th>Inelastic BBM</th>
<th>BBM with intra-block fracturing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Normalized bulking factor (%)</td>
<td>Normalized bulking factor (%)</td>
</tr>
<tr>
<td>Unsupported</td>
<td>Unsupported</td>
<td>Unsupported</td>
</tr>
<tr>
<td>1</td>
<td>100 36.1 37.6 19.3</td>
<td>1 100 68.6 57.4 34.3</td>
</tr>
<tr>
<td>3</td>
<td>100 53.7 43.9 34.0</td>
<td>3 100 71.7 62.7 49.7</td>
</tr>
<tr>
<td>5</td>
<td>100 64.9 48.0 43.4</td>
<td>5 100 73.6 65.8 58.8</td>
</tr>
<tr>
<td>7</td>
<td>100 72.7 50.8 49.9</td>
<td>7 100 74.8 67.8 64.7</td>
</tr>
</tbody>
</table>

9.6 Conclusions

Pillar spalling is a major hazard in the mining industry and in absence of rigorous support guidelines, accidents have continued to occur in recent years. Developing effective support guidelines requires a complete understanding of the pillar damage mechanisms as well as the rock-support interaction effects on ground behavior. To that end, this study employed the Bonded Block Modeling (BBM) technique to simulate the progressive damage process of hard rock pillars and investigate the effect of reinforcement on pillar strength and deformability.

Three different pillar BBMs with width to height (W/H) of 1-3 were developed and these models calibrated to match the stress-strain curves from previously calibrated FLAC$^{3D}$ models. Since only elastic blocks have been used to date for field applications, we tested both elastic and inelastic blocks in this study. Results indicated that it is necessary to consider inelasticity within the blocks in order to reproduce both the target stress-strain curves and the transition in damage mode from low confinement brittle fracturing along the periphery to confined shearing deeper within the pillar. The inelastic BBM was subsequently used for support analysis, whereby the
peak strengths and lateral deformations (quantified by the ‘bulking factor’) in the unsupported and supported models with varying rockbolt densities were compared. Unlike the W/H=2 and 3 models, the peak pillar strength remained relatively unaffected in the W/H=1 model with inclusion of supports; the residual strengths, however, increased with increasing support density. The bulking factors evolved as each pillar BBM was progressively loaded up to failure, and at any given loading stage, the bulking factors were lower in the models with higher support density. The exact trends differed as a function of the W/H ratio. These results indicate that the support influence varies depending on the geometry and loading conditions of a pillar.

Previous studies have shown that continuum models tend to underestimate the reinforcement effect of supports. As inelastic yield within blocks is also a continuum approximation of damage, it is possible that the support influence on ground behavior observed from the inelastic BBM might be a slight underestimation of reality. To test this hypothesis, an elastic BBM was developed with explicit intra-block fracturing capability (polygonal blocks sub-divided into numerous triangular blocks), and the support analysis was repeated with this model. Contrary to expectation, it was found that when more damage was allowed to develop explicitly within the blocks, the influence of supports increased in terms of peak strength but reduced in terms of displacement suppression. This finding was explained by the inability of triangular blocks to dilate in a realistic manner as compared to polygonal block assemblies.

9.7 Acknowledgements

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CHAPTER 10

INTEGRATION OF A THREE-DIMENSIONAL CONTINUUM MODEL AND A TWO-DIMENSIONAL BONDED BLOCK MODEL (BBM) FOR STUDYING THE DAMAGE PROCESS IN A GRANITE PILLAR AT THE CREIGHTON MINE, SUDBURY, CANADA

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10.1 Abstract

Bonded Block Models (BBMs) have shown potential in replicating rockmass behavior as well as the rock-support interaction mechanism, but their practical application is limited to two dimensions due to the high associated computational demand. To allow for the use of BBMs in simulating three-dimensional problems, this study proposes an integrated 3D continuum – 2D discontinuum approach, in the context of rock pillars. A cross-section of a granite pillar was simulated using a BBM with a load path from a calibrated mine-scale FLAC\(^3\)D model. Pillar support as employed in the mine was also incorporated at distinct stages during the simulation. The model was calibrated by varying the input parameters until the displacements at six locations within the pillar matched those measured by a multi-point borehole extensometer (MPBX) in the field.

The calibrated model was subsequently used to understand how the support and load path influenced the damage evolution in the pillar. The shear component of the load path was found to have a major effect on the severity and extent of the damaged regions. When the support density was increased in the model, the lateral displacements along the pillar walls were significantly
suppressed, but in a somewhat unpredictable manner. This was explained by the interaction between the supports and the damaged regions at the corners, which ultimately modified the stresses along the pillar periphery. The amount of displacement reduction obtained by increasing the support density illustrates the potential of BBMs to be used as a support design tool.

10.2 Introduction

Pillars are the most important load bearing member in underground mines and play a crucial role in sustaining the functional integrity of openings. With mining progressing to deeper depths, it becomes even more important to evaluate the integrity of such pillars, both from a localized skin-damage as well as a global failure standpoint. Although the complete collapse of a pillar is catastrophic and often violent in nature, its occurrence is rare; extensile fracturing and geometric bulking of the rockmass along the pillar periphery, on the other hand, is more common, and support should be designed accordingly to manage instabilities and potential fall of grounds resulting from large displacements. The extent and magnitude of stress-induced damage is controlled by a multitude of interacting factors, including but not limited to rock strength, stress path, support timing and density, etc. (Kaiser et al., 1996; Diederichs, 2003, 2007; Diederichs et al., 2004; Cai and Kaiser, 2014).

Advanced numerical modeling techniques now allow us to investigate complex rock mechanics problems, which are otherwise difficult to study using low (spatial) resolution in-situ data (e.g., from anchored extensometers, borehole pressure cells). Broadly speaking, numerical modeling techniques can be sub-divided into two categories: continuum and discontinuum (Jing and Hudson, 2002). The key difference between the two techniques is that discontinuum models allow for the formation and opening of explicit fractures while continuum models approximate...
this damage process using inelastic constitutive relationships. While continuum models may be more appropriate for larger scale (e.g. mine scale) applications based on their relative simplicity and faster run-times, if small-scale fracturing and damage phenomena are of interest (e.g. spalling), the use of a discontinuum approach may be more appropriate (Jing, 2003). This is discussed in more detail below.

Excavations in massive to sparsely fractured rockmasses at depth experience spalling, and such a behavior can be effectively simulated in continuum models using the Cohesion-Weakening-Frictional-Strengthening (CWFS) strength model (Hajiabdolmajid et al., 2002). In essence, this strength model respects the non-simultaneous mobilization of cohesive and frictional strength in rocks undergoing brittle fracturing (Martin and Chandler, 1994). While continuum models are undoubtedly capable of replicating the brittle rock damage process and the behavior of reinforced ground (Edelbro, 2009; Walton et al., 2014; Renani et al., 2016; Walton et al., 2016), a recent study by the authors (Sinha and Walton, 2019b) has shown that such models tend to significantly underestimate ground movements when supports are removed from the system. This is due to the inability of continuum models to accurately replicate large-scale deformation mechanisms, including separation and rotation of discrete blocks of rock. In the same study, Sinha and Walton (2019b) demonstrate how a polygonal Bonded Block Modeling (BBM) approach better captures the behavioral difference in the unsupported and supported ground conditions, and accordingly suggests the use of discontinuum models for design of pillar skin supports. With this in mind and given the computationally demanding nature of 3D discontinuum models, it is useful to establish an approach where a portion of a 3D mine-scale problem (e.g., asymmetrical damage in a pillar due to sequential stope extraction) can be analyzed using a two-dimensional discontinuum model setup for the purposes of support design.
Ultimately, it should be possible to optimize the support layout at a site by varying the support density until target (or safe) displacements are attained.

This study is an attempt to establish such an integrated approach; in particular, we chose to simulate a cross-section of a supported granite pillar in the Creighton Mine (Sudbury, Canada) using the polygonal 2D-BBM approach. Other discontinuum approaches and software packages are available and could be applied for the purpose described here (PFC$^{2D}$: Wanne et al., 2004; Cai et al., 2007; Kias and Ozbay, 2013; PFC$^{3D}$: Cundall et al., 2008; Mas Ivars et al., 2011; Zhang et al., 2015; Zhang and Zhou, 2016; Finite-Discrete Element Method or FDEM: Elmo and Stead, 2010; Li et al., 2019a; Vazaios et al., 2019), but the focus of this study is on the BBM approach. A 2D approximation is valid in this case since the pillar was long (length (L) x width (W) x height (H) is 28.125 m x 7.5 m x 5 m; L/W is therefore 3.75) and previous research has found the effect of increased length on pillar behavior to become negligible beyond L/W=3-4 (Sinha and Walton, 2019a).

The BBM was loaded in a realistic fashion by following a load path from a calibrated continuum FLAC$^{3D}$ model of the Creighton Mine. Appropriate input parameters for the BBM pillar model were ultimately constrained by matching the displacements at six locations within the pillar to those measured in the field by a multi-point extensometer. Post-calibration, hypothetical load paths were tested to better understand how the pillar damage process can be influenced by mining activities elsewhere in the mine. One alternate support pattern and an unsupported condition were also tested. Note that although some results of simulations focused on ground-support interaction are presented, the primary goal of this study is to establish the integrated continuum-discontinuum approach. In the BBM, no pre-existing fractures were considered, as no prominent fractures were observed in the field (Walton et al., 2016), but it is
possible to include Discrete Fracture Networks (DFN) in BBMs if necessary, as has been previously done by Preston et al. (2013), Farahmand et al. (2018), Vazaios et al. (2018) and Stavrou et al. (2019). In other words, this means that the BBM in this study corresponds to an un-jointed pillar where yield progressed via intact rock damage.

Polygonal BBM is one of the most common applications of discontinuum modeling and typically operates using Itasca’s Universal Distinct Element Code (UDEC). This approach represents a material domain as an aggregate of polygonal blocks (generated as a Voronoi Tessellation) that can detach once the contacts between the blocks are damaged. While the vast majority of the studies conducted using the polygonal BBM approach have focused on the laboratory-scale rock fracturing process (Lan et al., 2010; Kazerani and Zhou, 2010; Ghazvinian et al., 2014; Farahmand and Diederichs, 2015; Sinha and Walton, 2020a, Sinha et al., 2020a), there has been some success in simulating field-scale behaviors as well (Christianson et al., 2006; Preston et al., 2013; Bai et al., 2016; Farahmand et al., 2018; Sinha and Walton, 2018b).

Perhaps the most relevant study in this regard is that by Preston et al. (2013) who investigated the effect of pillar height on its ultimate strength. However, no support was considered and a constant velocity boundary was applied at the model edges to load the pillar. Bai et al. (2016) simulated a coal mine-entry housed in a water-rich environment using Voronoi BBM. Although Bai et al. (2016) considered support in the form of rockbolts, cables and steel strips, the model was restricted only to the development loading stage (no production related loading). Other studies like Kaiser and Cai (2013), Xue and Mishra (2015), Muaka et al. (2017) and Kaiser (2019) have developed excavation scale models, but these too were limited by simplistic loading conditions. In terms of reproducing displacements at multiple points as measured in-situ, the authors are unaware of any previous BBM-based study that has been
successful in this regard. This study is therefore the first such attempt to quantitatively reproduce field-measured displacements using BBM.

On the other hand, application of a load path from a 3D to 2D model (Shabanimashcool and Li, 2012) and coupling between continuum and discontinuum software (PFC$^{2D}$-FLAC$^{2D}$; Potyondy and Cundall, 2004; Cai et al., 2007; Saiang, 2010; Song and Hong, 2012; Zhang et al., 2017a; Jia et al., 2018; Zhang et al., 2019; PFC$^{3D}$-FLAC$^{3D}$; Khazaei et al., 2015; Zhao et al., 2018; Qu et al., 2019) are not novel concepts. In the case of PFC$^{2D}$-FLAC$^{2D}$ and PFC$^{3D}$-FLAC$^{3D}$ couplings, a region of the FLAC model is replaced by PFC elements, and there is transfer of unbalanced forces between the two regions at each solution step. An active coupling between UDEC-FLAC$^{3D}$ is not possible because of the difference in dimensionality. FLAC$^{3D}$ Version 6 now allows coupling with 3DEC, but the large computational demand of 3D-BBMs in 3DEC added to a mine-scale FLAC$^{3D}$ model continues to be prohibitive for practical application for most users. In this study, some simplifications were sought while integrating FLAC$^{3D}$ and UDEC to ensure that the model runtime was acceptable while replicating the important aspects of the load path.

10.3 Integrated continuum-discontinuum approach – Model setup

Creighton Mine in Sudbury, Canada is one of the 10 deepest mines in the world with active mining depths extending below 2.5 kms. The main orebody consists of nickel-copper mineralization along the contact between a felsic norite unit and the Huronian footwall rocks. Details on the geological environment and predominant structures at the Creighton Mine are provided in detail in the literature (Malek et al., 2008; Snelling et al., 2013). The particular location of interest for this study is the 7910 ft (2.4 km) level in the 461 orebody (Malek et al.,
2009), which has 10 access drifts extending from a footwall drift (Figure 10.1). The granite in this area is relatively massive with RQD near 100% and GSI approximately 75 to 85 (Walton, 2014).

![Diagram of extensometers and mining operations](image)

**Figure 10.1** Location of the extensometers with respect to the different drifts and stopes (after Walton, 2014). The drift and stope numbers do not correspond to the depth of mining. All drifts and stopes are at the 7910 ft (2.4 km) level. The different numbers in brown (1-6) indicate the different stages of mining – (1) Mine-by or crossing of the 6330 drift face past the extensometer locations, (2) 6287 Crown blast, (3) 6336 Raise blast, (4) 6336 Blast 1, (5) 6336 Blast 2, (6) 6336 Crown blast. Readers are referred to Walton et al. (2016) for more details on these excavation stages.

In November 2013, 6 multipoint extensometers (in 2 clusters of 3) were installed in a pillar in the 7910 ft (2.4 km) level. The purpose of these extensometers was to record the deformations associated with mining activities like drift development, extraction of stopes, etc. Unfortunately, 4 out of the 6 extensometers (2 in each cluster) were damaged during blasting operations, and deformation data could only be acquired from 2 extensometers. Although...
measurements from all 6 extensometers would have undoubtedly improved the data resolution, valuable information regarding the pillar behavior could still be obtained from the two functional extensometers. These two extensometers, termed as E1 and E2 (Figure 10.1), were 6.7 m and 6.9 m long and were installed from the left side of the pillar (6300 drift), when the 6330 drift face was 7.5 m from E2. The locations of the 6 anchors of E2 (data from this extensometer was used in this study) are presented in the next section, while those for E1 can be found in Walton et al. (2016). The data from E1 was not used in this study as it was closer to the stope (larger out-of-plane movements) and exhibited time-dependent behavior due to the increased presence of sulphide minerals at this location (Walton et al., 2016). Each of the 2 extensometers were operational for 258 days and recorded movements during 6300 and 6330 drift advance and extraction of the 6287, 6307 and 6336 stopes. For more details on the excavation sequence and mining method followed at the Creighton Mine, readers are referred to Walton et al. (2016).

The extensometer data was subsequently used by Walton et al. (2015b, 2016) to calibrate a mine-scale FLAC\textsuperscript{3D} model of the Creighton Mine. Figure 10.2 (top) shows the geometry and mesh size employed in this model (total model dimensions are 220 m x 160 m x 70 m). In the study by Walton et al. (2016), a combination of the CWFS strength model and a mobilized dilation angle model was employed for modeling a 2.5 m region in the vicinity of the pillar under consideration, while a CWFS strength model with zero dilation was used for a region 150 m x 100 m x 40 m centered on the pillar; regions further away from the area of interest were modeled as elastic. The orebody was modeled as perfectly plastic with input parameters from Ofoegbu and Curran (1992) and Vale (2013). Fixed displacement boundary conditions were imposed on all sides of the model, and pre-mining stresses of \(\sigma_1=96\) MPa (E-W), \(\sigma_2=72\) MPa (N-S) and \(\sigma_3=60\) MPa (vertical), corresponding to in-situ stress ratios of \(k_{EW}=1.6\) and \(k_{NS}=1.2\), were
applied. It is important to note that the pillars at the site were supported by 75 mm shotcrete, mesh and 3 rebar bolts (2 m long) along the pillar heights, but no such support was present in the FLAC\textsuperscript{3D} model. This means that the effect of support was implicitly accounted for by the derived rockmass parameters and the continuum nature of the model. The model was solved in 47 ‘stages’ to mimic the excavation sequence at the Creighton Mine.

To constrain the model input parameters, they were varied until the displacements in the model matched those measured by the two extensometers. Although data from only two extensometers was used for this purpose, the ability of the model to match displacements at 12 different locations (anchors) over an extended period of time, combined with the fact that the input parameters were constrained using actual laboratory tests, provides a high degree of confidence in the model results.

Given the asymmetric manner in which the stopes were positioned with respect to the instrumented pillar in Figure 10.1, one can expect the loads to vary across the pillar width and to not be purely vertical in nature. Additionally, there will be some strain along the long axis of the pillar, but the magnitude can be expected to be relatively small away from the stope. As this problem is three-dimensional in nature, to be able to model the pillar in 2D, some simplifications are necessary. The complete modeling approach, including the methodology for transitioning from the FLAC\textsuperscript{3D} model to the 2D BBM, is as follows:

(1) It was decided to model a cross-section of the pillar (i.e., AA’ in Figure 10.2 top) that passes through the extensometer further away from the stope (E2). Accordingly, the magnitude of differential movements along the pillar length was less than those at the other extensometer location, as indicated by the calibrated FLAC\textsuperscript{3D} model results.
Figure 10.2 Schematic of the integrated continuum-discontinuum approach that shows the minescale FLAC\textsuperscript{3D} model (after Walton, 2014) and the corresponding BBM pillar model with dimensions.
(2) Only the pillar is simulated in UDEC, as the focus of this study is on the local discontinuum behavior of the pillar; the complex interaction between the host rock and the pillar was considered to be accounted for by the FLAC$^{3D}$ model and the corresponding boundary conditions applied to the UDEC model. The corresponding BBM pillar is shown at the bottom in Figure 10.2.

(3) In order to apply the same load path to the BBM as experienced by the FLAC$^{3D}$ pillar section AA’, a strain-controlled approach was adopted. In particular, the displacements (horizontal and vertical) along the top and bottom of the pillar were recorded for each of the 47 stages and were applied to the BBM via a velocity boundary condition. Note that it is the difference between the displacements at two subsequent stages that was applied to the BBM as a velocity condition, and not the raw displacement magnitudes at each stage.

(4) To develop the BBM, the built-in Voronoi generator in UDEC was employed with an edge length of 0.1 m. This block size is considered to be small enough so as to not impose significant kinematic constraints on where fractures can develop within the model and also resulted in a manageable runtime (~3 days for all 47 stages). Each block was also discretized using multiple constant-strain triangular zones.

(5) The zones in the BBM were much smaller than the zones in the FLAC$^{3D}$ model (for comparison, FLAC$^{3D}$ zones were 0.3125 m, which is more than three times the edge length of the BBM blocks). Therefore, the displacements from the FLAC$^{3D}$ model could not be directly applied to the gridpoints (gridpoints are vertices of the zones) in the BBM pillar model. To resolve this issue, the addresses of all gridpoints along the top and bottom of the BBM pillar were extracted and classified into 25 groups, such that each
group corresponds to half the cartesian space on either side of a gridpoint in the FLAC\textsuperscript{3D} model. This classification permitted the displacements from the 25 gridpoints in the FLAC\textsuperscript{3D} pillar slice to be correctly applied to the 25 groups of gridpoints in the BBM pillar model.

(6) Instead of directly applying the displacement differences (extracted from the FLAC\textsuperscript{3D} model) as a velocity in the BBM, it was scaled-up 10 times so that the model would need to be stepped only 1/10\textsuperscript{th} second instead of 1 second to apply the appropriate displacements. It is important to recognize here that the ‘time’ in UDEC simulations is different from real time. For the current models, 0.1 seconds corresponded to approximately 111,170 timesteps. Once the model was stepped by 0.1 seconds, the boundaries were fixed and then solved until mechanical equilibrium (a solution ratio of $10^{-5}$) was attained. This loading mechanism was repeated for all 47 stages.

(7) Prior to excavation of the drifts at the modeled section, a stress boundary condition was applied to the lateral edges of the BBM pillar. These stresses were also extracted from the FLAC\textsuperscript{3D} model and were applied through a layer of elastic continuum zones. The contacts between the elastic zones and the lateral edges of the BBM were made indestructible to facilitate a smooth transfer of stress. While the direct application of stresses to the BBM pillar boundary might seem a simpler alternative, it resulted in extensive cracking of the block contacts right from the start of the simulation (even when the drifts were not excavated past the AA’ location in the FLAC\textsuperscript{3D} model). The issue was related to the slight differential nature of the boundary stresses being applied to the blocks along the pillar wall.
As soon as the drifts were excavated, the stress boundaries were removed and replaced by 3, 2 m long, 19 mm diameter rockbolts. The rockbolts were modeled using cable structural elements (Gao et al., 2015; Bai et al., 2016; Shreedharan and Kulatilake, 2016). A 75 mm thick shotcrete layer, modeled using beam structural elements, was also applied along the lateral boundaries of the model at pre-defined stages (see Figure 10.2 for the support installation timeline).

The ‘spraying’ option in UDEC was employed for installing the shotcrete. For ‘spraying’, a central point from which the simulated spraying occurs and the angular extent of the region to be lined are specified, and UDEC installs the beam elements along the excavation periphery according to these parameters. To ensure that the shotcrete was uniformly applied along the entire pillar edge, all minor/partial blocks that got dislodged during the initial loading process were manually deleted at Stage 11 (left side) and Stage 26 (right side) prior to ‘spraying’.

A constant out-of-plane (N-S) stress of 72 MPa ($\sigma_2$; Walton et al., 2016) was applied to the BBM. Variations in out-of-plane stresses might have some effect on the pillar behavior in reality, especially in the later stages, but any such effects are at least partially accounted for in the material parameters derived from model calibration. To verify this assertion, a continuum model was run in UDEC (same setup as the BBM but only 1 block), and similar displacement profiles to those from the fully 3D model at AA’ location could be obtained following slight parameter modifications. In particular, the peak cohesion and mobilized friction angle, as reported in Walton et al. (2016), had to be reduced by ~10%.
With all this in mind, the authors would like to highlight the difference in the loading mechanism in the FLAC$^{3D}$ model and in the pillar BBM. The FLAC$^{3D}$ model was actively perturbed by drift advancement and excavation of stopes which created unbalanced forces in the model. The BBM pillar, on the other hand, was perturbed by the equilibrium displacements of the FLAC$^{3D}$ model and in a sense, lags behind the FLAC$^{3D}$ model at each stage. This is unavoidable as an active coupling between a 2D and 3D model is not possible.

Figure 10.3 illustrates the complex nature of the load path applied to the BBM. The solid lines are the average displacements at each stage, while the error bars indicate the upper and lower bound magnitudes. A wider range on the error bars signifies greater variability in the displacements applied to the pillar top or bottom. Interestingly, the variability in the displacements, especially the vertical ones, was found to increase in discrete increments following production blasts (refer to the excavation stages marked by black dotted lines in Figure 10.3a). Even more interesting is the fact that both the top and bottom edge of the pillar were subjected to significant horizontal movements which also happened to be in the same direction (Figure 10.3b). Both horizontal displacements being negative implies that the pillar is moving towards the left (West); this is intuitive, as the stopes are located to the left (West) of the instrumented pillar (Figure 10.1). Additionally, the difference in the magnitude of horizontal displacements along the bottom and top edge indicates that there is some shear loading on the pillar. This shear component increases rapidly following the 6287 Crown blast. Studies by Jessu et al. (2018) and Garza-Cruz et al. (2018) have previously shown how shear loading can destabilize rock pillars; it is, therefore, important to properly understand the loading mechanism at a site and assign realistic boundary conditions when attempting to model rock pillars.
10.4 Model calibration

With the integration of FLAC$^3$D and UDEC established, the next task was to identify a set of BBM input parameters that could reproduce the displacement profiles measured in the field (i.e. the data from the extensometer located at AA’ or E2). Before describing the calibration methodology, it is useful to review the different components of a BBM and the choices available in UDEC to represent these components.

As previously discussed, BBMs are composed of detachable blocks, and each block is discretized using finite-difference zones (Itasca, 2014a). The interfaces between the blocks are called ‘contacts’ and they operate through the Coulomb slip model. When the local tensile stress or shear stress on a contact exceeds its tensile or shear strength (defined by cohesion and friction angle), the contact is considered to be damaged. Zones can either be elastic or inelastic, depending on the constitutive model assigned to them. Elastic zones only require Young’s modulus (E) and Poisson’s ratio (ν) as input parameters, while appropriate strength and post-
peak properties have to be defined for inelastic zones (e.g. cohesion, friction angle, tensile strength, dilation angle, etc.).

An elastic BBM is the simplest model representation, has the least number of input parameters and is relatively straightforward to adjust during calibration in terms of the relationships between input parameters and model outputs. All large-scale studies to date have employed elastic zones in their simulations. Given its wide acceptance, we started by using the elastic zone representation in our BBM pillar. Initial calibration efforts indicated that it might not be possible to match the anchor displacements, especially the anchors located deeper into the pillar, using this representation. These models had pronounced damage along the pillar edges, but the contacts located deeper in the pillar remained almost unaffected. An example is shown in Figure 10.4; this model corresponds to a contact cohesion of 39 MPa, contact tensile strength of 10 MPa and contact friction angle of 27.5° which degraded to 21.3° upon failure. The different colors in Figure 10.4a correspond to the 6 anchors of the multi-point borehole extensometer (MPBX) and the positions of the anchors with respect to the pillar center are shown in Figure 10.4c. As can be seen, the model was able to replicate the displacement at the 3.15 m anchor location but there was no/minimal movement at the other anchor locations. The reason is the lack of fracturing deeper inside the pillar as illustrated by the plot of failed contacts (Figure 10.4b).

It is known that microscopic heterogeneity or defects play an important role in the brittle rock damage process. Garza-Cruz et al. (2014) showed that when a tensile strength heterogeneity was introduced in the contact properties, a high macroscopic uniaxial compressive strength to tensile strength ratio (typical of brittle rocks) could be attained in elastic BBMs. Following the same methodology, we randomly assigned the contacts tensile strength values from a Weibull distribution; the Weibull distribution parameters were based on a statistical fit to the laboratory
tensile strength values of Creighton Granite (see Figure 10.5a). With the tensile strengths assigned stochastically, the cohesion was calculated following a constant ratio (cohesion/tensile strength), meaning that the cohesive strength also corresponded to a Weibull distribution. The contact friction angle was, however, homogeneous across the entire pillar, as was implemented by Garza-Cruz et al. (2018). When selecting parameter combinations for model calibration, care was taken to ensure that the tensile strengths were always smaller than the Mohr-Coulomb theoretical upper limit of cohesion / tan (friction angle).

![Image](image-url)

Figure 10.4 (a) Displacement profiles at the different MPBX anchor locations compared to the field measurements, (b) Location of failed contacts at the 47th stage, (c) location of the 6 anchors of the extensometer with respect to the center of the pillar. All displacements are reported with respect to the center of the pillar (i.e. the center of the pillar has zero displacement in this representation).

As with the homogeneous, elastic representation, it was possible to identify a set of input parameters that could match the displacements at 3.15 m anchor location (Figure 10.5b). In particular, this model (termed as ‘Model 1’) had a c/T ratio of 3.25 and a friction angle of 26°.
which degraded to 25° on failure. A significant mismatch in displacements was observed at all other anchor locations and this can again be attributed to the lack of contact damage deeper inside the pillar. If the contact strengths were lowered to induce damage near the pillar center, then the peripheral displacements were extremely large. To demonstrate this, the results for another model (termed as ‘Model 2’) calibrated to the 2.45 m anchor is shown in Figure 10.5b. Clearly, it is not possible to reproduce the displacement profiles at all anchor locations using an elastic block representation.

Figure 10.5 (a) Weibull distribution fitted to laboratory tensile strengths of Creighton Granite, and, (b) Displacement profiles at the different anchor locations in the Model 1 and 2 compared to the field measurements.

A convenient alternative is to use an inelastic constitutive model in the zones. There are two associated benefits to this: (1) the model run time does not increase drastically, and, (2) zone yield can approximate the finer-scale cracking deeper within the pillar where contact damage is suppressed by large confining stresses. The damage along the pillar periphery, however, can still be explicitly simulated by contact failures and block separations. A continuum approximation of the damage development within the pillar is acceptable as long as the deformations are not
significant (i.e. the process is not macroscopically discontinuous). The key to the successful implementation of this approach is the selection of zone and contact input parameters such that irreversible damage occurs primarily via contact failure along the periphery and primarily via zone yield within the pillar.

In this study, we assigned the CWFS model and Walton-Diederichs (WD; Walton and Diederichs, 2015a) mobilized dilation model to the zones, similar to the approach used for the entire pillar by Walton et al. (2015b, 2016). The strength and dilation model parameters were initially chosen from Walton et al. (2016) but were varied to match the displacement profiles. Responses of inelastic models are mesh-dependent and it is sometimes necessary to modify the parameters when the mesh size is changed.

The contacts were defined using peak and residual cohesion ($c_{\text{peak}}$ and $c_{\text{res}}$), peak and residual friction angle ($\phi_{\text{peak}}$ and $\phi_{\text{res}}$), and tensile strength ($\sigma_t$). The physical meaning of the individual contact parameters is not as straightforward as might be assumed. As for example, one might expect the $\phi_{\text{res}}$ to be related to the basic friction angle (Alejano et al., 2012) in some fashion, but BBM behaviors are heavily influenced by the geometric interlocking of the blocks, and consequently much smaller values of $\phi_{\text{res}}$ are typically required to obtain reasonable results (Christianson et al., 2006; Farahmand et al., 2018; Stavrou and Murphy, 2018; Dadashzadeh, 2020). As a result, the contact properties in this study were calibrated based purely on the capability of the model to match the field displacements. For support elements, the shotcrete properties of Chryssanthakis et al. (1997) and Malgrem and Norlund (2008) were used in the initial model, while those in Bahrani and Hadjigeorgiou (2017) and Zipf (2006) were referred to for determination of cable properties; ultimately, these were also varied as a part of the model calibration process.
As the contact properties had the greatest degree of uncertainty, these were modified over a much wider range than the others. For this type of BBM application, there is no well-established calibration methodology. A trial and error approach was used, and the greatest confidence was placed in the support and zone input parameters (meaning these were modified least from their initial values). The ranges over which the different parameters were modified in this study are as follows: (a) Contact - peak cohesion: 30-70 MPa, peak friction angle: 0°-50°, residual friction angle: 0°-30°, tensile strength: 8-12 MPa; (b) Zones – peak cohesion: 40-55 MPa, mobilized friction angle: 40°-50°, tensile strength: 8-14 MPa, plastic shear strain from peak to residual (or initial to mobilized): 0.005-0.025; (c) Shotcrete – modulus: 15-25 GPa, compressive strength: 25-40 MPa; (d) Cable – stiffness of grout: 400-2000 MN/m/m. In BBMs, due to block wedging, contacts are somewhat more prone to failure than the zones within the blocks; accordingly, contact cohesion values greater than the zone cohesion values were tested. Modifications to zone dilation parameters were also tested as a part of the calibration process, but ultimately those from Walton et al. (2016) were used.

During the back-analysis process, some unusual trends were noted. If the zones were made too strong, it increased the model displacements drastically at the outer anchor locations. This likely occurred due to block movements contributing more towards the pillar displacements. Secondly, it was found difficult to control the displacement profiles by small systematic changes to the input parameters. For example, when the contact cohesion was increased from 45.2 MPa to 45.3 MPa, the deformation at the outer anchors did not decrease. The behaviors were, however, consistent with expectation when large changes were introduced (for e.g., 45 MPa to 47 MPa). A closer look at the model results revealed interaction between the fractured contacts and the
yielded zones, which modified the locations of the damaged regions, thus impacting the model results in sometimes unpredictable ways.

During the model calibration phase, the authors identified a number of input parameter sets that produced similar data-model fits. Four such results are shown in Figure 10.6, termed as ‘Calibrated’, ‘Alternative 1’, ‘Alternative 2’ and ‘Alternative 3’. The only differences in the parameters for the three alternate models with respect to the ‘Calibrated’ model (Table 10.1) are presented in Table 10.2. Some of the differences in these model parameters are very small (0.05 MPa and 0.2°) and they affected different aspects of the data-model fit, further exemplifying the somewhat unpredictable influences of small parameter changes in these BBMs. Note that although the data from the MBPX is one-dimensional in nature, displacement measurements corresponding to 6 different locations over 47 different stages were considered in this study. Agreement with such a large amount of data helps to establish the reliability of these models.

The four parameter sets have different strengths and drawbacks with respect to their representation of specific pillar displacement behaviors. Alternative 1 best replicates the displacements of the outermost anchors, but the mismatch for the 2.45 m anchor is large. Alternatives 2 and 3 replicate the displacements at all anchors except the 3.15 m one. The ‘Calibrated’ model represents a balance between the two extremes - it captures the displacements at all anchors fairly well but slightly overestimates the 2.45 m anchor displacement (errors are on the order of ~1 mm) in the early phases of loading. Despite numerous trials, the 2.45 m anchor displacements could not be reduced any further without affecting the data-model fit for the other anchor locations. Ultimately, the authors decided to use the ‘Calibrated’ model to conduct the rest of the study.
Table 10.1 Calibrated input properties of zones, contacts, shotcrete and bolts.

<table>
<thead>
<tr>
<th>Zones - CWFS</th>
<th>Contacts</th>
<th>Beam (shotcrete)</th>
<th>Cables (bolts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak cohesion (MPa)</td>
<td>46.5</td>
<td>46.5</td>
<td>19</td>
</tr>
<tr>
<td>Residual cohesion (MPa)</td>
<td>0.5</td>
<td>0</td>
<td>35</td>
</tr>
<tr>
<td>Initial friction angle (°)</td>
<td>0.0</td>
<td>33</td>
<td>3</td>
</tr>
<tr>
<td>Mobilized friction angle (°)</td>
<td>42.0</td>
<td>20</td>
<td>0.5</td>
</tr>
<tr>
<td>Tensile strength (MPa)</td>
<td>12</td>
<td>10</td>
<td>55</td>
</tr>
<tr>
<td>Plastic shear strain from peak to residual (or initial to mobilized)</td>
<td>0.01</td>
<td>120000</td>
<td>0.86</td>
</tr>
<tr>
<td>Shear stiffness (GPa/m/m)</td>
<td>60000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

10.5 Calibrated model results

Figure 10.7a shows the vertical stress contour and damaged contacts for the ‘Calibrated’ model after the 47th stage. The presence of elongated, axial cracks along the periphery is readily evident. As one moves towards the center, the intensity of fractures reduces and is almost negligible 2 m into the pillar. Due to extensive cracking, there is also a loss of load carrying ability of the pillar periphery, which pushes the stresses deeper into the pillar (often called the
This boundary-relaxation-core-formation process in pillars was observed by Wagner (1974) in the field. It is interesting to note the asymmetric shape of the de-confined region on either side of the pillar (Figure 10.7b). The greater extent of damage on the left side is likely due to the stope being located to the left of the pillar and the left drift (i.e. Drift 6300) being developed earlier than the right one (i.e. Drift 6330).

Figure 10.6 (a-d) Displacement profiles for four input parameter sets.

Within the stressed ‘core’, all contacts are subjected to large compressive normal forces (as high as 6 MN for the 47th stage), and this reduces the likelihood of any tensile or shear
failures. The damage process at this point must be controlled mainly by the yield of zones, which in this case represents finer-scale microcracking (or low dilatant shearing). Such a change in failure characteristics as a function of position within the pillar is well documented in the literature (Chen, 1993; Esterhuizen and Ellenberger, 2007; Kang et al., 2015; Bai et al., 2019; Li et al., 2019a). For the ‘Calibrated’ model, the extent of plastic yield at the 47th stage is shown in Figure 10.7c. Similar plastic yield plots for other mining stages as well as for Alternative Models 1-3 can be found in the Supplementary Materials. Firstly, it can be observed that there is minimal zone yield in the periphery of the pillar, where extensive contact damage exists (see Figure 10.7a). Secondly, zone yield occurs across the entire diagonal of the model and includes regions well within the confined core. A cross-shear shape of the yielded zone in slender pillars can also be found in the continuum simulations of Sinha and Walton (2018a). The shape is not perfectly symmetrical in this case due to the complex load path applied to the pillar.

Table 10.2 Input parameters that are dissimilar in the ‘Calibrated’ model and the alternative models.

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alternative 1</td>
<td>Stiffness of grout: 1500 MN/m/m → 1700 MN/m/m</td>
</tr>
<tr>
<td>Alternative 2</td>
<td>$c_{\text{peak}}$: 46.5 MPa → 46.55 MPa</td>
</tr>
<tr>
<td></td>
<td>$\phi_{\text{res}}$: 20º → 19.8º</td>
</tr>
<tr>
<td></td>
<td>Stiffness of grout: 1500 MN/m/m → 1700 MN/m/m</td>
</tr>
<tr>
<td>Alternative 3</td>
<td>$c_{\text{peak}}$: 46.5 MPa → 46.75 MPa</td>
</tr>
</tbody>
</table>

A closer look at the model results revealed an interesting damage mechanism - it is not the contacts along the periphery that is damaged first but the zones at the pillar corners (see the highly damaged green regions in Figure 10.7c). Such an observation was also made by Chen (1993) for brittle potash pillars. The zone yield perturbs the stress field along the pillar periphery in a complex manner that allows contacts deeper within the pillar to undergo damage. This
mechanism was verified by analyzing the plastic shear strain contours and fractured contacts at each stage. Note that although the zone yield modified the stress field to some extent, the damage was mainly via contact failure at the outermost anchor locations (see Figure 10.7a and c).

Figure 10.7 (a) Vertical stress contours, (b) Horizontal stress contours, (c) Plastic yield contours in the ‘Calibrated’ model at 47th stage. $\varepsilon_{ps}$ is plastic shear strain, which is a proxy for damage.

It is important to track the evolution of pillar stresses to identify its load state on a complete stress-strain curve. To that end, the vertical stress was averaged over all zones in the model for each of the 47 stages and is plotted in Figure 10.8. The evolution of fractured contacts in the BBM is also shown for three particular stages. At stage 8, distributed fractures can be seen to develop along the left (West) pillar edge (prior to ‘Mine-by’); at this stage, the stress boundary was still present on the right margin. Subsequently following the ‘Mine-by’, fracturing was
induced near the right (East) pillar boundary, and some of the previously disconnected fractures coalesced along the left (West) pillar boundary. This suggests that the strength of the rockmass was likely locally reduced. With further loading due to mining activities, the intensity of failed contacts increased along both edges of the pillar. Similar progressive brittle damage or slabbing with formation of axial cracks along the pillar edge after ‘Mine-by’ has also been reported in other mining case studies (Gendzwill and Stead, 1992; Chen, 1993; Lajtai et al., 1994; Maybee, 2000; Styles et al., 2010).

From Figure 10.3a, it is easy to perceive the monotonic nature of the vertical strain path applied to the pillar. This in turn implies that the drop in the average vertical stress in Figure 10.8 is a result of mining-induced damage and not a reduction in strain. The peak vertical stress attained during the loading process, otherwise known as pillar strength, is 90.4 MPa for the Creighton BBM. Interestingly enough, the strength predicted by the Hedley-Grant equation (Hedley and Grant, 1972) for a 7.5 m wide, 5 m high quartzite pillar (the equation was based on quartzite pillar case studies) is 108.9 MPa. If this strength is normalized to the intact uniaxial compressive strength of the host rock, then one obtains 108.9/230 = 0.47. For the Creighton pillar BBM, the ratio of the peak strength to the intact uniaxial compressive strength is 90.4/200 = 0.45, which is within 5% of the value predicted by the Hedley and Grant equation. Note that this agreement in strength values was not a goal of the calibration process, but rather arose as emergent behavior from the model after calibration to the displacement results. The peak strength is a reflection of load capacity in the vertical direction (perpendicular to the MPBX orientation), and the agreement in strength therefore increases confidence in the model results relative to the displacement-based calibration which relied on purely one-dimensional data.
The original database of Hedley and Grant (1972) includes only long pillars, and it is known that longer pillars are generally stronger (Dolinar and Esterhuizen, 2007; Sinha and Walton, 2019a). Implicit in the Hedley and Grant equation, therefore, is the additional strengthening effect of length. The particular Creightion pillar being modeled in this case is also long, with a L/W of ~3.75. As discussed in Section 1, the pillar strengthening effect due to length flatten out beyond L/W=3-4 (Dolinar and Esterhuizen, 2007; Sinha and Walton, 2019a) and this permits the 2D BBM, which assumes a plane-strain condition and effectively simulates an infinitely long pillar, to attain a pillar strength consistent with the Hedley and Grant equation.

Figure 10.8 Average stress in the pillar models as a function of stage number.

Post-peak, the pillar starts to lose its load carrying capacity in a gradual fashion, and by the time the 47th stage is reached, the stresses have reduced by 20%. In contrast to the BBM, the FLAC3D model exhibited a slightly different post-peak stress trend, and a comparison of the stresses predicted by these two models is presented in Appendix L. It is well established that the
response of a pillar transitions from brittle to ductile as the W/H is increased (Mortazavi et al., 2009; Esterhuizen et al., 2010b; Sinha and Walton, 2018a; Li et al., 2019a). For hard-rock pillars, this transition point has been found to be around W/H of 2 (Mortazavi et al., 2009; Sinha and Walton, 2018a). As the simulated pillar is below this transition point, a softening response is naturally expected. Of particular importance in this regard is the ability of the pillar to carry large load levels beyond its peak strength. Current pillar design techniques only incorporate the peak strength in the design process but neglect any load carrying capacity of pillars post-peak. If the residual load carrying capacity is included in the design process, then it may be possible to reduce/optimize pillar dimensions. It is of course necessary to improve our understanding of the post-peak behavior before it can be implemented in the design process.

10.6 Analyzing the effect of support

With the reliability of the ‘Calibrated’ BBM established, this model was then used to examine the effect of alternate support patterns on the model response. Although this research topic is interesting and can provide valuable insight into the rock-support interaction mechanism, it will only be dealt with briefly in this study. Our intent is not to optimize support design for the Creighton Mine, but rather to demonstrate a procedure such that future studies could develop site-specific BBMs and test multiple support combinations to optimize support (including consideration of cost) at the site.

We tested two alternate support conditions – Unsupported (no support in the model) and 5 bolts + shotcrete. Relative to the support pattern at the site (i.e. 3 bolts + shotcrete), these two support conditions should encompass a wide range of ground behavior. The location of the bolts in the 3 bolts + shotcrete layout and the 5 bolts + shotcrete layout is shown in Figure 10.9a. Of
particular relevance here is to revisit the FLAC$^{3D}$ supported and unsupported model results of Walton et al. (2016). In that study, the effect of rockbolts and shotcrete was simulated using equivalent support pressures of 250 kPa and 1.5 MPa respectively, following the guidelines of Hoek (1999). It can be seen in Walton et al. (2016) that the model displacements with and without supports did not differ by more than 1 mm (at all anchor locations). Sinha and Walton (2019b) also highlighted this drawback (underestimation of support effect) of continuum models using hypothetical granite pillar models and a coal pillar model. The practical implication of this finding is that continuum models are not well suited for designing rock reinforcement in terms of its influence on ground behavior.

While there are no field data available to establish the realism of the deformations in the unsupported and 5 bolts + shotcrete BBM, it can at least be expected that the deformations will increase and decrease non-negligibly relative to the 3 bolts + shotcrete condition (unlike in the FLAC$^{3D}$ model). If such results are indeed obtained, it will qualitatively demonstrate the potential of BBM to be used as a support design tool. Nevertheless, further studies are required to assess if BBMs can also capture the finer-scale interactions between support and a deforming rockmass to increase our confidence in this modeling approach.

To identify the effect of support in the current BBM, it was decided to extract the horizontal displacement of gridpoints spaced at 0.1 m along the vertical edges of the pillar. This is a direct approach to evaluate the support effects since the displacements at pillar walls are aggregations of the movements occurring across the entire pillar. In mining environments as well, displacements on the pillar walls are used to identify impending instabilities or the need for additional support. For that purpose, the addresses of the gridpoints were extracted in the pre-
cycling condition (i.e. before running the model), and the corresponding horizontal displacements for the 47th stage are shown in Figure 10.9b.

Figure 10.9 (a) Location of the bolts in the 3 bolts/Calibrated layout and the 5 Bolts + shotcrete layout, (b) Horizontal displacements along the edges of the pillar for three support cases.

A reduction in deformation with increasing support density is apparent in Figure 10.9b. Note the asymmetric nature of displacements along the pillar height caused by the differential loading along the top and bottom edge of the pillar. While one would expect the deformation profiles with increasing support density to be similar in shape but translated towards lesser magnitudes, this is certainly not the case in Figure 10.9b. The greatest degree of displacement reduction was observed along the bottom left corner of the pillar, where the displacements dropped from 27 mm under unsupported conditions to 17 mm in the ‘Calibrated’ model. This change (37% drop) is much more than that simulated using a continuum model.

Some other trends in Figure 10.9b can be observed:

1. At the top left corner, the displacements in the unsupported and ‘Calibrated’ model are similar. The 5 bolts + shotcrete model, however, predicts more than 5 mm less
displacements at this location. The lack of difference in displacements between the 'Calibrated' and unsupported model is related to the greater depth of fracturing in the former model and the absence of support in the latter model. This is discussed further below in the context of the displacement results shown in Figure 10.10.

(2) The addition of 2 extra bolts did not have a significant effect on the displacements at the bottom left region of the pillar. It appears that the bottom rockbolt in the 'Calibrated' model was able to efficiently suppress the deformations around this region without the addition of further support.

(3) The simulated displacements reduce systematically from unsupported to 3 bolts + shotcrete to 5 bolts + shotcrete condition along the right pillar edge up to ~1.5 m from the pillar bottom.

(4) The change in displacements along the lower 1.5 m of the pillar right edge is minimal, as fracturing is insignificant at this location.

From these results, it is clear that the 5 bolts + shotcrete system performs in a superior fashion in suppressing surficial movements.

Figure 10.10 shows the displacement contours for the unsupported, 3 bolts + shotcrete and 5 bolts + shotcrete conditions at the 47\textsuperscript{th} stage. An interesting thing to note here is the difference in the damaged high displacement region along the left edge of the pillar in the 3 bolts + shotcrete case versus the unsupported and 5 bolts + shotcrete cases. For the 3 bolts + shotcrete case, the ‘< -10 mm’ and the ‘< -20 mm’ regions extend much deeper into the pillar than in other two models (more than 0.5 m for the ‘< -20 mm’ region). Such a deep damage region does not
exist in the 5 bolt + shotcrete case due to the presence of bolts closer to the top and bottom edges of the pillar. These bolts stiffen the rockmass at the corners and prevent the propagation of damage deeper into the pillar. In the 3 bolts + shotcrete case, the outer bolts are farther away from the top and bottom edge of the pillar; accordingly, the region around the center of the shallow fracture zone is stiffened, but this pushes the damaged zone deeper into the pillar, leading to greater pillar boundary displacements. The thinner high displacement region on the left side of the unsupported model can be explained by the lack of any support-induced stiffening in the system. The damage that initiated at the corners in the unsupported model had greater kinematic freedom to propagate along the pillar surface. To further verify that the variations in the damaged region are indeed related to the positioning of the outer bolts, another 3 bolts + shotcrete model was run, with the outer bolts installed at the same location as those in the 5 bolts + shotcrete case. From Figure 10.10, it can be seen that the depth of the damaged region reduced significantly and is similar to that in the 5 bolts + shotcrete case. These results indicate that the interaction between unsupported rock and support can be quite complex, and positioning of the supports can locally modify the depth of the damaged region.

When the displacements on the left edge of the pillar in the unsupported and 5 bolts + shotcrete cases are compared, one can discern the similarity in the shape of the high displacement regions (i.e. ‘>15 mm’ in the 5 bolts + shotcrete case and ‘>20 mm’ in the unsupported case). Only a reduction in the magnitude of displacement in this region signifies the clamping effect generated by the rockbolts (pinning the fractured slab to the pillar core). On the right edge, the high displacement regions are significantly suppressed in the 5 bolts + shotcrete case owing to its greater support density. There is only minor fracturing along the bottom right corner of the pillar (Figure 10.7a), and this is likely the reason why the displacements for all
three support cases are similar in this region in Figure 10.10. Lastly, it can be seen that the spatial extent of the ‘>20 mm’ region declines systematically with increase in support and this establishes the capability of BBM in reproducing the support effect.

Figure 10.10 Horizontal displacement contours for the unsupported, 3 bolts + shotcrete and 5 bolts + shotcrete, and, 3 bolts + shotcrete with the outer bolts further away from the center of the pillar cases at 47th stage.

The difference in deformation of reinforced ground versus unreinforced ground should become more apparent as the intensity of damage increases. In other words, as a pillar is continuously strained, new stress-fractures will be formed along the edge and the existing stress-fractures will continue to separate. If a bolt is structurally intact at this point, it would resist the deformations and suppress the volumetric bulking of the pillar. It therefore follows that the
lateral deformations along a supported and an unsupported pillar edge should diverge as the pillar is subjected to greater load levels.

This was tested in the current BBM by increasing the load along the model top and bottom, following two hypothetical schemes. In the first (H and V), the horizontal and vertical displacement increments for the 47th stage were applied to the model boundaries 40 times while bringing the model to static equilibrium following each increment. In the second one (only V), only the vertical boundary displacement increments were applied while fixing the horizontal boundary displacements. Both the unsupported and 3 bolt + shotcrete (Calibrated) cases were tested, and the resulting horizontal displacement contours at the end of the 87th stages (47 + 40 = 87) are shown in Figure 10.11.

Figure 10.11 Horizontal displacement contours for the unsupported and 3 bolts + shotcrete cases after 40 additional stages (following two different loading techniques; H=Horizontal and V=Vertical).
As expected, the discrepancy in deformation in the unsupported and 3 bolts + shotcrete model increased for both loading schemes. For the ‘H and V’ scheme, although the ‘-40 to -60 mm’ region continued to be larger in the 3 bolts + shotcrete case, the displacements were suppressed along the bottom left by over 50%. A growth in the extent of ‘>40 mm’ displacement region can also be observed along the right edge. For the ‘V’ loading scheme, a similar behavior was noted but with significant additional increase in the ‘>40 mm’ and ‘>60 mm’ regions along the right edge. Because the horizontal boundary displacement increments were not considered in the ‘V’ scheme, the contour patterns became somewhat symmetrical.

10.7 Analyzing the effect of hypothetical load paths

The response of numerical models is heavily dependent on the applied boundary conditions, and to be able to use such models to simulate the behavior of underground structures, it is important to assign a load path in accordance to how the structure is loaded in reality. It has been illustrated in Section 2 how the Creighton pillar slice was subjected to both non-uniform vertical displacements as well as horizontal shear along the top and bottom edges. To highlight the importance of assigning a realistic boundary condition while modeling rock pillars, we re-ran the ‘Calibrated’ model but with only the vertical displacement increments. The focus understandably is on the presence of horizontal shear movements along the edges of a pillar, something which has been neglected in many previous pillar-oriented studies. This scheme is similar to that employed in the last section (only V), with the key difference being that the vertical load path was applied right from the start of the simulation rather than after the 47th stage.
Figure 10.12a shows the horizontal displacement contour after the 47th stage. As can be seen, the damaged regions became much more symmetric and localized when the model was loaded via vertical displacement increments only. In particular, the depth of the ‘< -20 mm’ region was markedly reduced (~0.5 m), and the overall magnitudes are lower as well (see Figure 10.12a). The displacements along the right edge, however, increased; this can be identified by comparing the 20 mm displacement contour in Figure 10.10 and Figure 10.12a. It seems that the presence of horizontal displacements along pillar edges can disproportionately damage the two opposite walls of a pillar. For the current BBM, the displacement magnitudes as well as the extent of the damaged region shifted notably when the shear movements along the model top and bottom were omitted. One must therefore pay particular attention to boundary conditions when using numerical models to estimate support needs (i.e. volume of bulked material). A practical implication of these results is that support requirements on either side of a pillar might differ, depending on its location with respect to the stopes.

In addition to testing the effect of shear loading on a pillar, we also investigated how the shallow damage process might be influenced by a loading-unloading cycle. This investigation is of particular relevance to block cave mining, where a pillar (e.g. between the undercut and draw levels in block caving mines) might undergo cyclic loading during the caving operations (Garza-Cruz et al., 2014). Two different load paths were tested to accomplish this goal and are shown in Figure 10.12b. The horizontal component of the displacement increments was again neglected in the BBM to simplify the analysis.

Path 1 is an unloading-loading scheme, where the vertical strain is first lowered by 0.00035 at Stage 47, solved to equilibrium, then raised by 0.00035 and re-equilibrated. The idea behind testing an unloading-loading path was to understand if additional damage can occur even
when the pillar is not being monotonically loaded. Path 2 is a loading-unloading scheme, opposite to Path 1. To increase or decrease the applied vertical strain, the displacement increments at Stage 47 were multiplied by constant factors with the appropriate sign. This means that all the individual vertical displacement increments along the top and bottom edge were increased/decreased by the same proportion.

Figure 10.12 (a) Horizontal displacement contours in BBM with vertical displacement increment boundary at 47th stage, (b) Original and 2 alternate vertical strain paths (Path 1 and 2) tested on the BBM pillar, (c) Horizontal displacement contours in BBM after Path 1, and (d) Path 2.

The horizontal displacements corresponding to the final state of the simulations with loading Paths 1 and 2 are shown in Figure 10.12 (c, d). It can be observed that even when the pillar is not being exposed to a higher maximum strain, damage can still be induced by an unloading-loading cycle (Path 1). This is evident along both the left and right edge of the model.
(Figure 10.12c). When the pillar is over-streained (Path 2) instead, the displacements along both edges increase drastically (>30%; Figure 10.12d). Such a behavior is intuitive, as additional load would fail more contacts along the pillar boundaries, resulting in more bulking. A loading-unloading cycle during production activities (e.g. caving), therefore, might have the potential to severely damage a pillar and increase the strain on the pillar support.

When developing the cycling loading schemes in the BBM, some alternate load paths were also tested. Interesting results were obtained when the loading-unloading was conducted in a more gradual/stepwise manner at an earlier model stage (Stage 30). It was found that the damage and bulking was less significant in this model in comparison to a model subjected to a sudden loading-unloading path (i.e., Path 2). Such a difference was not noted at Stage 47, likely due to the rockmass being already substantially damaged at this point. This is interesting in the sense that it can be tied to real time excavation sequencing. If the production activities are designed such that the rockmass (relatively undamaged at this point) is allowed to equilibrate before the next blast, then the severity of rock damage could be minimized. The time for a rockmass to stabilize after load application will of course vary from one mining site to another.

The effect of loading-unloading on pillars was previously investigated by Garza Cruz et al. (2014) in 3DEC using elastic, tetrahedral blocks. In terms of the model findings, the key difference is that the bulking and the extent of the damaged zone was affected more during the unloading phase in Garza-Cruz et al. (2014), while in our study, the effect was found to be more pronounced in the loading (or re-loading) phase. Although the models in Garza Cruz et al. (2014) were unsupported, the potential for the presence of support causing this difference was ruled out by also running Paths 1 and 2 under unsupported conditions, which yielded similar results. The exact cause of the discrepancy is not well understood and could be attributed to a number of
factors, including the different loading-unloading mechanisms in Garza Cruz et al. (2014) and in this study, differences in the degree of rockmass damage prior to unloading and differences in block shapes and zone representations.

10.8 Conclusions

This study has presented an integrated 3D continuum – 2D discontinuum modeling approach to assist in the use of 2D Bonded Block Models (BBMs) for studying three-dimensional mining problems. In particular, the load path from a calibrated mine-scale FLAC$^{3D}$ model was applied to the BBM to simulate the damage process in a rock pillar (slice of pillar) in the Creighton Mine, Sudbury, Canada. Pillar support in the form of rockbolts and shotcrete was also considered. The discontinuum pillar model was ultimately calibrated by matching the displacements in the model to the MPBX extensometer measurements made at the site.

Post-calibration, two different support conditions were tested - unsupported and 5 bolts + shotcrete (the pillar at the site was supported by 3 bolts + shotcrete). A significant decrease in the lateral deformation along the edge of the pillar was noted as the support density was increased. Additionally, there was an interaction between the load path applied to the BBM and the supports that led to non-uniform deformation changes along the height of the pillar. It was concluded that the location of rockbolts can affect the extent of damage and magnitude of bulking along a pillar edge. The effect of shear movements along the pillar top and bottom boundary was subsequently assessed. It was found that shear loading can disproportionately damage the two vertical edges of a pillar. Lastly, to understand how caving-induced loading-unloading can affect pillar behavior, two different load paths were tested. Loading-unloading resulted in greater shallow damage than unloading-loading. Although the pillar was not over-
strained during the unloading-loading scheme, it still resulted in some additional cracking along
the pillar edge. In future, with advances in computational power, a similar integrated study could
be conducted with coupled FLAC$^{3D}$-PFC$^{3D}$ or FLAC$^{3D}$-3DEC model to better represent the
three-dimensional damage process in rock pillars.

10.9 Acknowledgements

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capacity.
CHAPTER 11

INVESTIGATION OF LONGWALL HEADGATE STRESS DISTRIBUTION WITH AN EMPHASIS ON PILLAR BEHAVIOR

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11.1 Abstract

Design of underground excavations requires a clear understanding of the stress transfer mechanisms that are associated with mineral extraction processes. In the longwall mining method, the redistribution of stresses is substantial given the large panel widths used, typically on the order of 300-400 m. While numerical models are a convenient tool for analyzing this global process, they are often limited by their inability to simulate the complex behavior of gob and coal. An alternative is to use field instrumentation to record actual stress patterns around mine openings. This study uses an extensive suite of borehole pressure cell (BPC) data from a US longwall mine to better characterize the global stress redistribution process during mining. A new methodology employing the Bieniawski’s (1984) strength gradient equation was used for converting the BPC measurements into rock pressure. It was observed that the Bieniawski’s (1984) gradient equation underestimates coal strength for points located deeper into the pillar. Comparisons with previous empirical and analytical equations are also presented for different stages of longwall loading. Very low front abutment load in comparison to the side abutment load was observed, along with a low abutment angle for this mine. This abutment angle value was verified from stress measurements made within the solid coal face ahead of the advancing
panel. Some other atypical trends were noted during data analyses, such as a monotonic drop in stress from the panel edge within the coal face; these trends were explained using an elastic FLAC$^3$D model. Finally, the load distribution in a pillar is presented in three-dimensions using interpolated bar graphs for different locations of the longwall face.

11.2 Introduction

With technological advances in the last few decades, longwall mining has grown to be the most productive coal mining technique in the United States. In contrast to the room and pillar mining method, longwall mining involves the complete extraction of the coal block, often termed as the ‘panel’, allowing the roof to collapse in the mined-out area (Peng and Chiang, 1984). Since these extracted areas are often hundreds of meters wide, substantial redistribution of stresses occurs within adjacent chain pillars with passage of the longwall face (Mark, 1987). Broadly speaking, the loads applied to longwall chain pillars can be divided into two parts (Mark, 1987): (a) Development load: This is the load associated with the development of the entries and can be estimated using the tributary area method; (b) Abutment load: This is the load that is associated with the extraction of the panel and is unique to the retreat mining nature of longwall. The abutment load can be further categorized into two load levels based on the location of the longwall face relative to the pillar in question. The abutment load when the face is at a position of 0 m relative to the pillar (i.e. face at the pillar) is called the ‘front abutment load’ and the abutment load long after the face has gone past the pillar is termed as ‘side abutment load’.

The load redistribution process in chain pillars during longwall advance has been a topic of research since the 1970s (King and Whittaker, 1971; Peng and Chiang, 1984; Wilson, 1972). Since then, numerous authors have tried to quantify mining-induced stress magnitudes using
field measurements (Schuerger, 1985; Allwes et al., 1985; Mark, 1987; Lu, 1985), analytical approaches (Carr and Wilson, 1982) and numerical models (Choi and McCain, 1980; Hsuing and Peng, 1985; Larson and Whyatt, 2013; Tulu et al. 2018). A proper understanding of the stress distribution mechanism at a global scale and its effect on the local behavior of chain pillars is critical in designing ground control systems for longwall gate entries. The large stresses that are typical in longwall mines can lead to pillar failure or violent rib bursts (Mark, 2016); an example of pillar rib degradation associated with headgate side abutment load is shown in Figure 11.1.

Analytical approaches for predicting stress levels in longwall chain pillars rely on assumptions about the internal damage mechanics of coal pillars. The most popular and well known method by Carr and Wilson (1982) uses a stress balance approach to distribute the abutment stress onto the chain pillar following an exponential decay function. The loss of load carrying capacity of the pillar rib and subsequent transfer of stress deeper into the pillar is controlled by the ‘Limit and Roadway Stability’ and the ‘Ultimate Limit’ concept introduced by Wilson (1972). However, as indicated by Mark (1987), the sequence of damage and load redistribution process assumed by Wilson (1972) is not that of a progressive failure, but a process where the initial yield zone does not expand until a certain magnitude of average stress builds up in the pillar core. These solutions also require input parameters that are hard to determine (for example, uniaxial compressive strength of failed coal and friction angle of coal at the field scale rather than the laboratory scale). More recently, Rezaei et al. (2015a, b) used energetic considerations (Salamon, 1984) to compute the stress concentration factors on longwall chain pillars. While the approach is novel, it considers the coal mass as an elastic isotropic material, meaning that any local stress redistribution due to damage is not accounted for within this analytical framework.
With the advent of advanced modeling techniques, numerous studies have been undertaken to understand the longwall gateroad loading process. Esterhuizen et al. (2010b) used a continuum-based model to simulate the abutment stresses in chain pillars for three different case studies. Yasitli and Unver (2005) attempted to model a longwall mine in Turkey that employed top-coal caving behind the face. Zhang et al. (2015) utilized FLAC$^3$D to study the stress changes in longwall pillars with various sizes. In all these studies, the authors unanimously highlight the difficulty of modeling gob - the gob undergoing continued consolidation with increase in load. This is further exacerbated by our limited understanding of coal mass behavior and how it is affected by the orientation and frequency of cleats. With all this in mind, it is clear that numerical modeling presents several challenges when it comes to studying the global stress transfer process in longwall mines.

A valuable alternative to the previously described methods is to use field data to empirically characterize the load redistribution process. The advantage of this method is its ability to account for the complex components of the system (and their interaction) without having to consider them explicitly. The first such systematic study was conducted by Mark (1987), who developed mathematical relationships for describing the abutment loads (front and side abutment load) and their distributions over the chain pillars; all mines considered in this study were located in the eastern US. Contrary to Carr and Wilson (1982), Mark (1987) found an inverse square relationship to best describe the decay of abutment stress in the pillars. Since then, numerous studies have employed borehole pressure cell (BPC) measurements (Mark and Iannacchione, 1992; Colwell et al., 1999; Larson and Whyatt, 2012; Hill et al., 2015) and tomography (Friedel et al., 1996; Luxbacher et al., 2008; Hosseini et al., 2013; Cai et al., 2014) to further our understanding of load transfer in longwall panels.
In this study, data from 44 BPCs are utilized for interpreting the load transfer mechanism in context of previously conducted analytical and empirical studies (Carr and Wilson, 1982; Wilson, 1983; Mark, 1987). The BPCs were installed by mine personnel and a contractor in two adjacent chain pillars in a Western US longwall panel. Stress measurements commenced 245 m in front of (outby) the face and continued until the face was 400 m ahead of (outby) the instrumented pillars. The spatial distribution of the BPCs within the chain pillars provided detailed information on the local pillar response as the global stress state in the ground was perturbed by the retreat of the longwall face. In order to ensure anonymity, the mine under consideration is referred to as ‘Mine A’.

![Figure 11.1 Extensive rib damage in an underground coal mine in the Western US.](image)

11.3 Site description and instrument location

11.3.1 Regional and site geology

Mine A is located in an asymmetric basin formed during the Laramide Orogeny (occurred approximately 75 to 45 million years ago). The entire basin covers an area of about
17,000 km² and is about 225 kms wide and 320 kms long. It is bounded in the west, north and east by a steeply dipping monocline and along the south by a zone of uplift. The formation housing the coal bed is of Upper Cretaceous age and was deposited as interbedded marine and non-marine strata during transgression and regression cycles. It is composed mainly of coal, mudstones, carbonaceous mudstones, siltstones and sandstones.

Multiple coal seams are present within this host formation. At Mine A, the seam dips at a very shallow angle (<3º) towards east-northeast and ranges in thickness from 2.5-4 m. More broadly, the seam exhibits a gently plunging synclinal structure with its axis aligned along the E-W direction. The majority of the immediate roof in the mine is composed of carbonaceous mudstones that slake and degrade in the presence of water.

11.3.2 Site description

The average depth of mining at the instrumented location is 250 m below ground surface. The particular panel under consideration has a width of 306 m. The adjacent development entries are about 6 m wide and 2.8 m high. Primary roof support consists of 22 mm diameter, 1.83 m long, grade 60 fully grouted rebars, spaced at 0.9-1.3 m across the entry and 1-1.4 m along the entry axis. Roof trusses are also installed on cycle in some areas, but are normally considered as secondary means of ground support. Table 11.1 lists the unconfined compressive strengths and Young’s modulus derived from laboratory testing on standard rock samples (ISRM, 1981) and the mean Q for different lithologies at the mine.
11.3.3 Instrumentation details

16 arrays of BPCs were installed in two chain pillars and the solid coal of the adjacent panel between X-Cut 10 and 11 at Mine A. Figure 11.2 shows the location of each linear instrumentation array and the distance of each BPC (marked by red dots) along its length. A hydraulic pump was used to pressurize each cell to a pressure of about 5.5 MPa. All BPC gauges and lines were secured to roof and rib mesh to protect them from potential hazards. BPC arrays 1 and 3 were installed at an angle of 30.7° to the rib in order to ensure adequate space for erecting permanent seals between Entry 2 and 3. Similarly, BPC arrays 2 and 4 were installed at an angle of 14.7° outby of the future seal location for clearance.

Table 11.1 Geomechanical characterization for different strata at Mine A.

<table>
<thead>
<tr>
<th>Lithology</th>
<th>Unconfined Compressive Strength (MPa)</th>
<th>Young’s Modulus (GPa)</th>
<th>Q values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal</td>
<td>5.4 – 11.4</td>
<td>2.3-7.8</td>
<td>13-23.7</td>
</tr>
<tr>
<td>Mudstones</td>
<td>1.3 – 5.5</td>
<td>0.78-1.76</td>
<td>5.5-9.4</td>
</tr>
<tr>
<td>Siltstones</td>
<td>5.1 – 51.2</td>
<td>6.4-18.6</td>
<td>11.3-18.2</td>
</tr>
<tr>
<td>Sandstones</td>
<td>29.9 – 50.3</td>
<td>8.8-20.1</td>
<td>23.4-51.7</td>
</tr>
</tbody>
</table>

To record the data generated by the 44 BPCs, two Roctest dataloggers were securely mounted directly above X-Cut 11 on the ground surface. The fiber optic cables were routed via existing 18 cm boreholes in the panel. Out of the 44 BPCs, 43 of them were operational throughout the monitoring period with the exception of the BPC in array 8 located closest to the rib surface. When installing the BPC arrays, care was taken to set them to within a very close margin of the design depth. Stress measurements commenced when the face was 245 m away and continued for 6 months until the face was 400 m outby of X-Cut 10.
11.4 Existing models for stress transfer in longwall mines

11.4.1 Abutment angle model and load transfer distance

The most common approach for estimating abutment stress is through the concept of ‘abutment angle’ ($\beta$), which defines the proportion of overburden load above gob that is transferred to the chain pillars (refer Figure 11.3). While the term ‘abutment angle’ has been also called ‘negative angle of draw’ by Choi and McCain (1980) and ‘shear angle’ by King and Whittaker (1971), its relationship to subsidence profiles is not straightforward (Heasley and Saperstein, 1986). Mark (1987) considered it as a mathematical approximation of physical reality rather than a true measurable characteristic of the gob. For subcritical panels, the abutment angle is related to the side abutment load ($L_{ss}$) as:
\[ L_{ss} = \gamma \left( \frac{H \cdot P}{2} - \frac{P^2}{8 \tan \beta} \right) \]  

(11.1)

where, \( H \) is the depth of cover, \( P \) is the panel width, \( \gamma \) is the unit weight of overburden, and, \( \beta \) is the abutment angle. For critical and super-critical panels, the equation for side abutment load \( (L_s) \) is as follows:

\[ L_s = H^2 \tan \beta \left( \frac{\gamma}{2} \right) \]  

(11.2)

The abutment load, in general, is characterized by the front abutment and the side abutment load. The side abutment load can be easily computed in two dimensions using Equations 1 or 2. The front abutment load, on the other hand, is difficult to quantify mathematically, since the stresses are unequally distributed in three dimensions within the solid coal ahead (outby) of the longwall face, the chain pillars and the gob depending on the effective stiffness of each of these components of the mine system. While numerical modeling can be used to study this process, selection of proper constitutive models to represent coal mass and gob behavior may be difficult. This is because of our lack of complete understanding of how coal masses and gob material behave under increasing levels of load and how this behavior can be appropriately represented using material constitutive relationships.

Empirical characterization on the basis of field measurements is often useful when dealing with problems that cannot be analyzed using basic mathematics. Mark (1987) found from previous field studies that the front abutment load is considerably lower than the side abutment load, and that the two can be related as \( L_f = F (L_s) \), where \( F \) is the ‘front abutment factor’, with a value less than 1. From a physical standpoint this is intuitive, since a significant portion of the front abutment load is carried by the solid coal face. Further analyses of the front abutment loads
by Mark (1987) revealed that F ranges from 0.38-0.68 with 0.5 being the mean value. He also concluded $\beta = 21^\circ$ to be a conservative estimate of abutment angle for US coal mining conditions. Subsequent studies, however, have found $\beta$ to decrease with increasing mining depth (Tulu and Heasley, 2011; Lawson et al., 2013).

![Diagram of abutment angle concept](after Choi and McCain, 1980) with the front and side abutment load conditions (after Colwell, 2018).

Figure 11.3 Abutment angle concept (after Choi and McCain, 1980) with the front and side abutment load conditions (after Colwell, 2018).

The abutment loads generated by the retreat of a longwall face can only be detected up to a finite distance from the edge of the panel. Peng and Chiang (1984) analyzed field stress measurements to develop a relationship between the load transfer distance (LTD) and mining depth (H), which is given by:
More recently, Larson et al. (2013) proposed a negative exponential relationship based on a large database of LTD observations from various mines. Field measurements made by Larson et al. (2013) indicated a much larger LTD in comparison to that predicted by Eq. 11.3. An important conclusion drawn in this study was that the relationship between LTD and H is non-unique and is dependent on the overburden geology. As a result, a single equation might not be applicable to every geo-mining condition.

### 11.4.2 Abutment load distribution

The distribution of the abutment load on longwall chain pillars is non-uniform in nature. From stress measurements made at five mines, Mark (1987) proposed a square decay relationship with distance from panel edge, given by:

\[
\sigma_{abutment} = \frac{L_s}{D^2} (D - x)^2
\]  

(11.4)

where, \(\sigma_{abutment}\) is the abutment stress level at a given position, \(L_s\) is the total side abutment load, \(D\) is the LTD (defined in Eq. 11.3) and \(x\) is the distance from the panel edge. An alternate analytical relationship was proposed by Wilson (1983):

\[
\sigma_{abutment} = (\hat{\sigma} - q) e^{\left(\frac{-x}{C}\right)}
\]

(11.5)

where, \(\hat{\sigma}\) is the peak abutment stress, \(q\) is the cover load, \(x\) is the distance from the rib into the solid coal, and \(C\) is a constant with units of distance. In absence of any yield zone, \(C\) can be estimated as follows (Mark, 1987):
\[ C = \frac{L_s}{\sigma - q} \]  

(11.6)

11.4.3 Depth of yield in coal pillars

To predict the depth of failure in pillars, Wilson (1983) proposed two analytical equations considering yield in the coal seam only and yield in the roof, floor and coal seam. The analytical equations are expressed as:

Yield in roof, floor and coal: \( x_b = \frac{M}{2} \left[ \left( \frac{q}{p+p'} \right)^{1/(k-1)} - 1 \right] \)  

(11.7)

Yield in coal only: \( x_b = \frac{M}{F} \ln \left( \frac{q}{p+p'} \right) \)  

(11.8)

where, \( x_b \) is the depth of yield, \( M \) is the height of the pillar, \( q \) is the vertical stress associated with rock cover, \( k \) is the triaxial stress factor, \( p \) is the restraint on the boundary (expressed as a stress), \( p' \) is the uniaxial strength of fractured coal, and, \( F \) is defined as follows:

\[
F = \frac{k-1}{k^{1/2}} \left( \frac{k-1}{k^{1/2}} \right)^2 \tan^{-1} \left( k^{1/2} \right) \]  

(11.9)

11.5 Results

11.5.1 Expected stress redistribution and stress cell data

In this paper, measurements from the 44 Borehole Pressure Cells are critically analyzed in context of previously proposed models (Carr and Wilson, 1982; Wilson, 1983; Mark, 1987) for stress redistribution during longwall mining to gain a better understanding of this process.
The chain pillar closest to the panel is termed as ‘Pillar A’ while the one closer to the solid coal barrier is termed as ‘Pillar B’. Since stresses are non-uniformly distributed within pillars (Wagner, 1974; Luxbacher et al. 2008), a sufficient spread of measurement points along the width of a pillar is required to accurately estimate the average stress. In this case, 7 BPCs are located along the width of Pillar A and B (refer to Figure 11.2), which made it possible to compute the average stress magnitudes with reasonable confidence. Figure 11.4c shows the average stress change in Pillar A and B as a function of the face location. To compute these stresses, the areas under the stress profiles for Pillar A and B were first determined using Surveyor’s formula (Braden, 1986) and then divided by the respective pillar widths. The corresponding equation for Pillar A is as follows:

$$\sigma_{A, avg} = \frac{0.5 \left| \sum_{i=1}^{n-1} x_i \sigma_{i+1} - \sum_{i=1}^{n-1} x_{i+1} \sigma_i + \sum_{i=1}^{n-1} x_i \sigma_{i+1} - x_n \sigma_1 \right|}{\text{Width of Pillar A}}$$  \hspace{1cm} (11.10)

where, \(\sigma_{A, avg}\) is the average stress for pillar A, \(x_i\) is the distance of different data points from panel edge, and, \(\sigma_i\) is the measured stress magnitude at the corresponding data points.

Stress in Pillar A started to increase rapidly when the face was 30 m behind (inby) the pillar (the leftmost point in Figure 11.4c) and continued to rise with further face movement. Interestingly, just after the passage of the longwall face, a temporary loss of load was detected in Pillar A (Figure 11.4a). Such an observation was also made in Mark (1987) while analyzing the stress measurements from Kitt Mine in West Virginia. The explanation proposed by Mark (1987) is the release of stress that occurs due to caving behind (inby) the face. With the approach of the longwall face, a cantilever is formed in the roof that is supported by the solid coal and the chain pillars. As the face advances, the cantilever breaks, temporarily relieving the load in the chain
pillar. Finally, as the face moves further and further away, additional deflection of the roof occurs which increases the stress in the pillars.

Figure 11.4 (a) Stress change profile for BPC 5A-7B, (b) Stress change profile for BPC 12A-14B, (c) Change in average BPC stress with face location for Pillar A and B.

The stress measurements shown in Figure 11.4c only represent the stress change recorded by the BPC and not the actual stress change in the surrounding rock. Generally, the two stress magnitudes can be related by a constant; the exact value of this constant is dependent on the instrument used, host rock elastic properties, pre-stress applied during installation, etc. (Babcock, 1980; Heasley, 1989; Lu, 1984; Tulu and Heasley, 2011; Mohamed et al., 2016b). Since it was
not possible to gather these details to determine the conversion factor, a different approach was undertaken in this study (similar to the approach used by Tulu and Heasley, 2011).

11.5.2 Conversion of BPC measurements into rock stress

Mark and Iannacchione (1992) estimated the stress gradient at ultimate load for different pillar strength formulas. In the same study, a comparison was made between yield stresses measured in the field from six different mines and empirical equations. It was found that the Bieniawski (1984) stress gradient provides the best estimate of the yield stresses within a pillar, although it diverges from the average field-measured curve for points located deeper in the pillar. The corresponding equation is as follows (Mark and Iannacchione, 1992):

\[
\sigma_v(x) = S_1 \left( 0.64 + 2.16 \frac{x}{h} \right)
\]

(11.11)

where, \( S_1 \) is the in-situ coal uniaxial strength, \( \sigma_v(x) \) is the coal stress at a distance \( x \) from the pillar edge, and \( h \) is the height of the pillar.

From the measured stress profiles at Mine A (see Figure 11.4a), it was observed that the three BPC closest to the rib (7B, 7A and 6B) yielded at stress levels of 5.15 MPa, 2.33 MPa and 3.39 MPa when the face was 228 m outby. Plugging the relevant \( x \) values into Eq. 11.11, with a mining height of 2.80 m and \( S_1 \) of 6.2 MPa (Mark and Barton, 1997), gives \( \sigma_{v,7B} = 16.0 \) MPa, \( \sigma_{v,7A} = 30.4 \) MPa and \( \sigma_{v,6B} = 59.5 \) MPa. Since the BPC exhibited a continued drop in stress beyond this point, it can be reasonably assumed that the coal strength was attained at this location. The conversion factor (CF) could then be calculated as the ratio of the strength obtained from Eq. 11.11 to the failure stress (development stress using tributary area method + stress
measured by BPC), resulting in three possible conversion factors: $CF_{7B}=1.21$, $CF_{7A}=2.90$ and $CF_{6B}=5.19$. The discrepancy between conversion factors may, in part, be due to the fact the tributary area method neglects variations in stress within individual pillars. Only $CF_{7B}$ was utilized for adjusting the BPC measurements for subsequent analyses. The choice of conversion factor is further justified in Section 11.5.5 through an inspection of BPC Array 16 stress data.

11.5.3 Calculation of abutment loads and abutment angle

With the BPC measurements converted into actual rock stress values, the next task was to determine the front abutment and side abutment loads at Mine A. As defined earlier, the calculation for front abutment load is performed using the stress data corresponding to a face location of 0. With further face advance, the average stress in Pillar A and B rises (see Figure 11.4c) and then flattens out at a distance of around 260 m from face. While the 260 m face location could be used to estimate the side abutment load, the 400 m location was chosen (as the final data location) for the purposes of side abutment load assessment. The authors would like to point out that there was a 21.6% and 36.3% increase in average stress for Pillar A and B respectively between face locations 115 m and 400 m. In the work by Mark (1987), due to extensive instrument damage, the side abutment calculation was performed at a face location of 115 m (outby). As a result, there may have been a significant underestimation of the side abutment loads in that study.

Figure 11.5 shows the stress distribution in Pillars A and B and the adjacent unmined coal for longwall face locations at 0 m outby (front abutment loading) and 400 m outby (side abutment loading). Two important observations can be readily made: (a) there is yield and subsequent loss of load carrying capacity in the first 5 m of the rib between the front and side
abutment loading conditions; (b) the proportion of load carried by the unmined coal and LTD increases as the longwall face advances. The two observations can be explained by a progressive damage mechanism where a gradual widening of the fractured rib transfers the load away from the panel edge. It is therefore evident that calculation of the side and front abutment loads requires a proper consideration of load carried by the unmined coal. It should be noted that the right-most data point in Figure 11.5 was obtained by fitting a straight line to the three stress measurement points in the unmined pillar. The error incurred by using a linear fit instead of an exponential decay or inverse square function is relatively small and can be neglected for practical purposes.

![Figure 11.5 Stress distribution in Pillar A, Pillar B and solid coal for front loading and side loading condition.](image)

The methodology followed for calculating the front and side abutment load is similar to what was proposed by Mark (1987). The front abutment load per unit length of the entry ($L_f$), in terms of the average pillar stress, can be represented as:
where, $\sigma_{A,f}$, $\sigma_{B,f}$ and $\sigma_{unmined,f}$ are the average stresses in Pillars A and B and the unmined coal when the face was at 0 m, respectively; $Ar_A$, $Ar_B$ and $Ar_{unmined}$ are the areas of Pillars A and B and the unmined coal (only the loaded portion of the unmined solid coal was considered in the calculation), and $C$ is the X-cut spacing.

The side abutment load, similarly, can be computed using the following equation:

$$L_s = \frac{\sigma_{A,s} Ar_A + \sigma_{B,s} Ar_B + \sigma_{unmined,s} Ar_{unmined}}{c}$$  \hspace{1cm} (11.13)

where, $\sigma_{A,s}$, $\sigma_{B,s}$ and $\sigma_{unmined,s}$ are the average stresses in Pillars A and B and the unmined coal when the face was 400 m outby, respectively. The respective data-derived input parameters for Eqs. 11.12 and 11.13 are listed in Table 11.2. In order to emphasize the need to include the load borne by the unmined solid coal in front and side abutment load calculations, two independent analyses were conducted. Table 11.3 presents the front abutment load, side abutment load, abutment angles and the front abutment factors from these analyses. The abutment angle was calculated by equating the side abutment load to Eq. 11.2 for super-critical panels – for Mine A, the ratio of the depth to the length of panel is 306/252=1.21, which is more than the critical value given by Wilson (1972) and Agioutantis and Karmis (2009). A unit weight ($\gamma$) of 25 kN/m$^3$ was also considered for this calculation (Mark, 1987; Mukherjee et al., 1994; Suchowerska et al., 2013; Rezaei et al., 2015a).

The front abutment factors (F) obtained are well below the typical value of 0.5 that is commonly applied in the mining industry. With this in mind, chain pillars designed considering
F=0.5 would be conservative in nature at Mine A, with the only downside being leaving more coal unmined in the pillars. The F factor has been found to be highly variable and mine specific (Hill et al., 2015); a conservative value, therefore, is better suited for preliminary pillar design. Such a low value of F is not an artifact of the choice of conversion factor since its effect is nullified when taking the ratio between the front and side abutment loads.

Table 11.2 Input parameters for calculating the front and side abutment loads.

<table>
<thead>
<tr>
<th>Input parameters</th>
<th>Values (using CF$_{7B}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Front Abutment</td>
</tr>
<tr>
<td>$\sigma_A$ (MPa)</td>
<td>2.27</td>
</tr>
<tr>
<td>$\sigma_B$ (MPa)</td>
<td>1.49</td>
</tr>
<tr>
<td>$\sigma_{\text{unmined}}$ (MPa)</td>
<td>0.16</td>
</tr>
<tr>
<td>$A_{rA}$ (m$^2$)</td>
<td>1489</td>
</tr>
<tr>
<td>$A_{rB}$ (m$^2$)</td>
<td>1128</td>
</tr>
<tr>
<td>$A_{r\text{unmined}}$ (m$^2$)</td>
<td>796</td>
</tr>
<tr>
<td>C (m)</td>
<td>53</td>
</tr>
</tbody>
</table>

Table 11.3 Front, side abutment load, front abutment factor and abutment angle for Mine A.

<table>
<thead>
<tr>
<th>Case</th>
<th>Front abutment (MN/m)</th>
<th>Side abutment (MN/m)</th>
<th>F</th>
<th>$\beta$ (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Considering load on unmined coal (CF$_{7B}$)</td>
<td>97.9</td>
<td>337.9</td>
<td>0.29</td>
<td>11.5</td>
</tr>
<tr>
<td>Not considering load on unmined coal (CF$_{7B}$)</td>
<td>95.5</td>
<td>301.3</td>
<td>0.32</td>
<td>10.3</td>
</tr>
</tbody>
</table>

A comparison of the results with and without consideration of the load on the unmined solid coal revealed a negligible effect on the front abutment load calculation and an 11% error in the side abutment load. The abutment angle also decreased by 1.2$^\circ$ when the load on the unmined coal was ignored. This agrees well with a 2.2$^\circ$ increase in abutment angle reported by Mark
(1987) when the assumed stress in the solid coal barrier in Kitt mine, WV, was incorporated into the calculation.

11.5.4 Distribution of abutment loads

11.5.4.1 Front abutment load

Seven data points were available for each chain pillar and three data points for the solid unmined coal along the pillar mid-point cross-section. Therefore, the average front abutment stress between every two adjacent data points could be computed. The area enclosed by the data points was first determined by the Surveyor’s formula (Braden, 1986) and then divided by the distance between them. The average stress obtained was then assigned a position at the center of the measurement point pair. Figure 11.6 shows the inverse square and the exponential decay fits to the average stress values calculated using this approach. The equation for determining $\sigma$ (Wilson, 1983) has not been used here, as it assumes the abutment to have reached the peak stress level. It was verified from the BPC measurements that none of the data points yielded under the front abutment load. The Wilson (Wilson, 1983) equation, therefore, is utilized here purely to test its capability in capturing the trend of the field stress data.

It was possible to fit the curves directly to the BPC measurements, but the approach of averaging adjacent data points is more consistent with how the decay equation was developed (Mark, 1987). Another advantage of this approach is its ability to limit the effect of excavation induced stress concentrations around the edges of the pillar.
Both equations appear to fit the stress data fairly well, with Mark’s (1987) decay function exhibiting a slightly higher $R^2$ value in comparison to Wilson’s (1983) decay function. The $L_f$ (instead of $L_s$, since the front abutment load is being analyzed here) served as an input parameter for Wilson’s (1983) equation, and was determined through a least-square curve fitting process for Mark’s (1987) equation. The best-fit value of $L_f$ was found to be 98.4 MN/m which is within 0.5% of the front abutment load computed directly from the stress data. The trend of the two equations in Figure 11.6 indicates some differences – Wilson’s (1983) equation predicts a lower peak abutment stress and better captures the stress levels in the unmined coal, but predicts an infinite LTD. Conversely, Mark’s (1987) equation predicts a higher peak abutment stress and underestimates the load on unmined coal pillars, but assumes the stress change to terminate at the LTD.

11.5.4.2 Side abutment load and yield depth

As the longwall face advanced beyond the monitored pillars, an increase in stress was observed across the two chain pillars and in the solid unmined coal of the adjacent panel. The load continued to increase until the face reached 228 m outby, at which point the stresses in BPC Array 7 and instrument 6B started to drop (Figure 11.7). The loss in load carrying capacity was compensated by an increase in the BPC stress levels recorded deeper into the pillars. Since the aforementioned BPCs showed a monotonic decrease in stress levels, it can be reasonably inferred that the first 13.7 m (halfway between 4th and 5th data point in Figure 11.7) of the rib has yielded. The extent of rib damage observed in this case is much larger than those reported in previous studies. For comparison, Mark (1987) reported 5.3 m of rib damage for a face location of 40 m outby while Colwell (2006) found 4 m of rib damage with face at 981 m outby. Note
that the extrapolated rightmost data point in Figure 11.5 for side abutment loading was located 92.4 m from the panel edge. This is 13.5% higher than the LTD calculated from Eq. 11.3.

![Figure 11.6](image)

Figure 11.6 Inverse square and exponential decay fit to average front abutment stress across the pillars.

The immediate floor in Mine A is composed mostly of soft mudstones, and it is probable that failure occurred in both the host rock and the coal seam. Equation 11.7 is, therefore, more appropriate for estimating the depth of yield. Plugging in $q=6.55$ MPa, $p=0$ MPa (no lateral restraint, considering the worst case scenario), $M=2.80$ m, $k=4$ (equivalent to a friction angle of $37^\circ$; Wilson, 1983) and $p^*=0.1$ MPa (Mark, 1987) gives a depth of yield equal to 4.24 m. The analytical solution significantly underestimates the depth of yield in this case. This is not surprising, as the mechanism of fracture initiation and propagation is far more complex than what can be captured using analytical approaches. In addition, the interaction between the host
rock and the pillar is unique for any given geo-mining condition, making it difficult to quantify this behavior using a generalized equation.

Figure 11.7 Stress distribution across the chain pillars and solid unmined coal for different face locations.

Wilson’s (1983) method also assumes the peak stress to occur at the interface between the yield zone and the elastic core. The sudden jump in the stress level 16 m into the pillar (5th data point from left in Figure 11.7) appears consistent with this assumption. As the stress within the first 11.4 m (4th data point from left in Figure 11.7) of the rib falls while those beyond 16 m rise with face advance beyond the 228 m outby location, it can be assumed that the edge of the yield zone lies somewhere between 11.4 m and 16 m. In absence of any measurement point within this zone, the center (i.e. 13.7 m) was chosen as the edge of the yielded coal. Based on Wilson’s (1983) hypothesis then, it could be postulated that the stress level around 13.7 m may be significantly higher than the measured value at 16 m.
In the post-yield stage, Wilson (1983) assumed the coal to carry the same elevated stress level as additional load is applied to the pillar. It appears that this assumption is erroneous; significant load transfer occurs from the yield zone to the elastic pillar core and the adjacent chain pillar, even when the pillar has not failed completely.

Mark’s (1987) decay equation and Wilson’s (1983) decay equation could only be fitted to the data points located 13.7 m or more from the panel edge. This is because stress decay occurs only within the elastic portion of the pillar. The sharp peaks in stress at the edges of the pillar in Figure 11.7 are believed to be purely elastic in nature (excavation-adjacent stress concentration) and caused significant distortion in the data-fitting process. In particular, the Ls value obtained by fitting Mark’s (1987) decay equation (considering the sudden peak points) was found to be significantly higher than the Ls computed from BPC data for chain pillars and the solid coal barrier. This is unacceptable, as only a portion of the BPC data (in the elastic portion of the pillar systems) was considered in the data-fitting process, which should have therefore yielded L values lower than the total L values for the entire pillar system. In order to eliminate this issue, these two peak stress points were removed from the analysis (but are presented in Figure 11.8 as green bars).

The procedure presented in Section 11.5.4.1 was followed, whereby the average stress between adjacent data points were computed and then assigned to its respective midpoint. This was performed for the datasets corresponding to a longwall face location of 228 m and 400 m ouby. Due to lack of any stress measurement between 11.4 m and 16 m within the pillar, the peak stress level at the interface between the elastic core and the yield zone was obtained by setting x = 0 in Mark’s (1987) decay function at a distance of 13.7 m into the pillar. For the Wilson (1983) fit, the portion of the side abutment load carried by the pillar was used to
determine ‘C’. \( \hat{\sigma} \), as obtained from the equation \( \hat{\sigma} = kq + S_1 \) (Wilson, 1983), for the face location of 228 m led to a poor fit to the data; as a consequence, it was allowed to vary during the fitting process.

Figure 11.8 Curve fit to the data points for longwall face locations of (a) 228 m outby, (b) 400 m outby. \( L_{s,yield}^{calc} \) and \( L_{s,elastic}^{calc} \) indicate the side abutment loads calculated for the yielded and elastic region from the BPC data; \( L_{s,yield}^{fitted} \) and \( L_{s,elastic}^{fitted} \) indicate the side abutment loads obtained after fitting the respective curves. Data values shown in green were not considered in the curve fitting process. MN/m (Mega-Newton per meter) is the unit of abutment load used in this study.

Wilson (1983) hypothesized an exponential rise in stress levels within the yield zone. Mark (1987), on the other hand, found that a straight line better characterizes the stress gradient from the rib to the elastic core. In this study, an attempt was made to fit both types of curve to the field data. An acceptable fit could not be obtained for the exponential curve using constants calculated from Wilson (1983) or when using the constants as fit variables. A straight line was deemed to be a simpler and better predictor of the BPC data in the yield zone. To fit the line, an intercept was manually selected on the vertical (stress) axis such that it satisfied both the peak
stress level obtained from Mark’s (1987) decay fit at 13.7 m from the pillar edge as well as the overall trend of the dataset.

Figure 11.8a shows the curve fit to the BPC data for the 228 m outby face location. A comparison was made between the proportion of side abutment load carried by the yield zone and the elastic pillar material as obtained from the curve fitting process and from the field data (see Figure 11.8a). The ability of the fit in capturing the trend of the BPC data as well as the excellent correspondence in the side abutment load values provides confidence in the data-fitting process.

For the 400 m outby face location, a bi-linear curve was chosen. The fitted curve and the proportion of load carried by the yield zone and elastic pillar material is shown in Figure 11.8b. The bi-linear nature of the stress gradient in the yield zone was also observed by the authors in a calibrated FLAC$^{3D}$ model of an Australian longwall mine (Sinha and Walton, 2020b). The mine is located at a depth of 480 m and has a panel width of approximately 195 m (roadway center to center). Further details on this study can be found in Sinha and Walton (2020b). In this case, a model was able to accurately match displacement and stress profiles for different locations of the longwall face. Figure 11.9 shows the model predicted stress profiles for three different locations of the longwall face. Given the dissimilarity in the geo-mining condition between Mine A and the Australian mine, only a qualitative comparison was performed here. To that end, normalized stress change profiles were used instead of raw stress change profiles.

As can be seen from Figure 11.9, the stress gradient in the yield zone changes from a linear to a bi-linear shape with the advance of the longwall face. Prior to the formation of a bi-linear shape, the slope of the linear section increases until it reaches a peak value. The skin of the
pillar, at this point, attains its residual strength, shedding the excess load deeper into the pillar (shown by the green line). The red and the green line in Figure 11.9 can be considered qualitatively similar to the face locations at 228 m and 400 m in Figure 11.8. With continued loss of load carrying capacity of the pillar rib, the yield zone finally expands to accommodate the excess stress. It is interesting to note that the slope of the second segment of the bi-linear curve (the portion located closer to the elastic core) remains almost constant during the rib extension process (refer to the red and green curves). Since no BPC measurements were available beyond the 400 m face location, this observation could not be validated at Mine A. During the entire loading process, the peak stress level at the interface between the yield zone and the elastic material was also found to increase in both the numerical model and the field data.

11.5.5 Stress distribution in coal ahead of (outby) the longwall face

BPC Array 16 was installed in the solid longwall face at distances of 15.2 m, 30.4 m and 51.8 m from the edge of the panel. Figure 11.10 shows the stress change profiles as a function of face distance; measurements were terminated when the face was about 7.5 m from the instruments. The BPCs started experiencing load when the face was about 230 m inby the instrumented location. Interestingly, BPCs located closer to the chain pillars consistently experienced higher stress levels than those located deeper within the solid coal. Intuitively, one might expect some of the load around the panel edge to be distributed over the chain pillars, resulting in lower stress levels in the solid coal face. That being said, it is unlikely that the data are faulty since that all three BPCs exhibited the same trend.

In the context of the data shown in Figure 11.10, some hypotheses were formulated and then tested using a schematic FLAC$^{3D}$ model in Section 11.6. Here, we argue that when the main
roof breaks behind (inby) the longwall face, the region of the gob closest to the chain pillars cannot compact to the same extent as the gob located away from the panel edge. The primary reason is the resistance offered by the chain pillars to reduce the flexure of the roof layers. This means that the effective stiffness of the gob material increases as one moves away from the panel edge. The excess stress near the panel edge (corresponding to the load not carried by the chain pillars) is transferred to the stiffer solid coal and the shield support rather than to the ‘soft’ gob material. Away from the panel edge, the gob becomes stiffer and can proportionately carry higher loads, reducing the amount of stress channeled to the solid coal face.

![Figure 11.9](image.png)

Figure 11.9 Normalized stress change versus distance from panel edge from a calibrated numerical model of an Australian longwall case study.

At a certain distance behind (inby) the panel face, the gob should be transferring almost the entire load of the overburden to the floor (Oyanguren, 1972; Wade and Conroy, 1980; Campoli et al., 1993; Abbasi et al., 2014; Li et al., 2015). The distance behind the face at which this occurs depends on the type of gob material and the depth of mining. A strong roof material
will compact faster and will be able to re-load to the in-situ stress more rapidly than a weak roof material. For Mine A, the behavior of the gob is unknown; however, from Array 7 stress data, it appears that the main roof collapsed about 10 m after the face went past the instruments. This showed up as a drop in the stress levels in Array 7 and 6B and in the average stress for Pillar A (see Figure 11.4). Based on this, the authors believe that the relatively low change in stress for the 51.8 m BPC shown in Figure 11.10 is related to it being very close to the edge of the abutment influence zone, with the unsupported overburden load from between the ‘fully compacted’ gob (that is >10 m behind) and the face being carried primarily by the compacted gob, the shield supports and first 7.5 m of solid coal ahead (outby) of the face. For such effective stress transfer between the three load bearing elements, the gob must have been a material that compacted relatively quickly.

Figure 11.10 Stress change profile as measured in the solid longwall face.

If indeed the 51.8 m BPC was located close to the edge of the abutment influence zone, it could be used to calculate the abutment angle. Tangent inverse of 51.8 m divided by the depth of
mining (252 m) gives an abutment angle of 11.6°, which is only 0.1° larger than the value computed using the corrected BPC data. For CF7A and CF6B, Eq. 11.2 yields β=26° and 41°, corresponding to abutment influence zones of 123 m and 219 m, respectively. Clearly, this is in disagreement with the data presented in Figure 11.10. The excellent correspondence obtained with CF7B not only confirms the abutment loading theory but also highlights the robustness of the selected conversion factor. Future studies can use a similar methodology for converting the BPC data to actual rock stress.

For an abutment angle of 21°, the edge of abutment would be located 97 m from the panel edge, in which case the BPC at 51.8 m would have experienced elevated load levels. The side abutment load would also have been overestimated by 89% (using Eq. 11.2). A key takeaway from this analysis is that chain pillars designed using a 21° abutment angle (used as default in the Analysis of Longwall Pillar Stability software) may be conservative in nature in some cases.

11.5.6 Three-dimensional (3D) stress redistribution over chain pillars

In the previous sub-sections, only the stress data along the center of the chain pillars was utilized for estimating the side abutment loads. The purpose was to ignore any three-dimensional stress arching effect to simplify calculations and interpretation. In order to fully utilize the dataset and obtain a better understanding of the stress redistribution process, a three-dimensional analysis was necessary. To that end, 3D bar charts were developed for chain Pillars A and B for four different face locations (see Figure 11.11). Stress measurements were available at specific points, which necessitated the use of an interpolation technique to populate stress values throughout the rest of the pillar. As no data points were available around the pillar corners, only
a diamond-shaped region could be populated with results from the ‘cubic’ interpolation scheme in MATLAB.

Figure 11.11 Two-dimensional stress distribution in Chain Pillar A and B for longwall face location of (a) 0 m outby, (b) 250 m outby, (c) 750 m outby, (d) 1300 m outby.

The distribution of stresses within longwall chain pillars are not only non-uniform but also asymmetric in nature. For chain Pillar A, the peak stress level for all four face locations was situated along the longer axis of the pillar. A probable reason is the three-dimensional arching of the mining-induced stresses over the crosscut that resulted in higher stress concentrations along the shorter edge. The extent of arching is dependent on the surrounding strata, gob characteristics and the integrity of the pillar rib. The 3D effect reduces as one moves towards the center of the pillar, ultimately simplifying to a 2D side loading condition.

With advance of the longwall face, a progressive loss of load carrying capacity was noted in the panel-side rib of Pillar A which channeled the excess stress deeper into the pillar as well as
to the adjacent chain pillar (i.e. Pillar B). Rib failure and subsequent load transfer was also noted along the shorter edge of Pillar A (see Figure 11.11, parts c and d).

11.6 Analysis of stress-transfer mechanism using a hypothetical continuum model

A half panel model with the geometry of Mine A was developed in FLAC\textsuperscript{3D} (Itasca, 2016a) (see Figure 11.12) with the goal of testing two aspects of the stress transfer mechanism hypothesized above: (a) Is the continued drop in stress from the panel edge (as indicated in Figure 11.10) due to the differential compaction of gob? (b) Can the unsupported overburden that extends between the uncompacted gob and the shield support be carried by different load-carrying areas such that no stress change is registered 10 m into the solid coal face (i.e. at the 51.8 m BPC)? The dimensions of the pillar and the entries were obtained from an AutoCAD map of Mine A, then rounded to the nearest even whole number. The stratigraphic sequence in the model was based on the Australian longwall case study discussed in section 11.5.4.2, as no core data was available for Mine A at the instrumented location.

It should be noted that due a lack of relevant geological and geotechnical data from Mine A, the model developed and discussed herein is schematic in nature, and no attempt was made to quantitatively match the field measured stress profiles. All rock layers were represented by an isotropic elastic constitutive model with parameters listed in Table 11.4 (Sinha and Walton, 2020b). A vertical stress corresponding to 236.5 m of overburden was applied to the top of the model, and the body stress for coal seam and 14 m of overburden was considered by setting the acceleration due to gravity at 9.81 m/s\textsuperscript{2}. Roller constraints were assigned to the bottom and sides of the FLAC\textsuperscript{3D} model.
Table 11.4 Thickness and rockmass elastic parameters for different layers in model (Sinha and Walton, 2020b).

<table>
<thead>
<tr>
<th>Layer</th>
<th>Thickness (m)</th>
<th>Rockmass Young’s Modulus (GPa)</th>
<th>Rockmass Poisson’s ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interbedded sandstone (roof)</td>
<td>9.2</td>
<td>12.0</td>
<td>0.26</td>
</tr>
<tr>
<td>Mudstone (roof)</td>
<td>4.8</td>
<td>10.0</td>
<td>0.26</td>
</tr>
<tr>
<td>Coal</td>
<td>2.8</td>
<td>3.0</td>
<td>0.25</td>
</tr>
<tr>
<td>Mudstone (floor)</td>
<td>2.0</td>
<td>12.0</td>
<td>0.26</td>
</tr>
<tr>
<td>Interbedded sandstone (floor)</td>
<td>6.0</td>
<td>12.0</td>
<td>0.26</td>
</tr>
<tr>
<td>Sandstone (floor)</td>
<td>6.0</td>
<td>15.0</td>
<td>0.26</td>
</tr>
</tbody>
</table>

The model behavior is expected to be influenced by the load carrying capacity of the gob, which required an explicit consideration of the gob loading mechanism within the model. A methodology similar to Esterhuizen et al. (2010b) was followed, whereby the excavated coal was replaced by an equivalent gob material that exhibits a hyperbolic hardening behavior (Salamon, 1990). The modulus of the gob material was varied per equation 11.14:

$$\sigma = \frac{a \cdot \varepsilon}{b - \varepsilon}$$  

(from Salamon, 1990)

$$E = \frac{d\sigma}{d\varepsilon} = \frac{a \cdot b}{(b - \varepsilon)^2}$$  

(11.14)

where, $\sigma$ is the stress, $\varepsilon$ is the strain, and $a$ and $b$ are two constants that were found to be 7.65 MPa and 0.442 for shales and 13.03 MPa and 0.427 for sandstones (Pappas and Mark, 1993). Esterhuizen et al. (2010b) tabulated the gob parameters ‘$a$’ and ‘$b$’ for four different overburden types. Since the different rock layers were modeled as elastic material, the roof deformation after mining was significantly lower than would be expected in reality, meaning much lower ‘$a$’ and ‘$b$’ parameter values had to be used for the gob to develop any appreciable stiffness in the model. A FISH function was called every 500 solution steps that modified the Young’s modulus of the
gob according to Eq. 11.14. With $a=5.9$ MPa and $b=0.05$, the gob achieved pre-mining vertical stress at a distance of $\sim 55$ m from the longwall face, similar to what was obtained by Abbasi et al. (2014) in modeling a longwall mine in Illinois. While it is not known whether the gob compacted completely within 55 m for Mine A, this was assumed in order to obtain realistic gob parameters. The longwall face was advanced in two 40 m steps whereby the coal elements were first replaced by the elastic gob elements and then solved until mechanical equilibrium was achieved.

Figure 11.12 Overall geometry of the hypothetical longwall model.

In practice, shield supports are used along the length of the longwall face to ensure safe extraction of the coal. Within the first few meters of the face, the coal is generally yielded (Song et al., 2017), lowering its effective stiffness. The powered shield support, being stiffer in comparison to the yielded coal, therefore should transfer a large proportion of the nearby in-situ load from the overburden to the floor. A consequence of this phenomenon is the reduction in the distance ahead of (outby) the face that experiences change in stress levels. The shield load
transfer mechanism was simulated in the elastic model by replacing the first 4 m of the excavated material by a material that was three times stiffer than the coal mass.

Figure 11.13 shows the vertical stress contour along the mid-height of the longwall panel with and without consideration of shield supports. The stress monitoring points and the profile locations (as in Figure 11.10) are also shown by black circles and black discontinuous lines, respectively. It can be seen that all the measurement points experienced significantly lower stresses when the shield supports were considered in the model. This explains why the 51.8 m BPC experienced negligible change in stress when the extent of the roof between the compacted gob and the shield supports was left unsupported. The weight of this unsupported region was ultimately borne by the shield supports, compacted gob and the first 8 m of the coal face.

![Figure 11.13 Vertical stress contour at mid-height of the coal seam (a) with shield support, and, (b) without shield support.](image)

Although subtle in the presented results (refer to the stress level at the monitoring points in Figure 11.13), profiles drawn at 8 m from the face showed an exponential decay in stress from the edge of the panel inwards. This explains the trend in data shown in Figure 11.10 and is consistent with the hypothesized stress transfer mechanisms. In order to understand why such a behavior was exhibited by the model, the modulus distribution of the gob material was analyzed.
It was found that along the edge of the panel, the gob had a lower modulus and consequently carried a lesser proportion of the overburden load. As discussed before, this is caused by the flexural resistance of the pillar-supported roof layers that prevent complete compaction of the gob material at the panel edge.

11.7 Discussion

Abutment angles around the globe have been found to vary between 6°-27° (Hill et al., 2015) with 21° being used as a conservative estimate for most US coal mines (Mark, 1987). Other studies have found the abutment angle to decrease with increase in mining depth beyond 274 m (Tulu and Heasley, 2011). For Mine A, the depth is around 250 m, and therefore might be expected to correspond to an abutment angle of roughly 21°. However, the values calculated from BPC measurements are significantly lower than this abutment angle. Relatively low values in the range of 5° - 15° were also reported by Hill et al. (2015) for Western US mines. A possible explanation is that the traditional abutment angle model may be overestimating the magnitude of abutment load for Western US mines, indicating that a larger than typical amount of load is transferred elsewhere (for example, to the gob) due to different caving characteristics.

An interesting observation made during the BPC data analysis is the simultaneous loss of load-carrying capacity of instrument 6B and Array 7 when the longwall face reached 228 m outby. It seems that instead of a progressive expansion of the damage zone as was proposed by Mark (1987), fracturing initiated in discrete episodes over thick sections of the pillar. The authors have not previously observed such episodial-type behavior; if indeed this phenomenon is real, further study is required to understand the governing mechanics. It is possible that fracturing initiated in Array 7 first and then progressed to 6B, but this might have been masked.
by the limited temporal resolution of the dataset. In any case, for the purposes of analyses in this paper, it was assumed that the entire 13.7 m of the rib yielded when the face reached 228 m outby.

Based on an understanding of coal damage mechanics, one would expect the yield strengths to be similar at equal distances from the pillar edge. However, it was found that the yield strength along the shorter edge was higher than those along the longer edge of the pillar (i.e. using BPC Array 7 and instrument 6B). The exact cause for this behavior is not immediately apparent and is difficult to explain. Because this trend was observed along all of the pillar edges, the likelihood of the data being faulty is thought to be extremely low. One possible reason for this unexpected behavior is damage induced in the rib due to the rapid unloading 10 m after the face went past the instrumented location (refer Figure 11.4a). It is important to reiterate here that this loading-unloading cycle was only observed in BPC Array 7 and instrument 6B. To date, no study has been conducted to understand the effect of this load cycle on the integrity of the coal rib. In addition to the moment applied by the flexure of the immediate roof on the coal rib (which opens the cleat planes along the roof), the authors believe that the loading-unloading cycle may have damaged the first 13 m of the rib.

A second possible reason for this behavior could be the effect of cleat orientation on coal strength. The cleats at Mine A are near-vertical and trend approximately NE-SW. The panel, on the other hand, is oriented North-South which renders either edge of the pillar equally vulnerable to damage formation along cleats. With the panel currently inaccessible for further inspection, it is unclear whether there was any local variation in the cleat angle. If the cleats were trending closer to N, then the likelihood of spalling would be higher along the entry side of the pillar.
Along the shorter edge, spalling would be constrained in the lateral direction, forcing fractures to propagate through intact coal.

Evidence of strength anisotropy in coal due to cleat orientation was found by Kim et al. (2018) and Song et al. (2018) during laboratory testing. Under actual field conditions, cracks can propagate rather freely along the pillar periphery (Diederichs, 2007). As discussed by Diederichs (2007), the crack propagation mechanism is highly sensitive to confinement levels and a small increase in confinement can suppress the spalling mechanism. The confining stress generated across cleat planes on adjacent pillar faces in Mine A could possibly explain the stark difference in the observed stress levels.

It is useful to note here that the intersection of Bieniawski’s (1984) stress gradient curve and the Mark and Iannacchione (1992) average stress gradient curve (derived from field measurements) lies at $x$ (distance into pillar) / $h$ (height of pillar) = 0.18 (Figure 11.14), which gives $x = 0.52$ m for Mine A. Beyond the intersection point, the average stress lies above the Bieniawski’s (1984) curve (Mark and Iannacchione, 1992). For Mine A, the damage due to the loading-unloading cycle or local variability in the cleat angle might have lowered the actual yield strength at $x = 2.52$ m (used for calculating $C_{7B}$) such that it matched the strength predicted by the Bieniawski’s (1984) curve. This implies that the actual (undamaged) stress gradient in the pillars is higher than Bieniawski’s (1984) curve would suggest.

Besides BPC array 7 and instrument 6B, the only other data point that exhibited yielding was 1A, located 1.72 m from the shorter edge of Pillar A. The yield strength at this point was estimated to be 24.9 MPa, which is approximately double of what is predicted by Bieniawski’s (1984) gradient equation. Clearly, the yield strengths are underestimated for points located
deeper within the pillar. The use of Bieniawski’s (1984) gradient equation should, therefore, be restricted to within \( x/h < -0.2 \) from the rib. While the gradient approach for calculating a conversion factor was successful in this case, it might not yield reasonable values under every condition. Where possible, the appropriateness of the conversion factor should be validated using a secondary approach, as was done in this study.

![Figure 11.14](image)

**Figure 11.14** Plot of average field measured stress gradient (Mark and Iannacchione, 1992) and Bieniawski’s (1984) stress gradient.

### 11.8 Conclusions

This study has presented an extensive suite of borehole pressure cell data from a Western US mine with the goal of advancing our knowledge of the stress redistribution process in longwall chain pillars. A new method was employed for converting the raw data into actual stresses experienced by the host rock. Although this method worked well for Mine A, care should be exercised when applying it to other geo-mining conditions.
The abutment angle and the front abutment factor at Mine A were found to be significantly lower than what is normally associated with US longwall mines. The low abutment angle of the order of 11º was verified using the BPC measurements taken in the solid coal ahead of (outby) the longwall face. The stress rise in the yield zone was found to be bi-linear in nature which agreed with a calibrated mine model (from a case study in Australia) recently developed by the authors.

Some other trends such as a drop in stress from the edge of the panel towards the center and negligible stress change about 51 m into the solid coal face were also observed. Using an elastic FLAC\textsuperscript{3D} model with appropriate consideration of gob loading and face shield supports, these observations could be explained. Finally, the global stress redistribution process was presented in three dimensions using 3D bar charts for different locations of the longwall face.

Understanding the stress transfer process along with the local damage progression in the entry side rib will ultimately help the mining industry in better assessing pillar stability and improving the methodology that currently exists for determining pillar dimensions. The results suggest that using the current design approach ($\beta=21^\circ$) without a proper estimation of abutment angle might lead to highly overdesigned coal pillars.

11.9 Acknowledgements

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CHAPTER 12

APPLICATION OF AN INTEGRATED 3D-2D MODELING APPROACH FOR PILLAR SUPPORT DESIGN IN A WESTERN US UNDERGROUND COAL MINE

A version of this chapter is being prepared for submission to an international journal and will have the following author list: Sinha S, Walton G.

12.1 Abstract

Design of rib support systems in U.S. coal mines is based primarily on local practices and experience. With rib collapses continuing to cause injuries and fatalities in recent years, it is necessary to develop a more robust rib support design approach. Discontinuum Bonded Block Models (BBM) represent a potential tool for support design, as they can reproduce both the rock fracturing process and the influence of reinforcement (bolts) on unsupported ground. Despite their strengths, discontinuum models are seldom used for mining design due to their computationally intensive nature. To overcome this issue, an integrated 3D continuum – 2D discontinuum approach was recently proposed, in which the mine-wide stress distribution process is modeled using a continuum software and the local deformational behavior in response to a strain path from the continuum model is simulated with a 2D discontinuum software. This study is an application of the integrated modeling approach to simulate the damage in a coal pillar rib in a Western US longwall mine, and an attempt to test the approach as a practical support design tool.

In June 2017, two multi-point borehole extensometers were installed in a longwall chain pillar to record ground displacements as the longwall face approached and crossed the instrumented location. The data from one of the extensometers was then employed to constrain
the model parameters of a panel-scale FLAC$^{3D}$ model. With the FLAC$^{3D}$ model calibrated, the boundary conditions (from the FLAC$^{3D}$ model) along the top and bottom of a slice of the pillar containing the extensometer was applied to a 2D BBM, and input parameters from a previously calibrated coal model was employed. Subsequently, two support schemes were tested to understand how the incorporation of support might affect rib deformation. It was found that the BBM was able to reproduce the unsupported rib deformation and depth of fracturing very well, but underestimated the effect of support. The latter observation was attributed to the absence of host rock layers in the BBM, which prevented the reduction of shear deformations along the roof-pillar and floor-pillar interfaces with increasing support density. Therefore, while the approach successfully reproduced the unsupported ground behavior, future research is necessary to establish the most appropriate method for incorporating roof and floor layers for prediction of ground-support interaction.

12.2 Introduction

Scientific research to improve the performance of coal pillar ribs, particularly through the use of support, has been ongoing for several decades (Horino et al., 1971; Guana, 1983; Dolinar and Tadolini, 1991; Smith, 1992; Colwell and Mark, 2005; Colwell, 2006; Guana and Mark, 2010). More recent endeavors by National Institute of Occupational Safety and Health (NIOSH) researchers have focused on minimizing rib hazard using numerical and/or empirical approaches (Mohamed et al., 2015, 2016a; Zhang et al., 2017b; Sears et al., 2018) and novel techniques like seismic monitoring (Slaker et al., 2018) and photogrammetry (Slaker and Mohamed, 2017). Despite these advances in our knowledge of pillar damage mechanisms and the rock-support interaction, rib-failure-related fatalities in underground coal mines have continued to occur. For
example, among incidents classified as “fall of face, rib or pillar”, the average fatality rate was 1.3 per year for 1996-2012 and about 40-50 injuries occurred every year from 2013-2017 (MSHA, 2019). These ongoing issues reflect the fact that there is a gap in our knowledge base that must be bridged before robust support guidelines can be formulated for improving worker safety.

### 12.2.1 Factors affecting rib spalling and existing rib support design approaches

Stress-driven rib damage, or spalling, is a complex phenomenon that is influenced by a multitude of interacting factors. Previous studies have attempted to identify the critical factors that cause rib spalling using statistical approaches (Guana and Mark, 2010; Jones et al., 2014; Mohamed et al. 2016a). Mines with large excavation heights (>2 m) and at deeper depths (>200 m) are typically more prone to stress-induced damage; other important factors include presence of stone bands, clay partings, and excavation orientation with respect to cleats (Jones et al., 2014; Mohamed et al. 2016a). In 2011, MSHA had also issued a Program Information Bulletin (PIB11-29) recommending the application of rib support in coal mines that are deeper than 210 m and whose mining height exceeds 2.1 m.

The actual support requirement in any given case depends largely on the stage of mining. For example, the extent and severity of rib damage will be less during entry development but can increase dramatically during longwall production (Colwell and Mark, 2005; Colwell, 2006). Although not explicitly stated in PIB11-29, cleat orientation is also an important factor and instability via bulking of thin slabs (Colwell and Mark, 2005; Jones et al., 2014) has been found to occur in coal mines when the angle between the roadway driving direction and the face cleat orientation is less than 30° (Paul, 1981).
Coal rib control techniques in US can be broadly sub-divided into (Mohamed et al., 2016a): (1) installing intrinsic support in the form of bolts with/without mesh, and, (2) installing external rib support systems in the form of mesh (steel or synthetic), props, pillar banding, etc. As indicated by Mohamed et al. (2016a), the design of both intrinsic and external support systems is based on local practices and site-specific experience. Some rough guidelines for rib support design are available for Australian coal mines (Colwell and Mark, 2005), but no design parameters for support systems have been established for US coal mines (Mohamed et al., 2016a).

The risk-based approach of Colwell and Mark (2005) is perhaps the first (and only) systematic rib support design technique to be developed for any country’s underground coal industry. In their approach, a RIBSUP value, which quantifies the density of support and includes rockbolts, faceplates and liner/mesh, is suggested depending on a mine’s depth of operation, height of rib and grindability of coal (a proxy for strength; Tiryaki, 2005). The tool is empirical in nature and is based on extensive field observations in Australian coal mines and as such is not employed in US mines.

Dolinar and Tadolini (1991) proposed a statistical technique for designing rib support schemes, but their approach requires parameters that are difficult to estimate, such as the width of the slabs formed by spalling. The approach of Dolinar and Tadolini (1991) also assumes that a single bolt placed on a reference line drawn along the center of the rib is sufficient to hold the entire height of the spalled slabs. This may not always be true, as the support requirement varies based on the weight of spalled slabs that must be supported. More importantly, this is an observational design approach (i.e. based on the general size distribution of the spalled slabs at the mine or a section of a mine), meaning that the parameters for a given mine are not
necessarily applicable to other mines. Empirical tools are undoubtedly valuable, but they are generally constrained by limitations of the database that was used for their development and it is often difficult to isolate/understand the effects of individual factors when the size of the data set is small or the data set is noisy. Numerical approaches have the potential for greater generalization given their fundamental physical basis and therefore represent a potentially viable alternative for support design.

12.2.2 Numerical approach for design of rib support

With recent advancements in numerical modeling methods and computational power, complex ground control problems are increasingly being studied using numerical approaches. Continuum models, which represent the ground as an equivalent continuous material and whose behavior is governed by constitutive relationships between stress and strain, are a popular tool for analyzing large-scale damage processes (Mohamed et al., 2016; Sears et al., 2018). For instance, Shabanimashcool and Li (2012, 2013) used a caving algorithm in FLAC\textsuperscript{3D} to replicate observed roof bolt loads and pillar loads due to longwall face advance at Svea Nord coal mine in Svalbard, Norway. Zhang et al. (2015) utilized FLAC\textsuperscript{3D} to study the stress changes in longwall pillars with various sizes. Basarir et al. (2015) studied the stress changes around entries due to approach of the longwall face for a Turkish lignite mine. Esterhuizen et al. (2010b) used FLAC\textsuperscript{3D} to simulate the abutment stresses in chain pillars for three different case studies. In all these studies, the coal pillars were modeled using either the strain-softening Mohr-Coulomb constitutive model or the softening Hoek-Brown model. A new rock yield criterion, called the progressive S-shaped yield criterion, that is based on the fundamental fracturing process of brittle rocks was employed in this thesis (Chapter 4) to model coal pillars. With its potential to simulate
coal damage established in Chapter 4, this study also employed the progressive S-shaped yield criterion to model coal pillars.

Despite their widespread application, a major limitation of continuum models is that they tend to significantly underestimate the reinforcement effect of installed support on ground behavior (Sinha and Walton, 2019b). The issue is attributed to the enforcement of continuous strain distribution (or inability for elements to detach) within these models. As an alternative, Sinha and Walton (2019b) tested the discontinuum bonded block model (BBM) approach; in this approach, a material space is represented by polygonal blocks, and the parameters of the blocks and contacts (interfaces between blocks) control the macroscopic behavior of the models. It was found that BBMs can reproduce realistic behavioral differences between unsupported and supported ground conditions that are consistent with previous empirical observations.

More recently, Sinha and Walton (2020d) employed the BBM approach to simulate the observed rib damage in a longwall chain pillar in the West Cliff Mine (Australia). In that model, the blocks were inelastic (blocks can yield) and were elongated along the pillar height in order to allow fractures to form preferentially along the cleat direction. After being calibrated to displacement and stress data from one site in the mine, the support in the model was modified to match the support pattern at an adjacent site, and pillar displacements similar to those recorded at this adjacent site were obtained. This finding broadly suggests that the elongated BBM approach is not only capable of reproducing observed pillar damage phenomena, but also the ground-support interaction mechanism, and therefore has the potential to be used as a support design tool. Prior attempts to simulate coal damage processes using polygonal BBM and other discontinuum modeling approaches are summarized in Chapter 6.
Discontinuum models are computationally expensive, and it is typically impractical to develop a 3D entry-scale model with a reasonable runtime, let alone a mine-scale model. Consequently, most prior BBM studies are restricted to 2D or 2.5D (e.g. 1-2 m thick models) (Bai et al., 2016; Farahmand et al., 2018; Yang et al., 2018; Garza-Cruz et al., 2019b; Bai and Tu, 2020), with simplified boundary conditions (e.g. purely vertical loading). In reality, underground structures like coal pillars are subjected to complex load paths that consist of both vertical and shear components (Garza-Cruz et al., 2018; Sinha and Walton, 2020c), heterogeneously distributed across their cross-sections. An integrated 3D continuum – 2D discontinuum modeling approach was recently proposed by Sinha and Walton (2020c), in which the mine-wide 3D stress distribution process is simulated using a continuum model (e.g. FLAC\textsuperscript{3D}) and the local deformational behavior of a structure of interest subjected to the strain path obtained from the FLAC\textsuperscript{3D} model is simulated using a 2D discontinuum model (UDEC). There are three major advantages of this approach: (1) Restriction of the discontinuum modeling to 2D ensures that individual model run-times remain low enough for practical use (e.g. on the order of a couple of days), (2) the essential influences of the complex load path that the structure under consideration is subjected to are accounted for, and (3) the inability of continuum models to replicate the effect of support on ground behavior is overcome, as the local stability and support design analysis is conducted using the 2D BBM. Of course, calibration of the FLAC\textsuperscript{3D} model and the BBM against field-measured attributes is necessary in order to ensure that any predictions of the effect of support on ground behavior can be considered reasonable approximations of reality.
12.2.3 Study Objectives and Methodology

This study attempts to answer three major research questions: (1) Is the integrated 3D continuum – 2D discontinuum modeling approach capable of simulating the damage in a coal pillar rib? (2) Can this modeling approach in its present form be applied for design of rib support schemes? (3) If not, what modifications might be necessary to improve the predictive capabilities of such a local BBM?

To address these research questions, the integrated modeling approach was employed to simulate the damage evolution observed in a longwall chain pillar in a Western US mine (referred to as Mine A). In June 2017, two multipoint extensometers were installed in a chain pillar (one from the entry and one from the cross-cut) and deformations were subsequently monitored as the longwall face approached and crossed the instrumented location. Based on a borehole log from the site, a FLAC$^3$D model was developed and ultimately calibrated to match the extensometer measurements. Subsequently, a slice of the coal pillar containing the extensometer in the cross-cut was modeled using an inelastic BBM with boundary conditions derived from the calibrated FLAC$^3$D model. The ultimate goal is to examine the simulated response of the pillar to the addition of support, as might be done in the context of a practical design analysis.

12.3 Site description and overall FLAC$^3$D model setup

The longwall mine under consideration is located in Montana and extracts coal from the Mammoth seam. The surface topography is mountainous, with the depth of operation varying between 91 m to 250 m. Two multipoint extensometers were installed horizontally at the mid-
height of one of the chain pillars when the longwall face was operating in the next panel (Figure 12.1). The instrumented panel was ~360 m wide and ~6500 m long. At the instrumented site, the overburden depth was ~250 m, height of the coal seam was ~3.5 m, the entry and cross-cut widths were ~5.5 m, and the pillars dimensions were 62 m x 22 m. No intrinsic rib supports were used in any part of this panel.

Based on a borehole log located close to the instrumented location (within 50 m) and a surface topography map, a FLAC$^{3D}$ model was developed that extended a half panel into the coal to be extracted, 111 m in the opposite direction (into solid unmined coal) and 3-4 chain pillars on either side of the instrumented pillar (Figure 12.1). A distance of 111 m is determined to be sufficient for the stresses to return to their in-situ level, based on the empirical load transfer distance (LTD) equation of Peng and Chiang (1984):

\[
LTD \ (m) = 5.13 \sqrt{H \ (m)}
\]  

(12.1)

LTD for instrumented location (m) = \(5.13 \sqrt{250 \ (m)} = 81 \ m < 111 \ m\)

The dimensions were chosen to permit the use of a finer mesh size in the pillar under consideration, while still maintaining a model runtime of less than one week to allow for iterative model calibration to be completed. The zones were ultimately graded away from the instrumented pillar to reduce the total number of model zones (Figure 12.1). The overall dimensions of the model are shown in Figure 12.1.

For the site under consideration, reliable rock mechanics data such as in-situ stress measurements and the strength and deformation properties of the roof and floor lithologies or coal seam were not available. Consequently, all these parameters had to be estimated. The roof
layers were simulated using the strain-softening ubiquitous joint constitutive (SUBI) model, with parameters taken from a paper by Tulu et al. (2017) that provides practical estimates of SUBI input parameters for various coal measure rocks. The layers below the coal seam were modeled as elastic, as no heaving issues or floor damage were observed at the site. Discontinuum interfaces were also placed on either side of the coal seam to allow the coal seam to slip relative to the immediate roof and floor layers. Slippage along lithological boundaries is probably important at the entry-scale, but is unlikely to have any major effect on the stress distribution process at the mine-scale (Esterhuizen et al., 2010b; Li et al., 2015). The presence of ubiquitous joints in the roof does, however, partially account for failure and slippage along beddings in the overburden.

Roller boundary conditions were placed along the model bottom and along all the lateral edges, while the top boundary was left unconstrained, as it represents the actual ground surface. In the absence of site-specific horizontal stress data, a horizontal to vertical stress ratio (k) of 1 was ultimately used, based on the following considerations: (1) The location of the mine on the World Stress Map (Heidbach et al., 2016) is near the midpoint between a normal faulting regime and a thrust faulting regime, which suggests that the vertical and horizontal stresses should be similar; (2) Mark (1991) analyzed stress measurements from 17 Western US coal mines and found that the maximum horizontal stress was significantly lower than in the Eastern US, and was approximately equal to the vertical stress in most cases.

In the FLAC\textsuperscript{3D} model, once the in-situ stresses were initialized, the entry was excavated and the model was brought to an initial state of equilibrium. After this, the position of the longwall face was advanced in 7 stages (Figure 12.1), and the model was brought to mechanical equilibrium after every stage. To advance the longwall face and simulate the strain-hardening
response of gob, the mined portion of the coal was replaced by a strain-softening material, whose cohesion was updated continuously as a function of the critical plastic shear strain (Esterhuizen et al., 2010b).

Figure 12.1 FLAC$^{3D}$ model setup showing the different lithologies, extensometer locations and model stages. SS=Sandstone, MS=Mudstone, Exto=Extensometer.
In the FLAC<sup>3D</sup> model, once the in-situ stresses were initialized, the entry was excavated and the model was brought to an initial state of equilibrium. After this, the position of the longwall face was advanced in 7 stages (Figure 12.1), and the model was brought to mechanical equilibrium after every stage. To advance the longwall face and simulate the strain-hardening response of gob, the mined portion of the coal was replaced by a strain-softening material, whose cohesion was updated continuously as a function of the critical plastic shear strain (Esterhuizen et al., 2010b).

The 7 locations of the longwall face used in the 7 model stages were selected based on the extensometer measurements (see Figure 12.2). The outermost head of the extensometer in the cross-cut/XC (Extensometer 1) started detecting movements when the longwall face crossed the instrumented location (XC 77) on September 7<sup>th</sup>. Displacements then increased rapidly between September 9<sup>th</sup> and September 11<sup>th</sup> from 5 mm to 120 mm, at which point the extensometer was damaged (Figure 12.2a). During this time, the longwall face advanced from XC 76 to slightly beyond the midpoint between XC 75 and 76; this longwall position, which bounds the extent of reliable extensometer data, is the 7<sup>th</sup> stage in the FLAC<sup>3D</sup> model. Figure 12.3 shows the state of the ribs at the location of Extensometer 1 on September 29<sup>th</sup>. The rib had collapsed completely and the extensometer heads are visible. The spalling and buckling of thin coal slabs is readily evident from this figure, and the damage in the upper part of the coal seam appears more severe than that in the lower part of the seam.

Interestingly, no deformation was recorded by Extensometer 2 until September 20<sup>th</sup>, when the longwall face had advanced by an additional 2 XCs (Figure 12.2b). Less damage manifestation in this entry (Figure 12.3) indicates that cleating along the XC and more stress transfer into the solid coal were responsible for such anisotropic damage development. During
subsequent field visits, the author observed pronounced damage in the XCs, but less damage in the entries throughout the mine. This suggests that excavation orientation with respect to cleats is an important factor controlling the integrity of the coal ribs. The XCs are oriented parallel to the face cleats at this mine because such a design choice allows the longwall face to be oriented parallel to the face cleats, which improves the mineability of the coal.

Figure 12.2 (a) Face position and displacements measured by the outermost anchor of the extensometer located in XC 77 (Extensometer 1) as a function of date; the extensometer data used for calibration is in solid black, while the data from an additional redundant extensometer located at the same position is shown by the dotted black line. (b) Displacements measured by the outermost (head) and two adjacent anchors of the extensometer located in the entry between XC 77 and XC 78 (Extensometer 2) as a function of date. The head is 0.2 m long and Anchors 1 and 2 of Extensometer 2 are located 0.3 m and 0.9 m from the end of the head. Anchors 1 and 2 of Extensometer 1 are spaced differently and located at 0.2 m and 0.5 m from the end of the head.

It would have been useful to extend the model boundary by 5 more XCs such that the input parameters could be constrained by both Extensometer 1 and 2 data, but such a model was found to be computationally very expensive. More importantly, since the damage in the entry and XC are somewhat dependent on the cleat orientation, an anisotropic rock yield criterion or modification to the isotropic progressive S-shaped yield criterion would be necessary to simulate
the damage at both these locations. This is beyond the scope of the thesis, but is an interesting research topic for future consideration. For the purposes of the current study, the boundary was restricted to 120 m beyond Stage 7 longwall face position and calibration was conducted using Extensometer 1 data. Although two extensometers were installed at the same location for redundancy (Figure 12.3), data from only one of them (i.e. Extensometer 1) has been used for calibration, as both extensometers recorded nearly identical displacements (Figure 12.2a). Naturally, the use of an isotropic yield criterion in the FLAC$^{3D}$ model led to more yield in the entries than occurred in reality and to compensate for this behavior, relatively strong coal parameters had to be used during the calibration process.

The other longwall face locations for Stages 1-6 were selected to span the timeframe when the Extensometer 1 was not detecting any movements to when the extensometer was completely damaged. Note that there are some temporary delays in deformation between September 9$^{th}$ and September 11$^{th}$ (Figure 12.2a), which could be related to either intermittent pauses in production or transient (i.e. time-dependent) crack propagation and dilation phenomena. The “face location” curve in Figure 12.2a does not include any short-term production pauses, as this curve was interpolated from discrete data points provided by the mine (i.e. a continuous face position record was not available; longwall face positions may have been recorded after every working shift). Since the extensometer was recording the displacements continuously in real time and the longwall face positions are only approximate, it would not be appropriate to advance the longwall face in the FLAC$^{3D}$ model by small increments and then compare model results against the entire deformation curve. This is why only 7 stages were considered; the model input parameters were ultimately constrained by matching the deformations at the rib surface to those measured by Extensometer 1 for each of these stages.
12.4 FLAC\textsuperscript{3D} model setup, calibration and results

Broadly speaking, there are 3 groups of parameters that had to be constrained in the FLAC\textsuperscript{3D} model – the progressive S-shaped parameters for coal, the gob parameters, and the overburden properties. Unlike the coal parameters that could be constrained independently, the last two groups had to be calibrated together as they mutually interact to control the surface subsidence and loading on the chain pillars. Accordingly, the gob parameters and overburden
parameters were calibrated first so that the loading on the chain pillars are realistic, followed by
calibration of the coal parameters. In general, the coal parameters had negligible effect on the
subsidence profile, and therefore the preliminary progressive S-shaped parameters determined in
Section 12.4.1 were used during the gob and overburden parameter calibration. Later, the
preliminary coal parameters were modified until the displacements from Extensometer 1 could
be reproduced.

12.4.1 Progressive S-shaped criterion and preliminary coal pillar parameters

The progressive S-shaped yield criterion consists of three major thresholds (see Section
4.3): (1) Yield threshold, (2) Peak threshold, and (3) Residual threshold; all three thresholds can
be sub-divided into a low confinement (spalling) section and a high confinement (shearing)
section. In this study, the yield criterion was slightly modified, in that the requirement that all
high confinement sections should coincide at a particular confining stress was omitted. Such a
modification was introduced based on Appendix F, which shows that the CI threshold does not
necessarily bend upward (compare Figures F.5 and F.8) for coal. Therefore, the bi-linear shape
of the Yield threshold was replaced by a straight line, which evolved to a bi-linear Peak
threshold and then degraded to the Residual threshold. The high confinement section of the
Residual threshold was assigned the same friction angle value as that of the high confinement
section of the Peak threshold. Figure 12.4 illustrates the modified yield criterion in $\sigma_1-\sigma_3$ space.
Note that this modification is unlikely to affect the conclusions drawn in Chapter 4, as the focus
of Chapter 4 was mainly on the damage development in the low confinement regime (or left side
of the progressive S-shaped yield criterion, which remains unchanged).
To determine a preliminary set of coal parameters that could be directly applied to the pillars in the mine-scale model, hypothetical pillar models with W/H = 2, 4, 6 and 8 geometries were loaded in compression up to failure and the peak strengths were compared against the empirical Mark-Bieniawski pillar strength equation (Mark and Iannacchione, 1992). Since the pillar at Mine A was long, these hypothetical FLAC$^3$D models were made only 1 element thick to mimic a plane-strain condition (Figure 12.5a). Interfaces were placed on either side of the coal seam, and the same zone size as in the instrumented pillar in the mine-scale FLAC$^3$D model was used. Loading was conducted via two elastic beams (Elastic modulus/E and Poisson’s ratio/ν of 8 GPa and 0.25, respectively); the bottom surface was fixed, and a very slow velocity of $5 \times 10^{-8}$ m/step was applied to the top surface to simulate a pseudo-static loading condition. The different W/H models had the same height of 2.5 m, but the width was varied to match the corresponding ratio. The interfaces at the top and bottom of the pillar had zero tensile strength, 0.1 MPa cohesive strength and a friction angle of 25° (Esterhuizen et al., 2010b) in both these hypothetical pillar models and in the mine-scale FLAC$^3$D model.
Following a manual iterative back analysis process, a set of progressive S-shaped criterion parameters was identified that could reproduce the transition from brittle for W/H=2 to ductile behavior for W/H=8 (Figure 12.5b; Das, 1986; Esterhuizen et al., 2010b) and also matched the target peak pillar strengths (Figure 12.5c). When computing the peak strengths per the Mark-Bieniawski equation, the length was set to infinite to approximate the plane-strain assumption in the hypothetical FLAC\textsuperscript{3D} models. Note that although the pillar at Mine A had a L/W (length to width ratio) of ~3, the progressive S-shaped yield criterion parameters obtained herein are still applicable to the mine-scale model, as the effect of length on peak strength becomes negligible beyond L/W of 3 (Dolinar and Esterhuizen, 2007; Sinha and Walton, 2019a).

The preliminary calibrated coal parameters are listed in Table 12.1.

Table 12.1 Progressive S-shaped yield criterion for coal obtained from hypothetical FLAC\textsuperscript{3D} pillar models.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield threshold (cohesion, MPa)</td>
<td>6.0</td>
</tr>
<tr>
<td>Yield threshold (friction angle, degrees)</td>
<td>0</td>
</tr>
<tr>
<td>Peak threshold; left side (cohesion, MPa)</td>
<td>0.1</td>
</tr>
<tr>
<td>Peak threshold; left side (friction angle, degrees)</td>
<td>50</td>
</tr>
<tr>
<td>Peak threshold; right side (cohesion, MPa)</td>
<td>9.5</td>
</tr>
<tr>
<td>Peak threshold; right side (friction angle, degrees)</td>
<td>25</td>
</tr>
<tr>
<td>Residual threshold; left side (cohesion, MPa)</td>
<td>0.05</td>
</tr>
<tr>
<td>Residual threshold; left side (friction angle, degrees)</td>
<td>22</td>
</tr>
<tr>
<td>$\epsilon_{ps}$ from yield to peak (millistrain)</td>
<td>9</td>
</tr>
<tr>
<td>$\epsilon_{ps}$ from yield to residual (millistrain)</td>
<td>45</td>
</tr>
<tr>
<td>Dilation angle (°)</td>
<td>15</td>
</tr>
</tbody>
</table>
12.4.2 Gob and overburden parameters

Gob is an aggregate of broken rocks, which when subjected to increasing load hardens in a hyperbolical manner (Pappas and Mark, 1993). The dramatic increase in load carrying capacity as a function of strain arises from the reduction in void space and an increase in surficial contacts between broken rock fragments that allows for better stress transfer. When the longwall face advances, the immediate roof layers before the face collapse and form the gob. The portion of the
gob closest to the longwall face is loose and has the lowest load carrying capacity, while the portions of the gob away from the face are compact, and in the limit, should carry loads equivalent to the in-situ stress (Abbasi et al., 2014; Peng, 2019). The ability of gob to carry larger loads is also observed as one traverses the panel across its width - closer to the chain pillar, the gob is relatively loose (less gob loading/strain due to stress transfer to the chain pillars) while towards the panel center, it is relatively compact (less arching away from the pillars).

Gob has been historically modeled in FLAC\textsuperscript{3D} using three different approaches – (1) The gob elements are elastic and their elastic modulus is varied as a function of axial strain to reproduce the expected hyperbolic behavior (e.g. Tulu et al., 2017), (2) The gob elements are modeled using the double-yield constitutive model, where the cap pressures are defined as a function of plastic volumetric strain (e.g. Li et al., 2015), and (3) The gob is modeled as a strain-softening material with cohesion being varied as a function of critical plastic shear strain to obtain a hardening response (e.g. Esterhuizen et al., 2010b). The third approach was employed in this study.

In this approach, the friction angle and tensile strength of all gob elements are set to zero and the cohesive strength (half of the major principal stress the element can carry) is defined for various critical plastic shear strain values that conform to a specific hyperbolic curve. During each solution cycle, all stress components except the vertical stress are initialized to zero such that the elements are loaded in uniaxial compression and the plastic shear strain develops only due to inelastic deformation in the vertical direction (e.g. due to direct overburden loading). When defining the evolution of cohesion, it must be recognized that the total strain in each element consists of both elastic and plastic components, but the desired plastic strain is not
equivalent to the critical plastic shear strain. This is because of the relationship between critical plastic shear strain and principal plastic strains ($\epsilon^p_1$ and $\epsilon^p_3$):

$$\epsilon^{ps} = \frac{1}{\sqrt{2}} \sqrt{(\Delta \epsilon^p_1 - \Delta \epsilon^p_m)^2 + (\Delta \epsilon^p_m)^2 + (\Delta \epsilon^p_3 - \Delta \epsilon^p_m)^2} \quad (12.2)$$

$$\Delta \epsilon^p_m = \frac{1}{3} (\Delta \epsilon^p_1 + \Delta \epsilon^p_3) \quad (12.3)$$

If $\Delta \epsilon^p_3$ is set to zero in this equation, then the relationship between plastic strain in the major principal strain direction (vertical, in this case) and $\epsilon^{ps}$ simplifies to the following:

$$\epsilon^{ps} = \frac{1}{\sqrt{3}} \Delta \epsilon^p_1 \quad (12.4)$$

In terms of the overburden, Tulu et al. (2017) provides matrix and ubiquitous joint parameters for three rock types - sandstone, limestone and shale, and each rock type is further sub-divided on the basis of uniaxial compressive strengths (UCS). Parameters were either directly chosen from the paper or were linearly interpolated/extrapolated if the parameter set to be tested was not explicitly listed in Tulu et al. (2017). These parameters were not modified during the calibration process. Note that care was taken to ensure that the strengths assigned in the models were consistent with the following strength relationships that would typically be expected: Sandstone (SS) > Interbedded SS and MS > Mudstone (MS).

To constrain the gob and overburden parameters, an attempt was made to reproduce a subsidence profile that is typical of Mine A. The mine was able to provide subsidence contours for five previous panels (Panels 1-5), but had no data for the panel under consideration (Panel 6), as the mine usually performs subsidence analysis after completion of every 5 panels. On the subsidence map, four cross-sections were drawn immediately next to the instrumented site in the
adjacent panel (Panel 5) and the average distance from the panel edge where the magnitude of
subsidence was 0.3 m (1 ft), 0.6 m (2 ft), 0.91 m (3 ft), 1.22 m (4 ft), 1.52 m (5 ft) and 1.83 m (6
ft) was identified. These numbers correspond to the contours on the map, and since the survey
was via aerial mapping, the subsidence precision was limited to 0.3 m (1 ft). The cross-sections
were restricted to the half of Panel 5 that was closer to the solid coal, analogous to measuring
subsidence along Section A’A” in Figure 12.1.

During the calibration process, it was observed that the gob parameters influenced the
load transfer distance and the maximum subsidence at the model top surface (softer gob resulted
in larger maximum subsidence). Overburden parameters mainly controlled the shape of the
subsidence profile, with weaker overburden layers resulting in sharper drops in subsidence along
the panel edge.

Figure 12.6 compares the subsidence values in the calibrated model to those obtained
from the subsidence map. Despite testing a variety of weaker overburden parameters, a sharper
decline in the subsidence profile (as indicated in the subsidence map) could not be attained. The
inability to accurately match the slope of the subsidence profile could be related to the simplified
overburden geometry employed in the FLAC\textsuperscript{3D} model, geological structures, the differences in
the topography above Panel 5 and 6, or other unknown factors. In particular, only one set of
SUBI properties was assigned to each rock type in the model, but in reality, different layers of a
given rock type (e.g. Mudstone) could have different properties. In the absence of pertinent rock
mechanical data, however, further increases in model complexity to more precisely match the
subsidence profile were not considered justified. Each lithology was considered to be uniform in
thickness and perfectly horizontal, which also might not reflect reality. Despite the imperfect
match obtained regarding the exact subsidence trough, the maximum subsidence slopes are
similar, and most importantly, the subsidence plateau at the panel center of ~1.8 m observed in the field was well reproduced in the calibrated model (Figure 12.6). The corresponding overburden and gob parameters are listed in Tables 12.2 and 12.3, respectively.

Figure 12.6 Comparison of model results and field measurements for subsidence.

The total stress-strain relationship defined by the gob parameters in Table 12.3 is shown in Figure 12.7. It can be seen that the gob in the calibrated FLAC$^{3D}$ model is much softer than the ‘Sandstone’ and ‘Shale’ gob curves estimated by Pappas and Mark (1993) using laboratory compression tests on a rock fragment gob analog. Such a large difference occurs because the gob in this study was modeled using zone elements with height equivalent to that of the coal seam. In other words, if the height of the caved region is considered to be 3 times the mining height (a reasonable assumption based on Su, 1991), then the total deformation due to gob compaction
was compositely modeled using one seam-height element in the current FLAC\textsuperscript{3D} model (i.e. one third height but three times the strain corresponds to an equivalent total displacement). Other studies have extended the gob to a few meters into the roof (Tulu et al., 2017) but this was difficult to implement because the zone sizes in the gob and roof are different. Since displacements are dependent on the height of the zone elements for a given strain, a softer gob was necessary to allow the surface to subside by 1.8 m. For example, if the strain is 0.1, then the corresponding deformations for 2.5 m and 7.5 m of gob will be 250 mm and 750 mm, respectively.

![Figure 12.7 Modeled gob compared to ‘sandstone’ and ‘shale’ laboratory gob curves from Pappas and Mark (1993).](image)

Interestingly, if the strain of the modeled gob curve is scaled by a factor of 0.33, which is analogous to having 3 gob elements stacked on top of one another (or a caving height of 3 times
the mining height), then the curve moves close to the ‘Shale’ curve. This is not surprising, given that there are several mudstone and interbedded mudstone and sandstone layers present in the overburden (Figure 12.1).

Table 12.2 Properties of overburden layers. MS=Mudstone and SS=Sandstone. Some properties were extrapolated/interpolated (after Tulu et al., 2017).

<table>
<thead>
<tr>
<th>Layer</th>
<th>( E ) (GPa), ( v )</th>
<th>Matrix</th>
<th>Ubiquitous joint (parallel to bedding)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cohesion (MPa)(^1)</td>
<td>Friction Angle (°)(^2)</td>
<td>Tensile Strength (MPa)(^1)</td>
</tr>
<tr>
<td>SS</td>
<td>10.5, 0.25</td>
<td>5.22</td>
<td>28</td>
</tr>
<tr>
<td>MS</td>
<td>7.59, 0.25</td>
<td>2.03</td>
<td>20</td>
</tr>
<tr>
<td>Interbedded SS and MS</td>
<td>8.31, 0.25</td>
<td>3.05</td>
<td>20</td>
</tr>
<tr>
<td>Alluvium(^3)</td>
<td>2, 0.28</td>
<td>0.12</td>
<td>10</td>
</tr>
</tbody>
</table>

\(^1\)degrades to 10% of peak over plastic strain of 0.005; \(^2\)no degradation; \(^3\)strain-softening material with no ubiquitous joints.

Table 12.3 Table of cohesion for gob elements.

<table>
<thead>
<tr>
<th>( \epsilon^{ps} )</th>
<th>0</th>
<th>0.09</th>
<th>0.10</th>
<th>0.15</th>
<th>0.22</th>
<th>0.28</th>
<th>0.35</th>
<th>0.41</th>
<th>0.48</th>
<th>0.54</th>
<th>0.58</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cohesion (MPa)</td>
<td>0.001</td>
<td>0.001</td>
<td>0.04</td>
<td>0.22</td>
<td>0.49</td>
<td>0.87</td>
<td>1.4</td>
<td>2.2</td>
<td>3.57</td>
<td>6.41</td>
<td>15</td>
</tr>
</tbody>
</table>

12.4.3 Calibration of the coal parameters and associated results

With the overburden and gob parameters constrained, the next task was to adjust the coal pillar parameters obtained in Section 12.4.1 to match the displacements measured by Extensometer 1 (Figure 12.2). Each model took about 4-5 days to complete on a machine with an 8-core 3.6 GHz Intel i9-9900K processor and 64 GBs of RAM. During model calibration, the
right side of the progressive S-shaped criterion was kept unchanged, as these parameters did not significantly impact the progression of the low confinement damage along the pillar periphery.

Changes were made iteratively to the cohesion of the Yield threshold, friction angle of Peak threshold, friction angle of the Residual threshold, and the dilation angle. Table 12.4 lists the final calibrated coal pillar parameters, and Figure 12.8 shows the model-predicted displacements. It can be seen that the model was able to reproduce the start (Stage 5) and the end (Stage 7) of the main displacement increase very well. A slight mismatch in displacements (~8 mm) occurred at Stage 6, but this difference is not practically significant given the uncertainty in the position of the longwall face on different measurement dates and the overall brittleness of the deformation process.

Table 12.4 Progressive S-shaped yield criterion for Mine A.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield threshold (cohesion, MPa)</td>
<td>7.5</td>
</tr>
<tr>
<td>Yield threshold (friction angle, degrees)</td>
<td>0</td>
</tr>
<tr>
<td>Peak threshold; left side (cohesion, MPa)</td>
<td>0.1</td>
</tr>
<tr>
<td>Peak threshold; left side (friction angle, degrees)</td>
<td>58.5</td>
</tr>
<tr>
<td>Residual threshold; left side (cohesion, MPa)</td>
<td>0.05</td>
</tr>
<tr>
<td>Residual threshold; left side (friction angle, degrees)</td>
<td>30</td>
</tr>
<tr>
<td>$e^{ps}$ from Yield to Peak (millistrain)</td>
<td>9</td>
</tr>
<tr>
<td>$e^{ps}$ from Yield to Residual (millistrain)</td>
<td>45</td>
</tr>
<tr>
<td>Dilation angle (°)</td>
<td>30</td>
</tr>
</tbody>
</table>

In terms of model parameters, significantly higher strengths with respect to those obtained in Section 12.4.1 (see Table 12.1) were required to achieve the observed displacement trends. The most prominent changes were in the Yield threshold cohesion (left side), the
Residual threshold (left side) friction angle and dilation angle values. A larger Yield threshold cohesion and Residual threshold friction angle suggest the need for delayed damage initiation and greater ability for the confined coal to carry load post-yield, respectively. A larger dilation angle, on the other hand, relates to greater inelastic deformation within the yielded elements and is mathematically explained by the following equation (Vermeer and de Borst, 1984):

$$
\dot{\varepsilon}_3^P = -\frac{\dot{\varepsilon}_1^P}{2} \left( \frac{1+\sin \psi}{1-\sin \psi} \right)
$$

where, $\dot{\varepsilon}_1^P$ and $\dot{\varepsilon}_3^P$ are the maximum and minimum plastic strain increments, and $\psi$ is the dilation angle. The function $1/(1 − \sin \psi)$ is strictly increasing for $\psi \in [0^\circ, 90^\circ]$, meaning that for larger dilation angle values, more extension in $\varepsilon_3^P$ is obtained per increment of compression in $\varepsilon_1^P$ or vice-versa.

Before presenting any discussion on why changes in coal parameters were required, the concepts of ‘depth of yield’ and ‘depth of collapsed area’ in the FLAC3D model need to be introduced. Figure 12.9b shows the critical plastic shear strains in the FLAC3D model at Stage 7 and Figure 12.9c shows the rib displacement as a function of distance into the pillar for Extensometer 1. The Residual threshold is attained in the FLAC3D model at $\varepsilon^{ps}$ of 0.045, and the corresponding contour could be considered to approximate the edge of the collapsed region (0.87 m). The point where $\varepsilon^{ps}$ becomes minimal / negligible (1.87 m) is the edge of the yielded region. From this figure, it is clear that the damage is much more localized in the field when compared to the FLAC3D model. The FLAC3D zone damage was also more pronounced in the upper parts of the pillar (Figure 12.3b), as was observed at Mine A (Figure 12.3a).
The increase in the coal strength properties and inability of the FLAC\textsuperscript{3D} to reproduce the ‘depth of yield’ and ‘depth of collapsed area’ can be explained using physical principles. An isotropic rock yield criterion cannot simulate the preferential fracture development in an anisotropic rock, and since the calibration goal was to match the deformations in the XC, large yield occurred in the entries as well. The greater entry yield transferred more stresses onto the chain pillars located ahead of the longwall face, thereby necessitating the use of stronger Yield threshold parameters to delay the initiation of damage. This is illustrated using vertical stress contours along a section passing through the middle of the coal seam (Figure 12.9a) – note the ‘red’ regions that correspond to limited load transfer through the pillars. Also note how the vertical stress increases away from the solid coal and the chain pillars, consistent with the previous discussion on gob loading.

Figure 12.8 Comparison of model results and field measurements for rib displacements.
Figure 12.9 (a) Vertical stress contour at Stage 7 along a horizontal section passing through the middle of the coal seam, (b) Critical plastic shear strain in the rib under consideration at Stage 7, (c) Rib displacement versus distance into the rib from Extensometer 1.
The Residual threshold friction angle controls how far yielding progresses into the pillar, and this had to be raised to 30° to restrict the spatial extent of the damaged region. Dilation angle also had to be increased to 30° to increase the inelastic minimum plastic strain component (or lateral deformation). A number of different combinations of these two parameters were tested, but a better match against the target displacements and ‘depth of yield’ could not be attained. This mainly occurred because of a competing feedback loop between dilation angle and the Residual threshold – larger dilation angle led to a larger ‘depth of yield’ (also refer to Figure 15 in Zhao et al., 2010a) and deformations at the rib surface, and a stronger Residual threshold decreased the ‘depth of yield’ and the surficial deformations. Understandably, the displacements at locations deeper within the rib is overestimated and it is due to this reason that the comparison against the data from other the anchors was excluded.

It is not surprising that the FLAC$^{3D}$ model was unable to reproduce the observed localized, highly dilatant discontinuum process (Figure 12.3), given that other authors have previously identified this drawback of continuum models. Lorig and Varona (2013) note “Numerical models, especially continuum models, are not usually capable of realistically simulating the disaggregation of the rock mass as it deforms”, and Corkum et al. (2012) state “Observed tunnel displacements in brittle rocks are far greater than would be predicted by plasticity theory / shear failure analysis methods, even using the highest possible dilation values (e.g., 45°). As a result, continuum numerical models based on plasticity theory are unable to match observed displacements. This is a significant limitation, particularly for the design of ground support systems and components”.

It is also hypothesized that part of the difficulty in matching the overall depth of collapse (~0.4 m; Figure 12.9c) is the size of the model zones (0.25 m) relative to this depth. The pillar
under consideration was discretized by zones that are already smaller than most previous mine-scale studies (e.g. Esterhuizen et al., 2010b; Basarir et al., 2015; Li et al., 2015; Klemetti et al., 2019; Feng et al., 2019; Basarir et al., 2019; Klemetti et al., 2020); in total, the current FLAC\textsuperscript{3D} model had 4.3 million zones (relative to the 1 million zone threshold deemed as “practical” by Klemetti et al., 2019 and 2020). A secondary reason as to why the FLAC\textsuperscript{3D} model over-predicted the ‘depth of collapsed area’ could be that the Residual threshold does not correspond to the collapsed material and a critical plastic shear strain larger than 45 mstrain should be used. In any case, despite these limitations in the FLAC\textsuperscript{3D} model calibration, it is important to recall that the ultimate goal of this model is to obtain a reasonable approximation of the loading path at the pillar boundaries to be applied to a 2D discontinuum model. As will be shown in the subsequent section, it might be sufficient to only apply an approximate strain path from a FLAC\textsuperscript{3D} model, even if the actual extent of damaged rockmass is not fully reproduced.

12.5 The integrated modeling approach – setup and results

12.5.1 Bonded Block Model (BBM) setup for Mine A

Following preliminary calibration of the FLAC\textsuperscript{3D} model, the integrated 3D continuum – 2D discontinuum approach introduced in Chapter 10 was employed to develop a local pillar model of Mine A. Similar to Chapter 10, a slice of the pillar containing Extensometer 1 was modeled in UDEC under a strain path derived from the mine-scale FLAC\textsuperscript{3D} model. The roof and floor layers were not modeled explicitly, but their effect is implicitly accounted for in the boundary conditions applied to the pillar.
Figure 12.10 shows the BBM setup – the model has two continuum sections on either side of the 5 m wide Voronoi section. A 5 m discontinuum section is sufficient in this case as the depth of yield does not extend beyond 2 m (Figure 12.9c). The geometrical details of the Voronoi section from Chapter 6 were used directly such that the input parameters in Table 6.3 could be used as a starting point for model calibration. Specifically, each block had an average edge length of 0.05 m (effective block width is ~0.07-0.08 m), elongated in the vertical direction by a factor of two to mimic cleat planes (see Section 6.3.2) and was discretized using multiple constant-strain triangular zones. An inelastic constitutive model (specifically the Cohesion-Weakening-Frictional-Strengthening or CWFS model) was assigned to all zones in the BBM such that damage near the pillar periphery is explicitly represented by contact failure, while finer-scale damage occurring deeper within the pillar is approximated by a combination of contact failure and zone yield (see Section 9.3.3).

![Figure 12.10 BBM geometry and setup.](image)

The model sequencing approach used is similar to that applied in Chapter 10 and is briefly described below:
(1) In-situ stresses corresponding to those in the FLAC³D model at the instrumented location were applied to the entire BBM and the model was brought to mechanical equilibrium. Next, the entry was excavated and the unbalanced forces were relaxed in 10 stages using UDEC’s built-in ZONK function. This gradual relaxation is necessary in order to avoid unrealistic yielding/fracturing along the entry due to a sudden increase of the unbalanced forces in the model.

(2) In order to apply the same load path to the BBM as was experienced by the calibrated FLAC³D pillar section, a strain-controlled loading approach was adopted. In particular, the displacements (horizontal and vertical) along the top, bottom and left edges of the pillar were recorded for each of the 7 stages, segmented into multiple sub-stages and then applied to the BBM via a velocity boundary condition. The segmentation was performed to avoid applying a large displacement to the BBM in one step, which could result in unrealistic fracturing. The number of sub-stages was based on the displacement difference between two consecutive stages (when the difference was large, a larger number of sub-stages was used). To segment a stage, the difference in a given displacement component from the previous stage to the stage of interest is split equally into the required number of sub-stages. Ultimately, 20 total sub-stages were considered in the final BBM such that the displacement increase (horizontal and vertical) between any two sub-stages did not exceed 5 mm at any point along the pillar boundaries (Figure 12.10). Note that it is the differences between the displacements at two subsequent stages (or sub-stages) that were applied to the BBM as velocity boundary conditions, and not the raw displacement magnitudes.
(3) The zones in the Voronoi section of the BBM were much smaller than the zones in the FLAC\textsuperscript{3D} model. Therefore, the displacements from the FLAC\textsuperscript{3D} model could not be directly applied to these gridpoints (gridpoints are vertices of the zones). To resolve this issue, the addresses of all gridpoints along the top and bottom of the BBM pillar were extracted and classified into 60 groups, such that each group corresponds to half the cartesian space on either side of a gridpoint in the FLAC\textsuperscript{3D} model. This classification permitted the displacements from the 60 gridpoints in the FLAC\textsuperscript{3D} pillar slice to be correctly applied to the 60 groups of gridpoints in the Voronoi section of the BBM. The continuum portion of the model had the same zone size as that of the FLAC\textsuperscript{3D} model, and the displacements could therefore be directly applied to these gridpoints.

(4) Instead of applying the displacement differences as a velocity in the BBM, it was scaled-up 10 times so that the model would need to be stepped for only 1/10\textsuperscript{th} of a second of model time (~30,000 solution steps) instead of 1 second to apply the appropriate displacements. Once the model was stepped by 0.1 seconds, the boundaries were fixed and then model stepping continued until mechanical equilibrium was attained. This loading mechanism was repeated for all 7 stages (including all 20 sub-stages).

Although there are 20 sub-stages in the BBM, model calibration was conducted only for the 7 stages considered in the FLAC\textsuperscript{3D} model. This is because the displacements in the FLAC\textsuperscript{3D} model increased from Stage 1 to 7, meaning that the BBM was also loaded per a monotonically increasing displacement boundary condition. Since the FLAC\textsuperscript{3D} model did not account for any pause in longwall production, the BBM simulated a scenario where the longwall face was advancing continuously from sub-stage 1 to 20. In reality, there were intermittent production halts that resulted in breaks in the deformation curve recorded by Extensometer 1 (Figure 12.2a).
Due to the discrepancy between the actual and modeled loading conditions at the different measurement points, it is only appropriate to compare the displacements at face positions used during the FLAC$^{3D}$ model calibration.

12.5.2 Model results for unsupported BBM

An inelastic BBM requires the definition of two groups of parameters – *Zone parameters*: elastic modulus, Poisson’s ratio, peak and residual cohesion, peak and residual friction angle, peak and residual tensile strength, critical plastic shear strain from peak to residual, and *Contact parameters*: normal ($j_{kn}$) and shear stiffness ($j_{ks}$), peak and residual cohesion ($c_{peak}$ and $c_{res}$), peak and residual friction angle ($\phi_{peak}$ and $\phi_{res}$), peak and residual tensile strength ($\sigma_{t,peak}$ and $\sigma_{t,res}$). The key difference between the two groups is that the drop from peak to residual can be controlled by the critical plastic shear strain parameter for zones, but is instantaneous for the contacts.

A manual back-analysis approach was planned, whereby the input parameters from Chapter 6 were to be modified through trial and error until the BBM rib displacements matched those recorded by Extensometer 1. Interestingly, such a calibration was not required, as the base parameter set from Chapter 6 could match the field measured displacements very well (Figure 12.11). This is in contrast to the Creighton Mine case study (Chapter 10), where the BBM parameters from a different chapter focusing on the same rock type (under different loading conditions) could not reproduce the extensometer measurements (see Appendix N for more details). The direct applicability of Chapter 6 parameters (see Table 12.5) in this case can be ascribed to the similar strain paths and coal brittleness in the two studies. Although it would have been possible to slightly improve the data-model fit by modifying the input parameters, this was
not conducted as the marginal benefits of such an effort would be minimal. The mismatch in displacements with respect to the field data is less than 7 mm at all stages (Figure 12.11).

![Figure 12.11 Comparison of model rib displacements and field measurements.](image)

Figure 12.12 shows how the fractures and displacement distributions evolved with continued loading in the BBM. At Stage 4, there were some fractures along the top 1/3 of the pillar, and peak displacements were limited to ≤ 40 mm. At Stage 6, the spatial extent of the ‘>10 mm’ contour widened and a single fracture was noted to propagate towards the pillar bottom. Such axial cracks are typical in excavations undergoing spalling (Diederichs, 2007). At Stage 7, the extent of the ‘>10 mm’ contour increased dramatically and extensive damage in the form of axial cracking and buckling along the top half of the pillar occurred. Buckling of thin slabs was also noted in the field (Figure 12.3) and is a common mode of damage manifestation in coal mines (Smith, 1992; Jones et al., 2014). Of course, the thickness of the spalled slabs at Mine A
are relatively smaller than those in Figure 12.12, but it is not feasible to develop an excavation-scale BBM with block sizes on the order of one cm. The separation of blocks in this case is therefore a representation of the total dilatancy occurring across a specific thickness of spalled coal. Choosing very large block sizes will naturally lead to incorrect results, and a balance must be established between the computational requirement of a BBM and the mechanism being modeled (refer Appendix M). In the current BBM, there are 5 blocks across the detaching slab (Figure 12.12), and this is believed to be acceptable. As a point of comparison, Garza-Cruz et al. (2019b) used 4-6 cm edge length tetrahedral blocks to model rock beds that are 0.3-0.5 m thick; if the width of the blocks is 7-8 cm (typically ~50% larger than the nominal edge length), then the number of elements across the bed thickness is 4-7. Additionally, Itasca suggests using at least 4 zone elements be used across the thickness of a beam (Kim, 2020); if the zones are considered to be analogous to stiff Voronoi blocks and the failed slab is considered to a beam, then this criterion is also satisfied with 5 blocks across the slab.

Table 12.5 Zone and contact parameters from Chapter 6.

<table>
<thead>
<tr>
<th>Zones - CWFS</th>
<th>Contacts</th>
</tr>
</thead>
<tbody>
<tr>
<td>E (GPa)</td>
<td>3.0 c</td>
</tr>
<tr>
<td></td>
<td>$c_{\text{peak}}$ (MPa)</td>
</tr>
<tr>
<td></td>
<td>$c_{\text{res}}$ (MPa)</td>
</tr>
<tr>
<td></td>
<td>$\phi_{\text{peak}}$ (°)</td>
</tr>
<tr>
<td></td>
<td>$\phi_{\text{res}}$ (°)</td>
</tr>
<tr>
<td></td>
<td>$\sigma_1$ (MPa)</td>
</tr>
<tr>
<td>Tensile strength (MPa)</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Normal stiffness (GPa/m/m)</td>
</tr>
<tr>
<td>Critical plastic shear strain</td>
<td>0.035</td>
</tr>
<tr>
<td>from peak to residual</td>
<td>$\text{Shear stiffness (GPa/m/m)}$</td>
</tr>
</tbody>
</table>
Figure 12.12 Horizontal displacements in the BBM at Stage 4, 6 and 7 and rib displacement versus distance into the rib in the model and in the field. The ‘depth of collapse area’ and ‘depth of yield’ at Stage 7 are also shown.
The expanded view of the fractured portion of the BBM indicates that the width of the detaching slab at the pillar mid-height is about ~0.4 m wide, which is in fact consistent with field observations (Figure 12.9c). To better quantify the ‘depth of yield’ and ‘depth of collapsed area’, lateral displacements of all gridpoints along a horizontal line passing through the pillar mid-height were extracted from the BBM. Since gridpoints move during the simulation process, their addresses were extracted before running the model, followed by importing them back at Stage 7 to query their respective displacements. A comparison of the BBM displacements with those measured in the field on September 11th is shown in Figure 12.12. The ‘depth of yield’ is close to 1.4 m while the ‘depth of collapsed area’ is about 0.7 m. It is important to note here that the depths at which the changes in the displacement gradients occur match well, even though the specific gradients aren’t exactly the same. Overall, the results are promising and illustrate the capability of BBMs to simulate large-strain discontinuum processes like spalling, buckling, etc.

The contribution of zone yield and contact failure towards the deformation and damage in the pillar BBM is illustrated in Figure 12.13. Blue regions correspond to yielded zones with $\varepsilon_{ps}>0.005$, while pink regions correspond to $\varepsilon_{ps} \leq 0.005$. It can be seen that failure along the pillar periphery (highly dilatant) occurred mainly via contact failure and within this region of explicit fracturing, the number of yielded zones is limited. For regions deeper within the pillar (minimal dilation), zone yield is more prevalent. The concentration of yielded zones is greater towards the upper margin because the parent FLAC$^{3D}$ model had a similar yield pattern (Figure 12.9b). The presence of significant shearing along the model top boundary distorted the blocks and zones in the BBM, and since blocks cannot translate or rotate deeper within the pillar, failure manifested in the form of zone yield.
The rib photographs in Figure 12.3 were taken on September 29th when the longwall face had advanced past XC 71. Stage 7 corresponds to September 11th when the longwall face was between XC 75 and 76 (Figure 12.1). It is believed that the rib had not collapsed on September 11th, which is supported by the fact that Extensometer 1 was still operational and the outermost anchor had not yet attained its ultimate capacity of 120 mm at this point. To ascertain if the fractured slab in the BBM would continue to detach and ultimately collapse, an additional stage, corresponding to the removal of all remnant coal in the FLAC3D model (i.e. face just past XC 74), was considered. The gridpoint displacements were again extracted from the FLAC3D model and applied to the BBM in 3 sub-stages after completion of Stage 7.

Figure 12.13 Fractured contacts and yielded zones at Stage 7.

A comparison of the rib displacements at Stage 7 and after the additional stage is shown in Figure 12.14. As expected, the fractured slab continued to displace outward and the spatial
extent of the ‘>100 mm’ contour increased significantly. More fracture development and separation within the detached slab can also be observed. An interesting observation with respect to the top left panel in Figure 12.3 is that in both the model and in the field, a lower slab is visible and the coal above this slab is heavily fractured (Figure 12.14). Although the exact location of this transition might not be quantitatively captured by the BBM, the similarity is intriguing, given that the parent FLAC\(^3\)D model was not fully calibrated. Perhaps it is only more important to apply a realistic strain path to the BBM and any mismatch in the FLAC\(^3\)D model with field data/observations can be partially accounted for by the BBM material parameters derived from calibration.

Figure 12.14 Rib displacements at Stage 7 and after an additional stage in the BBM.
One might hypothesize that with further loading, the top portion of the rib in Figure 12.14 would collapse completely. However, this will likely not occur because the loading scheme assigns a boundary condition to the top of the slab and impedes its motion. A slab which is collapsing will have no applied boundary forces and will displace under gravity loading, but in the current model, a fixed velocity is assigned to all gridpoints along the bottom and top surface, meaning that the creation of a ‘truly free’ slab that is in contact with either boundary is not possible.

12.6 Effect of support in the calibrated BBM

With the unsupported BBM calibrated to the extensometer measurements, the influence of support on the rib displacements can be tested using this model. Specifically, two support patterns were tested: a 3 Bolt pattern, and a 5 Bolt pattern. In both cases, the outermost bolts were positioned 0.5 m from the pillar ends and the remaining bolts were then spaced equally along the pillar height. Bolts were modeled using the built-in rockbolt structural element and were 16 mm in diameter, 1.2 m long and sub-divided into 24 segments such that there was at least one structural node in each block. A 20 mm long steel faceplate was also attached to each rockbolt end as a liner structural element, an E of 200 GPa and ν of 0.3, and a rock-to-faceplate friction angle of 25°. Because the block geometry, zone and contact properties, and rockbolt and faceplate setups were taken directly from Chapter 6, the same rockbolt properties were used as well (Table 6.3). It is known that entry relaxation in brittle geomaterials like coal occurs very close to the development face (Mohamed et al., 2016b), and so the rockbolts were installed in the BBM after the entry excavation stage (see Section 12.5.1).
Figure 12.15 shows the simulated rib displacements in the Unsupported, 3 Bolt and 5 Bolt BBMs. With the incorporation of 3 Bolts, the displacements dropped from 122 mm to 92.1 mm (a 24.5% drop) at Stage 7, and with 5 bolts, the displacements further reduced to 83.5 mm (a 31.5% total drop). A drop in displacement is natural, as rockbolts help in pinning the spalled slab to the intact portion of the pillar. The effect of rockbolts on the displacement distribution across the entire rib is illustrated in Figure 12.16. There is a systematic decrease in the spatial extent of the ‘>100 mm’ contour with increasing support density, and the opening of the fracture that was propagating towards the pillar bottom in the unsupported case is also markedly suppressed. It is interesting to note how the location of buckling towards the pillar top is modified by the presence of rockbolts (compare the Unsupported and 3 Bolt cases in Figure 12.16); in the 5 Bolt case, the total length covered by the faceplates is large (0.2 m x 5 = 1 m), and this prevented the occurrence of buckling.

Changes in displacements on the order of 25-30% are much larger than could be simulated in a continuum model, but are lower than were originally expected. Although we have no field data for different support cases at Mine A (unlike the West Cliff Mine case) to allow us to draw definitive conclusions, the fact that the West Cliff Mine BBM and the BBM in the current study employed the same input parameters and encountered similar loading conditions means that the bolt-induced displacement reductions in the two cases should likely be similar. With respect to the differences in the depths of mining at Mine A and West Cliff Mine and the specific loading conditions being compared, it should be noted that in both BBMs, the unsupported lateral displacement at the mid-height of the pillar was ~0.1 m, and the damage via buckling was most pronounced in upper part of the pillar; given that passive support (with equivalent properties) was considered in both studies, this suggests that similar degrees of
support resistance to ground motion should be generated. Considering these similarities, an approximate comparison of the support effect in the two case studies is justified. With this in mind, the notably lower influence of support relative to the West Cliff Mine BBM, where the rib displacements dropped from 116 mm to 59 mm with 2 bolts (49%) and to 29 mm with 4 bolts (75%), indicates that the influence of the support may be underestimated in this study.

![Graph showing comparison of field measurements and model rib displacements](image)

Figure 12.15 Comparison of field measurements and model rib displacements for the Unsupported, 3 Bolt and 5 Bolt cases.

It is hypothesized that the discrepancy in simulated support effect between the two cases is due to the loading scheme employed in the integrated modeling approach considered for this study. In the models outlined above, the same strain path as of the unsupported BBM (with large shear displacements along the roof-pillar interface) was applied to all the supported BBMs, irrespective of the number of rockbolts installed. This loading approach effectively assumes that
any reinforcement of the pillar does not affect the magnitude or distribution of shear
displacements at the roof-pillar interface. From an examination of the lateral displacements along
the top of the pillar in the West Cliff Mine BBMs (see Figure 12.17), it is confirmed that
rockbolts can have some influence on the magnitude of shear slip across the roof/coal interface,
which means that the assumption required for the application of the predictive modeling
approach tested in this study is likely invalid.

Figure 12.16 Horizontal displacements in the Unsupported, 3 Bolt and 5 Bolt BBM.

Figure 12.18 illustrates how the horizontal displacements imposed at the top of the pillar
BBM prevent the rockbolts from significantly reducing rib displacements. In the initial BBM
loading stages, although the rockbolts generated some resistive force and prevented blocks from
separating, the horizontal forces continued the outward movement, which ultimately caused widespread tensile zone yield. Such tensile yield was not observed in the unsupported BBM, as there was no restriction on block movements, meaning that extensive yield was not necessary to accommodate the imposed boundary displacements.

Figure 12.17 Horizontal displacements of gridpoints along the top of the pillar in the West Cliff Mine case study BBMs (from Chapter 6). The out-of-plane spacing of all bolts are 1 m.

Figure 12.18 Zones yielding in tension in the Unsupported and 5 Bolt BBM at Stage 7.
12.6.1 Discussion and practical implications

Based on the results from this Chapter and Chapter 10 (Creighton Mine case study), it seems that extracting the pillar boundary displacements from a FLAC$^{3D}$ model and applying it to a discontinuum model is a valid option for simulating the damage observed at a site. The approach, however, is not valid for predictive support modeling purposes. The degree to which a given model’s predictions may be inaccurate is believed to depend on how much lateral movement is occurring along the pillar edges as well as the degree to which the installation of support will inhibit that lateral movement. In the Creighton Mine case study, there was no weak interface between the pillar and host rock, and the overall damage was much less in comparison to that at Mine A (displacements of ~25 mm versus ~120 mm at the pillar margin). As a result, displacement decreases on the order of 30% were obtained with the installation of rockbolts, even when the overall displacements available for mobilizing the passive bolt resistance were only in the range of 20-30 mm; note that for comparison to the Mine A and West Cliff BBM results, these values correspond to models considering rockbolt support only (unlike the models presented in Chapter 10 that included both rockbolts and shotcrete). On the contrary, the deformation and damage at Mine A was much larger, and this would be expected to lead to more bolt-induced displacement reduction, given that rockbolt loads tend to increase with increasing ground movement until they are broken. It appears that the discrepancy between the simulated and actual support effect when using the proposed modeling approach is particularly large in coal mining environments, where slippage along the coal-host rock interface is common (Iannacchione, 1990; Li et al., 2015). Perhaps modeling some portion of the roof/floor using a similar scheme for the application of local boundary conditions will allow for this limitation to
be overcome, but more research is necessary to establish how much of the roof/floor lithology needs to be included in such local BBMs.

12.7 Conclusions

This study is an application of a 3D continuum – 2D discontinuum modeling approach to simulate the damage in a Western US longwall chain pillar with the goal of demonstrating its applicability to the design of rib support schemes. To that end, two multi-point borehole extensometers were installed in a chain pillar in June 2017 and the rib deformations monitored as a function of the adjacent longwall face position. The deformation measurements from one of these extensometers was subsequently employed for calibrating a mine-scale FLAC\textsuperscript{3D} (continuum) model, followed by the extraction of the boundary deformations along the pillar slice containing the extensometer for application to a 2D Bonded Block Model (BBM). Interestingly, the BBM input parameters from a previous chapter, also focusing on coal pillars, were found to be directly applicable to the current BBM, meaning that no model calibration was required. Two different rockbolt support schemes were ultimately tested to understand how the incorporation of support might affect rib deformation.

It was found that the FLAC\textsuperscript{3D} model could reproduce the rib deformations very well, but failed to match the depth of fracturing. The BBM, however, exhibited a close correspondence with both the deformation and the depth of fracturing, which suggests that a perfect replication of the observed depth of yield is not necessary to obtain a reasonable approximation of the boundary conditions to be applied to the BBM. The effect of rib support is believed to have been underestimated in the BBM, and this is attributed to the lack of interaction between the condition of the rib and the pillar boundary conditions. In this case, the imposed boundary conditions
damaged the rib in a similar manner irrespective of the support density because the loads were applied directly to the pillar boundary. It is therefore suggested to include a few lithological layers above and below the coal seam such that the effect of rib supports on minimizing shearing along the coal-host rock interface can be directly simulated.

12.8 Acknowledgements

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CHAPTER 13

CONCLUSIONS AND FUTURE WORK

This thesis is focused on advancing continuum and discontinuum models of brittle rock damage and rock-support interaction through laboratory and field-scale simulations. FLAC$^{3D}$ and UDEC software were utilized for this purpose. The research involved extensive simulation calibrations against laboratory and field-measured attributes to better understand the mechanistic capabilities of the two modeling methods and thereby their applicability to various scientific investigations and underground mine design scenarios.

In the following sections, specific conclusions from each of the main Chapters are presented first, followed by discussion on the broader findings and implications of this thesis. Lastly, some recommendations for future research are provided, and the peer-reviewed scientific contributions of the author are summarized.

13.1 Conclusions from each Chapter

13.1.1 Chapter 2: Application of the progressive S-shaped yield criterion for modeling rock pillars

- The progressive S-shaped yield criterion was developed based on the fundamental damage mechanisms of brittle rocks and was found to replicate the small-scale damage processes in rock while exhibiting emergent pillar behaviors that are consistent with what has been observed in the field (i.e. hourglassing and progressive localization of stress along the pillar mid-section). Both low confinement extensile-spalling along the
periphery and high-confinement shear failure deeper within the pillar are emergent behaviors when using this yield criterion.

- Some of the empirical classifications relating pillar strength to the width to height (W/H) ratio have been based on databases that consisted of a variety of rocks with significantly different strength properties. Such an approach may inappropriately combine the behavior of several rock types with different mechanical characteristics, such as brittleness. A segregation was thus performed according to the rock UCS and the trend was found to vary depending on the UCS range considered for data filtering.

- Two end-member UCS cases, one hard rock and one moderate strength brittle rock, were selected to demonstrate that the pillar model can match the trend for both these rock types. The overall trend was also convex in shape, which is consistent with previous empirical findings. These results indicate that the progressive S-shaped yield criterion has wide applicability.

13.1.2 Chapter 3: Effect of yield criteria, dilatancy, heterogeneity and length to width ratio on rock pillar behavior

- The pillar models using the progressive S-shaped yield criterion were found to perform better than the brittle Hoek-Brown, Cohesion-Weakening-Frictional-Strength (CWFS), strain-softening Mohr-Coulomb and the ultimate S-shaped criteria in reproducing the relationship between pillar strength and W/H.

- Dilation angle had a major effect on the overall model behavior. Applying a dilation angle of zero delayed the damage progression and led to higher peak strengths. Although
the use of a mobilized dilation model is recommended, for the granite considered, a constant non-zero dilation angle could produce behaviors similar to those obtained using a mobilized dilation model.

- Incorporation of stochastic heterogeneity in strength properties had some effect on the peak strengths of the W/H=1 and W/H=3 pillar models, but the overall responses were similar to those from the fully homogeneous models (i.e. brittle for W/H=1 and pseudo-ductile for W/H=3). For the W/H=2 model, the macroscopic behavior fluctuated between brittle and pseudo-ductile for the different parameter realizations, indicating that 2 might be the critical W/H value corresponding to the brittle to ductile transition for the granite considered. The mean strengths for all three W/H cases were similar to those from the deterministic models.

- A modest strength increase was noted with increase in pillar length to width ratio (L/W). The effect, however, was minimal for slender pillars and for pillars with L/W>4.

13.1.3 Chapter 4: Application of the progressive S-shaped yield criterion for modeling damage in a coal pillar rib at West Cliff longwall mine (Australia)

- A FLAC$^3$D model employing the progressive S-shaped yield criterion was able to simulate the damage evolution in a coal pillar that was triggered by the approach of the longwall face. In particular, displacement profiles and stress measurements from the field were reproduced by the model for multiple locations of the longwall face. Overall, a continuum representation of coalmass behavior using the progressive S-shaped yield criterion was found to be viable.
• It is useful to consider multiple independent types of field measurements when calibrating complex numerical models. This was emphasized by presenting the results from a semi-calibrated model that was calibrated to the rib displacement data but not to the stress data, and this model ultimately failed to reproduce the observed stress trends.

13.1.4 Chapter 5: Rock-support interaction assessment using continuum and discontinuum models

• Continuum models can replicate stress and deformation changes in supported rockmasses, but they significantly underestimate the effect of support on the ground behavior when support is explicitly considered in such models. The main reason for this behavior is the lack of separation of failed elements and the enforcement of strain-continuity throughout the simulation domain, in contrast to discontinuum models.

• BBMs with Voronoi blocks produced more realistic changes in pillar wall displacements with incorporation of supports in comparison to triangular (or Trigon) block models. The difference was attributed to fractures localizing as non-dilatant shear in Trigon models and in extension in the Voronoi models.

• Field data and Voronoi model results were synthesized to develop a conceptual model for rock-support interaction. This conceptual model was sub-divided into three segments, based on the effectiveness of a support scheme: (1) the inadequate support region, (2) the maximum gain region, and (3) the overdesigned region. The Voronoi model was identified to capture the entire range of rock-support interaction behavior, while the FLAC³D model only simulated ground behaviors lying within the overdesigned region.
(i.e. after the rockmass has already been effectively reinforced). Voronoi block models, therefore, have the potential to be used as a tool for pillar support design.

13.1.5 Chapter 6: Modeling the support effect in context of the West Cliff Mine longwall case study

- A Voronoi BBM with inelastic elongated blocks was calibrated to reproduce pillar displacements and stresses measured at the West Cliff longwall mine in Australia. The blocks were elongated by a factor of two to simulate the strength anisotropy of coal as documented in the literature.

- When the support installed in the calibrated BBM was modified to match the support pattern at an adjacent site, pillar displacements similar to those recorded at this adjacent site were obtained. This suggests that the modeling approach used is not only capable of reproducing the pillar damage phenomena, but also the ground-support interaction mechanism.

- Removal of support from the calibrated BBM almost doubled the rib displacements, which is much larger than what was obtained in the previous unsupported continuum models of the same site. This further demonstrates the inability of continuum models to replicate large-strain processes and the support effect.

- An isotropic (not elongated) Voronoi BBM was also calibrated to the stress and displacement data, but did not produce the correct result when the support pattern was modified, as this geometric representation did not properly simulate the behavior of anisotropic ground. In particular, buckling was observed in the elongated block models,
while a greater degree of shearing along the block contacts was noted in the isotropic models.

13.1.6 Chapter 7: Study of BBM complexity for simulation of laboratory-scale damage of a granitic rock

• A comprehensive model complexity analysis was conducted to reproduce the rock mechanical attributes of a granitic rock. The different model representations (in order of increasing complexity), based on prior BBM studies, tested are as follows: homogeneous elastic blocks, homogeneous inelastic blocks, heterogeneous (or mineral-based) elastic blocks, and heterogeneous (or mineral-based) inelastic blocks.

• Homogeneous, elastic BBMs can only match the unconfined and low confinement laboratory peak strengths, but generally fail to reproduce confined peak strengths and the microfracture evolution process. Their application should be restricted to studying rock behavior under low confining stresses only.

• The elastic mismatch between mineral blocks in heterogeneous, elastic BBMs allows better reproduction of the microfracturing process, but due to the elastic nature of the blocks, high confinement peak strengths cannot be exactly reproduced. The lack of damage within the blocks (i.e. elastic blocks) implies that damage can only develop along grain contacts and no intra-granular fracturing can take place.

• Within the heterogeneous, elastic BBM category, it was found that models with homogeneous contact properties produced behaviors similar to those with heterogeneous contact properties based on mineral types; this suggests that mineralogically controlled
stiffness heterogeneity is more significant than strength heterogeneity in controlling the microcracking process. As the homogeneous contact property model type requires many less input parameters, it is recommended for applications focusing on low to moderate confinement conditions.

- Introduction of inelasticity within blocks to approximate intra-granular fracturing allowed reproduction of high confinement peak strengths, post-peak behaviors and dilatancy as observed in the laboratory, but it was necessary to incorporate heterogeneity as well so that the microfracturing process and the associated damage thresholds (i.e. Crack Initiation or CI and Crack Damage or CD) could also be reproduced.

- Increase in model complexity is clearly associated with an increase in the number of input parameters, but such modifications were necessary to improve model performance in terms of the number of calibration targets that could be replicated.

13.1.7 Chapter 8: Investigation of the rock fracturing process in a granite using an inelastic heterogeneous laboratory-scale BBM

- When the constant dilation angle model used in the previous Chapter was replaced with a confinement and plastic shear strain dependent dilation angle model, the heterogeneous inelastic BBM was able to better reproduce the post-peak portion of the stress-strain curve and the residual strength data from the laboratory. The capability of grain-based models to accurately reproduce post-peak behaviors of a rock for multiple confinement levels was demonstrated for the first time in this study.
• The calibrated BBM also exhibited a CWFS behavior: the specimen-scale cohesion declined and the friction angle increased as a function of plastic shear strain in the early stages of loading, followed by a region of constant cohesive strength but continued loss of frictional strength. This behavior is consistent with the experimental findings of Martin and Chandler (1994) and further illustrates the capabilities of advanced block-based models.

• The 2D Digital Image Correlation (2D DIC) technique was employed to understand how strain heterogeneity evolved with progressive fracturing of the synthetic specimen. The minor principal strains ($\varepsilon_{33}$) were found to be more sensitive to the microcracking process. Additionally, the heterogeneity of the strain field, as quantified by the standard deviation of $\varepsilon_{33}$, exhibited a trend that was consistent with that observed in laboratory tests.

• Comparison of the strain heterogeneity plot from a physical laboratory test and from the numerical simulation indicated that in cases where the point of axial stress-axial strain non-linearity does not coincide with the point of volumetric strain reversal for unconfined BBM simulations, the axial stress-axial strain non-linearity approach should be used for determination of the CD threshold.

13.1.8 Chapter 9: Investigation of pillar damage mechanisms and rock-support interaction using BBM

• At the field scale, inelastic blocks are necessary to reproduce the stress-strain response and the damage modes of both slender (W/H=1) and squat pillars (W/H=3). In such
models, large dilatant extensile fracturing along the periphery occurs primarily via contact failure and the minimally dilatant confined shearing inside the pillar occurs primarily via zone yield.

- Pillar wall supports did not affect the peak strength of the W/H=1 pillar BBM but raised the residual strength. In contrast, inclusion of support affected the peak strength of W/H=2 and 3 pillar BBMs, and the effect was greater in the W/H=3 model. The increased effect of support on the strength of larger W/H pillars is thought to be related to the presence of wider (confined) cores that allow for support-induced confinement to be generated.

- Bulking factor, a metric for the volumetric changes associated with progressive damage, evolved as each pillar BBM was loaded up to and beyond its peak strength. The bulking factors were lower in the models with higher support density, but the exact trend varied as a function of the W/H ratio. The effectiveness of a given support scheme, therefore, varies depending on the geometry and the loading conditions of a pillar.

- The rock-support interaction in the inelastic BBM was compared to that in a BBM where damage within each block was simulated explicitly via additional contact failure (intra-block fracturing) rather than an inelastic constitutive model. Allowing each individual block to break along pre-defined pathways increased the influence of support on the peak pillar strength in comparison to the inelastic BBM, but reduced its effect on displacement suppression. Such a finding contradicts the laboratory testing results of Alejano et al. (2017) and therefore suggests that the continuum inelastic representation of smaller-scale
damage within individual blocks is more appropriate for the study of rock-support interaction than the approach that allows explicit intra-block fracturing.

13.1.9 Chapter 10: Development of an integrated 3D continuum – 2D discontinuum modeling approach for studying the pillar damage process at Creighton Mine (Canada)

- A modeling approach that integrates continuum and discontinuum models was developed, in which the load path from a 3D mine-scale continuum model is applied to a 2D BBM to simulate a supported pillar slice. This approach was successful in reproducing the displacements measured by a six-anchored borehole extensometer in a field case study. Inelastic blocks were necessary to reproduce the displacements recorded at locations deeper within the pillar as well as at locations closer to the periphery. Elastic block models could only match the peripheral displacements, as geometric interlocking prevented fractures from forming deeper within the pillar.

- Increasing the support density in the calibrated BBM relative to what was actually installed in the case study suppressed the lateral deformations significantly, while in the mine-scale continuum model, the addition of support did not have any effect on the lateral displacements. This further confirms the inability of continuum models to simulate the effect of support on ground displacements.

- In the calibrated BBM, the horizontal and vertical components of the applied load path interacted with the supports, leading to non-uniform deformation changes along the pillar height. It was shown that the location of the rockbolts could affect the extent of damage
and bulking along a pillar edge. A comparison of models run with both vertical and shear loading and with only vertical loading indicated that shear loading can disproportionately damage the two vertical edges of a pillar. It is therefore important to apply the correct load path in numerical models when used for stability assessment of underground structures.

- A loading-unloading stress path resulted in greater shallow damage to the pillar than an unloading-loading stress path. Even though the pillar was not over-strained during the unloading-loading scheme, it still resulted in some additional fracturing along the edges. This indicates that repeated cyclic loading due to production activities can damage pillars.

13.1.10 Chapter 11: Investigation of mine-scale stress distribution and its effect on coal pillar behavior using Borehole Pressure Cell (BPC) data

- Through an analysis of BPC data, the abutment angle and the front abutment factor for the mine under consideration were found to be much lower than what are generally associated with US longwall mines. In particular, the abutment angle was ~11°, which is 10° lower than the current design recommendations.

- The stress rise in the yielded zone of the chain pillar with distance into the rib is initially linear, but eventually becomes bi-linear in shape; this trend is consistent with the results from the continuum model of West Cliff Mine (Chapter 4).

- The Mark-Bieniawski pillar strength equation was employed for converting the BPC stress to actual rock stress, but it was shown that this equation might be underpredicting
the strength of coal located deeper within the pillar (i.e. under highly confined conditions). This stress conversion approach must therefore be used with caution when applying it to other geo-mining conditions.

- The progressive damage of the chain pillar closer to the gob and subsequent redistribution of stresses to the next chain pillar was visualized with actual field data. Damage in chain pillars can initiate very early on due to stress relaxation caused by the breakage of the cantilever in the roof, as was observed at this site at the 0 m outby longwall face location.

13.1.11 Chapter 12: Application of an integrated 3D-2D modeling approach for pillar support design in a Western US underground coal mine

- The integrated modeling approach was used to simulate the damage in a longwall chain pillar located in a Western US longwall mine. A panel-scale FLAC$^{3D}$ model was developed and calibrated to displacements measured in the field from a multi-point extensometer; this extensometer was installed specifically for the purposes of this study by the author. Subsequently, a local pillar BBM was developed and subjected to boundary conditions derived from the FLAC$^{3D}$ model. No calibration was necessary, as the coal BBM input parameters from Chapter 6 were able to reproduce the extensometer measurements.

- The FLAC$^{3D}$ model could match the rib deformations very well, but failed to reproduce the depth of fracturing. The BBM, however, exhibited a close correspondence with both the deformation and the depth of fracturing, which indicates that a perfect replication of
the observed depth of yield is not necessary to obtain a reasonable approximation of the boundary conditions to be applied to the BBM.

- Two different rockbolt schemes were tested, and the model was found to underestimate the effect of support on rib damage and deformation. This was attributed to the absence of host rock layers in the BBM, which prevented the reduction of shear deformations along the roof-pillar and floor-pillar interfaces with increasing support density.

- For practical application of this approach for support design in coal mines, it is recommended to include a few lithological layers above and below the coal seam such that the effect of rib supports on minimizing shearing along the coal-host rock interface can be directly simulated.

13.2 Broader discussion, conclusions, and associated implications

13.2.1 Progressive S-shaped criterion and continuum models

Continuum modeling is the most commonly applied tool for studying the stress-induced damage process around excavations, but many of these models are phenomenological in nature and/or highly simplified relative to in-situ conditions. The progressive S-shaped yield criterion developed as part of this thesis has the advantage of being based on brittle rock damage mechanisms observed in the laboratory and in the field, combined with the fact that it accounts for the confined shear mechanism as well, which has received limited attention in previous excavation-focused studies. Such a yield criterion has the potential to reproduce both deformations along the pillar periphery and the peak strength, with the marginal benefit relative
to existing approaches being greatest for relatively large W/H pillars. Besides stability analysis and pillar design, the progressive S-shaped criterion is also applicable to design of deep geological repositories, where determination of the complete extent of the excavation damage zone (spanning from inter-connected macro-fractures closer to excavation periphery to diffused, disconnected fractures deeper within) is critical (Perras and Diederichs, 2016). The fractures within the excavation damage zone increase the permeability of the rockmass and could thus facilitate the dispersion of toxic radionuclides (Lajtai and Bielus, 1986). However, the proposed yield criterion is not necessarily the best choice for simulation of anisotropic geomaterials (e.g. coal), and some modifications are therefore necessary to model these materials, such as the inclusion of ubiquitous joints along cleats, fabric, or other structures.

Kaiser et al. (2000) sub-divided rockmass dilation into three components: (1) dilation due to fracture formation and growth, (2) dilation due to shearing along fractures, and (3) growth of voids between geometrically incompatible fractured blocks. Walton (2014) demonstrated that a continuum model employing a mobilized dilation model is sufficient for simulating the first two components of dilatancy. The third phase of dilatancy (i.e. large-strain rockmass bulking) is not a continuum process and is very sensitive to the applied confining pressure (Kaiser et al., 1996; Kaiser et al., 2010). Walton (2014) further indicates that as long as there is any support pressure on the excavation wall (strain boundary), the fractured rockmass can be considered to be numerically equivalent to an aggregate of coarse gravels and thereby modeled using a continuum approach. Implicit in this discussion is an acknowledgement that continuum models can only accurately simulate reinforced ground, and therefore the high sensitivity of confining pressure (due to supports) on the geometric bulking process cannot be reproduced in these models; instead, the effect of support is implicitly accounted for within the rockmass parameters. The
inability of continuum models to reproduce large-strain processes is also evident from Read (2004): *Continuum codes did not capture the observed transition from continuum to discontinuum behavior (i.e., large dilation leading to buckling and slabbing).* Accordingly, the difference in deformation between an unsupported and supported continuum model is far lower than what is observed in reality (Chapter 5), implying that such models should not be used to predict the effects of support on rockmass behavior.

### 13.2.2 Discontinuum model setup (laboratory and excavation scale)

It is difficult to directly link field-scale input parameters to laboratory parameters that are available prior to excavation (Walton, 2019) and consequently, it is not possible to develop a field-scale model (continuum or discontinuum) with good predictive capability prior to extensive calibration. BBMs have the potential to overcome this issue, as their emergent macroscopic behavior depends on fundamental micro-parameters and basic physics, meaning that it might be theoretically possible to calibrate a laboratory-scale BBM and then upscale it for studying excavation-scale problems. Studies like Dadashzadeh (2020) and Farahmand et al. (2018) have shown some promise in this regard, but the transformative potential of such a predictive approach has not yet been fully established. Accordingly, there is a need to further refine these upscaling procedures such that it can be applied to a wide range of ground conditions.

The behavior of excavation-scale BBMs is dependent on the block size. The size is typically chosen to be small enough such that there is a large number of potential failure pathways and there is limited kinematic constraint on fracture development and propagation (Christianson et al., 2016; Mayer and Stead, 2017). Field-scale models have historically used block sizes ranging from 6.5 cm to 50 cm (Damjanac et al., 2007; Preston et al., 2013; Bai et al.,
Unlike finite difference zones, which are a purely mathematical construct used to discretize differential equations, block size interacts with the input parameters assigned to the excavation-scale model to influence its emergent behavior. Very large block size will naturally lead to incorrect results, but unlike zone size, there is not necessarily a limiting block size below which the model results will become independent of block size and “converge”. This is illustrated using the Creighton mine BBM in Appendix M and is also partly evident from the results of Gui et al. (2016).

Gui et al. (2016) studied the effect of Voronoi block size on the results of simulated UCS and Brazilian tensile strength tests; although the authors stated that the strengths are linearly related to the block size, the actual data points did not show a definitive trend ($R^2$ of 0.6 for UCS and $R^2$ of 0.4 for tensile strength). The stress-strain curves also did not show any consistent trend. Such a finding is consistent with the bifurcation and strain localization phenomena observed in experimental research (Sture and Ko, 1978; Sulem and Vardoulakis, 1995; Riedel and Labuz, 2007). The author believes that the randomness in the results can be partly attributed to the stochastic positioning of the Voronoi blocks in space. Accordingly, the calibrated input parameters should be considered specific to the block size chosen and the block size itself should be treated as an input parameter that is constrained by observations from the field (i.e. thickness of spalled slabs).

In laboratory-scale BBMs of polycrystalline rocks, block stiffness heterogeneity corresponding to different mineral phases plays a crucial role in generating local tensile stresses within the model and thereby allows the accurate reproduction of the microfracturing process. At the excavation-scale, however, the block size is typically two orders of magnitude larger and the blocks themselves do not have any direct physical significance. As the blocks are of a scale...
similar to laboratory specimens (~100 mm), one might expect it to be more important to represent the macroscopic changes at the specimen-scale rather than the micro-damage process within each specimen sized block. Naturally, stiffness heterogeneity is not as relevant at the excavation-scale, although contact strength heterogeneity has been used by some authors in the past (Garza-Cruz et al., 2014; 2018). Introduction of this type of complexity (i.e. contact strength heterogeneity) was not found to be necessary in this study (Chapter 10). Inelasticity within blocks was determined to be a more important attribute in the excavation-scale (e.g. pillar) models. This is because of the potential for multiple failure mechanisms to occur in pillars with squatter geometry (extensional fracturing in the “inner shell” around the periphery and shear damage in the confined “outer shell”; Valley et al. 2011). Block yield at this scale corresponds to the finer-scale, minimally dilatant damage that cannot be explicitly represented by the block structure. The fact that block inelasticity is needed for simulating confined rock damage in the pillar models and high confinement rock attributes in the laboratory-scale BBMs (where it approximates the intra-granular fracturing process at the grain scale) indicates consistent system behavior across multiple scales.

It is also possible to simulate different failure mechanisms using different scales of bonded-blocks (elastic blocks within blocks, also called sub-tessellation) and some associated laboratory-scale results are presented in Appendix E. This approach, however, was determined to be inappropriate for modeling rock pillars, possibly because of the less complex nature of UDEC’s contact constitutive models in comparison to its zone constitutive models. Contacts in UDEC undergo an instantaneous drop from peak to residual strength, while the strength drop in zones can be controlled via the critical plastic shear strain parameter. Since sub-tessellated models typically have a large number of blocks (or sub-blocks), each block is only sub-divided
into 5-6 sub-blocks to reduce the computational cost (Gao et al., 2016; Wang and Cai, 2018), and this enforces some kinematic constraints on where fractures can develop. A continuum inelastic representation of smaller-scale damage within individual blocks is therefore desirable for BBM applications.

An important finding of the small-scale (Chapter 7) and excavation-scale (Chapter 9) complexity analysis is that the appropriate model representation always depends on the phenomena of interest to be reproduced. For example, at the laboratory-scale, if the focus is on the micro-damage process at low to moderate confinement conditions only, then considering only block elastic stiffness heterogeneity is sufficient, but if post-peak behaviors or high-confinement attributes are to be reproduced, then block inelasticity is critical. Similarly, at the excavation-scale, if slender pillars or shallow damage around a tunnel are of concern, then a homogeneous elastic block model might be sufficient, but if the focus is on slender and wider/squatter pillars that develop confined cores at their centers, then inelastic blocks are necessary.

The ability to reproduce certain mechanisms, therefore, depends on model setup decisions, and because these micromechanical models are physics-based, it is generally possible to identify physical reasons whenever an aspect of rock behavior cannot be replicated by a given model. This has been discussed in great detail in Chapters 7-10. When opting for a more complex model representation, it is important to first understand if such a choice has any relative phenomenological benefits. Chapter 8 presents an example of this consideration. In this case, the calibration of the inelastic heterogeneous BBM was initially conducted with three different sets of inelastic parameters (corresponding to the three constituent mineral phases). Later, it was found that when the same model was run with one set of inelastic properties (derived by area-
weighted average of the inelastic strength parameters for the three mineral grains), it performed equally well in replicating the calibration targets (see Figure 13.1). Since both models behaved similarly in reproducing the calibration targets, the homogenized inelastic zone property model was ultimately employed for the Chapter. In this particular case, the model complexity level was reduced, as there was no apparent phenomenological benefit of using three different sets of inelastic parameters. Since rock mechanics problems are generally data-limited, researchers must always strive to identify the lowest model complexity level possible that can simulate the target mechanics (Starfield and Cundall, 1988).

It was shown in Chapter 5 that 2D Trigon block structures under-predict rock dilatancy in comparison to 2D Voronoi block structures. Although not tested as a part of this thesis, there is some anecdotal evidence that suggests that in 3D, tetrahedral block geometries replicate dilation in a realistic manner and Voronoi structures tend to overestimate bulking (Azocar, 2016; Walton and Sinha, 2020). This is thought to be caused by the higher dimensionality, which increases the degree of freedom (translation and rotation) of each constituent block. Perhaps it is for this reason that most of the 3D BBM excavation-scale modeling by Itasca has been conducted using Tetrahedral blocks (e.g. Garza-Cruz and Pierce, 2014; Bouzeran et al., 2017; Garza-Cruz et al., 2019a, 2019b).

13.2.3 Numerical support design approach

This thesis has demonstrated the capability of 2D Voronoi BBMs to reproduce the rock-support interaction mechanism both in terms of macroscopic bulking and local deformation (Chapters 6 and 10). For the latter, it was quantitatively shown that BBMs can reproduce specific ground behaviors under various support conditions. Rockbolts (and faceplates), in the models as
well as in the field, operate by pinning and suppressing the movement of spalled blocks and they also delay the inward propagation of fractures (Chapter 6 and 9). This delay can be attributed to the fact that the blocks are held in place by the small support-generated confinement, which in-turn redirects any additional confinement generated by dilating fractures to deeper within the pillar. In absence of support, this dilatancy would lead to further disintegration of the already broken rockmass.

Figure 13.1 (a) Comparison of model and laboratory peak and residual strength, (b) Comparison of model and laboratory CD thresholds, and (c) Comparison of model and laboratory normalized peak dilation angle. ‘Homo Inel.’ refers to the BBM with only one set of inelastic parameters while ‘Het Inel.’ refers to the BBM with three different sets of inelastic parameters.
A rockbolt load profile in homogeneous ground (Freeman, 1978) can be sub-divided into a “pickup segment” and an “anchor segment”. The “pickup segment” refers to the section of the bolt closer to the excavation periphery that picks up the load from the deforming rockmass, while the “anchor segment” refers to the section of the bolt seated deeper into the rock that resists the deformation. The point of connection is called the “neutral point” where the axial load is the highest. In a jointed rockmass, the loading is generally much more complex and includes multiple “neutral points”, depending on which joints are separating along the rockbolt (Bjornfot and Stephansson, 1984). The rockbolt logic in UDEC was found to be capable of reproducing the aforementioned loading mechanism and this is illustrated by the bolt load profile in Figure 13.2 (corresponds to the BBM presented in Section 6.4.2). It can be seen that the axial loads are high towards the periphery where the displacements are large, and they drop to almost zero deeper within the pillar. It is this deeper segment that resists the bolt deformation (into the entry) and transfers the stresses back into the rockmass. Some local peaks in the axial load profile can be observed in the top bolt, which is consistent with the concept that a fractured system could have multiple “neutral points” (Bjornfot and Stephansson, 1984). Overall, it appears that the finer-scale interactions between the rockbolt and ground are being reproduced appropriately.

The integrated 3D continuum – 2D discontinuum approach presented in this thesis is a variation of the directly coupled approaches (e.g. FLAC^{3D}-PFC^{3D} or FLAC^{2D}-PFC^{2D}), and is of great practical value as it allows detailed analysis of individual excavations and/or underground structures without the need to simulate multiple adjacent excavations or the entire mine using a discontinuum model. The reduction in computational need is dramatic, thereby allowing users to utilize small blocks without making the simulation time impractical. Previous excavation-scale studies explicitly modeling the immediate roof and floor lithologies using BBM have used
Trigon blocks as large as 0.3 m (Yang et al., 2018) in the coal seam to obtain acceptable runtimes, but such large blocks might be inappropriate for studying the rock-support interaction and rockmass bulking process.

![Figure 13.2 Bolt load profiles and rockmass displacements adjacent to the rockbolts in the West Cliff Mine BBM from Section 6.4.2.](image)

When employing the integrated modeling approach for design purposes, care should be taken to consider the potential load path dependence of BBM input parameters. BBMs are micromechanical physics-based models and any parameter set obtained through calibration might therefore be considered an intrinsic micro-scale descriptor of the rock type considered. Additional analyses (Appendix N), however, revealed that BBM input parameters determined for a given loading condition may not necessarily perform well in reproducing the behavior of structures subjected to a different load path. It is therefore suggested that if a calibrated BBM is being used as a predictive tool, then its application should be restricted to loading conditions similar to the ones to which it has been calibrated to. Note that the parameters from Chapter 6 could be directly used in Chapter 12 for reproducing the field-measured displacements. This was
possible because of the similarity in the loading conditions of the two BBMs (and more broadly, the case studies).

Although more evaluation of the interaction between rockmass and support using BBMs is necessary, the 3D continuum - 2D discontinuum coupling approach (Chapters 10 and 12) considering a few host rock layers, could be employed by mining consultants for design of pillar support schemes. The need to include roof and floor lithologies is particularly important in those geological settings where large slip along the pillar boundaries can occur (e.g. for coal pillars). Once local BBMs have been calibrated for a few sites with the same rock type, then different support patterns and loading conditions can be tested and some broad support recommendations or design charts could be developed (e.g. support pressure versus mining depth, rib displacement versus support pressure for different depths; see Figures 5.15 and 5.16). This will be of great practical value, given that the current support designs are mostly based on site-specific experience (Larson and Dunford, 1996; Colwell, 2006; Mohamed et al., 2016a).

The findings, in general, have the potential to aid in pillar design and pillar support design in hard brittle rocks and in coal under different loading conditions. The findings can also be extended and applied to other underground structures (e.g. tunnels) and rock types that exhibit spalling.

13.3 Recommendations for future research

The research presented in this thesis improves our understanding of the capabilities of continuum and discontinuum models as applicable to various mine design scenarios. The
following represents a summary of research questions that were not actively pursued as part of this thesis, but can be considered for future continuation of this line of research:

- The strain softening ubiquitous joint constitutive model in FLAC\textsuperscript{3D} can be utilized for simultaneously modeling damage along and across the direction of anisotropy in anisotropic geomaterials, such as coal. The guidelines for selection of ubiquitous joint input parameters in Sainsbury and Sainsbury (2017) can be considered as a starting point. Besides anisotropic rocks, the effect of discrete structural features like faults and large-scale joints on the rock damage process can be studied using DFNs in FLAC\textsuperscript{3D} (Cao et al., 2018).

- The progressive S-shaped yield criterion is useful for short-term stability analysis of underground structures. If time-dependent deformation processes (constant strain ‘stress relaxation’ or constant stress ‘creep’) are of concern, then elastic or visco-plastic constitutive models could be used (Paraskevopoulou, 2016). Theoretically, the progressive S-shaped yield criterion could also be used for long-term simulations, following the modeling scheme of Li and Konietzky (2015). Li and Konietzky (2015) coupled the growth of sub-critical cracks with time (based on stresses in each finite difference zone) and the yielding of the zones using the FISH programming language to predict the long-term response of underground structures.

- The heterogeneous inelastic BBM representation could be used for studying the behavior of artificially jointed specimens (Arzua et al., 2014; Walton et al., 2018c), the effect of thermo-mechanical loading (Woodman et al., 2020), the effect of long-term loading (Chen and Konietzky, 2014), etc. Specifically, in the thermo-mechanical study of
Woodman et al. (2020), contact damage was shown to increase with thermal loading, which led to a reduction in the peak strengths (5 MPa confinement) of homogeneous elastic BBMs. Although the laboratory test results exhibited a reduction in the peak strength and post-peak modulus with greater thermal load (also see Peng et al., 2016; Zhao et al., 2019), the model was able to only replicate the former observation. Interestingly, the heterogeneous inelastic BBM in Appendix H exhibited both the aforementioned observations with increase in specimen pre-damage; if thermal loading is considered to only affect some of the contacts prior to deviatoric loading, then this further highlights the potential applicability of the heterogeneous inelastic modeling approach.

- There is scope for significant improvement in laboratory-scale simulation of coal using either a strain-softening Mohr-Coulomb or strain softening ubiquitous joint (ubiquitous joints oriented along the face cleat direction) constitutive model within an elongated Voronoi block structure. The ubiquitous joints, if used, will serve to approximate the finer scale cracking along cleats that cannot be explicitly represented by the block structure.

- DFNs can be incorporated in excavation-scale BBMs to model lower GSI rockmasses or cases where prominent joints/fractures are present in the field (Preston et al., 2013; Farahmand et al., 2018; Vazaios et al., 2018; Stavrou et al. 2019). The synthetic rockmass modeling approach has found wide applications in recent years; studies like Farahmand et al. (2018) and Stavrou et al. (2019) have even used the DFN-BBM approach to investigate scale effects in rocks. Although not studied in great detail for this thesis, some preliminary model runs revealed a marked scale effect in the inelastic heterogeneous BBM, even without the inclusion of any discrete pre-existing fractures or
flaws (Figure 13.3). This could be caused by the increased prevalence of weaker Biotite-Biotite contacts within the volume of the specimen (weak link concept), but more evidence is required before any definite conclusions can be drawn.

- The effect of production-related cyclic loading on the integrity of pillars should be studied in more detail, as it has not received significant attention in the literature. In particular, the effect of loading-unloading (or vice-versa) on rockmasses damaged to various extents could be an interesting topic for future research.

Figure 13.3 Geometry and unconfined stress-strain curves for three model sizes. The edge length of each Voronoi block and the input parameters are identical to those in Section 8.3.3.
• As out-of-plane movements were neglected in the current study, a similar study could be conducted in future using 3D BBMs.

• The capability of BBMs to capture the finer-scale interactions between rockmass and support needs to be further assessed. This could be done using two approaches: (1) Conduct laboratory compression tests on large blocks with and without supports and monitor the deformational changes (axial and lateral) during the loading process; this data could then be used to evaluate the capabilities of BBMs. (2) Supports of various densities could be installed in neighboring pillars in one or more mines and the pillar deformations could then be monitored over time with multi-point extensometers. Assuming similar geological and loading conditions, any difference in measured displacements could be tied directly to the ground-support interaction mechanism. If a BBM, calibrated to the deformations from one pillar, can reproduce the displacements at the other locations with only support modification, then that will also establish the potential of BBMs to be as a support design tool. The second approach was employed in Chapter 6, but it must be repeated for other rock types and mining sites to gain more confidence in the modeling method.

13.4 Contributions

The most significant scientific contributions of this thesis are summarized as follows:

• A new rock yield criterion was developed that considers both low confinement brittle fracturing and high confinement shear damage
• The ability of BBM to quantitatively replicate support influence on the behavior of rock undergoing stress-driven fracturing was demonstrated for the first time

• A comprehensive model complexity analysis was presented using laboratory-scale and excavation-scale BBMs to document how the capabilities of this modeling approach are influenced by decisions regarding model representation

• The ability of micromechanical models to accurately reproduce post-peak behaviors, dilatancy, residual strengths and the Cohesion-Weakening-Frictional-Strengthening behavior as observed in the laboratory was demonstrated

• 2D DIC was implemented in the analysis of numerical models for the first time

• A conceptual framework for understanding support effect in continuum and discontinuum models was developed

• An integrated continuum-discontinuum modeling framework was developed to aid in the practical design of pillar support

13.4.1 Thesis-related publications

13.4.1.1 Journal articles – published


**13.4.1.2 Journal articles – submitted or under preparation**

1. **Sinha S** and Walton G. Modeling the behavior of a coal pillar rib using Bonded Block Models (BBMs) with emphasis on ground-support interaction. *Tunnelling and Underground Space Technology*. 2020; Under review.


**13.4.1.2 Fully refereed conference papers**


**13.4.2 Additional publications**

**13.4.2.1 Journal articles – published**


### 13.4.2.2 Fully refereed conference papers


Allwes RA, Listak JM, Chekan GJ and Babich DR. The effects of a retreating longwall on a three-entry gate road system. USBM RI 8966. 1985; 19 pp.

Amitrano D and Helmstetter A. Brittle creep, damage, and time to failure in rocks. Journal of Geophysical Research: Solid Earth. 2006; 111(B11).


Bourcier M, Bornert M, Dimanov A, Héripré E and Raphanel JL. Multiscale experimental investigation of crystal plasticity and grain boundary sliding in synthetic halite using


Colwell M and Mark C. Analysis and design of rib support (ADRS) – A rib support design methodology for Australian collieries. Proceedings of the *24th International Conference on Ground Control in Mining*, Morgantown, WV. 2005; 24:12–22.


Dolinar DR. Load capacity and stiffness characteristics of screen materials used for surface control in underground coal mines. Proceedings of the 25th *International Conference on Ground Control in Mining*, Morgantown, West Virginia. 2006; 152-158.

Dolinar DR and Esterhuizen GS. Evaluation of the effect of length on the strength of slender pillars in limestone mines using numerical modeling. Proceedings of the 26th *International Conference on Ground Control in Mining*, Morgantown, WV. 2007; 304-313.


Esterhuizen E, Mark C and Murphy MM. The ground response curve, pillar loading and pillar failure in coal mines. Proceedings of the 29th International Conference on Ground Control in Mining, Morgantown, West Virginia, USA. 2010a; 19-27.

Esterhuizen E, Mark C, Murphy MM. Numerical model calibration for simulation coal pillars, gob and overburden response. Proceeding of the 29th International Conference on Ground Control in Mining, Morgantown, WV. 2010b; 46-57.


Ghazvinian E. *Fracture initiation and propagation in low porosity crystalline rocks: Implications for excavation damage zone (EDZ) mechanics*. PhD Thesis, Queen’s University, Canada. 2015.


Guana M. Angle bolts control rib side at No. 4 Mine, Brockwood, Alabama. Proceedings of *SME-AIME Fall Meeting and Exhibit*, Salt Lake City, UT. 1983; Preprint number 83-310.


Heasley KA and Saperstein LW. Practical subsidence prediction for the operating coal mine. Proceedings of the 2nd Workshop on Surface Subsidence due to Underground Mining, West Virginia University, Morgantown, West Virginia. 1986; 54-67.


Larson MK, Lawson HE and Tesarik DR. Load transfer distance measurements at two mines in the Western US. Proceedings of the 34th *International Conference on Ground Control in Mining*, Morgantown, West Virginia. 2015.


Singh M and Singh B. Laboratory and numerical modelling of a jointed rock mass. Proceedings of the 12th *International Conference of International Association for Computer Methods and Advances in Geomechanics* (IACMAG), Goa, India. 2008.


Sinha S and Walton G. Integration of a three-dimensional continuum model and a two-dimensional Bonded Block Model (BBM) for studying the damage process in a granite pillar at the Creighton Mine, Sudbury, Canada. Journal of Rock Mechanics and Geotechnical Engineering. 2020c; In press.
Sinha S and Walton G. Modeling the behavior of a coal pillar rib using Bonded Block Models (BBMs) with emphasis on ground-support interaction. *Tunnelling and Underground Space Technology*. 2020d; Under review.


Su DWH. Finite Element Modeling of Subsidence induced by underground coal mining: the influence of material nonlinearity and shearing along existing planes of weakness. Proceedings of the 10th International Conference on Ground Control in Mining, Morgantown, WV. 1991; 287 -300.


Walton G, Diederichs MS and Punkkinen A. The influence of constitutive model selection on predicted stresses and yield in deep mine pillars - A case study at the Creighton mine, Sudbury, Canada. Geomechanics and Tunneling. 2015b; 5:441-449.


Woodman J, Ougier-Simonin A, Stavrou A, Vazaios I, Murphy W, Thomas ME and Reeves HJ. Laboratory experiments and grain based discrete element numerical simulations


Yang SQ, Jing HW, Li YS and Han LJ. Experimental investigation on mechanical behavior of coarse marble under six different loading paths. *Experimental Mechanics*. 2011; 51:315-334.


APPENDIX A

MODELING THE EFFECTS OF BOLTS ON THE BEHAVIOR OF ROCK PILLARS AND ASSOCIATED INSIGHTS ON RIB REINFORCEMENT

This paper has been published in the proceedings of the 36th International Conference on Ground Control in Mining (Sinha and Walton, 2017b). It is reprinted with permission from the Society for Mining, Metallurgy & Exploration with some minor variations.

A.1 Abstract

Rib failures are a major hazard in underground mining. When pillars are subjected to loading, extensile cracks are generated parallel to the excavation boundary, which can manifest as rib falls. A common mitigation measure is to install supports, especially bolts, normal to the rib surface. The type and spatial distribution of these supports along the pillar is often based on experience rather than comprehensive analysis. Since conducting field experiments is often impractical and expensive, a convenient alternative is to perform 3D numerical modeling. In this study, FLAC$^{3D}$ has been used to capture the interaction of rib bolts and the failure processes governing the stability of pillars. An associated goal is to determine whether, at all, a continuum software is capable of modeling the effect of supports. The modeling involved two important steps: selection of a proper constitutive model, and, explicit simulation of rockbolts. Based on precursory work in the last decade, an S-shaped strength criterion was chosen that can describe the tensile fracture mechanism of rock at low confinement and shear failure mechanism of rock at high confinement. The built-in pile structural element in FLAC$^{3D}$ is a simplified representation of the complete bolt system. Hence, the different components of a rockbolt (i.e., steel, grout,
steel-grout interface, and grout-rock interface) have been explicitly modeled within the rockmass and then tested for its ability to replicate the ground-support interaction behavior.

A.2 Introduction

Rib failure-related hazards and injuries have plagued the mining industry for decades (Mohamed et al., 2016; Iannacchione and Prosser, 1998). In highly stressed environments, excavation parallel cracks are the primary initiators of skin instability. In absence of any support, this failure can indirectly destabilize the excavation by increasing the exposed span of intersections and roadways. Over the last two decades, there have been numerous injuries (fatal and non-fatal) due to rib failures in underground coal/non-coal mines.

In the field of rock mechanics, it is imperative to have an understanding of the controlling mechanisms of a problem before a satisfactory solution can be devised. Often, such problems are intangible and cannot be investigated using laboratory or field experiments. Pillar support design falls under this category, where a knowledge of applied stresses, global strength, and damage processes is required to assess reinforcement needs. The stability of a pillar may be improved by either varying the dimensions or by installing horizontal bolts on the rib surface. The bolts prevent the unravelling of the fractured rock skin, which, in turn, increases the confinement and minimizes the inward propagation of the extensile cracks. The amount of confinement generated by the bolt itself per unit volume of rock is negligible in comparison to the ground stresses; it only upholds the structural integrity of the spalled skin and prevents geometric bulking of the rockmass (Hoek et al., 1995; Kaiser et al., 1996).
Extensometers and stress-cells have the capability to measure the stresses and extent of damage in the peripheral portion of a pillar (Walton et al., 2015b). However, this is an expensive affair and garners only local information (i.e., at the anchor locations). A more robust design alternative is to use 3D numerical models calibrated to such measurements (Walton, 2014). The success of numerical simulations in replicating ground behavior is heavily dependent on the quality of input parameters and the constitutive relationship used.

In this study, an improved rock constitutive model is introduced. It is then numerically implemented in a FLAC\textsuperscript{3D} pillar model to demonstrate the capability of reproducing well-accepted failure behaviors. Subsequently, an explicit bolt (discretely simulates the different components of a bolt) calibrated to a pull test is developed and incorporated in the pillar model. The goal is to capture the ground-support interaction in its entirety and analyze the degree of reinforcement that a bolt generates in the continuum model. For comparative purposes, the built-in pile structural element of FLAC\textsuperscript{3D} has also been tested.

A realistic constitutive model for rock pillars must account for two fundamental behaviors: brittle failure (or spalling) at low confinement (near pillar ribs) (Diederichs, 2007; Martin, 1997) and semi-brittle shear at higher confinement (deeper into the pillar) (Diederichs, 2007). The spalling behavior can be represented by a cohesion-weakening-friction-strengthening (CWFS) model where the friction and cohesion are mobilized as a function of the plastic shear strain (a variable that quantities damage to the system) (Hajiabdolmajid et al., 2002; Martin, 1997; Walton, 2014). The shear behavior, on the other hand, can be described by a shear-yield model like Mohr-Coulomb (Hudson and Harrison, 1997) or Hoek-Brown (Hoek et al., 2012; Hoek and Brown, 1980). Neither of the two models can independently represent the damage
processes that occur in a pillar; a comprehensive failure criterion for the entire range of confinement must, therefore, require amalgamation of the two behaviors.

Only in the last decade, an S-shaped criterion was conceived by Diederichs (2007) and formalized by Kaiser and Kim (2015). This criterion has a strong theoretical basis and combines the CWFS at low confinement and shear behavior at higher confinement but has historically neglected the progressive nature of the damage process. It is not sufficient to capture only the tri-linear shape of the ultimate envelope; the complete strength envelope must be defined for all material states. The reason is that the complex interrelationships between the mobilization of cohesion, friction, and dilation ultimately control pillar behavior. It is, therefore, evident that a failure criterion must respect the tri-linear shape and, at the same time, satisfy the inherent evolving nature. With this in mind, the authors have formulated a progressive S-shaped criterion (Sinha and Walton, 2017a), which is discussed in the next section of this paper. It must be noted here that this study uses rockmass properties similar to the Creighton Granite (Walton et al., 2015b). Although this conceptual study is focused on hard rocks, the authors expect the progressive S-shaped criterion to apply to coal mass as well (given that coal is fairly brittle exhibiting similar mechanical characteristics). To assess the validity of the stated proposition, attempts are currently being made to back-analyze the West Cliff coal case study (Mohamed et al., 2016).

Rockbolts have been extensively used as a primary support element in underground mines in the US for about 40 years. A major proportion of these bolts are passive and full column grouted with resin or cement (Sinha, 2016). The mechanistic behavior of ribs bolts is identical to that of a roof bolt. Differential displacement within the rock (vertical cracks) locally loads the bolt in tension. The bolt, in turn, generates resistance to the dilation of the cracks;
additionally, it provides skin support to the raveling surface slabs. As the pillar is perpetually loaded, the cracks try to further dilate, increasing the load along the bolt. If the load exceeds the capacity of the system, the bolt ruptures. Failure can occur either in the metal, grout, or at the interfaces of metal-grout or grout-rock. Numerical representation of a bolt must, therefore, include all these components.

FLAC\textsuperscript{3D} has a rockbolt logic under pile structural element, which is often used to model bolts (Itasca, 2016c). The bolt-rock interaction is portrayed via a spring-slider system rather than through a discrete grout and associated interface. Whether this simplistic representation still captures every behavioral aspect of a bolt is not fully understood. A bigger concern, however, is whether a continuum software can model the rock-support interaction effect. A continuum model, by virtue of its definition, cannot introduce true discontinuities into the modeled system. As a consequence, an unsupported failed slab would still be structurally intact and exert a confining force on the neighboring elements. This functionally implies that the surfaces are already supported as a consequence of the continuum formulation. In such a scenario, a bolt (either explicit or pile) may not have any additional effect on the stability of the model. This hypothesis is evaluated in this study by comparing the results of an unbolted, explicitly bolted and pile-element-bolted model. Discontinuum softwares like UDEC and 3DEC, on the other hand, have the capability of allowing fractures to develop and coalesce producing true unconfined spalled surfaces. A bolt may perform more efficiently in such situations by limiting the bulking of rockmass and controlling lateral displacements. The authors understand that the discussion regarding support interaction in continuum model is not valid for mine scale coal simulations where interfaces (or discontinuities) are introduced between the roof layers. In such
cases, the bolts generate a clamping effect and minimizes the bed separations between adjacent strata (Chugh and Sinha, 2015; Sinha, 2016).

A.3 Progressive S-shaped failure criterion

Current pillar designs are largely empirical and do not explicitly account for the micro-mechanical damage processes that occur within a pillar. There are two basic empirical guidelines: design charts (Lunder and Pakalnis, 1997; Potvin et al., 1989; Pakalnis and Vongpaisal, 1993) and strength curves that relate the aspect ratio (width to height) to the pillar strength (Hedley and Grant, 1972; Lunder and Pakalnis, 1997; Sjoberg, 1992, Kimmelmann et al., 1984). Both of these have been developed by statistical correlation to observational failure behavior of pillars in the field. When using such methods for design purposes, it is essential to compare the geo-mining condition of the case studies that were used in developing the approaches to the existing condition at the site.

Numerical modeling can be used as a convenient alternative, but this requires the selection of a suitable constitutive relationship. The strength criterion used in this study has been built on the precursory works of Diederichs (2007) and Kaiser and Kim (2015) and accounts for the evolutionary nature of the envelope by relating the different segments to plastic shear strain (Figure A.1). This criterion, referred to as progressive S-shaped failure criterion in this study, is composed of three important envelopes:

(1) Yield Envelope: The left portion corresponds to the Crack Initiation (CI) threshold, while the right portion is Mogi’s Line (Mogi, 1966). CI marks the initiation of stable cracks and is typically associated with inelastic lateral extension (Diederichs, 2007; Martin, 1997).
CI can be determined from laboratory testing using acoustic emission techniques (Eberhardt et al., 1998), volumetric stress-strain curves (Brace et al., 1966), or axial and lateral stress-strain curves (Diederichs, 2007; Nicksiar and Martin, 2012).

(2) Mobilized Envelope: The yield envelope evolves to the peak envelope over a pre-defined plastic shear strain (0.01 in this case). The left portion corresponds to the spalling limit, while the right portion follows the Crack Damage (CD) threshold. The CD threshold is commonly known as the long-term laboratory strength (Martin and Chandler, 1994) and can be identified from acoustic emission techniques or the axial stress-strain curve. The spalling limit, on the other hand, describes the ultimate strength of spalled ground at low confinement.

(3) Residual Envelope: The peak envelope is gradually degraded to a residual envelope over a pre-defined plastic shear strain (0.05 in this case).

![Figure A.1 Progressive S-shaped strength criterion.](image)
As can be observed from Figure A.1, in the lower confinement regime ($\sigma_3<45$ MPa), the behavior is identical to CWFS, while, at higher confinement ($\sigma_3>45$ MPa), the cohesion and friction are mobilized simultaneously similar to a classical shear strength model. The tri-linear shape of the upper envelope conforms well to Diederichs (2007) and Kaiser and Kim (2015). Note that both the choice of plastic strains and the demarcation between higher and lower confinement regime was based on the hard rock case study of Creighton mine (Sudbury, Canada) (Walton et al., 2015b).

The progressive S-shaped criterion was implemented in FLAC$^{3D}$ using the bilinear strain-softening ubiquitous joint constitutive model. An 8 m cubic pillar model was developed and loaded through two elastic beams on either side using displacement boundary conditions. Very high strength properties were assigned to the ubiquitous joints for the purposes of this study to ensure that they would not have any influence on the observed pillar behavior.

The model could replicate some of the well-documented failure pillar behaviors, like the hourglass shape of the core (Krauland and Soder, 1987; Esterhuizen, 2006) (Figure A.2) and progressive localization of stress along the mid-height of the pillar (Figure A.3) (Wagner, 1974). As a further check on the reliability of the model, the pillar strength (normalized to the uniaxial compressive strength) for different w/h ratio was plotted alongside the empirical pillar strength database; results showed that the model-derived curve could demarcate the stable and failed pillar case histories. The reader is referred to Sinha and Walton (2017a) for further details on this study and parameter selection. Overall, results show that the progressive S-shaped criterion is capable of capturing microscopic damage processes while exhibiting an emergent pillar behavior consistent with what has been observed in the field.
Figure A.2 Concentration of vertical stress around the core of the pillar with W/H of 3.

Figure A.3 Development of vertical stress along the mid-height of the pillar with increasing damage to the system.
A.4 Modeling rock-support interaction

The progressive S-shaped failure criterion is capable of replicating rock pillar behavior. This section attempts to capture the rock-bolt interaction effect using the developed pillar model. The built-in pile structural element in FLAC\textsuperscript{3D} has the capability of modeling bolt-rock interaction using a spring-slider system. These are essentially two-noded, straight elements of finite length with six degrees of freedom (Itasca, 2016c). However, the pile structural element is a simplification of a bolt system (Crockford, 2012), so it is of interest to also study a more explicit representation of this system.

The bolt model developed here explicitly simulates the different components of a bolt system: rock, bolt, grout, rock-grout interface, and grout-bolt interface (Windsor, 1997). Since this is a material representation rather than a structural one, it is the hypothesis of the authors that this would be sufficient in replicating the rock-support interaction effect (if it can be captured at all in a continuum model). The bolt model was first calibrated to a pull test and then incorporated into the pillar model. For comparison, a similar approach was followed using a pile structural element. It must be mentioned here that no pull test data was available to the authors for the Creighton mine; as a result, the pull test curve from Luke (2016) was selected with an increased ultimate strength to account for the mechanical behavior of Creighton Granite. Note that the goal of this study is not to precisely model the rock-support interaction effect for a particular geomining condition, but to examine whether this interaction can be captured in a continuum model. Once established, this bolt model can be calibrated to site-specific pull tests and then used as a design tool for locating rib supports.
A.4.1 Explicit bolt model

A.4.1.1 Geometric setup

The rock, grout, and steel rod, along with their interfaces, were modeled independently in the bolt model. In order to replicate a drilled borehole with a fully grouted cylindrical bolt, a 2.4 m long, 24 mm steel rod was placed through the rock with 10 mm grout annulus. The rod was modeled as perfectly elastic, while the grout was assigned a perfectly plastic Mohr-Coulomb constitutive model. The interfaces are characterized by Coulomb sliding with their strength and mechanical behavior being governed by friction, cohesion, tensile strength, normal, and shear stiffness parameters. Table A.1 lists the properties of the different components of the bolt model.

Table A.1 Properties of different components of the bolt model.

<table>
<thead>
<tr>
<th>Input parameter</th>
<th>Model assigned value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grout (after Crockford, 2012)</td>
<td></td>
</tr>
<tr>
<td>Young’s Modulus</td>
<td>4.8 GPa</td>
</tr>
<tr>
<td>Strength (UCS)</td>
<td>22 MPa</td>
</tr>
<tr>
<td>Cohesion</td>
<td>6.215 MPa</td>
</tr>
<tr>
<td>Friction</td>
<td>30°</td>
</tr>
<tr>
<td>Tensile strength</td>
<td>2.92 MPa</td>
</tr>
<tr>
<td><strong>Bolt</strong></td>
<td></td>
</tr>
<tr>
<td>Young’s Modulus</td>
<td>200 GPa</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.28</td>
</tr>
<tr>
<td><strong>Grout-Bolt Interface (after Crockford, 2012)</strong></td>
<td></td>
</tr>
<tr>
<td>Shear stiffness</td>
<td>8 GPa/m/m</td>
</tr>
<tr>
<td>Cohesion (Peak/Residual)</td>
<td>2 MPa/0 MPa</td>
</tr>
<tr>
<td>Friction (Peak/Residual)</td>
<td>45°/25°</td>
</tr>
</tbody>
</table>
Table A.1 continued.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Grout-Rock Interface (after Crockford, 2012)</strong></td>
<td></td>
</tr>
<tr>
<td>Tensile Strength</td>
<td>4 MPa</td>
</tr>
<tr>
<td>Shear stiffness</td>
<td>9 GPa/m/m</td>
</tr>
<tr>
<td>Cohesion (Peak/Residual)</td>
<td>4 MPa/0 MPa</td>
</tr>
<tr>
<td>Friction (Peak/Residual)</td>
<td>$53^\circ/32^\circ$</td>
</tr>
<tr>
<td>Tensile Strength</td>
<td>4.5 MPa</td>
</tr>
</tbody>
</table>

The mesh element sizes were selected in proportion to the dimension of the rod and the grout and were graded to ensure smooth transfer of stresses and displacements in the entire model. Figure A.4 shows the geometrical setup of the final bolt model. In the strain-dependent constitutive relationships in FLAC$^{3D}$, the pre- and post-peak behaviors are heavily dependent on the mesh element size (Itasca, 2016c). To ensure that the disparities in the behavior of the different models tested are not an artifact of this mesh dependency, the same geometric setup has been used throughout the study. In the case where a pile element is used instead of the explicit bolt or where no bolt is installed, the interfaces were deleted and the bolt and grout were assigned properties same as that of the rock.

A.4.1.2 Pull test

Figure A.4 shows the geometric layout that was used for conducting the pull test on the bolt model. A displacement boundary condition was applied to the surface of the rod and the developed unbalanced forces at the model grid points were recorded using a FISH function. To ensure that the perturbations were being introduced in a stable fashion, the applied velocity was set at $10^{-8}$ m/step. Figure A.5 depicts the load-displacement curve obtained from the pull test. The authors acknowledge the fact that field pull tests are force-controlled rather than
displacement-controlled. In FLAC³D, both methods yield similar load-displacement curves. The displacement-controlled method, however, yields more data points in the non-linear portion of the curve once yielding initiates in the system.

Figure A.4 Geometry of the explicit bolt model: (a) 2-point perspective view of the entire model, (b) 2-point perspective view of a longitudinal section of the model, (c) Magnified 2-point perspective view of the longitudinal section illustrating the geometry of the bolt, (d) Rock-grout (green) and bolt-grout (blue) interface.

A.4.2 Pull test with pile structural element

For comparative purposes, a pile structural element was calibrated to the load-displacement curve obtained from the bolt model. The same geometric layout was used;
however, the interfaces were deleted, and the rod and grout were assigned the same properties as the rock. Loading of the pile was again conducted through a constant velocity of $10^{-8}$ mm/step. Figure A.5 compares the load-displacement curves obtained from the two-pull test, and Table A.2 lists the calibrated pile element parameters. A logical concern may be the slight disparity in the stiffness of the two curves. Such a difference could lead to dissimilar ground behavior in underground mines. As is demonstrated in the subsequent section, however, the reinforcement effect generated by the pile, as well as the explicit bolt, is minimal. Therefore, the differing stiffness of the two models will not have a notable effect on the final inferences of this paper.

Table A.2 Calibrated properties of the pile element.

<table>
<thead>
<tr>
<th>Input parameter</th>
<th>Model assigned value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s Modulus</td>
<td>200 GPa</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>0.28</td>
</tr>
<tr>
<td>Perimeter</td>
<td>0.13823 m</td>
</tr>
<tr>
<td>Shear stiffness</td>
<td>$5 \times 10^3$ MPa/m/m</td>
</tr>
<tr>
<td>Shear Cohesive Strength</td>
<td>$10^6$ Pa/m</td>
</tr>
<tr>
<td>Tensile Yield Strength</td>
<td>0.3 MN</td>
</tr>
</tbody>
</table>

A.4.3 Effect of bolt in a continuum pillar model

Pillar skins are typically damaged by extensile fracturing and can be visually illustrated by tensile strain in continuum models. These fractures are located along the ribs where the level of confinement is very low. In reality, a passive grouted bolt can minimize the growth of near-vertical cracks by generating confinement and reducing their progressive development towards...
the center of the pillar. Numerically, this should show up as a reduction in the localized tensile
strains near the ribs.

Figure A.5 Load-displacement curve obtained from pull tests.

In an attempt to capture the bolt support mechanism, three models were run:

- Model 1: No bolt is installed in the pillar.
- Model 2: Three explicit bolts are installed in the pillar.
- Model 3: Three pile elements are installed in the pillar.

The inclusion of the three bolts and the associated mesh in a complete pillar model resulted in an order of magnitude increase in the number of zone elements. To compensate for the associated increase in the simulation time, it was decided to take advantage of the symmetry
and analyze only a quarter portion of the model. Figure A.6 shows the geometry of the quarter model with two elastic beams on either side. The elastic beams (8m x 8m x 8m) were constrained on all sides by rollers while the pillar (4m x 4m x 8m) was constrained along only the two planes of symmetry. Two routine FISH functions recorded the average stress and the differential displacement at the two ends of the pillar. This was then imported into Excel to generate the overall stress-strain curve for the pillar.

Figure A.6 Quarter pillar model developed for this study: a) On the left is a front view of the entire model, b) On the right is a 2-point perspective view of a longitudinal section of the model. The perspective view gives a clear idea of the extent and location of the three bolts relative to the model boundary.
A.4.4 Model results

Figure A.7 shows the average stress-strain curves for the three models. In the elastic domain, the three curves coincide perfectly with a slope of about 82 GPa (the slight mismatch with the rockmass elastic modulus of 80 GPa may be related to the mesh geometry). As previously hypothesized, the difference in the peak strengths are negligible, indicating that neither the explicit bolt nor the pile has a significant reinforcement effect on the stability of the pillar. Surprisingly, the explicit bolt model shows the least strength (~82 MPa) in comparison to the no bolt (~83 MPa) and pile element model (~83 MPa). The authors think that this could be a combined effect of the mesh gradation and the differing constitutive models assigned to the rock (bilinear strain softening ubiquitous joint), grout (strain softening), and bolt (elastic) in the explicit bolt model. As a contrast, the pile element and no bolt models had the bilinear strain softening ubiquitous joint constitutive model assigned to all its elements.

Normal displacements are good indicators of the potential location and extent of spalling. Figure A.8 provides a visual illustration along longitudinal sections of the models. The range of values in the color scale have been made consistent to facilitate a direct comparison. It is evident that the explicit bolt is more effective in reducing the relative normal displacements along the rib. A closer look at the contours show that the explicit bolt generates a local clamping effect (concave bend) and reduces the surficial deformations. However, the differences in the pile and explicit bolt model in comparison to the no bolt model are minor and, therefore, do not affect the global stability of the pillar. This is confirmed from the similar stress-strain curves in Figure A.7. Finally, a plot of unbalanced forces indicates that the pile elements and the explicit bolts were being subjected to 8–9 tons of tensile load.
Figure A.7 Global stress-strain curve for the quarter pillar model.

Figure A.8 Contour plots of normal displacement along longitudinal sections of the three models.

A.5 Conclusion

Rib instabilities are a major cause of accidents in underground mines in the US. Appropriate support types installed at strategic locations can help in minimizing this impending
hazard. Numerical modeling is a convenient tool, which is often used for identifying critically unstable areas and assessing their reinforcement needs. The success of such modeling is largely dependent on the choice of a constitutive relationship. The constitutive relationship must account for the different damage mechanisms under a particular stress environment.

This study puts forward a progressive S-shaped failure criterion that has the capability of modeling the macroscopic behavior of pillars while accounting for the microscopic rock damage processes. When implemented in a pillar model in FLAC$^{3D}$, it could demonstrate some of the well-documented behaviors seen in the field.

The pile structural element in FLAC$^{3D}$ is a simplification of the actual bolt system. Hence, an explicit bolt model was developed and incorporated in the pillar model to assess the ground-support interaction effect. It must be noted here that the explicit bolt model was calibrated to demonstrate the modeling issues of a fully grouted bolt. However, the logic (and related issues) should equally extend to other types of support/bolts. To specifically investigate into other support type behavior, a similar calibration technique can be followed. It was hypothesized that continuum models, as a consequence of their formulation, do not possess the capability of properly reproducing ground-support interaction throughout the pillar damage process. Results from the pillar model showed a negligible reinforcement effect of the pile elements and the explicit bolts on the overall stability of the pillar. This validates the authors’ hypothesis in the study and further indicates that discontinuum modeling is required when attempting to simulate the interaction between a support and fractured ground. Endeavors are currently being made to replicate this work using the grain-based Voronoi logic in UDEC.
A.6 Acknowledgements

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APPENDIX B

DELETION CODE IN FLAC$^{3D}$ AND ASSOCIATED UNSUPPORTED AND SUPPORTED BBM RESULTS

B.1 Introduction

In absence of explicit information on the support condition of pillars in the empirical pillar strength database, it was initially assumed that the empirical pillar strength database corresponded to moderately to highly supported pillars in the field. Accordingly, the FLAC$^{3D}$ models of Chapters 2 and 3 that were calibrated to the empirical pillar strength database also were considered to correspond to supported ground conditions. Given that the stress-strain curves corresponding to supported pillar conditions are not appropriate calibration targets for unsupported BBMs, a deletion approach was developed in FLAC$^{3D}$ to mimic the loss of spalled material from the boundary of unsupported pillars, with the ultimate aim of determining the stress-strain behaviors of unsupported pillars. This appendix presents the details of the deletion code, the FLAC$^{3D}$ results from the deletion code and the associated unsupported and supported BBM results.

B.2 Deletion code in continuum models

Failure in pillars typically initiates along the boundaries when the local tensile stresses exceed the grain-contact strength. Once created, the microcracks propagate in the direction of $\sigma_1$ due to the low confining stresses present adjacent to excavations. The interaction of these extended fractures leads to the formation of thin slabs that pose a serious hazard to mining personnel. A combination of mesh and rockbolts (and sometimes shotcrete) is commonly used to
prevent the collapse and/or geometric bulking of the spalled slabs. These supports operate by inhibiting the collapse of the ‘baggage zone’ which generates confinement, thereby inhibiting the inward propagation of the stress-induced fracturing process.

A pillar that is unsupported would allow physical separation of the fractured blocks. A direct consequence is the generation of newly unconfined surfaces, which triggers additional spalling. A potentially viable approach for capturing this phenomenon in continuum pillar models is to incrementally delete boundary blocks as the pillar is progressively damaged. To implement such an approach, a set of criteria based on a framework relatable to the actual physics of rock fracturing is needed to identify the target blocks for deletion.

An appropriate set of criteria is thought to consist of three components: (1) Spatial component: blocks should be at the boundary of the pillar; (2) Confining stress component: blocks should be under a tensile minimum principal stress; and, (3) Damage component: blocks should be damaged beyond some user-defined level of plastic shear strain. The parameter plastic shear strain ($\varepsilon_{ps}$) is a well-established damage metric in the field of rock mechanics. The inclusion of a damage criterion is necessary to represent an equivalent degree of fracturing associated with spalling failure. An additional reason is that many boundary elements are subjected to tensile stresses during simulation, but are not necessarily damaged to an extent that would qualify them as spalled blocks capable of physically separating from the pillar. $\varepsilon_{ps} = 10$ millistrains, which corresponds to the spalling limit (left side of peak envelope) in Chapters 2 and 3, was used as the damage threshold above which zones were considered to consist of spalled blocks.
The granite pillar models from Chapter 3 with L/W of 4 (the comparable BBMs in UDEC are run in plane-strain mode, which effectively assumes infinite pillar length) were re-run with the deletion code such that all elements located on the boundary under tension and with $\varepsilon_p > 10$ millistrains were incrementally deleted during the loading phase. Since the search and deletion algorithm is computationally intensive, it was called every 1000 solution steps. Ultimately, the W/H=1, 2 and 3 models were run for 450,000, 550,000 and 825,000 solution steps, respectively, to delineate the complete stress-strain curves.

Figure B.1 compares the model-predicted stress-strain curves with and without the ‘deletion’ code for W/H=1, 2 and 3. It can be seen that there is a drastic change in the peak strength as well as the degree of ductility in the W/H=2 and 3 pillars. Specifically, the drops in peak strength for the W/H=1, 2 and 3 pillar are 0.07%, 21.8% and 47.5%, respectively. The effect of ‘deletion’ was found to be relatively insignificant for the W/H=1 model peak strength, since failure in such slender pillars is governed by cross-shear planes where internal confining stresses do not play a major role. A decrease in the residual strength was also observed, which is consistent with the experimental findings of Alejano et al. (2017). Although Alejano et al. (2017) looked into the effect of cable straps on the pre- and post-peak behavior of laboratory-scale granite samples, the overall objective (i.e. comparing the mechanical behavior with and without supports) is similar to this study, and therefore the results can be compared qualitatively. The support testing in Alejano et al. (2017) was conducted on W/H=0.5 samples, which falls within the ‘slender’ category for pillars. A consistent peak strength with and without supports and a near-zero residual strength without cables in Alejano et al. (2017) and with the deletion code for W/H = 1 (in this study) suggests the stress-strain curves presented in Figure B.1 represent
reasonable approximations of actual unsupported pillar behavior, assuming the original models without ‘deletion’ correspond to a supported pillar condition.

![Stress-strain curves with and without the ‘deletion’ code for W/H=1, 2 and 3.](image)

Figure B.1 Stress-strain curves with and without the ‘deletion’ code for W/H=1, 2 and 3.

The behavior of squatter pillars, on the other hand, is controlled by a confined core that develops due to dilation of the peripheral fractured region. The removal of surficial blocks therefore restricts the amount of dilation-induced confinement that can be generated in the core, and this results in lower strength and reduced pre-peak hardening in the W/H = 2 and 3 models. The unconfined tests in Alejano et al. (2017) considered only samples with W/H = 0.5, and the results as such are not comparable to the W/H = 2 and 3 pillar models.
The notably higher effect of the deletion code on \( W/H = 2 \) and 3 leads to two potential conclusions: (1) Prevention of bulking can stabilize pillars through confinement generation (thereby improving the load carrying capacity of the core); and, (2) The effect of pillar support on pillar strength and ductility increases with increasing \( W/H \) ratio. The typical hourglass geometry for the \( W/H = 2 \) and 3 models at peak strength is shown in Figure B.1. The wider base of the hourglass for \( W/H = 3 \) in comparison to \( W/H = 2 \) is consistent with the findings of Chapter 2 and corresponds to its higher load carrying capacity.

### B.3 Bonded Block Models (unsupported and supported)

With the stress-strain responses for unsupported pillars established, the next task was to develop and calibrate unsupported BBMs to these curves. The same model geometry and loading conditions as those in Chapter 5 were used. Initially, elastic block models were tested, but it was found impossible to match the stress-strain curves for all three \( W/H \) geometries with only 1 set of input parameters (further discussion on this topic can be found in Appendix J and Chapter 9). Subsequently, inelastic BBMs were considered, and with a CWFS strength model for the zones, it was possible to reproduce the stress-strain response for all three \( W/H \) geometries using a single set of parameters. The calibrated zone and contact parameters are listed in Table B.1, and a comparison of the stress-strain curves from the calibrated BBMs and from the deletion FLAC\(^{3D}\) models is shown in Figure B.2. As can be seen, the model was able to match the peak strengths, pre-peak hardening as well as the post-peak response very well. A remarkable similarity was also noted in the fractured portions of the BBMs (Figure B.2) and the final geometries of the ‘deletion’ FLAC\(^{3D}\) models (Figure B.1).
Following calibration to the ‘deletion’ stress-strain curves, we tested a variety of different support combinations in the BBMs. In particular, rockbolts (parameters in Table B.1) and shotcrete (parameters in Table B.1) were considered. Rockbolts were modeled using the built-in cable structural element, while the beam element was employed for modeling shotcrete.

Table B.1 Zone, contact, shotcrete and cable parameters used in BBM.

<table>
<thead>
<tr>
<th>Zones - CWFS</th>
<th>Contacts</th>
<th>Beam (shotcrete)</th>
<th>Cables (bolts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak cohesion (MPa)</td>
<td>46.5</td>
<td>65</td>
<td>19</td>
</tr>
<tr>
<td>Residual cohesion (MPa)</td>
<td>0.5</td>
<td>0</td>
<td>35</td>
</tr>
<tr>
<td>Peak friction angle (°)</td>
<td>0.0</td>
<td>30</td>
<td>3.8</td>
</tr>
<tr>
<td>Residual friction angle (°)</td>
<td>42.0</td>
<td>10</td>
<td>0.5</td>
</tr>
<tr>
<td>Tensile strength (MPa)</td>
<td>12</td>
<td>10</td>
<td>60</td>
</tr>
<tr>
<td>Plastic shear strain from peak to residual</td>
<td>0.012</td>
<td>120000</td>
<td>0.86</td>
</tr>
</tbody>
</table>

Given the assumption that the undeleted FLAC\textsuperscript{3D} models correspond to supported pillar behavior and the ‘deletion’ FLAC\textsuperscript{3D} models correspond to unsupported pillar behavior, it was initially hypothesized that when a typical amount of support is incorporated in an unsupported
BBM calibrated to the ‘deletion’ FLAC$^{3D}$ models, the resulting behavior would tend to be
similar to that of the undeleted FLAC$^{3D}$ models. However, despite numerous trials, a match to
the undeleted FLAC$^{3D}$ stress-strain curves (especially for W/H=3) could not be obtained when
support was added to the BBMs. Example results in context of the W/H=3 BBM are shown in
Figure B.3. ‘3 bolt’ and ‘4 bolt’ correspond to support patterns consisting of 3 and 4 rockbolts
along each side of the pillar, ‘3 bolt + shot’ corresponds to 3 rockbolts and a 75 mm thick
shotcrete liner on both sides of the pillar, and ‘3 bolt + shot (1)’ corresponds to 3 rockbolts and a
150 mm thick shotcrete liner; the shotcrete and rockbolt parameters were taken from Chapter 10
and Chapter 5, respectively. The large discrepancy in the peak strength between the supported
BBMs and the undeleted (‘FLAC No Del’ in Figure B.3) FLAC$^{3D}$ curve can be readily observed
in Figure B.3.

Figure B.2 ‘Deletion’ stress-strain curves compared to calibrated inelastic BBM stress-strain
curves.
Figure B.3 Stress-strain curves for different supported W/H=3 cases.

There could be two possible reasons for the discrepancy between the apparent support effect as represented in the two model types (FLAC$^{3D}$ – undeleted versus with deletion; BBM – unsupported versus supported):

(1) The deletion code overestimates the strength drop for unsupported conditions. This is quite plausible, as in reality, broken blocks may aggregate at the base of pillars and generate some confinement on the pillar wall. In the ‘deletion’ code, the spalled blocks were removed entirely, thereby neglecting this potential confinement feedback mechanism. Furthermore, as not all spalled blocks detach immediately, the use of the spalling limit as a deletion threshold represents a lower-bound estimate of what might be an appropriate $\varepsilon_{ps}$ threshold for block deletion.
(2) The rock-support interaction mechanism is underestimated in BBM. This is considered less likely to be true, given that BBMs were able to match the reinforcement effect for the West Cliff Mine case study (Chapter 6) and also matched the bulking factor data of Kaiser et al. (1996) (Chapter 5).

Because of the inability of the supported BBMs to replicate the large strength difference between the deletion and undeleted stress-strain curves from FLAC$^{3D}$, we decided to re-think our assumption that the empirical database (and therefore the calibrated undeleted FLAC$^{3D}$ models) corresponds primarily to supported pillar conditions. A closer reading of some of the associated literature suggests that it is in fact likely that the majority of the pillars in the database were unsupported or lightly-supported:

- Hedley and Grant (1972) – “All the mines (12 in total) use rockbolts as means of artificially supporting the roof”. Since this study explicitly reports roof support but makes no mention of pillar support, it can be reasonably assumed that the pillars in these mines were (mostly) unsupported. A picture of a failed pillar from Elliot Lake mine (one of the 12 mines in Hedley and Grant, 1972), published in a recent study (Renani and Martin, 2018b), also appears to indicate a lack of pillar support.

- Sjoberg (1992) – Sentences like “However, in a pillar geometry, development of failure can more readily lead to a global collapse. Support can probably arrest some of the instabilities, but this has to be studied in more detail” and “Even so, it is important to point out that spalling remains a significant problem at the mine. It can most probably be solved with better support strategy or by destressing, but all this requires better
understanding of the governing mechanisms” seem to suggest that the pillars considered in this study were unsupported as well.

It appears that at least some pillars considered in this study were supported, however, based on this description of a specific pillar: “Stresses were immediately redistributed to the sill pillar above the upper drift, also causing this pillar to fail, but in a more controlled manner. The damage was limited also due to heavy reinforcement which kept the fractured pillar in place”.

It is possible that out of the two types of pillars considered in this study (sill pillar and bench-type pillar), only some of the sill pillars were supported. In any case, there are indications that most pillars in this study were unsupported.

- Hudyma (1988) – Phrases like “Sloughing of large slabs from pillar walls as far as center of pillar”, “fracturing due to pillar failure….cause severe pillar sloughing and eventually complete pillar disintegration”, and “development in pillars will likely need full friction artificial support such as cable bolts and grouted rebar, as pillar fracturing could substantially affect development stability” again indicates that the failed pillars considered in the study had no support.

With all that in mind, it was concluded that it is in fact likely that the majority of the pillars in the empirical pillar strength database were unsupported or lightly-supported, and that the undeleted FLAC$^{3D}$ models calibrated to this database therefore correspond to unsupported pillar behavior. Accordingly, the BBM was re-calibrated to the undeleted stress-strain curves and the support analysis presented above was repeated using these calibrated models for W/H of 1, 2 and 3 (Chapter 9). Note that although the ground-support interaction curve in Chapter 5 indicates that continuum models implicitly represent a supported condition, it might be better stated that
continuum models can represent both supported or unsupported conditions (depending on the model parameter inputs selected), although post-peak strength loss and bulking under unsupported conditions may be underestimated.

The application of the continuum FLAC$^{3D}$ models as a representation of unsupported pillar behavior for calibration purposes in this case can be justified based on two key factors: (1) It is more important that the peak strength of the unsupported BBMs match those from the (unsupported) pillar strength database, and (2) Although continuum models can’t be used to represent the support effect, they can reproduce supported or unsupported conditions with appropriate parameter calibration. It therefore follows that irrespective of whether the empirical pillar database is supported or unsupported, the conclusions of Chapters 2 and 3 are still valid as the continuum models were calibrated directly to the empirical database. Specifically, the S-shaped criterion can be considered applicable for modeling pillars of various W/H, and the effects of dilatancy, meso-scale parameter heterogeneity and pillar length on pillar behavior that were reported can be considered representative.

Given the uncertainty in the interpretation of the text from the original pillar strength works as presented above, there is still a possibility that the empirical pillar strength database and the calibrated FLAC$^{3D}$ models represent support pillar behavior. In that case, the larger discrepancy between the unsupported and supported conditions in the FLAC$^{3D}$ models (i.e. undeleted versus with deletion) in comparison to the BBMs is almost certainly related to the specific implementation of the deletion code that was used. In particular, the strain limit threshold could be raised or some modifications could be introduced to better represent the spalling process (for example, removal of the zones satisfying the deletion criteria and adding these zones to the bottom edges of the pillar per an angle of repose). An example is shown in
Figure B.4, where the deletion strain limit threshold has been raised from 10 millistrains to 50 millistrains (corresponding to the residual envelope of the progressive S-shaped yield criterion) for the W/H=3 model. As expected, the peak strength increased dramatically and is closer to the stress-strain curve from the FLAC$^{3D}$ model without deletion.

![Stress-strain curves for the W/H=3 model without deletion and with deletion at plastic shear strain of 0.01 and 0.05.](image)

Figure B.4 Stress-strain curves for the W/H=3 model without deletion and with deletion at plastic shear strain of 0.01 and 0.05.
APPENDIX C

EFFECT OF DAMPING MODE IN LABORATORY AND FIELD-SCALE UNIVERSAL DISTINCT ELEMENT CODE (UDEC) MODELS

This paper has been published in the proceedings of the 54th US Rock Mechanics/Geomechanics Symposium, Golden, Colorado (Sinha et al., 2020b). It is reprinted with permission from the co-authors and ARMA with some minor variations.

C.1 Abstract

In practical rock engineering, Universal Distinct Element Code (UDEC) is one of the most widely used two-dimensional software packages for simulating the damage process in rocks. Over the years, this software has been used to study grain-scale fracturing in laboratory specimens, spalling around tunnels and roadways, shearing and separation along discontinuities in jointed rockmasses, etc. One of the lesser discussed topics in context of UDEC modeling is the damping mode employed for such simulations. For static analysis, the ‘local’ (default) and ‘combined’ damping modes are generally used, but their effects on the emergent model response as well as their suitability to a particular problem are not well documented in the literature. To help bridge this gap, this study contrasts the responses of a laboratory-scale model, a hypothetical granite pillar model and a hypothetical coal mine entry model with ‘local’ and ‘combined’ damping modes. It was found that in small-scale simulations, the results using ‘local’ and ‘combined’ damping modes were fairly similar, but the differences were significant in the large-scale models. In particular, ‘local’ damping mode tended to suppress large deformations, predicted high pillar strengths and increased the model run-time significantly. Although it is difficult to establish which of the two damping modes leads to more realistic excavation-scale
behavior, it is the opinion of the authors that ‘combined’ damping should be employed for excavation models where large-deformations and block separations are expected.

C.2 Introduction

Advances in numerical modeling techniques now allow us to study complex rock mechanics problems that are otherwise difficult to study using empirical and analytical approaches. Although numerical models are powerful and have the potential to mimic the mechanical responses of complex systems, it is critical to understand that such models will yield meaningful results only when the input parameters are realistic. Besides the material input parameters, other modeling components like constitutive models, boundary conditions, excavation schemes, damping modes, etc. also play an important role in controlling the emergent behavior of the models.

The current study is focused on the Itasca software UDEC, which is an explicit Discrete Element Method (DEM) code first introduced in the 1980s (Itasca, 2014a). This two-dimensional software is extremely powerful and versatile and allows users to conduct both continuum and discontinuum (jointed rockmass and/or bonded block) analyses. Over the years, UDEC has been used to simulate large scale phenomena like shearing and separation of rockmasses along discrete discontinuities (Singh and Singh, 2008; Saeidi et al., 2013; Walton et al., 2018a), stress-induced damage around excavations (Bai et al., 2016; Farahmand et al., 2018) as well as small-scale phenomena like microfracturing in laboratory rock specimens (Lan et al., 2010; Gao and Stead, 2014; Farahmand and Diederichs, 2015). While the aforementioned studies (as well as others) have examined various fundamental and practical aspects of UDEC, damping mode has continued to remain one of the lesser discussed topics in the literature. This study is therefore
focused on understanding the effect of damping mode on UDEC model behaviors. The authors believe that this study will ultimately provide some guidance to future researchers in choosing a damping mode for a particular application.

In UDEC, a rock specimen or a rockmass is represented by an aggregate of discrete blocks that can detach and separate once the ‘contacts’ between the blocks are damaged. The calculation alternates between application of a force-displacement law at the contacts (determines the force at the contacts due to relative shear and/or interpenetration of blocks) and Newton’s second law at each blocks (determines the motion of each block based on the net force acting on it). If the blocks are discretized using constant-strain triangular ‘zones’, then motion is first calculated at the gridpoints (vertices of zones), followed by application of constitutive relationships to define the stresses in the zones. Damping is introduced in the motion equations to achieve static equilibrium as quickly as possible.

The default damping mode in UDEC for static analysis is ‘Local’. In this mode, the damping forces applied to each gridpoint are proportional to the magnitude of unbalanced forces and the direction is opposite to the gridpoint velocity such that energy is always dissipated. The two other damping modes in UDEC are ‘Auto’ and ‘Combined’. ‘Auto’ and ‘Local’ modes have been previously found to converge to the same solution by Cundall (1987). Accordingly, only the ‘Local’ and ‘Combined’ damping modes were tested in this study. The UDEC manual (Itasca, 2014a) suggests using the ‘Combined’ damping mode when there is constant motion in one direction; the ‘Local’ damping mode tends to be effective only when the velocity changes sign locally during the simulation. The main mathematical difference is that in ‘Combined’ mode, the direction of damping force is related to the velocity as well as the derivative of unbalanced force.
The derivative of unbalanced force is independent of rigid-body motion and serves to better dampen a model that is undergoing unidirectional movements.

In terms of modeling physical phenomena like spalling of pillar ribs and detachment of roof layers in a mine entry (generally unidirectional), it is not well understood which damping mode should be used. To better understand how the choice of damping mode might influence model behavior, three different model setups were considered in this study: (a) Laboratory-scale model of a granite specimen, (b) Field-scale bonded block model of a granite pillar, and, (c) Field-scale rockmass model of a hypothetical coal mine entry. All three model types were run with ‘Local’ and ‘Combined’ damping modes and the results are compared. Note that it may be possible to achieve similar behavior with both damping modes for different sets of parameters but this was not pursued in this study. The comparisons made here between damping modes are for single sets of input parameters.

C.3 Laboratory-scale Bonded Block Models

This section deals with micromechanical modeling of Creighton Granite using the polygonal Bonded Block Modeling (BBM) approach. Creighton Granite is a granitic rock from Sudbury, Canada. It has an average grain size of 2.25 mm and is composed of approximately 15% Biotite, 30% Quartz and 55% Na-Feldspar (Sinha and Walton, 2019d). Geomechanical characterization of this rock was previously conducted by Walton et al. (2016) based on uniaxial and triaxial (confining stress range of 0-60 MPa) compression test data from laboratory. Besides the peak strengths, Crack Initiation (CI; Martin and Chandler, 1994) thresholds, Crack Damage (CD; Martin and Chandler, 1994) thresholds and volumetric changes in the rock specimens were also examined in detail by Walton et al. (2016).
Recently, the authors attempted to model this rock type using Voronoi BBMs (Sinha and Walton, 2019d; Sinha and Walton, 2020a; Sinha et al., 2020a). In that study, different representations of blocks, zones and contacts were tested to identify the optimum model complexity level that could capture all attributes/laboratory-derived properties of Creighton Granite. In particular, different combinations of homogeneous and heterogeneous (corresponding to different mineral grains and associated mechanical properties) blocks, homogeneous and heterogeneous contacts and elastic and inelastic zones were tested. Elastic zones have been generally used in the past, but we considered inelastic zones to approximate the intra-granular fracturing process in rocks. The model considered here is the outcome of this comprehensive complexity analysis and corresponds to the model representation that captured all attributes of Creighton Granite. Some details of the model setup are discussed below.

To conduct the UCS simulation, a rectangular model of 120 mm x 55 mm was developed and was loaded via a constant velocity along the top surface (Figure C.1). The bottom was constrained via roller boundaries. Triaxial simulations were also conducted using the same model setup, but with a stress boundary along the lateral edges of the model. As previously stated, all blocks and contacts in the homogeneous models were assigned a single set of properties. In heterogeneous models, three sets of properties, corresponding to the three mineral grains, were assigned to the blocks in areal proportions as defined above. This also includes six sets of contact properties corresponding to the different mineral grain associations. The mineral elastic properties were chosen from Bass (1995) and Mavko et al. (2009). For the inelastic zone representation, a strain-softening constitutive model was considered to mimic the loss of load carrying capacity of damaged/microfractured mineral grains.
It is important to record the axial stress and strain and lateral strain during the simulations so as to be able to compare the model-derived attributes to those from laboratory. For that purpose, the displacements along all gridpoints along the top/bottom edges and 10 pairs of gridpoints along the lateral edges of the model were tracked – these were later converted to axial and lateral strains by dividing with the length and width of the model, respectively. Axial stress was computed by dividing the sum of the unbalanced forces at all gridpoints along the model top with the width of the model. The extractions were performed once every 1000 solution steps using user-defined FISH functions.

Figure C.1 UDEC model setup for uniaxial/triaxial simulations (after Sinha and Walton, 2019d).

The particular model chosen in this study for testing the effect of damping mode has heterogeneous blocks and contacts with inelastic zones. Although the three mineral grain types should ideally have different strength parameters, it has been shown by Sinha and Walton
that only one set of strain-softening parameters is sufficient to capture all measured attributes of Creighton Granite. Accordingly, a further increase in model complexity (i.e. 3 sets of strain-softening properties) could not be justified. The calibrated micro-properties are documented by Sinha et al. (2020a). The models in Sinha and Walton (2019d), Sinha and Walton (2020a) and Sinha et al. (2020a) were calibrated using ‘Local’ damping mode, and the resulting models were shown to match the peak and residual strengths (confined and unconfined), average stress-strain curves, CI (confined and unconfined), CD (confined and unconfined) and normalized peak dilation angle as a function of confinement (Sinha and Walton, 2020a; Sinha et al., 2020a).

The calibrated model was re-rerun with the ‘Combined’ damping mode and the results are presented here. Figure C.2a shows the two sets of stress-strain curves for the $\sigma_3 = 0$-60 MPa simulations. Both damping modes yield more or less similar stress-strain responses for the $\sigma_3 = 20$-60 MPa simulations; for $\sigma_3 = 0$-10 MPa, the drop in stress levels in the post-peak regime is more sudden with ‘Combined’ damping mode. This likely occurred due to the failure being governed majorly by damage to block contacts and separation of blocks in the $\sigma_3 = 0$-10 MPa simulations. ‘Combined’ damping has a tendency to slightly underdamp the system (Itasca, 2014a) and accordingly, there is faster block separation and specimen de-confinement. This, however, does not occur at higher confinements, where block movements are constrained and zone yield (mimicking intra-granular fracturing) starts to play an appreciable role in the damage process. The faster block separation in ‘Combined’ damping simulations can also be observed in the volumetric strain – axial strain plot in Figure C.2b (similar volumetric strain levels in ‘Local’ and ‘Combined’ $\sigma_3 = 0$-10 MPa simulations but at lower axial strains in the ‘Combined’ ones).
Figure C.2 Comparison of (a) stress-strain curves, (b) Volumetric strain curves using ‘Local’ and ‘Combined’ damping modes.

Figure C.3 shows the fracture pattern at the end of the UCS simulations using ‘Combined’ and ‘Local’ damping modes. There is visibly greater block detachment in
‘Combined’ damping. Both models exhibit axial cracking, which is the dominant mode of failure in brittle rocks under unconfined compression, but the fracture patterns post-simulation are somewhat different. It seems that for micromechanical modeling, using either damping mode would yield similar results, although ‘Local’ damping allows for a more gradual post-peak behavior and volume change to be represented. It is to be noted here that the simulations were terminated earlier in the ‘Combined’ damping models due to block overlap errors. This is also an indication of more energy and oscillations in the models during the course of the simulations. More specifically, the tests terminated earlier in the $\sigma_3 = 40$ & $60$ MPa simulations because of its confined nature – the lateral confining stresses limited the uni-directional movements in the models and resulted in premature block overlap.

Figure C.3 Fracture pattern in the UCS model post-simulation using ‘Combined’ and ‘Local’ damping modes.
C.4 Field-scale granite pillar model

This section deals with a hypothetical granite pillar model and is an extension of a previous work conducted by the authors (Sinha and Walton, 2018b). The goal of that study was to reproduce the stress-strain responses of W/H (width to height) = 1, 2 and 3 granite pillars using elastic BBMs. In particular, the calibration targets (i.e. stress-strain curves) were selected from FLAC\textsuperscript{3D} granite pillar models (Sinha and Walton, 2018a). The elastic block pillar models could match the stress-strain responses of W/H=1-3 pillar but with 3 different input parameter sets, meaning that the parameters were not fundamental to the material being modeled.

Following that work, the authors tested the inelastic zone representation in the BBM pillar. In particular, the Cohesion-Weakening-Frictional-Strengthening (CWFS; Hajiabdolmajid et al., 2002) model was employed in the zones. The CWFS strength model is based on the findings of Martin and Chandler (1994) and essentially allows the cohesional strength to degrade first, followed by frictional mobilization. Contrary to the elastic block models, it was possible to identify a single set of input parameters that could match the stress-strain responses for all three W/H. Those models had an initial cohesion and friction angle of 46.5 MPa and 0º, respectively, which evolved to 0.5 MPa and 42º over a plastic shear strain ($\varepsilon_{ps}$, damage quantifier) of 0.01. For the purposes of this study, only the W/H=2 models (elastic and inelastic) have been considered. All original calibrations were conducted using the ‘Combined’ damping mode.

Figure C.4 shows the BBM setup for the W/H=2 pillar. The pillar is 8 m wide and 4 m high, with rounded corners to allow for a smooth transfer of stresses. Two elastic beams (mimicking the roof and the floor in underground mines) were placed on either side of the pillar and loading was conducted by assigning a velocity boundary condition to the upper and lower
edge of the top and bottom beam, respectively. The model was stepped until the peak and the post-peak behavior was completely captured. It took about 15-20 hours for the elastic and inelastic pillar simulations to complete. The calibrated W/H=2 model was subsequently rerun with the ‘Local’ damping mode and the results are compared in this section.

Figure C.4 Hypothetical granite pillar model setup in UDEC.

Figure C.5 shows the stress-strain curves from the elastic and inelastic pillar models ran with ‘Local’ and ‘Combined’ damping modes. These stresses and strains were determined by averaging the stresses over all zones in the pillar and by tracking 10 points along the top and bottom of the pillar, respectively. As expected, the stress-strain curves for ‘Combined’ damping
mode were similar and attained a peak strength of ~90 MPa (Figure C.5). Post-peak, the models exhibited a pseudo-ductile behavior as might be expected for such a moderate W/H pillar in brittle rock (Mortazavi et al., 2009; Sinha and Walton, 2018a; Li et al., 2019a). A surprisingly large mismatch in the pillar response was noted when the models were run with the ‘Local’ damping mode. In particular, the elastic block model continued to strain-harden and ultimately the simulation was terminated due to a block overlap error at stress level of ~280 MPa (>300% of the strengths in ‘Combined’ damping models). The inelastic pillar model exhibited a similar strain-hardening behavior but eventually attained a peak strength of ~190 MPa. Such large mismatches are concerning, especially when the only difference between these models is the damping mode.

Figure C.5 Comparison of the pillar stress-strain curves using ‘Combined’ and ‘Local’ damping modes.

Figure C.6 shows the fracture and yield zone patterns in the models post-simulation. For the elastic models, the damage was much more intensive with ‘Combined’ damping. Large volumetric bulking can be observed along the pillar walls, with the fracturing (white areas)
having progressed all the way to the center of the pillar. Although a direct comparison of the fracture patterns in the ‘Local’ and ‘Combined’ damping models cannot be made (one has been strained in the post-peak while the other has not yet attained peak strength), it is at least apparent that ‘Local’ damping mode results in significantly less block movement. The kinetic energy is quickly damped in such models, which leads to lesser fracture opening and block separation, an excessively confined core and greater peak strengths.

Figure C.6 Fracture and yield zone patterns in elastic and inelastic pillar models using the ‘Combined’ and ‘Local’ damping modes.
The inelastic models also exhibited a similar behavior but with even less fracture dilation with ‘Local’ damping. Part of the reason is that as the surficial blocks were prevented from detaching, there was confinement build-up within the model and this increase mobilized the frictional strength on the contacts. As a result, failure was forced to initiate in the zones (recall that the initial CWFS yield envelope had zero friction angle) and continued to progress deeper with loading. As can be seen in Figure C.6, when the ‘Local’ damping simulation was terminated, there was pervasive zone yield across the entire pillar. On the contrary, with ‘Combined’ damping, there was significant fracture dilation along the periphery and zone yield near the center. For blocks that have detached from the pillar wall, there was minimal or no zone yield within them, as damage at these locations occurred purely through contact failures. The ‘Combined’ damping model behavior is closer to reality and exhibits the highly dilatant damage process at the pillar periphery and finer-scale damage (low dilatancy due to confined state) deeper within the pillar core.

To ascertain if the strain-hardening in ‘Local’ damping models were truly attributed to its excessively damped nature, the elastic BBM was re-run but with damping factors of 0.2, 0.4 and 0.6 (the default is 0.8 for ‘Local’). Interestingly, the stress-strain response with damping factor of 0.2 was similar to that of the ‘Combined’ damping model but transitioned back to strain-hardening when the damping factor was raised to 0.4 or 0.6. Additionally, the model displacements at the pillar wall with damping factor of 0.2 were 30% greater than the displacements obtained with damping factors of 0.4 and 0.6. ‘Local’ damping indeed absorbs kinetic energy at a faster rate and slows down movement in the model. A relevant philosophical question to be asked here is whether damping in natural systems is as high as those chosen by default in UDEC. In any case, it seems that ‘Combined’ damping might be preferable when
attempting to model large dilatant phenomena, like pillar bulking. Additionally, this damping mode will yield more reasonable results when attempting to capture the clamping/reinforcement effect of supports (e.g., rockbolts, mesh, shotcrete, etc.; Sinha and Walton, 2019b; Sinha and Walton, 2020c)

C.5 Field-scale coal mine entry model

The last section illustrates a coal mine entry model with a horizontally layered lithology in the roof. In this model, sub-vertical jointing was incorporated in the form of Discrete Fracture Networks (DFNs) (Figure C.7). As it was not computationally feasible to include explicit joints spaced at few centimeters (as observed in reality), closely spaced weakness planes like beddings were accounted for implicitly using a ubiquitous joint constitutive model. The model chosen here is a part of a large matrix of models ran with various combinations of DFN and block parameters to represent different geological environments (Abousleiman et al., 2019). Abousleiman et al. (2019) established the realism of these models by comparing the results with the empirical Analysis of Roof Bolt Systems tool (Mark et al., 2001b). Originally, all simulations were run with the ‘Combined’ damping mode. For the purposes of this study, a model was selected such that there was block detachment in the roof, and it was subsequently run with the ‘Local’ damping mode. Before contrasting the results, the model setup is briefly described below.

A 2.5 m high, 6 m wide entry located at a depth of 200 m below ground surface was modeled using the setup shown in Figure C.7. Only 30 m of the roof was explicitly simulated and an in-situ stress gradient corresponding to a 200 m excavation depth was applied. A k-ratio (ratio of horizontal to vertical stress) of 0.5 was chosen. A newly derived rock yield criterion, termed as the progressive S-shaped criterion (Sinha and Walton, 2018a), was applied to the coal
seam, while the floor was modeled as elastic. First, in-situ stresses were initialized in the model, then the entry was excavated and a stress corresponding to 70% of in-situ stress was applied to excavation boundary and the model equilibrated, followed by removal of the boundary stresses and re-solving until a mechanical solution ratio of $10^{-5}$ was attained. This model is different from the two previous ones in the sense that the previous models were loaded monotonically, while in this case, the model was solved for static equilibrium following excavation. Additionally, this model used blocks to define physical blocks bounded by pre-existing rockmass fractures as opposed to potential fracture growth pathways.

![Figure C.7 UDEC model setup for simulating a coal-mine entry.](image_url)
The vertical displacement contours after the models (‘Combined’ and ‘Local’) achieved the target solution ratio are shown in Figure C.8. Interestingly, ‘Local’ damping mode suppressed the detachment of the roof slabs and estimated >50% lower displacements at the immediate roof in comparison to the ‘Combined’ damping mode. More specifically, the three immediate roof layers detached in the ‘Combined’ damping model (displacement >30 mm) while only one layer exhibited relatively high displacements (15-20 mm) in the ‘Local’ damping model. The 5-10 mm displacement region, however, extended to similar heights in the two models, indicating that the model responses in regions with slightly higher confining stress are identical when either damping mode is used (a similar observation was made in the triaxial simulations in Section C.3).

Figure C.8 Displacement profiles in the coal-mine entry model using ‘Combined’ and ‘Local’ damping modes.
While it is difficult to ascertain which of the two responses is closer to reality for the particular set of input parameters, it can at least be inferred that ‘Combined’ damping is useful when large movements are anticipated in the model. When high damping factors are used, the system is brought to equilibrium quickly and this allows less movement (e.g. fracture separation) to occur within the model. The difference in solution time can be verified by checking the number of solution steps in the two models – the ‘Combined’ damping model took 71821 steps to reach the target solution ratio, while the ‘Local’ damping model took 47774 steps. The greater number of solution steps in the ‘Combined’ damping model was likely related to the layers detaching from the entry roof (more kinetic energy) which would require more steps to reduce the solution ratio.

Although only one entry model is shown in this study, the authors would like to highlight the reason for choosing ‘Combined’ damping mode in Abousleiman et al. (2019). The ‘Local’ damping models exhibiting block detachment (unlike the model discussed above where there is no detachment) took a significantly longer time to run than the ‘Combined’ damping ones, because of the increased effort to bring the roof layers to equilibrium prior to detachment and the inability of ‘Local’ mode to dampen the system efficiently once uni-directional block movements initiated. For these model parameter sets, the separation and detachment occurred much earlier in the ‘Combined’ damping model and accordingly, unbalanced forces could be reduced quickly. The goal of Abousleiman et al. (2019) was to understand how the interaction between the roof and the pillar affected the global stability of the model, and the detachment of the roof layers was therefore integral to be able to study this phenomenon. The faster runtime for models exhibiting roof detachment was the primary motivation for choosing ‘Combined’ damping mode. The practical value of this discussion, in combination to those from the previous sections, is that the
damping mode can be influential not only from a mechanistic standpoint (i.e. capturing large dilation/separation phenomena) but also from a computational standpoint.

### C.6 Conclusions

This study has tried to understand the effect of damping mode on UDEC simulations in context of three model setups – a laboratory-scale micromechanical model of a granite, a field scale model of a granite pillar and a field-scale model of a coal mine entry. The following conclusions can be drawn:

1. Damping mode does not have a particularly strong influence on the behavior of micromechanical models, although ‘Combined’ damping mode results in faster deconfinement and volume increase under low confinement conditions. ‘Local’ damping allows for a more steady post-peak response, damps the model faster and prevents block overlap errors in triaxial simulations. Accordingly, ‘Local’ damping mode might be the preferable choice for studies attempting to simulate rock behavior (specimen-scale) over a wide range of confinements.

2. ‘Combined’ damping mode results in much lower pillar strengths (W/H=2 tested here) in comparison to ‘Local’ damping mode. The discrepancy is higher when elastic instead of inelastic zones are used. ‘Local’ damping mode tends to overdamp the system, prevents block separation along the pillar walls and generates excess confinement within the pillar. This confinement mobilizes frictional strength on the contacts and prohibits its failure, which in turn triggers failure to initiate within the zones. Understandably, when the zones
are elastic and cannot yield, the model continues to strain-harden, achieving very high peak strengths.

(3) ‘Combined’ damping mode allows block separation at the pillar walls and might be preferable for simulating processes like geometric dilation and bulking of ribs.

(4) In the entry-scale model, roof layer detachment was impeded by the ‘Local’ damping mode. Up to three layers detached when ‘Combined’ damping mode was employed. This reiterates the greater damping potential of ‘Local’, which ultimately leads to lower displacements and greater stability in the model.

(5) ‘Local’ damping models exhibiting roof detachment took significantly longer to attain equilibrium (or reduce unbalanced forces) in comparison to ‘Combined’ damping models. Accordingly, when attempting to model such unstable phenomenon, ‘Combined’ damping might be preferred.

C.7 Acknowledgements

The research conducted for this study was funded by the National Institute for Occupational Safety and Health (NIOSH) under Grant Number 200-2016-90154. The authors would like to extend their gratitude for the financial support. Special thanks to Dr. Mark K. Larson and Dr. Bo-Hyun Kim for reviewing this manuscript prior to submission and providing valuable suggestions.
APPENDIX D

EFFECT OF BOLT NODE SPACING ON BBM BEHAVIOR

Rockbolts and cable structural elements in UDEC are sub-divided into segments, with a nodal point located at each end of a segment. Shear resistance is represented by a spring/slider connection between the structural nodes and the block zones in which the nodes are located. The interaction between the structural elements and the finite difference zones therefore occurs specifically at these nodal points.

There are no well-established guidelines for selecting the number of nodes for a particular problem. Studies like Bouzeran et al. (2017) and Garza-Cruz et al. (2019b) have used a segment length of 0.1 m (where \# of nodes = 1 + [Total Length of Bolt / Segment Length]) for 3D BBM simulations with tetrahedral blocks. From a theoretical perspective, the selection of bolt node spacing is important for two reasons: (1) The block contacts are potential failure pathways in a BBM and if the node spacing is not sufficiently small, then the influence of support with respect to the suppression of block separation (local strain concentrations) could be underestimated/neglected; (2) As discussed previously, the interaction between the rockmass and structural elements occurs at the nodes. The spacing, therefore, influences how bolt elements are strained due to the deformation of the rockmass around them, which ultimately controls their yield/failure. Bolt node spacing is not as important in continuum models, as the rockmass strain distributions are relatively smooth.

To understand the combined effects of fracture position along the bolt length, zone gridpoint locations relative to bolt nodes and number of nodes on rockmass-support interaction, a simple model, as shown in Figure D.1, was developed. In this model, the blocks are elastic (E=8
GPa and ν= 0.28) and the left block is fixed while the right boundary is translated via a slow velocity of 0.01 m/s. The model setups tested and some associated results are presented below:

(1) When the fracture is placed exactly at the bolt midpoint, increasing the number of nodes (11, 21, 31 segments, or 22, 22, 32 nodes) did not affect the peak axial load. The peak axial load (related to the strain in each segment) after 30000 solution steps when there was 3.26 mm of fracture opening was found to be 78.1 kN in all three models (see Figure D.1).

Figure D.1 Axial load distribution in the 22 node (or 21 segment) model. The peak load was the same in the 22 and 32 node models.

(2) To ascertain if the locations of the zone gridpoints with respect to the bolt nodes have an effect on the bolt-rockmass interaction, three sets of models were run with the fracture placed at the bolt mid-points (similar to Figure D.1). The three sets of models had the following numbers of bolt segments: 13, 23, 33; 15, 25, 35; 17, 27, 37. These numbers
were chosen to change the locations of the nodes that define the bolt segment containing
the fracture with respect to the zone gridpoints. All these models resulted in the same
peak axial load of 78.1 kN. It is therefore concluded that the relative positions of the zone
gridpoints and bolt nodes in a symmetrical case do not affect the bolt loads.

(3) In the 21 segment case, when the bolt was translated in increments of 0.01 m to the right
so as to modify the location of the fracture with respect to the bolt, the loads remained
constant as long as the fracture was in between the two central nodes. Each time the bolt
was sufficiently translated such that the fracture entered a new bolt segment, however,
the bolt load dropped (to 70.6 kN after the first transition and 63.2 kN after the second). It
therefore appears that changing the location of the fracture with respect to the bolt has a
significant influence on the bolt load.

(4) When the fracture was slightly offset (0.07 m) with respect to the bolt center (Figure D.2)
and the model was run with 11, 21 and 31 segments (or 12, 22, 32 nodes), the obtained
peak axial loads were 64.01 kN, 70.6 kN and 68.1 kN, respectively. Note that the peak
axial loads did not show any consistent trend as a function of the number of segments
considered. It appears that the addition of nodes and associated changes in the locations
of the nodes immediately bounding a fracture affects the bolt-rockmass interaction only
when the fracture is asymmetrically located with respect to the bolt length.

(5) To further determine if zone gridpoint locations with respect to node locations have some
effect in this asymmetric case, the model with 21 bolt segments (or 22 nodes) and 0.07 m
fracture offset from the bolt center was re-run with different zone sizes. The changes in
peak axial bolt loads with changing zone size were minimal; when the zone size was
doubled or halved, differences of the order of ~3-6 kN in peak axial loads were obtained. For smaller changes in zone size, the peak axial loads changed by less than 1 kN. Based on this, it appears that for the asymmetric case, node positioning relative to fractures impacts the rockmass-support interaction more significantly the relative positions of zone gridpoints and bolt nodes.

Figure D.2 Fracture located asymmetrically with respect to the bolt length.

While these simple models provide some sense of what discretization factors influence bolt-rockmass interaction, the effects of node density and position are expected to be much more pronounced in a BBM where fracturing occurs simultaneously at multiple points along the length of the bolt. In this simple model, the support resistance is much lower than the driving force, but in an excavation-scale BBM, the feedback loop between the structural element loads and rockmass loads has the potential to lead to much larger differences than those presented here.

To understand the influence of bolt node spacing on the behavior of excavation-scale BBMs, the West Cliff Mine model from Chapter 6 and the hypothetical BBM from Chapter 9
were re-run with different number of bolt nodes (i.e. different node spacing). In the calibrated West Cliff Mine model (0.05-0.08 m wide blocks), the bolts were 1.2 m long and were subdivided using 25 nodes (24 segments). The node spacing (0.05 m) was chosen to ensure that there was at least one node in each block. The model was subsequently run with 13 nodes, 37 nodes, 49 nodes, 61 nodes and 81 nodes and the corresponding rib displacements as a function of stress changes 4 m into the rib are shown in Figure D.3. It can be seen that the profiles differ in a seemingly random manner and even with a reduction in bolt node spacing, the stress-displacement profile does not converge to a single solution. One might expect this behavior to be related to more strain localization on segments with smaller node spacing (strain $\propto 1/\text{length}$) causing early element rupture, but this is not supported by the 49 node and 61 node curves, as the 49 node model shows significantly less displacement than the 61 nodes model at the end of simulation.

Figure D.3 Rib displacement versus stress change in models with different number of nodes.
To illustrate that the erratic influence of bolt node spacing shown in Figure D.3 is not related to the elongated block shape used in the West Cliff case, the hypothetical granite pillar case from Chapter 9 was considered. Specifically, one half of the pillar was simulated with input parameters from Table 9.2 and 2.4 m long bolts were sub-divided into 24 segments, 48 segments and 72 segments. The blocks in this case were 0.1-0.12 m wide. The axial load on the bolts and the corresponding rock displacements for a particular loading stage (average axial strain of 0.0034 close to peak strength; refer Figure 9.3) is shown in Figure D.4. Similar to the West Cliff Mine model, the displacement profiles vary without any consistent trend. In other words, more bolt nodes did not necessarily imply more displacement reduction or less displacement reduction.

Figure D.4 Axial load and lateral displacements in the hypothetical granite pillar models with different number of bolt nodes.
Although more study is necessary to better understand this interaction between BBMs and structural elements, the author believes that the observed random behavior could be related to the specific interaction algorithm used. In particular, UDEC employs an interpolation scheme to compute the incremental displacements ($\Delta u$) at the node points (Itasca, 2014b):

$$\Delta u = W_1 \Delta u_1 + W_2 \Delta u_2 + W_3 \Delta u_3$$

(D.1)

where, $\Delta u_1, \Delta u_1$, and $\Delta u_3$ are the incremental displacements of the gridpoints of the zone in which the bolt node is located, and $W_1, W_2,$ and $W_3$ are the weighting factors. The weighting factors are calculated as (refer Figure D.5):

$$W_1 = \frac{A_1}{A_T}$$

(D.2)

Forces that are generated at the nodes ($F_p$) are distributed back to the gridpoints according to the same weighting factors:

$$F_i = W_i \cdot F_p$$

(D.3)

where, $F_i$ is the force applied to the gridpoints, $i=1, 2, 3$.

The location of a node within a zone, therefore, has an effect on the amount of resistive force that is applied to the host gridpoints. With the results from the simple models in mind, it seems that this effect is most dominant when the zone is adjacent to a block boundary (i.e. a potential fracture). The breakage of a block contact locally modifies the boundary conditions of the zones contacting it, and if a structural element node is present inside these zones, then the bolt strains and displacements will also be modified. Given that the block arrangement in BBMs are stochastic in nature, increasing or decreasing the node density has the potential of introducing nodes closer to or further away from block boundaries. It is perhaps for this reason that damage
progresses differently in BBMs with different node spacing. Until this phenomenon is better understood, it is the author’s suggestion that at least one bolt node must be positioned within each block and the node spacing should not be changed in subsequent model runs.

Figure D.5 Interaction between the gridpoints of a zone and a nodal point of a reinforcement element (Itasca, 2014b).
APPENDIX E

SMALL-SCALE BBM WITH EXPLICIT INTRA-GRANULAR FRACTURING

In order to allow for explicit intra-granular fracturing, a small-scale BBM was developed in which each of the mineral blocks were further sub-divided into sub-blocks. Sub-blocks were generated by determining the centroid of each block and then incorporating crack elements between the centroid and the block vertices. A similar methodology was followed by Gao et al. (2016) for creating sub-tessellations in their Voronoi models. As each sub-block was further discretized using constant strain-triangular zones, a criterion was set in order to prevent creation of very small sub-blocks – smaller sub-blocks would require smaller zones, which would ultimately increase the model run-time. In particular, the crack insertion procedure was skipped whenever the distance between the centroid and a vertex was less than 0.75 mm. Figure E.1 shows the final geometry of the UCS model. Care was taken to not modify the block structure or the location of each mineral block within the model such that a direct comparison with the heterogeneous elastic BBM in Chapter 7 was possible.

For this study, the same inter-grain contact properties as those in the heterogeneous, elastic BBM were selected and only the intra-grain contact properties were modified. The goal was three folds: (a) Determine if the high confinement peak strengths could be reproduced if intra-granular fracturing was considered explicitly, (b) Discern if the artificial hardening in the stress-strain curves for the heterogeneous, elastic BBMs could be reduced by providing additional pathways for fracture development, and, (c) Identify whether behaviors similar to the heterogeneous, inelastic BBM could be obtained using such an approach. Note that each sub-
block in this BBM was modeled as elastic, meaning that damage can occur only along the inter-
(i.e. grain boundary) and intra-grain contacts.

Figure E.1 Geometry of small-scale BBM.

Table E.1 lists the calibrated set of intra-grain contact properties. Very high values had to be selected to allow damage to initiate first at grain boundaries, followed by grain damage at higher stress levels. All intra-grain contacts were assigned the same properties in order to reduce the potential for parameter non-uniqueness and also because Sinha and Walton (2020a) found a single set of inelastic properties (for all grains in the model) to be sufficient for capturing various pre- and post-peak geomechanical attributes. When calibrating the models, it was found that the intra-grain contacts had a predisposition to fail in tension. Consequently, tensile strength that was
much higher than the cohesive strength had to be selected. Such a combination of intra-grain properties can also be found in Hoffman et al. (2015a, b).

Table E.1 Calibrated parameter set of the intra-grain contacts.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$j_{kn}$ (GPa/m)</th>
<th>$j_{kn}/j_{ks}$</th>
<th>$j_{coh}$ (MPa)</th>
<th>$j_{fric}$ (°)</th>
<th>$j_{rfric}$ (°)</th>
<th>$j_{tens}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>800000</td>
<td>2</td>
<td>210</td>
<td>72.5</td>
<td>5</td>
<td>380</td>
</tr>
</tbody>
</table>

Figure E.2a shows the model-predicted peak strengths as compared to the laboratory data. It can be seen that the BBM was not only capable of reproducing the tensile strength but also the peak strengths for the entire range of confinement tested. Also note the non-linear shape of the BBM strength envelope; such a non-linear shape could only be obtained using inelastic BBMs (Sinha and Walton, 2020a). Figure E.2b shows the stress-strain curves for $\sigma_3=0$-60 MPa simulations with and without intra-granular fracturing capability. The stress-strain responses are smoother and indeed more realistic for the current set of models (with intra) in comparison to the ones without intra-grain contacts. Based on these results, it can be understood that enabling damage to develop within the grains themselves significantly improved the phenomenological capabilities of the BBM. The confinement-dependent dilatancy phenomena was also well captured by this model (Figure E.2c).

The specimen Young’s modulus for the current models are lower (~10%) than the ones without intra-grain contacts (Figure E.2b). This occurred because of the more number of contacts in the model. Note that the joint normal and shear stiffness of the intra-grain contacts were much higher than those at the grain boundaries, yet a drop in the elastic modulus was obtained. Further increasing the stiffnesses might have resulted in higher modulus but only at the expense of longer runtime (timestep is related to the stiffness of various components in an UDEC model).
The post-peak responses could not be obtained for all models due to contact overlap and block deformation errors. This problem occurred with almost all parameter sets ran as a part of this study. It seems that the post-peak behaviors can be better reproduced when using inelastic zones within the blocks.

Figure E.2 (a) Comparison of peak strength as measured in laboratory and as obtained from the BBM, (b) Comparison of stress-strain curves of models with and without intra-granular fracturing capability, (c) Volumetric strain versus axial strain from the BBM.
APPENDIX F

ISSUES IN DETERMINING THE CRACK INITIATION (CI) THRESHOLD UNDER CONFINED CONDITIONS

This paper has been published in the proceedings of the 54th US Rock Mechanics/Geomechanics Symposium, Golden, Colorado (Sinha et al., 2020c). It is reprinted with permission from the co-authors and ARMA with some minor variations.

F.1 Abstract

It is important to accurately identify the stress at which microfractures initiate in rocks under compression when studying phenomena like excavation damage zone (EDZ) formation around deep underground structures, rock burst dynamics, stability of rock slopes, etc. This stress threshold, popularly termed as the point of Crack Initiation (CI), can be determined from strain measurements or acoustic emissions monitored during laboratory compression tests. In terms of strain-based approaches, the reversal of crack volumetric strain and the point of non-linearity in the axial stress-lateral strain curve are commonly used to identify CI. There is, however, a lack of understanding as to whether these approaches are applicable/effective for CI determination under higher confinements. To shed some light on this topic, this study utilizes the two aforementioned strain-based approaches to determine CI over a wide range of confinements in context of a granitic rock, a limestone and a coal. We discuss difficulties associated with using the two approaches for cases when the lateral strain curve is non-linear from the start of the tests. Ultimately, it was found that both approaches yield similar CI estimates at low confinements, but the discrepancy between the results obtained using these approaches increases as a function of
confinement. In particular, the stress at the point of axial stress-lateral strain non-linearity can be 1.5 times higher than the stress at the point of crack volumetric strain reversal.

F.2 Introduction

When brittle rocks are loaded in compression under a high ratio of major to minor principal stress ($\sigma_1/\sigma_3$; e.g. unconfined or low confinement conditions), they undergo progressive damage starting with the initiation of extensile microcracking, followed by microcrack interaction and coalescence that ultimate leads to the formation of a failure plane at peak strength. These distinct phases of damage are illustrated on a typical stress-strain curve in Figure F.1. The stress levels at which extensile microcracks initiate and interact/coalesce are popularly known as Crack Initiation (CI) threshold and Crack Damage (CD) threshold, respectively. Unlike the peak strength (Hudson et al. 1972; Diederichs and Martin, 2010), CI and CD are true characteristic material parameters, meaning that they do not depend on the loading conditions employed in the test (Martin and Chandler, 1994).

CI and CD have broad design applications, especially for deep underground structures (e.g. nuclear repositories) being constructed in massive to sparsely fractured rockmasses. As a consequence of the AECL Mine-By Experiment (Martin and Read, 1996; Reed, 2004) and Aspo Pillar Experiment (Andersson and Martin, 2009; Andersson et al., 2009), CI and CD have been long treated as the lower and upper bound in-situ compressive strength, respectively (Diederichs, 2003). In particular, extensile cracking/spalling can initiate adjacent to an excavation boundary (low confinement conditions) when the tangential stresses exceed the CI threshold. Characterizing the CI under high confinements (i.e. for locations away from the excavation wall), is equally important as characterizing the CI under low confinements. This is because of
the potential for connected microcracks to form pathways for leakage of radioactive contaminants in the Excavation Damage Zone (EDZ) (Ghazvinian, 2015; Perras and Diederichs, 2016). As one moves away from the excavation wall, the microcracks become less dilatant in the EDZ, but they can still affect the flow and transport properties of the rockmass (Ghazvinian, 2015). Another reason for characterizing CI is its relevance to pillar design: under low $\sigma_3$, CI plays a major role in controlling the surficial spalling process and thereby the integrity of the openings.

The CI threshold marks the onset of stable extensile microcracking in a rock specimen and can be identified by monitoring the acoustic emissions or the axial and lateral strains during compression tests (Eberhardt et al., 1998). For the acoustic emission technique, CI corresponds to the point beyond which there is a systematic increase in crack emissions following an increase in applied stress, which is also equivalent to the point of nonlinearity in the lateral strain – axial stress curve (Eberhardt et al., 1998; Diederichs and Martin, 2010). In terms of the strain-based approaches, CI can be determined directly as the point of non-linearity in lateral strain - axial stress curve or as the point of crack volumetric strain reversal (CVSR) (Diederichs and Martin, 2010; Ghazvinian, 2010). The CVSR approach has been used extensively over the years to determine the CI for different rock types tested over a wide range of confining stresses (Andersson et al., 2009; Zhao et al., 2013; Walton et al., 2017; Peng et al., 2018b; Kim et al., 2018). While most of these previous studies report CI, they do not discuss the subjectivity associated with determination of these points (it will be shown in this study that the CI determined using the CVSR and Inverse Tangent Lateral Stiffness or ITLS approaches could be different by over 150%), especially using the strain-based approaches under high confining stresses. This study is therefore an attempt to identify the issues that one might encounter while
determining the CI from confined laboratory test data. For that purpose, the complete stress-strain curves for a granite (Walton, 2014), a limestone (Walton et al., 2017) and a coal (Kim et al., 2018) were examined. For the sake of completeness, the data obtained from the unconfined compressive test were also included in the analysis.

Figure F.1 Different phases of damage on a typical stress-strain curve.

F.3 Strain-based methodologies for determination of CI

In this study, we consider the CVSR and ITLS (which is used to identify the point of lateral strain non-linearity; Ghazvinian, 2010) approaches only. In principle, both these approaches should yield the same CI, as the inelastic volume change is related to the lateral dilation of microcracks (at this point, there is no inelastic axial strain).

The CVSR approach predicts the CI to be at the point of reversal of crack volumetric strain. The crack volumetric strain \( \varepsilon_{\text{v,crack}} \) can be determined by subtracting the elastic volumetric strain from the total volumetric strain \( \varepsilon_\text{v} \), accordingly to the equations given by Martin and Chandler (1994):
\[ \epsilon_v = \epsilon_{axial} + 2\epsilon_{lateral} \]  
\[ \epsilon_{v,crack} = \epsilon_v - (\sigma_1 - \sigma_3) \frac{(1-2\nu)}{E} \]

where, \( \epsilon_{axial} \) and \( \epsilon_{lateral} \) are the axial and lateral strains, \( \sigma_1 \) and \( \sigma_3 \) are the major and minor principal stress, \( E \) is the Young’s modulus and \( \nu \) is the Poisson’s ratio. As the crack volumetric strain is dependent on the elastic constants (see Eq. F.2), it is important to first obtain reliable estimates of \( E \) and \( \nu \) before employing the CVSR approach to determine CI (Eberhardt et al., 1998; Ghazvinian, 2010). The crack volumetric strain is sensitive particularly to the Poisson’s ratio (Eberhardt et al., 1998; Ghazvinian, 2010), and it is often very difficult to determine \( \nu \) from confined rock test data. The issue lies in the mismatch in the stress ranges corresponding to the linear segments of the axial and lateral strain data. In other words, the linear segment of axial strain – axial stress curve might exceed the CI while the linear segment of lateral strain - axial stress curve might include some degree of crack closure in the axial strain space. The occurrence of the former situation results in overestimation of \( \nu \), while the latter one leads to underestimation. Note that the ISRM suggested methods do not discuss this difficulty nor does it provide any solution to this problem (Bieniawski and Bernede, 1979; Fairhurst and Hudson, 1999). Recently, in one case the authors found the Poisson’s ratio to be 260% higher when using the second approach in comparison to the first one (Sinha and Walton, 2020a). Given the subjectivity in \( \nu \) determination, the authors computed the crack volumetric strain (CVS) for a granite (tested under 60 MPa confinement) with \( \nu \) of 0.1, 0.2 and 0.3 and the corresponding graphs are shown in Figure F.2. As can be seen, the point of CVSR shifts significantly to the right with an increase in Poisson’s ratio. The amount of shift is not linearly related to \( \nu \) and is controlled by the shape of the stress-strain curve.
To avoid these issues with the CVSR approach, Ghazvinian (2010) proposed the ITLS approach. The advantage of the ITLS approach is that it relies solely on the shape of the axial stress – lateral strain curve for identifying CI. The inverse tangent lateral stiffness can be calculated using the following equation (Ghazvinian, 2010):

$$\epsilon_i \Delta = \frac{\Delta \epsilon_{\text{lateral}}}{\Delta \sigma_i}$$  \hspace{1cm} (F.3)

where, $$\Delta \sigma_i = \sigma_{i+8} - \sigma_{i-8} \ (i = 1, 2, 3 ...$$) and $$\sigma_i$$ is the i\(^{\text{th}}\) axial stress data point, $$\Delta \epsilon_{\text{lateral}} = \epsilon_{\text{lateral},i+8} - \epsilon_{\text{lateral},i-8} \ (i = 1, 2, 3 ...)$$ and $$\epsilon_{\text{lateral},i}$$ is the i\(^{\text{th}}\) lateral strain data point.

![Figure F.2 Effect of ν on the point of CVSR.](image_url)

The recommended bin size of 16 can be modified depending on the resolution of test data. In this study, a bin size of 16 was employed following a moving average analysis to reduce the noise in the ITLS values. The point where the ITLS starts to deviate from linearity is the CI threshold. Although a more robust and repeatable methodology would have been to analyze the derivative of the axial stress – lateral strain curve, this was not done here as the purpose of this

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paper is to compare the CVSR and ITLS approaches rather than to establish a new methodology for determination of CI.

F.4 Creighton Granite

The granite selected for analysis is from Creighton mine located in Sudbury, Canada. Walton et al. (2016) previously characterized this rock on the basis of unconfined and confined compression test data. These tests were conducted on 120 mm x 55 mm circular specimens in a stiff loading system at CANMET laboratory in Canada. Creighton Granite has an average unconfined compressive strength (UCS) of 203 MPa and a tensile strength of 9 MPa. A Hoek-Brown fit (Hoek and Brown, 1980) to the compression and tensile strengths yielded $m_I$ of 20.9 (Walton, 2014), which corresponds to high strain burst potential per Diederichs (2007). Sinha and Walton (2020a) subsequently utilized the axial stress, axial strain and lateral strain data from unconfined and triaxial ($\sigma_3$=0-60 MPa) tests to determine the CI (ITLS approach) and CD thresholds and re-evaluate the Poisson’s ratio. For the purposes of this study, CIs were determined using the CVSR approach and have been compared to those in Sinha and Walton (2020a).

Figure F.3 shows the ITLS-based and CVSR-based CIs. Note that the ITLS-based CIs have been translated to the right by 1 MPa for visualization purposes. A linear fit is employed in this case as well as in the subsequent sections because such a trend was previously observed from micro-seismic data in Lac du Bonnet granite (Martin, 1997). The ITLS and CVSR provide close estimates of CI at $\sigma_3$=0 MPa (although ITLS-based CI was found to be consistently higher than CVSR-based CI under unconfined conditions and was statistically different with a p<0.0001 for Student’s t-test) but diverge for increasing confining stress. While this might seem counter-
intuitive to the trend associated with the CD threshold, i.e. volumetric strain reversal occurs at a greater stress level than the point of non-linearity in axial stress – axial strain curve in confined tests (Diederichs, 2003), it must be recalled that there is no inelastic axial deformation in the specimen at this point. The divergence is likely related to the low estimation of Poisson’s ratio, which in this case was based on the initial linear section of the axial stress – lateral strain curve. When Poisson’s ratio is estimated from the initial portion of the axial stress – lateral strain curve and that curve is non-linear right from the start of the test, inelastic volume change in the specimen (or CI) must correspond to a relatively low stress level.

Figure F.3 CI thresholds of Creighton Granite determined using the CVSR and ITLS approaches. When no linear region could be identified in the axial stress - lateral strain curve, especially in the high confinement tests, the region immediately ensuing the hydrostatic loading phase was selected for computing the Poisson’s ratio. This is illustrated in Figure F.4a and b, in context of $\sigma_3=60$ MPa triaxial stress-strain curves (this is the highest level of confinement at which the rocks were tested). The methods followed for selecting the CI using the ITLS and
CVSR approaches for this test are also illustrated in Figure F.4c and d. The ITLS is an inverse measure of the slope of the axial stress – lateral strain curve and a linear increase in ITLS therefore implies a decrease in slope. From Figure F.4c, it can be seen that there is no constant ITLS section, meaning that the slope of the axial stress – lateral strain started to change as soon as the deviatoric stress was applied to the specimen. The lack of a linear region is the primary source of the difficulty in estimating a Poisson’s ratio in triaxial compression tests and the subjectivity involved in the CVSR approach.

The selection of CI in this case was based on the change in linearity of the ITLS values (Figure F.4c), which is mathematically equivalent to a change in the curvature of the axial stress – lateral strain curve. A similar approach was followed by Ghazvinian (2010) for determining the CI for Stanstead Granite. The exact mechanism by which the slope of the axial stress – lateral strain curve can decrease continually without the occurrence of additional irreversible damage (extensile microcracking) is not well understood. It is possible that the vertical pre-existing cracks that close during the hydrostatic loading phase start to open as soon as the deviatoric stress is applied to the specimen, manifesting in a non-linear lateral response from the start of deviatoric loading. The fact that the CI determined from acoustic emissions corresponds well with that from the ITLS approach in Ghazvinian (2010) further supports this proposition.

The crack volumetric strain reversal approach predicts a CI of 120 MPa that is less than 50% of the ITLS-based CI value. As discussed above, this is caused by the low estimate of \( \nu \). One can expect the CI to increase with increase in \( \nu \) (Ghazvinian, 2010); increase in \( \nu \) reduces the elastic component of volumetric strain (refer equation F.2) and raises the crack volumetric strain to the limit when crack volumetric strain becomes equal to volumetric strain at \( \nu \) of 0.5.
Figure F.4 (a) Stress-strain curve for a $\sigma_3 = 60$ MPa triaxial test, (b) Zoomed in-view of the stress-strain curve in (a) to show the region selected for determining Poisson’s ratio in cases where the axial stress – lateral strain curve is non-linear from the start of the deviatoric loading phase. The regions are also highlighted by dotted maroon lines. (c) ITLS versus axial stress, (d) CVS versus axial strain.

F.5 Utah Coal

In this section, we utilized the compression test data on Utah coal, analyzed and documented previously by Kim et al. (2018). Uniaxial and triaxial tests were conducted on 44 mm diameter specimens with different cleat orientations (with respect to the longitudinal axis of the specimen) but only the $0^\circ$ cleat data were considered here. In addition to the $0^\circ$ dataset in
Kim et al. (2018), some high confinement data ($\sigma_3 > 12 \text{ MPa}$) were also included for broadening the scope of this study. The CI thresholds were determined using the CVSR and ITLS approaches and the results can be found in Figure F.5. Similar to Figure F.3, a divergent trend was noted, with the ITLS approach estimating a 55% higher CI than the CVSR approach at $\sigma_3 = 110 \text{ MPa}$. Slight mismatch was observed in the CI values under unconfined conditions (statistically different means). The Poisson’s ratio was again based on the initial linear section of the axial stress – lateral strain curve, where available; otherwise, it was based on the region immediately following the hydrostatic loading phase (as in Figure F.4b).

![Figure F.5 CI thresholds of Utah coal determined using the CVSR and ITLS approaches. The ITLS data points have been translated to the right by 1 MPa for better visualization.](image)

The ITLS versus axial stress and the CVS versus axial strain for a $\sigma_3 = 11.2 \text{ MPa}$ triaxial test are shown in Figure F.6 to further emphasize the discrepancy. A continued increase in the ITLS value immediately past the hydrostatic loading phase can be noted. The point where the ITLS departed from linearity is the CI, which in this case is at $\sigma_1 = 51.7 \text{ MPa}$. The CVSR, on the other hand, occurred at 22.8 MPa – a 56% drop with respect to the ITLS-based CI. With regards
to Figure F.6a, no identifiable change in the ITLS slope occurred at 22.8 MPa that can be related to systematic extensile cracking. A similar behavior was observed in all other confined tests.

![Figure F.6](image)

Figure F.6 (a) ITLS versus axial stress, (b) CVS versus axial strain for a $\sigma_3 = 11.2$ MPa triaxial test.

The identical trends in the CI data for Creighton Granite and Utah coal might be related to their brittle character. A popular index for measuring material brittleness is the Hoek Brown
curve fit parameter $m_i$ (Diederichs et al., 2007; Kahraman et al., 2018) – a larger value implies more brittleness. For Creighton Granite, $m_i$ is equal to 20.9 (Walton, 2014) while for Utah Coal, it ranges from 15 to 20 for cleat angles of 0° - 45° (Kim et al., 2016). Such high brittleness for both rock types imply that the brittle-to-ductile transition (Mogi, 1966) would occur at large confining stresses (Kim et al., 2019); consequently, fully ductile stress-strain behaviors were not observed in any of the aforementioned tests (even at $\sigma_3=110$ MPa in coal).

**F.6 Indiana Limestone**

Indiana Limestone is a Mississippian age carbonate rock, with a grain size of approximately 0.3-0.5 mm and a mean porosity of 14.8%. Walton et al. (2017) previously analyzed the stress-strain curves of Indiana limestone ($\sigma_3=0-60$ MPa) to understand how the post-yield strength and dilatancy evolves across the brittle-ductile transition. That study found $\sigma_3=30$ MPa, equivalent to $\sigma_1/\sigma_3\approx 5$, to mark the transition between the brittle and ductile domain, and accordingly, ductile/strain-hardening responses were obtained in the $\sigma_3>30$ MPa tests. When deformations become increasingly ductile, pore collapse plays a more important role than microcracking, leading to an initial compactant stage in carbonate rock deformation (Wong and Baud, 2012). However, as the specimen is loaded continually, the behavior ultimately becomes more dilatant at large strains.

For the purposes of the current study, CI was re-evaluated from the Indiana Limestone stress-strain curves using the ITLS and CVSR approaches. Figure F.7 graphically illustrates these two approaches in the context of a UCS test and two triaxial tests with the confining stresses at 20 MPa and 50 MPa. It follows from the previous discussion that the UCS and $\sigma_3=20$ MPa tests are in the brittle regime while the $\sigma_3=50$ MPa test is in the ductile regime. From the
ITLS approach, two different CIs were determined (termed as 1 and 2) – one corresponds to the termination point of the plateau while the other corresponds to the point where the ITLS becomes non-linear. Recall that it is the second point (or 2) that was considered as CI in the two previous sections. In the CVSR approach, the CVS curves from the triaxial tests had to be translated in order to align the peak point with 0 crack volume strain (refer to Figure F.7d, f).

The ITLS (1) point seemed to align with the CVSR point in the UCS and $\sigma_3=20$ MPa tests, but a mismatch was noted in the $\sigma_3=50$ MPa test. In particular, two peaks were observed in the $\sigma_3=50$ MPa test, with the first peak occurring at a stress level close to the ITLS (1) point. Double peaks were also noted in the $\sigma_3=40$ MPa and $\sigma_3=60$ MPa tests (Walton et al., 2017), and in such a scenario, the global maximum was chosen as the CI. The double peaks were not observed in the low confinement tests (see Figure F.7b, d) implying that this is related to the ductile deformation processes.

Strangely enough, a near-perfect elastic change in the lateral strain occurred right after the hydrostatic loading phase (‘plateau’ before ITLS (2)). This ‘plateau’ was found in all the confined tests, even at $\sigma_3=2$ MPa. It was in this section that the Poisson’s ratio was determined in Walton et al. (2017) and in this study. Since the ‘plateau’ region is preceded by compaction (closure of pre-existing cracks, pore collapse, etc.), it is possible there was a significant drop in porosity due to hydrostatic loading and subsequent application of deviatoric loading produced ‘true’ elastic response. With the discussions in previous sections in mind, the rise of ITLS at (1) and the continued linear response could be related to the re-opening of the pre-existing cracks. The fact that the slope of ITLS is steeper in the hydrostatic loading phase than in the linear region supports this notion – the closed pores cannot re-open with as much ease as can the pre-existing cracks.
Figure F.7 ITLS versus axial stress for (a) $\sigma_3 = 0$ MPa, (c) $\sigma_3 = 20$ MPa, (e) $\sigma_3 = 50$ MPa triaxial tests, and CVS versus axial strain for (b) $\sigma_3 = 0$ MPa, (d) $\sigma_3 = 20$ MPa, (f) $\sigma_3 = 50$ MPa triaxial tests.
The predominance of pore closure in the ductile regime can be readily observed from the \( \sigma_3 = 50 \) MPa test result (Figure F.7e, f). Once the ITLS (1) stress level was attained in this test, there was a slight drop in the CVS value (dilation) triggered by an increase in lateral strain. Following that, the ITLS increased (lateral dilation) in a linear fashion but the crack volume continued to decrease (CVS increased). This is counterintuitive but can be explained by the greater contribution of pore collapse towards the crack volume strain than the opening of pre-existing cracks (Wong and Baud, 2012). It is only when the ITLS has started to increase in a non-linear fashion, i.e. formation and dilation of new extensile cracks, that the crack volume strain exhibits dilatancy. Clearly, the mechanisms involved in the deformation of a porous, less brittle \( m_1=7.1 \); Walton et al., 2015a) carbonate rock in the ductile regime is different than those in the brittle granite and coal.

The CIs determined from the two approaches are summarized in Figure F.8 (a modified Boltzmann sigmoid curve was fitted to CVSR, as in Walton et al., 2017). The following observations can be made: (a) ITLS (1) occurred at a lower stress level than ITLS (2) and CVSR; (b) Even at \( \sigma_3=0 \), ITLS (2) consistently occurred at a higher stress level in comparison to CVSR; (c) Prior to 30 MPa confining stress, ITLS (2) estimated greater CIs than CVSR but the trend reversed as soon as the specimens were loaded past the brittle-ductile transition. This is likely attributed to a change in deformation mechanisms as discussed above. Based on these results, it seems that ITLS might be a better approach for determining CI under high confinements, especially because it considers the lateral response of the specimen only. At CI, cracking occurs mainly along the major principal stress direction (Diederichs, 2007) and any lateral strain-based approach should therefore yield accurate CI estimates. On the contrary, CVSR involves the entire volume of the specimen and therefore has the potential to erroneously estimate CI if the
volume is influenced by inelastic deformation mechanisms occurring along the longitudinal axis of the specimen.

Figure F.8 CI thresholds of Indiana Limestone determined using the CVSR and ITLS approaches. The ITLS data points have been translated to the right by 1 MPa for better visualization.

F.7 Conclusions

This study has compared the CI thresholds determined using the crack volumetric strain reversal (CVSR) approach and the inverse lateral tangent stiffness (ITLS) approach for a granite, a coal and a limestone. A summary of the CIs determined using the two approaches is provided in Table F.1.

The following conclusions were drawn:
(1) For the coal and the granite, the ITLS approach yielded higher CI thresholds in comparison to the CVSR approach, and the discrepancy increased with increase in confinement.

(2) In cases where the axial stress – lateral strain curve was non-linear right from the start of the test (mostly in triaxial tests), the ITLS (mathematically equivalent to the inverse of stiffness) was initially linear with a positive slope. The point where the ITLS departed from linearity was chosen as CI. The initial linear increase in ITLS, indicating an expansion of the specimen diameter, was thought to be caused by reopening of the pre-existing cracks under deviatoric loading.

(3) For the confined Indiana limestone tests, a constant ITLS section was observed after the hydrostatic loading phase. This was followed by an upward linear section, ultimately becoming non-linear at higher stresses. Although the end of the constant section (start of inelastic lateral strain) should ideally be the CI threshold, we considered both this point as well as the start of non-linearity to be possible CIs.

(4) When the Indiana Limestone specimens were loaded in the ductile regime, two peaks were observed in the crack volume strain values; accordingly, the global maximum was chosen as CI. The CVSR-based CI was much higher than the two potential CIs obtained using the ITLS approach. In the brittle regime, however, the ITLS approach (the point of non-linearity) predicted higher CIs than the CVSR approach in Indiana Limestone, similar to the trend observed in the coal and the granite. The reversal in trend across the brittle-ductile transition likely occurred due to a change in deformation mechanisms.
(5) From Table F.1, it can be seen that there is a significant discrepancy in the CIs determined using the two approaches. If acoustic emission or ultrasonic techniques are unavailable, the authors recommend using the ITLS approach, as it is a more direct methodology for CI determination. The ability of the ITLS approach to estimate CIs similar to those from acoustic emission was previously demonstrated by Ghazvinian (2010).

Table F.1 CIs determined using the two approaches for the three rock types.

<table>
<thead>
<tr>
<th>Methodology</th>
<th>Creighton Granite</th>
<th>Utah Coal</th>
<th>Indiana Limestone</th>
</tr>
</thead>
<tbody>
<tr>
<td>CVSR</td>
<td>(1.18 \sigma_3 + 74.2)</td>
<td>(1.12 \sigma_3 + 9.9)</td>
<td>(91.7 + 1.69 \sigma_3 - 69.7 / (1 + e^{(\sigma_3-20)/4}))</td>
</tr>
<tr>
<td>ITLS (1)</td>
<td>(2.77 \sigma_3 + 98.1)</td>
<td>(1.56 \sigma_3 + 25.7)</td>
<td>(1.11 \sigma_3 + 18.2)</td>
</tr>
<tr>
<td>ITLS (2)</td>
<td>(2.77 \sigma_3 + 98.1)</td>
<td>(1.56 \sigma_3 + 25.7)</td>
<td>(1.91 \sigma_3 + 37.9)</td>
</tr>
</tbody>
</table>

F.8 Acknowledgements

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APPENDIX G

EFFECT OF PLATEN FRICTION ON THE BEHAVIOR OF LABORATORY-SCALE BBMS

Although different testing apparatus and approaches have been employed to minimize the influence of the end constraint in laboratory compression tests (Labuz and Bridell, 1993; Ojo, 1993; Bobet, 2001), it is practically impossible to exclude this effect completely (Brady, 1971). Previous studies have tried to quantify the extent of end constraint effect using experimental and numerical approaches. Some of the important conclusions that were drawn in these prior studies are as follows:

- Increase in platen-rock friction angle increases the peak unconfined compressive strength but the strengthening effect reduces with increasing L/D ratio (length to diameter) (Tang et al., 2000; Mogi, 2006; Hemami and Fakhimi, 2014; Xu and Cai, 2015; Gao et al., 2018).

- Lower L/D (or squatter) specimens have a larger region under confinement than higher L/D (or slender) specimens. This is illustrated by the schematic in Figure G.1. Since the strength of rocks increases with confinement, it is apparent why the peak strengths of squatter specimens are more sensitive to the platen-specimen friction angle than slender specimens.

- The critical L/D beyond which the effect of platen-specimen friction angle is minimal is ~2-2.5 (Tang et al., 2000; Mogi, 2006; Hemami and Fakhimi, 2014; Gao et al., 2018).

- Increase in platen-specimen friction angle affects the post-peak response of rocks but this effect is less in stiff loading systems (Hemami and Fakhimi, 2014). Aspect ratio has a
greater control on the post-peak behavior and the response transitions from brittle to ductile as the L/D is reduced (Das et al., 1986; Tang et al., 2000; Hemami and Fakhimi, 2014).

Figure G.1 Horizontal confined zones due to end effect in specimens with different aspect ratios in uniaxial compression tests (Gao et al., 2018).

With all that in mind, we tested a platen-based loading mechanism to understand how the ideal loading mechanism (specimen loaded directly without a platen) might have influenced the response of the heterogeneous, inelastic BBM. In particular, the platens were assigned an elastic modulus and Poisson’s ratio of 200 GPa and 0.3, respectively, while the contacts between the specimen and the platens were assigned a zero cohesion and tensile strength and a friction angle of 10° (Gaffney, 1976; Fakhimi and Hemami, 2015). Loading was conducted by assigning the same velocity as those employed in the ideal models to the upper and lower edge of the platens.
Three simulations were run with confining stresses of 0, 10 and 20 MPa and the results are contrasted below.

As can be seen in Figure G.2, the peak strengths with and without platens are very similar; this is expected given that the synthetic specimens have a L/D=2.18, which falls within the range of critical L/D identified in other studies. The fracture patterns in the unconfined and $\sigma_3$=20 MPa BBM post-simulation with and without platens are shown in Figure G.3. Again, these are identical, but with lesser fracture dilation in the UCS simulation with platens. The platens probably generated some constraints along the specimen edges and this led to lesser block movements. In any case, for both model types, the mode of fracture formation transitioned from axial cracking to shear banding with increase in confinement. From these results, it seems that the effect of platens might not be very significant in the BBMs.

![Figure G.2 BBM Peak strengths for $\sigma_3$=0, 10 and 20 MPa, with and without platens.](image)
A slight mismatch was noted in the post-peak behaviors with and without platens but this is not of particular concern because of two reasons: (a) The post-peak behavior is controlled by the residual strength and residual contact parameters. Accordingly, one can identify another set of residual parameters that will yield similar post-peak behaviors. (b) The goal of Chapter 7 was to test the phenomenological capabilities of a BBM towards modeling a granitic rock. Employing either loading mechanism would have ultimately led to similar conclusions (but with different parameter sets).

Figure G.3 Fracture pattern in the $\sigma_3=0$ and 20 MPa BBM simulations, with and without platens.
SIMULATING LABORATORY-SCALE DAMAGE IN GRANITE USING BONDED BLOCK MODELS (BBM).

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H.1 Abstract

With recent advances in numerical modeling techniques, Bonded Block Models (BBMs) are increasingly being used to study damage processes in rocks. In this modeling technique, a material space is represented by an aggregate of polygonal blocks, which interact along their contacting edges. In this study, a BBM of Creighton Granite was developed and the input parameters were calibrated to match a wide range of laboratory-derived attributes. Special attention has been paid to the post-yield behavior while accounting for the heterogeneous nature of the rock matrix. The calibrated BBM was then employed to quantify the strength loss that is associated with specimen damage (for example, due to core extraction). The results were found to be consistent with laboratory testing data and previous numerical simulations.

H.2 Introduction

Brittle geomaterials under compression fail due to microcrack initiation, subsequent crack growth and ultimate coalescence into macroscopic failure planes. In spite of the applied stress field being compressive in nature, the initial damage occurs in an extensile mode (Diederichs, 2007). The reason is the geometric, material and contact heterogeneity between
constituent grains that generates local tensile stresses within the specimen. The stress levels corresponding to the initiation and coalesce of microfractures during a compression test are commonly known as the Crack Initiation (CI) threshold and the Crack Damage threshold (CD) (Martin, 1993; Diederichs, 2007). This microfracturing process is ubiquitous across the sample and as such cannot be tracked using conventional specimen-resolution laboratory measurement techniques.

Bonded Block Models (BBMs) are increasingly being used to generate simulated grain or block structures for investigation of damage processes in rocks. BBMs represent a material space using an aggregate of polygonal blocks that can interact along their contacting interfaces. Depending on the stress magnitudes and the strength assigned, contacts can fail and facilitate block separation. The input parameters corresponding to the blocks and contacts are generally varied until an acceptable match is obtained between the model behavior and some laboratory-derived attributes. The BBM approach has been previously used to study damage processes in granite (e.g., Lan et al., 2010; Noorani and Cai, 2015), sandstone (e.g., Kazerani and Zhao, 2010) and marble (e.g., Norouzi et al., 2013).

It is generally understood that BBMs “calibrated” to a limited number of attributes may only be able to capture a limited range of physical phenomena. In particular, most studies calibrate their model parameters against the peak strength (and in some cases pre-peak damage thresholds) but neglect the post-peak response of the rock. In this study, the authors have developed a BBM of a granitic rock, with calibration efforts extending beyond what has been conventionally considered in literature. The novel aspects of the BBM are its ability to capture the non-linear shape of the strength envelope for 0-60 MPa confinement and the sample-scale dilatancy as measured in laboratory. The model was subsequently used to assess the effect of
pre-damage and the results compared to pertinent laboratory data and numerical simulations by Bahrani (2015).

A number of approaches have been used over the years to model granitic rocks in BBM – (a) homogeneous, elastic blocks (Ghazvinian et al., 2014), (b) heterogeneous, elastic blocks (Lan et al., 2010; Farahmand and Diederichs, 2015), (c) homogeneous, inelastic blocks (Noorani and Cai, 2015). Intuitively, more complex models are capable of reproducing a larger number of laboratory-derived attributes than simpler models. We argue that a true analogue for a granitic rock must consider both aspects: material heterogeneity (that controls grain-boundary fracturing at low confinement) and inelastic block yield (approximates intra-granular fracturing at high confinement). The formation of grain-boundary cracks at low loads, followed by development of intra-granular cracks has been illustrated by Eberhardt et al. (1999c) using SEM. When a confining stress is applied to the rock sample, the generation of tensile microcracks (primarily intergranular) is restricted, forcing the fractures to develop through intact grains. This has been confirmed through grain-based models (GBM) (Hofmann et al., 2015; Peng et al., 2018a).

**H.3 BBM simulation of Creighton Granite**

**H.3.1 Laboratory data**

The granite selected for this study is from the footwall of the 2410 m level in the Creighton mine (Walton, 2014). Geomechanical characterization was previously performed by Walton (2014) on the basis of laboratory tests conducted by CANMET in Canada. The relevant properties selected for calibration of the BBM micro-parameters are UCS, Triaxial strength (10-60 MPa), Brazilian Tensile Strength (BTS), CI (unconfined condition), CD (unconfined...
condition) and dilation angle. The CI threshold can be identified using acoustic emission techniques (Eberhardt et al., 1998), axial/lateral stress-strain curves (Diederichs, 2007) and crack volumetric stress-strain curves (Martin and Chandler, 1994). Walton (2014) determined the CI threshold to be 85-100 MPa using the reversal of crack volumetric strain. The CD threshold, on the other hand, can be identified from axial-strain curves (Diederichs, 2007) and volumetric stress-strain curves (Martin and Chandler, 1994), and was found to be 80% - 90% of the UCS.

A slight confinement dependency of the Young’s modulus (E) was noted, with values ranging from 52.7 GPa under unconfined conditions to 79.7 GPa for $\sigma_3 = 60$ MPa. This variability is in part associated with unloading of the samples during core extraction from the mine. Since BBMs are zero porosity systems (no pre-existing open cracks), this mechanism cannot be easily incorporated in the modeling method. The upper bound E is probably closer to the intact rock modulus; however, since the laboratory-derived attributes correspond to rock specimens with different degrees of pre-damage, a better approximation is to consider the average modulus.

A modal estimation of constituent mineral prevalence led to the following percentages: Quartz = 30%; Plagioclase = 55% and Biotite = 15% (see Figure H.1a for a representative sample of Creighton Granite). The grain size distribution of Creighton Granite has not been previously studied in detail, so it was not possible to quantitatively match the size of constituent grains. On the basis of pictures available to the authors, the size was estimated to be around 1 – 3 mm with irregularly shaped mineral grains. The irregularity in the BBM block shape was introduced by selecting a low value of the ‘iteration’ parameter in the UDEC Voronoi generator. The block edge length was then varied until a satisfactory size gradation was obtained. Figure
H.1b shows the distribution of equivalent block diameter (diameter of a circle with the same area as that of the block) in the final BBM.

H.3.2 Numerical modeling: Synthetic specimen generation and parameter calibration.

The applicability of the BBM for simulation of brittle damage and deformation in crystalline rocks was tested through a series of UCS, Triaxial and Brazilian tests in the explicit DEM software UDEC. Figure H.2 shows the overall model geometry for the Brazilian test and the UCS / Triaxial test. The Brazilian sample is 55 mm in diameter and is loaded using two metallic beams along either edge. The UCS / Triaxial sample is 120 mm in height and 55 mm in width. In both sets of models, the bottom edge is restrained using roller boundaries while a constant velocity is applied along the top edge. For the triaxial case, a hydro-static stress corresponding to the confinement level to be tested is applied before assigning a velocity to the model top. The velocities selected are 0.005 m/sec for Brazilian test and 0.01 m/sec for the UCS / Triaxial test. These values are small enough so as to simulate a pseudo-static loading condition in the model.

The tensile stress in the Brazilian model is calculated based on the total reaction force (P) along the upper platens as follows: $\sigma_t = \frac{2P}{\piDt}$, where D and t are the specimen diameter and width, respectively. In the UCS / Triaxial models, axial stress is calculated every 1000 solution steps by dividing the total reaction force at the top surface by the width of the sample. The axial and lateral strains are computed as the difference in displacements along two opposite edges divided by the distance between them. 10 points pairs equally spaced along the longer edge of the model were selected for determination of the lateral strain, while all gridpoints along the shorter edges were utilized for the axial strains.
Figure H.1 (a) A representative sample of Creighton Granite (post Brazilian test), (b) Distribution of equivalent block diameter of the developed BBM sample.

Figure H.2 Overall geometry of (a) the Brazilian test model, (b) the UCS / Triaxial model.
BBM blocks can be modeled either as a rigid entity or a deformable material in UDEC. Deformable blocks require discretization into constant-strain finite-difference triangular zones (Itasca, 2014a). In this study, the blocks were set as deformable and a strain-softening (Mohr-Coulomb; MC) constitutive model assigned to them. In conjunction with the MC yield criterion, the mobilized Walton-Diederichs (WD; Walton and Diederichs, 2015a) dilation model was also employed in order to account for the confinement and plastic shear strain dependency of dilation angle in the block zones. The motivation for using the WD model within the mineral blocks was based on a logical extrapolation – if a physical mechanism (such as confinement-dependency of dilatancy) applies to a rock specimen, then it should apply to its constituents as well. The micro-parameters were varied until the BBM could reproduce all the attributes. Table H.1 and H.2 list the calibrated block and contact micro-parameters, respectively.

Table H.1 Calibrated parameters for the BBM blocks.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td><strong>Inelastic Block Parameters</strong></td>
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<tr>
<td>Peak Cohesion ($c_{\text{peak}}$; MPa)</td>
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<tr>
<td>Residual Cohesion ($c_{\text{res}}$; MPa)</td>
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</tr>
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<td>Peak Friction ($\phi_{\text{peak}}$)</td>
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<tr>
<td>Residual Friction ($\phi_{\text{res}}$)</td>
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</tr>
<tr>
<td>Peak Tensile Strength ($\sigma_{t,\text{peak}}$; MPa)</td>
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<tr>
<td>Residual Tensile Strength ($\sigma_{t,\text{res}}$; MPa)</td>
<td>0.1 (over $\epsilon_{\text{ps}} = 0.05$)</td>
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<td><strong>WD Model Parameters</strong></td>
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<tr>
<td>Low Confinement peak dilation parameter ($\beta_0$)</td>
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<tr>
<td>Low Confinement peak dilation parameter ($\beta_0$)</td>
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</tr>
<tr>
<td>Dilation decay plastic shear strain parameter ($\epsilon_{\text{ps}^*}$)</td>
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</tr>
</tbody>
</table>
Table H.2 Calibrated micro-parameters for the block contacts.

<table>
<thead>
<tr>
<th>Contacts</th>
<th>$j_{kn}$ (GPa/m)</th>
<th>$j_{ks}$ (GPa/m)</th>
<th>c$_{peak}$ (MPa)</th>
<th>$\phi_{peak}$</th>
<th>$\sigma_{t,peak}$ (MPa)</th>
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<td>Q-B</td>
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<td>149500</td>
<td>60</td>
<td>55</td>
<td>18</td>
</tr>
<tr>
<td>P-B</td>
<td>230000</td>
<td>149500</td>
<td>60</td>
<td>53</td>
<td>18</td>
</tr>
</tbody>
</table>

*Q=Quartz; P=Plagioclase; B=Biotite

**H.4 Model results and discussion**

Figure H.3a shows the model-derived axial stress – axial strain and axial stress – lateral strain curves for UCS and $\sigma_3 = 10, 20, 40$ and $60$ MPa conditions. Based on the modal estimation of mineral types and the elastic block parameters from Farahmand and Diederichs (2015), a specimen modulus of $\sim 67$ GPa was obtained. Under unconfined conditions, the sample is free to deform laterally and this results in very large lateral strain values. The deformation is constrained as a confining stress is applied, causing the axial stress – lateral strain curve to decline in a steeper manner. The residual strength increased with an increase in the confining stress and is consistent with the laboratory data for Creighton Granite (Walton, 2014).

Interestingly, the post-peak drop modulus was similar across the entire range of confining stresses tested. Brittle rocks are found to behave in a similar manner (Arzua and Alejano, 2013) and this could be attributed to the low porosity grain structure, heterogeneity in samples and imperfections. The authors, however, recognize that at 60 MPa confinement, the post-peak behavior should be more ductile than what has been presented. The inability of polygonal BBM
to properly capture the post-peak behavior of simulated rocks is a well-known limitation and is evident from the fact that most studies (e.g. Lan et al., 2010; Farahmand and Diederichs, 2015) only report the strain-strain curve up to or slightly beyond the peak.

Figure H.3b compares the model-predicted UCS and triaxial strengths to those measured in the laboratory. The calibrated model captures the non-linear trend of the peak strength over the entire range of confining stresses tested. To our knowledge, this is the first time that polygonal BBM (Voronoi Tessellation) has been successfully able to replicate the convex shape of the Hoek-Brown envelope. The reason is possibly due to the use of an inelastic constitutive model in the mineral blocks.

Figure H.3 (a) Axial stress – lateral strain and axial stress – axial strain curves for the UCS and triaxial tests, (b) Comparison of model strengths from laboratory and BBM.

Figure H.4a shows the tensile stress versus axial strain for the Brazilian test model. The peak strength is 10.4 MPa, which is only 0.1 MPa lower than the average tensile strength for Creighton Granite. The fracture pattern post-testing is also highlighted using red lines and is consistent with those in Figure H.1a.
In BBM, CI and CD are commonly identified as the point of tensile and shear fracture initiation along the block-contacts (Ghazvinian et al., 2014; Farahmand and Diederichs, 2015). A review of the existing literature indicated that there is no standard methodology to identify CI and CD from fracture plots. The authors thereby compared the CI derived from crack volumetric strain reversal to the normalized (with the total number of contacts) number of tensile fractures and found that the intersection of the initial linear portions of the tensile crack curve coincides with the CI value. Similarly, the intersection of the initial linear portions of the shear crack curve also coincided with CD determined from the non-linearity of axial stress-axial strain curve. Using this approach, the BBM CI and CD values were determined to be 89 MPa and 177 MPa (88.5% of UCS), respectively.

Figure H.4 (a) Tensile stress versus axial strain for the BTS model. The fracture pattern is indicated by red lines. (b) Comparison of BBM-predicted normalized peak dilation angle with laboratory measurements.

H.4.1 Macroscopic dilation of BBM specimen

The BBM has been shown to match the UCS, triaxial strength, tensile strength, CI and CD thresholds. In addition, the macroscopic dilation behavior of the samples was analyzed using
axial and lateral strains recorded from the numerical simulations. At the sample scale, the macroscopic dilatancy can be computed using the equation (Vermeer and De Borst, 1984):

\[
\sin(\psi) = \frac{\varepsilon_{1p}^p}{-2\varepsilon_{1p}^p + \varepsilon_{vp}^p}
\] (H.1)

where, \(\psi\) is the dilation angle, \(\varepsilon_{1p}\) and \(\varepsilon_{vp}\) are the maximum and volumetric inelastic strain increments, respectively. The only other parameter needed to quantify the evolution of dilation angle during the loading phase is the plastic shear strain. The plastic shear strain can be calculated using one of two approaches: (1) Extracting the irrecoverable strains recorded at unloading points during cyclic tests, and, (2) Using elasticity theory and the definition of CD and CD from Diederichs and Martin (2010). Since it is time-consuming to perform loading-unloading tests in the numerical models, the second approach was employed in this study. Mathematical details on how to calculate plastic shear strain and the variables in equation H.1 (i.e. \(\varepsilon_{1p}\) and \(\varepsilon_{vp}\)) are presented by Walton and Diederichs (2015a).

The instantaneous dilation angles and corresponding plastic shear strains were computed for all the BBM simulations, and the WD model fitted using an optimization algorithm in MATLAB. Figure H.4b shows the peak dilation angles normalized to the UCS simulation. As can be observed, the BBM closely replicates the normalized peak dilation angles from laboratory testing (shown in black), which further validates the robustness of the model. Note that as far as the authors are aware, this is the first time that the confinement-dependent volume changes that occur during rock damage have been quantitatively matched by a BBM.
H.5 Effect of pre-existing cracks

With the robustness of the presented BBM having been established, an investigation was conducted to analyze the effect of pre-damage on the model response. Previous studies on Lac Du Bonnet granite have found the density of stress-induced fractures to increase as the samples are retrieved from greater depths (Eberhardt et al., 1999d). Lim and Martin (2010) studied the termination pattern of the fractures within intact rock. They found the fractures to be non-persistent with rock bridges in-between them. Based on this observation, Bahrani (2015) concluded that the damaged samples could be considered as analogues for non-persistently jointed rockmasses.

Additional tests on Lac Du Bonnet granite have shown the strength of damaged specimens to increase rapidly as a function of confinement at low confining stresses in comparison to intact specimens (Bahrani, 2015). At higher confinement, the slopes of the undamaged and damaged strength envelopes are similar. Such behavior was also noted in tests on marble (Yang et al., 2011) and limestone (Ribacchi, 2000). The demarcation between the high and low confinement regime, called the critical confining pressure, has been suggested to be between $\sigma_3 \sim \frac{UCS}{20} - \frac{UCS}{6}$ (Bahrani, 2015).

With all that in mind, a study was conducted to see if the calibrated BBM could exhibit behavior that is consistent with the laboratory findings. A broader implication is the establishment of BBM as a tool to study the mechanical behavior of non-persistently jointed rockmasses. To that end, different proportions of block contacts were assigned zero strengths to correspond to varying levels of pre-damage. The same assumptions as those in Bahrani (2015) were employed: (1) Pre-damage only affects the block contacts and not the mineral grains;
therefore, all blocks were assigned the strain-softening parameters listed in Table H.1. (2) The damaged block boundaries have zero tensile strength and zero cohesive strength. (3) Pre-damage does not have any impact on the frictional properties of the block contacts. Therefore, the same friction angle was assigned to the contacts in the damaged and intact specimens.

Three realizations were tested for each model, with a different starting seed for the pre-damaged contacts. This was done to average out any anomalous effects of a specific fracture geometry on the model behavior. Again, unconfined and confined tests corresponding to $\sigma_3 = 10$ MPa, 20 MPa, 40 MPa and 60 MPa were conducted. Figure H.5a compares the stress-strain curves from two pre-damaged UCS simulations (10% and 30%) to the stress-strain curve from the intact BBM. The following observations can be readily made: (1) The peak strength reduces with an increase in sample pre-damage. The pre-damaged contacts act as stress-concentrators, thereby inducing failure relatively early in the loading process. (2) The post-peak drop modulus decreases with an increase in extent of sample pre-damage. This is consistent with numerical simulations by Bahrani (2015) and the laboratory test results of Walton et al. (2018c) on artificially jointed granites. (c) A closer look at the initial section of the stress-curve reveals a modest modulus softening. This is consistent with the findings of Bahrani (2015) for low damage levels ($\leq 30\%$); softening intensifies for higher levels of damage ($\geq 50\%$).

Figure H.5b shows the intact and pre-damaged (10% and 30%) strengths from the unconfined and confined BBM simulations. Note that each pre-damage strength data point is averaged over three independent model runs. As observed by Bahrani (2015), the strength of the damaged BBM increases notably with confinement at near unconfined conditions. At higher confinements, the slope of the damaged and the intact model were similar. A confining stress of 10 MPa represents a good approximation of the critical confinement level and is in accordance
with $\sigma_3 = \text{UCS} / 20$ as proposed for Lac Du Bonnet granite (Bahrani, 2015). Note that the percentage drops in strength at higher confining stress are larger than those in Bahrani (2015) and are likely related to the choice of a frictional hardening type joint behavior in Bahrani (2015) in comparison to a softening joint behavior used in this study.

Figure H.5 (a) Axial stress-axial strain curves for the undamaged and pre-damaged UCS models, (b) Comparison of the model-predicted strengths for the undamaged and pre-damaged models.

Piecewise equivalent friction angles were also computed for $\sigma_3 \leq 10$ MPa and $\sigma_3 > 10$ MPa, and a $16^\circ$ difference was noted. In comparison, the friction angle was found to change by $13^\circ$ ($\phi_e = 63^\circ$ to $50^\circ$) for Lac Du Bonnet granite (Bahrani, 2015). The $\phi_e$ between the undamaged and 30% damaged model differed by $4.5^\circ$ for $\sigma_3 \leq 10$ MPa and is consistent with the results of Bahrani (2015), where $\phi_e$ did not change substantially until 60% or more of the contacts were damaged. The mechanistic reason for the rapid strength increase at low confinement (thereby higher friction angle) is due to a high dilation angle contribution of the fractures. As the confinement is increased, the dilation is suppressed and the equivalent friction angle becomes comparable to that of the intact rock. The generally good correspondence between the calibrated
model results and those in Bahrani (2015) provides further confidence in the capabilities of the BBM.

### H.6 Conclusions

This study has presented a bonded block model of Creighton Granite that reproduced the non-linear shape of the strength envelope and exhibited realistic dilational behavior. The multi-minerallic nature of the rock was considered by incorporating material and contact heterogeneity in the BBM. Additionally, the blocks were allowed to yield via a strain-softening constitutive model to approximate intra-granular fracture development. The BBM also matched the peak strengths (unconfined and confined), tensile strength, CI (unconfined) and CD (unconfined).

The BBM was subsequently utilized to assess the effect of pre-damage. For 0 – 60 MPa confinement, the peak strength was found to reduce with increases in the degree of damage. At lower confinements, the effective friction angle was higher, but the friction angle was identical to that of the undamaged specimen for $\sigma_3 > \text{UCS} / 20$. The results are generally consistent with laboratory tests on Lac Du Bonnet granite and numerical simulations by Bahrani (2015). Future endeavors will focus on investigating other aspects of jointed rockmass behavior using BBM.

### H.7 Acknowledgements

The research conducted for this study was funded by the National Institute of Occupational Health and Science (NIOSH) under Grant Number 200-2016-90154.
APPENDIX I

APPLICATION OF MICROMECHANICAL MODELING TO PREDICTION OF IN-SITU ROCK BEHAVIOR

This paper has been published in the proceedings of the 52nd US Rock Mechanics/Geomechanics Symposium, Seattle, Washington (Sinha and Walton, 2018b). It is reprinted with permission from ARMA with some minor variations.

I.1 Abstract

With advances in numerical modeling techniques, Voronoi Tessellations are being increasingly used to generate simulated grain structures for investigation of small-scale damage processes in rocks. In a Voronoi model, a material is represented as an aggregate of polygonal blocks that interact through contacting interfaces. Typically, a set of input micro-parameters is calibrated to numerically replicate the macroscopic mechanical behavior of the rock under study. The calibration is mostly restricted to attributes estimated from laboratory testing, such as damage threshold levels, uniaxial compressive strength and tensile strength. While the potential of Voronoi Tessellations in modeling small-scale damage processes has been extensively tested, its utility in capturing field-scale behavior is largely unexplored. This study attempts to bridge the gap through development of a calibrated laboratory scale model of Creighton Granite followed by upscaling it to an 8 m wide pillar. An assessment of the Voronoi model’s abilities and shortcomings were investigated through a qualitative and quantitative comparison of the model’s macroscopic behavior against empirically validated continuum model results recently published by the authors and documented pillar behaviors as seen in the field.
1.2 Introduction

In recent years, the Discrete Element Method (DEM) has been increasingly used for the study of micromechanical damage processes in intact rocks. The development of this approach was driven by the inability of continuum models to explicitly simulate fracture opening and separation – a mechanism by which damage localizes primarily along grain contact boundaries. In addition, continuum model results are heavily dependent on the choice of a constitutive relationship.

In DEM models of intact rock, the material is represented by an aggregate of circular or polygonal blocks that can interact through the contacting interfaces. The calculation procedure oscillates between Newton’s law of motion and a force-displacement contact law, eliminating the need to assign a macroscopic material constitutive model. The macroscopic behavior of the simulated model depends on the choice of micro-parameters. Specifically, the micro-parameters can be classified into two groups: (a) Block properties, and, (b) Contact properties. The combined effect of both sets of parameters control the emergent macroscopic behavior of the model.

The formal pioneers of this modeling approach are Diederichs (1999) and Potyondy and Cundall (2004), who developed the Bonded Particle Method (BPM). In BPM, a material is simulated using circular disks/spheres that can interact through parallel contact bonds. Although novel and structurally homologous to rock mineral assemblages, it suffers from some limitations, such as inability to simultaneously match the material compressive and tensile strength, a linear failure envelope, low friction angles and reduced tendency of incipient fractures to propagate (Diederichs, 1999; Potyondy and Cundall, 2004). Subsequent developments like cluster particle
models (Potyondy and Cundall, 2004), the flat-joint model (Potyondy, 2012) and clumped particle models (Cho et al., 2007) were able to partially overcome some of the limitations associated with the traditional BPM. The main issue, however, remains the high inherent porosity and lack of particle interlocking due to the circular/spherical shape of constituent blocks.

The Voronoi tessellation represents an improvement over conventional BPM and utilizes triangular/polygonal blocks to characterize the material domain. As a result, particle interlocking issues are avoided. In terms of the mechanistic difference between polygonal and triangular blocks (also called trigons), it was pointed out by several authors (e.g. Ghazvinian et al., 2014; Mayer and Stead, 2017) that models with triangular elements have a predisposition towards shear failure due to the availability of potential linear failure pathways. Laboratory tests of rocks have shown damage to initiate in a tensile mode at about 25-40% of the uniaxial compressive strength (Diederichs, 2007). As a result, polygonal Voronoi blocks are better suited for studying the deformation and failure mechanisms in intact crystalline rocks.

The current study is focused on assessing the capabilities of the Voronoi tessellation approach in simulating the damage processes in field-scale engineering structures in rock. To that end, a small-scale synthetic model of Creighton Granite was developed, with parameters calibrated against the Uniaxial Compressive Strength (UCS), Brazilian Tensile strength (BTS), Crack Damage Threshold (CD) and Crack Initiation Threshold (CI). An additional check for model consistency was accomplished through a comparison of the evolution of dilation angle against those measured in laboratory, which is the first such comparison known to the authors. With its abilities examined, the small-scale model was subsequently enlarged to correspond to an 8 m wide pillar with W/H varying from 1-3. The overall behavior of these pillar models was
compared against recently developed and empirically verified continuum models results (Sinha and Walton, 2018a) and documented pillar behaviors as observed in the field (Krauland and Soder, 1987).

Several authors have attempted to capture the behavior of large-scale structures using DEM. Garza-Cruz et al. (2014) used a 3-dimensional tetrahedral grain-based model to study the stability of tunnels in brittle rocks. Coggan et al. (2012) and Gao and Stead (2014) used a similar technique to investigate the roadway stability in coal mines. Christianson et al. (2006) performed triaxial tests on a 1m x 1m lithophysal tuff sample with varying degree of porosity for aiding the design of a nuclear waste repository at Yucca Mountain, Nevada. Azocar (2016) attempted to replicate the brittle fracture processes observed in the URL Mine-by Experiment test tunnel through a comparative analysis of tetrahedral and polygonal block models. Interestingly, most of the previous studies that successfully replicated field-scale behaviors have utilized tetrahedral/triangular blocks even though they tend to underestimate the degree of tensile damage.

Upscaling a laboratory size model to a field-scale structure is cumbersome and computationally challenging if the block size is to be kept unchanged. Most studies in the past have used a larger element size, although a good understanding of the effect of block size on the emergent model behavior is not fully understood. For this study, the block size was increased by 40 times (~2.5 mm to ~10 cm) when transitioning from the lab scale to the pillar scale (10 cm longest dimension to 8 m longest dimension) to obtain an acceptable runtime for the pillar models. As will be later shown in this paper, the block size exerts a major influence on the model results, often overshadowing the effect of micro-parameters. Very low block-contact strength properties were required to obtain stress-strain curves comparable to those expected.
In addition to the biases involving selection of a constitutive relationship, continuum models also tend to underestimate the ground-support interaction effect (Sinha and Walton, 2017b). As discussed by Sinha and Walton (2017b), the root cause is the inability of such models to allow separation of failed boundary elements, thereby implicitly accounting for the effect of supports that act primarily as a strain-boundary for damaged material. In contrast, DEM softwares possess the capability of block detachment as fractures develop and coalesce in the model. A rib bolt, under such circumstances, would function more efficiently by limiting the bulking of rockmass. Clearly, a DEM approach is more effective in assessing support needs over conventional continuum approaches.

I.3 DEM simulation of Creighton Granite

I.3.1 Laboratory data

The granitic rock selected for this study is from the Main Orebody at Creighton Mine, located at the southern edge of the Sudbury Igneous Complex (SIC) in Canada (Walton, 2014). The rock was geomechanically characterized by Walton (2014) on the basis of laboratory tests, and the relevant properties are listed in Table I.1. The authors would like to point out that the granite in the mine is fairly massive, meaning the laboratory derived intact properties can be considered representative of the overall rockmass (Kaiser et al., 2000; Carter et al., 2008; Walton et al., 2016).

In-situ brittle failure initiates at a stress level corresponding to the Crack Initiation Threshold (CI) observed in laboratory tests (Martin, 1997; Diederichs, 2007). Given that this study has the ultimate goal of modeling mine pillars, where failure initiates through surficial
spalling, CI was chosen as a key calibration parameter. Away from the excavation boundary, the
dilation-induced confinement suppresses the formation of tensile cracks, causing the failure
mode to transition from tension to shear. The Crack Damage (CD) Threshold represents the
long-term laboratory shear strength and it was also considered as a point of calibration. With no
CD values for the Creighton Granite reported by Walton (2014) or Walton et al. (2016), a
calibration target for CD corresponding to 80-90% of the mean UCS was selected.

Table I.1 Geomechanical properties of Creighton Granite.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniaxial Compressive Strength (UCS)</td>
<td>200 MPa</td>
</tr>
<tr>
<td>Young’s Modulus</td>
<td>52 GPa</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>0.1</td>
</tr>
<tr>
<td>CI (unconfined)</td>
<td>85 MPa</td>
</tr>
<tr>
<td>Brazilian Tensile Strength (BTS)</td>
<td>10.5 MPa</td>
</tr>
</tbody>
</table>

To further constrain the calibration process, a representative stress-strain curve was
developed through mathematical averaging of 20 individual stress-strain curves. The following
steps, in sequence, were followed: (1) First, each of the 20 curves was normalized such that the
peak stress occurred at (1, 1); (2) The average peak strength (i.e. UCS) and the axial strain
corresponding to the peak strength was computed; (3) The normalized axial strain range (0-1.1)
was sub-divided into 220 bins and the stresses within each bin were averaged to generate the
average normalized curve (see black line in Figure I.1a); (4) The normalized curve was rescaled
back such that the peak occurred at the stress and strain levels computed in step 2; (5) The initial
non-linear portion of the curve, corresponding to crack closure, was replaced by a linear section
(red dashed line in Figure I.1b) and translated to the left such that the corrected curve
commenced from (0,0). The removal of the non-linear section was necessary because Voronoi models represent zero porosity systems with no pre-existing cracks. Figure I.1 illustrates the methodology followed for obtaining the averaged laboratory stress-strain curve.

Figure I.1 Methodology for obtaining a representative stress-strain curve for Creighton Granite from 20 individual curves.
I.3.2 Numerical modeling: Specimen generation, calibration and results

The applicability of Voronoi Tessellation for simulation of brittle microfracturing in crystalline rocks was tested through a series of UCS and Brazilian tests in the explicit DEM software UDEC. Explicit numerical softwares rely on a time-march algorithm that assumes a limited speed for transmission of disturbances within a material (Jing and Stephansson, 2007). Details of the mathematical equations and their implementation are beyond the scope of this paper and can be found in Jing and Stephansson (2007).

For the UCS test, a cylindrical (rectangular in two dimensions) 55 mm x 120 mm specimen with 2.5 mm blocks was selected (Figure I.2). The block size is consistent with what has been previously used for modeling granitic rocks (Kazerani and Zhao, 2010; Nicksiar and Martin, 2014). The sample was loaded from the top using a servo-controlled velocity boundary condition. The built-in servo function modifies the applied velocity as a function of the unbalanced forces in the model. The upper bound of the velocity was set at 0.005 m/sec (Fabjan et al., 2015). The steady post-peak portion of the laboratory stress-strain curves implied that the loading system was very stiff; as a result, the platens were omitted to simulate an infinitely stiff loading mechanism.

The Brazilian sample was a circular disc with 55 mm diameter, loaded through two platens on either side. The block size and the loading mechanism are similar to the UCS model. Figure I.2 shows the geometry of the UCS and the Brazilian models. For the UCS model, the axial stress was computed every 1000 solution steps by dividing the cumulative reaction force at the top surface by the width of the model. A similar methodology was used for computing the stresses in the BTS model.
Axial and lateral strains were also computed for the UCS model as the difference in displacement along two opposite edges divided by the distance between them. While every gridpoint on the top and bottom surface were used for axial strain calculation, only 10 monitoring points on either edge of the model were utilized for computing the lateral strains (see Figure I.2). This reduced the simulation time drastically. In case of the BTS model, the total reaction force was converted to tensile stress using the equation:

$$\sigma_t = \frac{2P}{\pi Dt}$$

(1.1)

Where \(P\) is the total reaction force, \(D\) and \(t\) are the specimen diameter and width, respectively.

Figure I.2 Geometric setup of the UCS and the Brazilian test model.

The CI and CD thresholds can be identified as the points of non-linearity in the lateral and axial stress-strain curve, respectively (Lajtai, 1974; Diederichs and Martin, 2010; Nicksiar
and Martin, 2012). A simpler approach, however, is to track the development of tensile and shear fractures in UDEC models. The stress level corresponding to the initiation of tensile fracturing can be designated as CI (Nicksiar and Martin, 2014) while the point where the shear crack accumulation begins to accelerate represents the CD (Farahmand and Diederichs, 2015). CD can also be identified as the point of reversal in the axial strain-volumetric strain curve (Diederichs and Martin, 2010).

While it may be simpler to just track the number of tensile and shear fractures using built-in FISH variables, users must take note of the fact that tensile fractures can transition to shear fractures and vice-versa, as the model equilibrates towards a quasi-static solution. It is therefore necessary to track the evolution of fractures in addition to their aggregate number in the models. A comparison using the UCS model revealed that the number of tensile fractures could be underestimated by as much as 20% near the peak strength when simply using built-in FISH variables to track aggregate numbers of different crack types.

The contact and block micro-parameters were modified to match the macro-properties listed in Table I.1, following the procedure suggested by Ghazvinian et al. (2014):

1. The Young’s Modulus (E) and Poisson’s ratio (ν) of blocks were selected according to the macroscopic E and ν of Creighton Granite.

2. The contact normal to shear stiffness ratio controls the macroscopic Poisson’s ratio. Therefore, kn/ks was varied until the desired Poisson’s ratio was obtained.

3. The normal stiffness (kn) was adjusted to match the Young’s modulus.
(4) With the elastic parameters constrained, the contact tensile strength was adjusted to match the CI threshold.

(5) Finally, different combinations of contact cohesion and friction angle were tested to match the Brazilian tensile strength, CD threshold and peak strength. The calibrated set of input parameters and the resulting macro-properties are listed in Table I.2.

Table I.2 Contact micro-parameters and model-predicted macroproperties.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
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<tbody>
<tr>
<td><strong>Contact Input Properties</strong></td>
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<tr>
<td>Contact cohesion (peak) - $c_{\text{peak}}$</td>
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<tr>
<td>Contact cohesion (residual) - $c_{\text{res}}$</td>
<td>0 MPa</td>
</tr>
<tr>
<td>Contact friction angle (peak) - $\phi_{\text{peak}}$</td>
<td>0º</td>
</tr>
<tr>
<td>Contact friction angle (residual) - $\phi_{\text{res}}$</td>
<td>47º</td>
</tr>
<tr>
<td>Dilation angle - $\psi$</td>
<td>5º</td>
</tr>
<tr>
<td>Contact tensile strength - $\sigma_t$</td>
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<td>Normal Stiffness ($k_n$)</td>
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<tr>
<td>Shear Stiffness ($k_s$)</td>
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<tr>
<td><strong>Emergent Macroscopic Properties</strong></td>
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<tr>
<td>Uniaxial Compressive Strength (UCS)</td>
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<tr>
<td>CD (unconfined)</td>
<td>185 MPa</td>
</tr>
<tr>
<td>Brazilian Tensile Strength</td>
<td>10.3 MPa</td>
</tr>
</tbody>
</table>

Figure I.3a shows the stress-strain curve obtained from the Voronoi model. A striking resemblance in both the pre-peak and post-peak response could be observed when compared to
the average laboratory curve. The CI, CD and the tensile strength are also consistent with previously reported laboratory test results for Creighton Granite.

Figure I.3 (a) Stress-strain curve and other damage thresholds for Voronoi UCS, (b) Volumetric versus axial strain.

To assess the model’s ability in capturing the lateral dilational behavior, volumetric strain was computed using the equation:

\[ \varepsilon_v = \varepsilon_y + 2\varepsilon_x \]  

\[ \text{(I.2)} \]
where, $\varepsilon_y$ is the axial strain, and $\varepsilon_x$ is the lateral strain. Upon plotting the volumetric strain versus the axial strain (see Figure I.3b), the CD threshold appears to coincide fairly well with the point of stress-reversal as expected for a crystalline rock under unconfined conditions (Martin, 1997; Diederichs and Martin, 2010; Ghazvinian, 2010). Further analysis on the model’s macroscopic dilatational behavior is presented in the next section.

The fracture pattern in the UCS model is shown in Figure I.3b while the tensile stress-strain curve and the fracture pattern for the BTS model is shown in Figure I.4. The UCS model fails via tensile splitting along the left edge and some form of macro-shear (through coalescence of tensile fractures) along the right edge. These observations conform closely with what was observed in the laboratory UCS samples post-testing. In the BTS model, a through-going fracture was identified that terminated at the two opposite platens.

![Tensile stress-strain curve with fracture pattern for the BTS model.](image)
I.3.3 Dilatancy of the Voronoi model

Dilatancy is the volumetric expansion of a yielding rock or rockmass. It is strongly related to the post-yield stress-strain response of laboratory specimens. At the excavation scale, it controls bulking and the interaction of support with the rockmass. With respect to the ultimate goal of investigating the behavior of large-scale structures, it is crucial to assess the model’s potential to exhibit a realistic dilatant behavior.

The macroscopic dilatancy of a specimen can be quantified using the parameter, dilation angle ($\psi$), which relates the maximum ($\varepsilon_1^P$) and minimum ($\varepsilon_3^P$) principal inelastic strain increments (see Eq. I.3). Several studies have found the dilation angle to be a function of confining stress ($\sigma_3$) and plastic shear strain ($\gamma^P$) (Alejano and Alonso, 2005; Zhao and Cai, 2010; Walton and Diederichs, 2015a). Since only UCS tests have been simulated here, only the plastic shear strain dependency of the dilation angle will be considered. The plastic shear strain is a variable that quantifies system damage and can be computed from internal variables using Eq. I.4.

$$\sin(\psi) = \frac{\varepsilon_v^p}{-2\varepsilon_1^p + \varepsilon_p^p} \tag{I.3}$$

$$\gamma^P = \varepsilon_1^p - \varepsilon_3^p \tag{I.4}$$

The inelastic principal strains were calculated following the methodology proposed by Walton et al. (2014) and Walton and Diederichs (2015a), which utilizes the elasticity theory and accepted definitions of CI and CD (Diederichs and Martin, 2010). A brief discussion of the approach and associated equations (Eqs. I.5-I.7) are presented below.
When calculating the maximum and minimum plastic strains using Equations I.5 and I.6, the middle terms represent a correction to ensure zero plastic axial strain at CD and zero plastic lateral strain at CI, in accordance with the definitions of CI and CD per Diederichs and Martin (2010). The last term signifies the elastic component that must be subtracted to obtain the respective plastic strain values. Once obtained, the plastic volumetric strains are computed using Eq. I.7. It is noted here that the lateral strain in the laboratory was measured using a chain along the center of the sample. In contrast, the lateral strain in the model was monitored at 20 locations along the edge, which could yield slightly different results.

\[
\varepsilon_1^p = \varepsilon_1 - \varepsilon_1(CD) - \frac{\sigma_1 - CD}{E} \tag{I.5}
\]

\[
\varepsilon_3^p = \varepsilon_3 - \varepsilon_3(CI) - \nu \frac{\sigma_1 - CI}{E} \tag{I.6}
\]

\[
\varepsilon_v^p = \varepsilon_1^p + 2\varepsilon_3^p \tag{I.7}
\]

To provide a basis for comparison between the Voronoi model results and the laboratory data, median, upper bound and lower bound dilation angle models (Walton and Diederichs, 2015a) to dilation data from individual UCS tests were developed. These three results (shown by bold black and green lines in Figure I.5a) were utilized in testing the model’s ability of reproducing a realistic dilatant behavior. As can be seen from Figure I.5b, the model-predicted instantaneous dilation angles are well within the upper and lower bounds of the laboratory data. The dilation model parameters for the laboratory median fit and the fit to the dilation angle values extracted from the Voronoi model (see Figure I.5b) are listed in Table I.3.

While the fit to the Voronoi model dilation angle values appropriately characterizes the overall trend in the dataset, the pre-mobilization curvature parameter is substantially higher than
the laboratory median fit. The cause for this discrepancy is not immediately evident and requires further study. Additionally, the decay in the post-mobilization portion of the curve was also found to be smaller, and could be related to the different lateral strain measurement scheme or the model’s inability to replicate shearing of asperities. Despite these minor discrepancies between the modeled dilation behavior and that observed from laboratory testing, the small-scale Voronoi model was able to reproduce the overall trends in the dilation angle observed in laboratory UCS tests.

Table I.3 Walton and Diederichs (2015a) dilation angle model (termed “WD”) fit parameters.

<table>
<thead>
<tr>
<th>Parameter and Influence</th>
<th>Median Laboratory WD Fit</th>
<th>Voronoi WD Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>α – Pre-Mobilization curvature</td>
<td>0</td>
<td>0.08</td>
</tr>
<tr>
<td>γm – Plastic shear strain at peak ψ</td>
<td>0.0003</td>
<td>0.001</td>
</tr>
<tr>
<td>γ* – Decay Rate</td>
<td>0.008</td>
<td>0.02</td>
</tr>
<tr>
<td>Ψ peak – Peak dilation angle</td>
<td>70º</td>
<td>64º</td>
</tr>
</tbody>
</table>

I.4 Pillar models

With the micro-parameters constrained against laboratory derived attributes, the next logical step was to upscale the model to simulate an 8 m wide pillar. A key issue when attempting to model large structures using Voronoi is the selection of an appropriate block size. Since using the same block size as in the small-scale Voronoi models would be computationally restrictive, 0.1 m Voronoi blocks were utilized. Three separate models were developed, corresponding to W/H of 1, 2 and 3. The overall geometry and loading mechanism (elastic beam on either side) was chosen to correspond to the continuum pillar models of Sinha and Walton (2018a). Figure I.6 shows the geometry for W/H=1 pillar model.
Figure I.5 (a) Walton and Diederichs (2015a) dilation angle models (termed “WD”) obtained from laboratory UCS tests of Creighton Granite with the median fit and fit bounds indicated, (b) dilation angle values extracted from the Voronoi UCS model versus plastic shear strain compared to dilation angle trends in the laboratory.

It is known that Voronoi model outputs are dependent on the block size chosen (Ghazvinian et al., 2014; Fabjan et al., 2015; Insana et al., 2016). Therefore, when the micro-parameters constrained against laboratory-scale attributes were used directly in the pillar models, excessively high peak strengths were obtained. An example is shown in Figure I.7, where the W/H=1, 2 and 3 pillar
models were run with the strength properties from the small-scale elastic, homogeneous BBMs in Chapter 7. Similar results were obtained when the contact properties in Table I.2 were used for pillar simulations. The main issue lies in the high residual friction angle of the contacts (47° in Table I.2 and 35° in Chapter 7) that increases the interlocking of an already highly interlocked block structure. To obtain micro-parameters which resulted in reasonable macroscopic pillar behavior, the pillar models were calibrated against the stress-strain responses obtained from the continuum (FLAC$^{3D}$) pillar models, independent of the small-scale BBMs.

Figure I.6 Voronoi pillar model setup (W/H=1).

Sinha and Walton (2017b) explained how continuum models implicitly account for the presence of support acting as a strain boundary to prevent the gradual degradation of pillars due
to loss of spalled material. Calibrating the Voronoi micro-parameters against the continuum model results, without any modification, would therefore be inappropriate.

Figure I.7 Stress-strain curves for W/H=1, 2 and 3 pillar BBM with contact strength properties from Chapter 7.

To mimic the pillar degradation associated with the loss of spalled material at the edges of pillars, the authors developed a ‘deletion’ code whereby all boundary elements in the FLAC\textsuperscript{3D} pillar models under tension and exceeding a pre-defined plastic shear strain limit (damage level associated with spalling) were incrementally deleted as the model was loaded through a constant downward velocity. FLAC\textsuperscript{3D} models calibrated to in-situ pillar strength observations for supported pillars were re-run using the ‘deletion’ code; the resulting stress-strain curves from these sets of models were considered more appropriate for direct comparison with the UDEC models, which explicitly allow for the separation of spalled material from the pillars.

Figure I.8 compares the stress-strain curves from FLAC\textsuperscript{3D} with those obtained in UDEC after model calibration. The associated micro-parameters are also listed in Table I.4. Although
these calibrated curves match quite well, the three UDEC model calibrations required independent parameter input sets, meaning that the micro-parameters used are not fundamental to the material being modeled. Specifically, very low friction angle and high peak cohesion for the contacts was necessary to replicate the brittle behavior of W/H=1 pillar. On the other hand, a higher friction angle and lower peak cohesion was required to initiate faster yield and continued pre-peak hardening. As a consequence of the model’s inability to predict expected pillar strengths using a single set of parameters, it is believed that use of homogeneous elastic Voronoi blocks may be too simplistic to capture the complete range of expected pillar behavior.

Table I.4 Contact micro-parameters for different W/H models.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>W/H=1</td>
</tr>
<tr>
<td>$c_{\text{peak}}$ (MPa)</td>
<td>135</td>
</tr>
<tr>
<td>$c_{\text{res}}$ (MPa)</td>
<td>0</td>
</tr>
<tr>
<td>$\phi_{\text{peak}}$</td>
<td>0º</td>
</tr>
<tr>
<td>$\phi_{\text{res}}$</td>
<td>5º</td>
</tr>
<tr>
<td>$\Psi$</td>
<td>5º</td>
</tr>
<tr>
<td>$\sigma_t$ (MPa)</td>
<td>10</td>
</tr>
<tr>
<td>$k_n$ (GPa/m/m)</td>
<td>23000</td>
</tr>
<tr>
<td>$k_s$ (GPa/m/m)</td>
<td>16000</td>
</tr>
</tbody>
</table>

Figure I.9 shows the major principal stress contour for W/H=3 pillar model at peak strength. The hour-glassing phenomenon, exhibited by the highly stressed core, is an evidence of the progressive damage mechanism typically associated with pillars. In the W/H=1 model, no confined core was generated since the failure was due to the formation of a cross-shear plane (refer Figure I.10; the plot was generated at a strain of 0.0015). The spalling and bulking of the
rib was found to be more pronounced in the W/H=3 model, although some corner failures were seen in the W/H=1 model. The macro-failure mechanism changes from brittle to ductile with W/H=2 acting as the transition point (refer to the pre-peak hardening in the stress-strain response for W/H=2 model).

Despite the inability of Voronoi to capture the entire range of pillar behavior using a single set of micro-parameters, each of the calibrated models themselves can be used for a variety of purposes (e.g. support effect testing), as long as they demonstrate the basic pillar failure mechanisms and reproduce realistic stress-strain response. This is a major advantage over conventional continuum models that implicitly account for the effect of skin support. Use of a discontinuum modeling approach is therefore critical to a successful pillar support design (bolts and/or mesh).

Figure I.8 Stress-strain curves from UDEC and FLAC$^{3D}$. 
Figure I.9 Major principal stress contour for W/H=3 model.

Figure I.10 Major principal stress contour for W/H=1 model in its post-peak portion.
I.5 Conclusions

While the Voronoi Tessellation approach has been found to be effective in simulating small-scale damage processes in brittle crystalline rocks, its ability in modeling field-scale structures is still largely unexplored. This study was performed with a focus on assessing the abilities and shortcomings of Voronoi in simulating large underground structures, like mine pillars.

To that end, a synthetic small-scale model of Creighton Granite was generated, with micro-parameters constrained against laboratory-derived attributes (UCS, BTS, CI and CD). Additionally, the overall pre- and post-peak stress-strain curves and the macroscopic dilatational behavior exhibited by the model were consistent with laboratory data. The model was subsequently enlarged to simulate three 8 m wide pillars with W/H of 1, 2 and 3. Because Voronoi model results are dependent on the block size selected, the contact properties had to be recalibrated. The average stress-strain response from continuum pillar models, recently developed by the authors, was used for this purpose.

A single set of micro-parameters was found to be inadequate in capturing the range of behavior that is expected from slender (brittle) and squatter (semi-ductile to ductile) pillars. This suggests that the approach of using homogeneous, elastic Voronoi blocks does not allow for the fundamental mechanics of field-scale damage processes to be captured. Additionally, this work serves as a cautionary tale that an impressive laboratory-scale model calibration does not ensure that the modeling approach used (let alone the specific parameters selected) can be universally applied, let alone used for predictive purposes.
I.6 Acknowledgements

The research conducted for this study was funded by the National Institute of Occupational Health and Science (NIOSH) under Grant Number 200-2016-90154. The authors would like to extend their sincere gratitude for this financial support. The authors would also like to thank Mark Christianson from Itasca for his valuable inputs and troubleshooting assistance.
APPENDIX J

ADDITIONAL PILLAR BBM RESULTS WITH THE ELASTIC BLOCK REPRESENTATION

This section is an appendix to the paper Sinha and Walton (2020e) that has been submitted to the journal International Journal of Rock Mechanics and Mining Sciences.

To demonstrate that it was indeed not possible to obtain a single set of elastic BBM parameters that could match the stress-strain curves for all 3 W/H geometries, some model results for W/H=1 and 3 BBMs are presented here. These two geometries were chosen for illustration as they represent the upper and lower bound of W/H tested in this study and also because the pillars corresponding to these two geometries have different failure modes – W/H=1 fails primarily by tensile fracturing while W/H=3 fails primarily by shearing. Note that a much larger number of models were actually run for all three W/H geometries to thoroughly explore the parameter space but only some of those are shown for illustrative purposes.

Out of the many different parameter combinations tested as a part of the calibration process for the W/H=1 BBM, 9 key combinations and their peak strengths are shown in Table J.1 (Model 1 is the base parameter set). The corresponding stress-strain curves can be found in Figure J.1. For this model, the target peak strength per the empirical database is 67 MPa and the target stress-strain response is indicated by the black line in Figure J.1. It can be seen from Table J.1 and Figure J.1 that an increase in both the $\sigma_{t,\text{peak}}$ and $c_{\text{peak}}$ (compare Models 1 and 6) was necessary in order to attain a peak strength of ~70 MPa. Individually raising $c_{\text{peak}}$ (compare Models 1 and 2) or $\sigma_{t,\text{peak}}$ (compare Models 1 and 7) or changing other input parameters like $\varphi_{\text{peak}}$ (Model 3) did not produce the desired effect. Additionally, in models where $\sigma_{t,\text{peak}}$ was 40 MPa,
the post-peak strengths were relatively high (Figure J.1), and a lowering of $\phi_{\text{res}}$ to 5° was necessary in order to reproduce the target stress-strain curve (compare Models 6 and 8).

Table J.1 Different parameter sets tested for the W/H=1 elastic BBM. Target W/H=1 strength is 67 MPa.

<table>
<thead>
<tr>
<th>Model</th>
<th>$c_{\text{peak}}^*$ (MPa)</th>
<th>$\phi_{\text{peak}}$ (°)</th>
<th>$\phi_{\text{res}}$ (°)</th>
<th>$\sigma_{\text{t,peak}}^*$ (MPa)</th>
<th>Peak strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-Base</td>
<td>80</td>
<td>30</td>
<td>10</td>
<td>17.5</td>
<td>39</td>
</tr>
<tr>
<td>2</td>
<td>120</td>
<td>30</td>
<td>10</td>
<td>17.5</td>
<td>48</td>
</tr>
<tr>
<td>3</td>
<td>120</td>
<td>40</td>
<td>10</td>
<td>17.5</td>
<td>59</td>
</tr>
<tr>
<td>4</td>
<td>140</td>
<td>30</td>
<td>10</td>
<td>17.5</td>
<td>54</td>
</tr>
<tr>
<td>5</td>
<td>120</td>
<td>30</td>
<td>10</td>
<td>25</td>
<td>53</td>
</tr>
<tr>
<td>6</td>
<td>120</td>
<td>30</td>
<td>10</td>
<td>40</td>
<td>69</td>
</tr>
<tr>
<td>7</td>
<td>80</td>
<td>30</td>
<td>10</td>
<td>40</td>
<td>53</td>
</tr>
<tr>
<td>8</td>
<td>120</td>
<td>30</td>
<td>5</td>
<td>40</td>
<td>58</td>
</tr>
<tr>
<td>9 - Calibrated</td>
<td>120</td>
<td>34</td>
<td>5</td>
<td>40</td>
<td>68</td>
</tr>
</tbody>
</table>

Figure J.1 Stress-strain curves for Models 1-10 with W/H=1.
For the W/H=3 geometry, the base parameter set overestimated the target peak strength and any increase in $c_{\text{peak}}$ (Model 3) only increased the mismatch (Figure J.2a). $\varphi_{\text{res}}$ had a substantial effect on the model results and the peak strength reduced to 151 MPa (Model 2) when the model was run with the calibrated parameters for the W/H=1 geometry. Note how all the input parameters of Model 2 are larger than Model 1 in Table J.2 except $\varphi_{\text{res}}$, yet the strength reduced by 31%. Given that Model 2 followed the target stress-strain curve up to \(~150\) MPa (Figure J.2a) and then deviated (due to low $\varphi_{\text{res}}$), one might wonder if it is possible to attain the target peak strength by just increasing $\varphi_{\text{res}}$.

Table J.2 Different parameter sets tested for the W/H=1 elastic BBM. Target W/H=1 strength is 67 MPa.

<table>
<thead>
<tr>
<th>Model</th>
<th>$c_{\text{peak}}^*$ (MPa)</th>
<th>$\varphi_{\text{peak}}$ ($^\circ$)</th>
<th>$\varphi_{\text{res}}$ ($^\circ$)</th>
<th>$\sigma_{\text{t,peak}}^*$ (MPa)</th>
<th>Peak strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-Base</td>
<td>80</td>
<td>30</td>
<td>10</td>
<td>17.5</td>
<td>219</td>
</tr>
<tr>
<td>2</td>
<td>120</td>
<td>34</td>
<td>5</td>
<td>40</td>
<td>151</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>30</td>
<td>10</td>
<td>17.5</td>
<td>235</td>
</tr>
<tr>
<td>4</td>
<td>120</td>
<td>34</td>
<td>8</td>
<td>40.0</td>
<td>194</td>
</tr>
<tr>
<td>5-Calibrated</td>
<td>50</td>
<td>46</td>
<td>8</td>
<td>17.5</td>
<td>201</td>
</tr>
</tbody>
</table>

To that end, we ran one more model with $\varphi_{\text{res}}$ of $8^\circ$ (Model 4). While this model could reproduce the target strength within 10 MPa, large mismatches occurred along the majority of the stress-strain curve. The corresponding W/H=1 model had a peak strength of 73 MPa. To further emphasize the effect of employing such large $c_{\text{peak}}$ and $\sigma_{\text{t,peak}}$, the stress-strain curves for the W/H=2 BBM with parameter sets 2 and 4 in Table J.2 are shown in Figure J.2(b). Clearly, the target trend is not being reproduced and the mismatch only increases with increasing $\varphi_{\text{res}}$ (Models 2 and 4). Based on this, it seems that it might be possible to reproduce only the target strengths, but not the complete stress-strain curves with a single set of input parameters, because
W/H= 2 and 3 require lower $c_{\text{peak}}$ and higher $\phi_{\text{res}}$ while W/H=1 requires higher $c_{\text{peak}}$ and $\sigma_{c,\text{peak}}$, and lower $\phi_{\text{res}}$.

![Stress-strain curves](image)

Figure J.2 (a) Stress-strain curves for Models 1-5 with W/H=3, (b) Stress-strain curves for W/H=2 with model parameter sets 2 and 4 in Table J.2.

Since the peak strength points were directly taken from the empirical database, it is more certain that the strength points are correct than the stress-strain curves from the FLAC$^{3D}$ models calibrated to these strength points. It is, however, reiterated that the different yield thresholds of the progressive S-shaped criterion are based on laboratory test data and the results from the FLAC$^{3D}$ models as such should provide a reasonable approximation of reality. The implication of using only the peak strength data points is that it raises the non-uniqueness potential of these BBMs dramatically. In particular, 3 data points would be used to constrain 4 input strength parameters when the shape of the stress-strain curve is not considered; employing the actual stress-strain curves from the FLAC$^{3D}$ model improves the calibration significantly as the model behaviors in this case are being constrained against multiple points along the entire stress-strain curves. This is particularly important in the inelastic BBM that has a larger number of input parameters. Calibration to the stress-strain curve is also desirable, as it allows for a more direct
comparison between the continuum and the discontinuum model in terms of their ability to capture the support effect (Chapters 5 and 9).
APPENDIX K

BULKING FACTOR CALCULATION DETAILS

This section is an appendix to the paper Sinha and Walton (2020e) that has been submitted to the journal *International Journal of Rock Mechanics and Mining Sciences*.

There is some subjectivity involved in the manner in which $d_f$ and $u_{df}$ were estimated in this study. In the initial stages of analysis, $d_f$ was identified manually, but it became increasingly difficult to determine $d_f$, especially in the supported models, where either the sudden jump in displacement was preceded by a section with inconsistent curvature or there was no sudden jump at all. To overcome this issue, we developed a criterion to automatically detect $d_f$ based on the average slope between every three adjacent displacement data points, referred to as $disp_{slope}$ (all data points were spaced at 0.1 m). $d_f$ was defined to correspond to the maximum distance from the pillar edge where a particular threshold value of this average slope is exceeded. Based on visual inspection of multiple lateral displacements and slope profiles, a $disp_{slope}$ threshold of 60 mm/m was selected for this study. The black open circles in Figure 9.8 were automatically identified using this threshold approach.

Determination of an appropriate $u_{df}$ also required some interpretation, particularly in cases where blocks have detached from the pillar surface. Some examples of how this separation process appears in the displacement profiles is provided in Figures 9.7 and 9.8 (heights of 2 m, 2.5 m, 3 m and 3.5 m). Since bulking factor is a continuum concept, in that it assumes the system to behave as a continuous body, utilizing the large displacements of the detached blocks is inappropriate. This is reflected in the fact that the bulking factors in Kaiser et al. (1996) for unsupported conditions were based on observations in the floor, where gravitational forces
prevented any significant block detachment. Since our focus is on pillar behavior rather than the floor, a different approach was required.

We compared various displacement profiles and model plots and determined that when the displacement difference between two adjacent data points was greater than ~50 mm (corresponds to a strain threshold of 0.5), this tended to correspond to blocks that the authors would judge to have fully separated. To simplify the bulking analysis, we disregarded all peripheral displacements beyond the point at which the threshold was exceeded and considered the displacement gradient to be 0.5 m/m in this remaining space. Physically, this implies that the region consisting of separated blocks is effectively replaced with an equivalent continuum region having a constant strain of 0.5. This is undoubtedly a simplified approach, but in absence of any well-defined methodology for calculating bulking factors in discontinuum models, such an approach serves as a useful starting point.

When the two approaches outlined above were employed for calculating the bulking factors at the four analysis stages (Table 9.4), an issue was encountered. In particular, as the lateral displacements and depths of fracturing are smaller in the early phase of loading, the corresponding $d_f$ values were also smaller. With continued loading, fractures propagated deeper into the pillar and $d_f$ increased. In context of Eq. 9.1, there are therefore two competing elements – the difference in displacements at the pillar wall and at $x = d_f$, and $d_f$, both of which increase with loading and ultimately control the bulking factor. Intuitively one would expect the bulking factor to either increase or remain constant with loading, but in some of our models, the bulking factor decreased due to the $d_f$ increasing at a faster rate than $u_w - u_{df}$. To resolve this issue, we computed $d_f$ only for the last analysis stage (Stage 4) and used this value for the rest of the
loading stages. This led to a systematic increase in the bulking factor values as the pillar was progressively strained. It should be recognized here that Kaiser et al.’s (1996) BF estimates do not acknowledge the progressive nature of BF as damage develops with continued loading.

Given the dependence of the BF calculation on user-selected parameters, a sensitivity analysis was conducted on the strain gradient threshold and $\text{disp}_{\text{slope}}$ threshold, and the corresponding W/H=2 unsupported model BFs are shown in Figures K.1 (a, b). The BFs increased with increases in both the strain gradient threshold and the $\text{disp}_{\text{slope}}$ threshold; this is expected, as the first threshold raises $u_w - u_d$ while the latter lowers $d_f$. The mean values appear to follow an approximately linear trend with a positive slope. Similar analyses were also conducted for the supported models, but the changes were minimal when the strain gradient was varied, while a reduced effect was noted with changes in the $\text{disp}_{\text{slope}}$ threshold. Supported models tended to be more continuous along the pillar edge and hence the application of the strain gradient threshold was mostly limited to the unsupported models. This is illustrated by the displacement profiles along the pillar mid-height for the W/H=2 unsupported, 3 bolt, 4 bolt and 5 bolt BBMs (Figure K.1c).

Although the sensitivity analysis shows that the strain gradient threshold and the $\text{disp}_{\text{slope}}$ threshold has some effect on the computed BF, their influence is not practically significant. Accordingly, if for example a $\text{disp}_{\text{slope}}$ threshold of 50 mm/m is ultimately judged to be more appropriate than our final analysis value of 60 mm/m, then such a choice does not invalidate the results presented in this study.
Figure K.1 Sensitivity of the (a) strain gradient threshold, (b) $disp_{slope}$ threshold on the computed bulking factor for the W/H=2 unsupported model, (c) Horizontal displacement profiles (left side) along the pillar mid-height for the W/H=2 BBMs at different loading stages. Strain gradient of 0.5 and $disp_{slope}$ threshold of 60 mm/m was employed for the final analysis.
This section is an appendix to the paper Sinha and Walton (2020c) that has been published in the journal *Journal of Rock Mechanics and Geotechnical Engineering*. It is reprinted with permission from Elsevier with some minor variations.

The ability of the BBM to match the displacement profiles as measured in the field was previously demonstrated in Section 10.5. To determine how closely the stresses matched in the two models, we averaged the vertical (Y) stresses over all zones in the BBM and over all zones in the pillar slice being considered in the FLAC$^{3D}$ model and plotted them for each loading stage (Figure L.1). A 1:1 line is also shown in Figure L.1; a perfect match is obtained when all the points lie on the 1:1 line.

Figure L.1 Average vertical stress in the FLAC$^{3D}$ pillar slice and in the BBM at each loading stage. The discrepancy in stresses later in the loading process is indicated by the green ellipse.
As can be seen, the vertical stresses were located very close to the 1:1 line for the majority of the loading stages, but ultimately deviated from linearity when the $\sigma_{yy}$ in the FLAC$^{3D}$ model was ~92 MPa (Stage 34). In particular, the pillar exhibited some softening in the BBM while it strain-hardened in the FLAC$^{3D}$ model. This discrepancy in the later loading stages could be attributed to two factors:

(1) Since a 3D to 2D simplification was used, any change in the out-of-plane direction (normal or shear stresses) was essentially neglected. To minimize the impact of this simplification, we purposely chose to model the pillar cross-section (AA’) that passed through the extensometer further away from the stope.

(2) It is possible that at higher levels of damage, continuum and discontinuum formulations may have different load-displacement relationships. This can occur because the FLAC$^{3D}$ model was simulating reinforced ground without supports, while the effect of reinforcement was being explicitly considered in an otherwise unsupported BBM. In other words, this suggests that the effect of reinforcement was implicitly accounted for in the rockmass strength properties of the continuum model, but not in the BBM. Of course, the interaction between the supports and the rockmass in the BBM should ideally yield a behavior close to that of the FLAC$^{3D}$ model, but since both models were able to match the target displacements, the observed discrepancy in average pillar stresses reflects the uncertainty in the back-analyzed stress condition at any given damage state. Further studies are required to better understand the extent of mismatch in the load-displacement response that one could obtain when simulating the stress damage process of reinforced ground using continuum and discontinuum models.
APPENDIX M

EFFECT OF BLOCK SIZE ON BBM BEHAVIOR IN CONTEXT OF THE CREIGHTON MINE CASE STUDY

In a Bonded Block Model (BBM), a material space is represented by an aggregate of polygonal blocks and each of these blocks are discretized by multiple constant strain triangular zones in UDEC. There are no well-established guidelines for selection of a block size, but it is generally chosen to be small enough such that there is a sufficient number of potential failure pathways to allow for fractures to develop without any kinematic constraints (Gao and Stead, 2014; Christianson et al., 2016; Mayer and Stead, 2017). An approximate relationship between the number of model elements and simulation time is provided in the Itasca manual (Itasca, 2014a): “The solution time for a UDEC run is a function of both the number of rigid blocks or gridpoints in deformable blocks, and the number of contacts in a model. If there are very few contacts in the model, then the time is proportional to $N^{3/2}$, where $N$ is the number of rigid blocks or gridpoints in deformable blocks. This formula holds for elastic problems. The runtime will vary somewhat, but not substantially, for plasticity problems.” In most BBMs, the number of gridpoints (vertices of zones) are similar to or larger than the number of contacts, meaning that reducing the block size can have a significant effect on model solution times. Therefore, solution time needs to be considered when evaluating the appropriate block size of BBMs for practical applications.

In Chapter 10, a block edge length of 0.1 m was used in the calibrated Creighton Mine BBM. To understand how block dimensions could affect the response of the BBM, models with four alternative edge lengths (namely 0.05 m, 0.075 m, 0.2 m and 0.3 m) were tested using the
calibrated model parameters. The zone sizes were also scaled according to the block edge length such that the number of zones in each block (or zone density) for all four model setups were similar to the calibrated BBM. This also implies that the zones in the bigger block models were larger than the zones in the smaller block models. As strain is more prone to localize in smaller zones (Itasca, 2014a), the critical plastic shear strain for degradation of cohesion and mobilization of friction in the CWFS strength model should be scaled accordingly (i.e. the rate of degradation needs to be slower for smaller zones and faster for larger zones). The propensity of strain localization in UDEC is related to its zone size because this is the smallest shear band width that can be resolved by the software. In any case, Itasca (2016b) suggests an approximate linear relationship between the zone size and critical plastic shear strain. For the purposes of this study, we ran the four BBMs with and without linear scaling of the critical plastic shear strain.

Figures M.1 and M.2 compare the displacement profiles measured in the field and those from the BBMs. Note that the models in this appendix were simply re-run with the parameters from Table 10.1 (except for critical plastic shear strain scaling) and no calibration was conducted. As can be seen, there is no consistent trend between model displacements and block size, but the displacement magnitudes are comparable (discrepancies in the order of 5-10 mm). The lack of any consistent influence of block edge length on the model results is further evident from Figure M.3 which shows the displacements at different anchor locations as a function of the block size for the last loading stage (47th stage). This finding highlights a key characteristic of BBMs: unlike finite difference zones that represent the discretization used to solve a differential equation, the block arrangement in a BBM is not a purely mathematical discretization, and hence the models do not converge to a particular solution as the blocks are made smaller. With that said, obviously very large block size will yield incorrect results (e.g. due to kinematic restrictions
imposed by the block arrangement), but there is not necessarily a block size below which the results will become independent of block size (unlike in the case of zone size sensitivity). Rather, block size interacts with the input parameters assigned to a field-scale model to influence model behavior, and the input parameters for a calibrated model should therefore be considered specific to the block size used. Further research is needed to fully understand this interaction between material parameters and block size, as such a relationship is necessary to develop any kind of upscaling procedure from laboratory-scale models to field-scale models.

Figure M.4 shows the distribution of plastic shear strain and the fractured contacts for the unscaled BBMs. It can be seen how the number of fractured contacts reduces with increasing block size, and the contribution of zone yield to the overall pillar yield mechanism is more significant in larger block models where there are limited options for damage to develop explicitly along block contacts. This further confirms the need to use blocks that are small enough such that the target mechanisms (explicit cracking along the periphery and zone yield deeper within the pillar) are appropriately represented.
Figure M.1 Comparison of field measured and BBM (critical plastic shear strain unscaled) displacements. The different colors correspond to the different anchors of the MPBX. Refer to the caption of Figure M.3 and Chapter 10.
Figure M.2 Comparison of field measured and BBM (critical plastic shear strain scaled) displacements. The different colors correspond to the different anchors of the MPBX. Refer to the caption of Figure M.3 and Chapter 10.
Figure M.3 Plot of the anchor displacements versus block size for the 47\textsuperscript{th} model stage in the (a) Unscaled, (b) Scaled BBM. The numbers on the right side of (b) corresponds to the location of the 6 anchors of the extensometer with respect to the center of the pillar. Refer Chapter 10 for more information.
Figure M.4 Plastic shear strain and fractured contacts in the four unscaled BBMs and in the calibrated BBM.
APPENDIX N

UNDERSTANDING THE EXTENT TO WHICH BBM INPUT PARAMETERS ARE TRUE MICRO-SCALE ROCK DESCRIPTORS

BBM zone and contact parameters are typically constrained by matching certain model attributes, such as displacements and/or stresses, to those measured in the field. Because these models are physics-based, the parameter set obtained through calibration should ideally be an intrinsic (i.e. material specific) micro-scale descriptor of the rock type considered. Chapters 9 and 10 provide a unique opportunity to study the fundamental nature of BBM input parameters as both these Chapters focused on simulating the same rock type (Creighton Granite) but used different calibration targets. The BBMs in Chapter 9 (Hypothetical Pillar) were loaded predominantly in the vertical direction and were calibrated to empirical pillar peak strength estimates for multiple pillar geometries, while the BBMs in Chapter 10 (Creighton Mine) were subjected to both vertical and horizontal (i.e. shear) loading and were calibrated to displacements measured using a multi-point borehole extensometer.

To test the degree to which the calibrated parameters obtained are indeed an intrinsic representation of the micro-properties of Creighton Granite, the calibrated parameters from the Hypothetical Pillar case were used to run the BBM in the Creighton Mine case and vice-versa. One important caveat is that while the original BBMs from Chapters 9 and 10 have the same block edge length (0.1 m), the zone sizes are different. More specifically, the Chapter 10 BBMs had about 8 zones per block while the Chapter 9 BBMs had 12-15 zones per block. Since the number of zones within a block controls the number of contacts (Itasca, 2014a), different
parameters are generally required, and it is therefore not correct to directly interchange the parameters from the two Chapters.

To that end, the Hypothetical Pillar calibration from Chapter 9 was adjusted to match the target stress-strain curves from FLAC\textsuperscript{3D} (see Figure N.1) using a zone size consistent with the BBMs in Chapter 10. Table N.1 lists the calibrated BBM input parameters; note the similarity in the parameters for the two cases considered. Having established a calibrated set of parameters for each case using consistent block and zone sizes, the Creighton Mine BBM was run with the Hypothetical Pillar parameters, and the Hypothetical Pillar BBMs were run with Creighton Mine parameters. Again, note that the Hypothetical Pillar case presented in this appendix is equivalent to the case presented in Chapter 9, but with a different zone size.

![Figure N.1 Pillar stress-strain curves for W/H=1-3 from FLAC\textsuperscript{3D} and from UDEC.](image)
The results of these model runs are shown in Figure N.2. It can be seen that while the parameters for the Creighton Mine BBM could reproduce the expected W/H=1 stress-strain curve from the Hypothetical Pillar BBM very well, the discrepancy increases with increasing W/H (Figure N.2a). This is not surprising, as the Creighton Mine pillar had a relatively low W/H of 1.5.

Table N.1 Calibrated zone and contact parameters for the Hypothetical Pillar case (equivalent to Chapter 9) and Creighton Mine case (Chapter 10).

<table>
<thead>
<tr>
<th>Hypothetical Pillar case</th>
<th>Creighton Mine case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zones</td>
<td>Contacts</td>
</tr>
<tr>
<td>Peak cohesion (MPa)</td>
<td>50</td>
</tr>
<tr>
<td>Residual cohesion (MPa)</td>
<td>4</td>
</tr>
<tr>
<td>Initial friction angle (°)</td>
<td>0</td>
</tr>
<tr>
<td>Mobilized friction angle (°)</td>
<td>44</td>
</tr>
<tr>
<td>Tensile strength (MPa)</td>
<td>12</td>
</tr>
<tr>
<td>Plastic shear strain from peak to residual (or initial to mobilized)</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure N.2 (a) Comparison of the stress-strain curves from FLAC$^{3D}$ and those from BBMs with Creighton Mine case parameters, and (b) Comparison of the rib displacement profiles obtained from the field and those from BBM with Hypothetical Pillar case parameters.

The Hypothetical Pillar parameters performed poorly when employed in the Creighton Mine BBM (Figure N.2b). In this case, the discrepancy is hypothesized to be related to the following factors: (1) The main focus of the Hypothetical Pillar case is on the global stress-strain response and the macroscopic reduction in bulking with incorporation of support, while the focus of the Creighton Mine case is the local suppression of spalled blocks. Recalling that slight modifications in input parameters resulted in changes in the rib displacement profiles in the Creighton Mine case (Section 10.4) and given that the focus of the Hypothetical Pillar case is not on the local deformational behavior, poor performance of the Hypothetical Pillar parameters in replicating the Creighton Mine displacements is not surprising. (2) The calibration performed for the Creighton Mine case in Chapter 10 had zone, contact and support parameters available for modification. It is possible that some combination of the Hypothetical Pillar parameters and adjusted support parameters could yield the target displacement response. (3) The calibrated parameters are hypothesized to be dependent on the load path considered to some extent. If this
is true, then it complicates the use of calibrated models for predictive purposes under different loading conditions.
APPENDIX O

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Sankhaneeel Sinha <sankhaneeelsinha@mymail.mines.edu>

Co-author_Permission_Sinha
2 messages

Sankhaneeel Sinha <sankhaneeelsinha@mymail.mines.edu>  Thu, Jul 9, 2020 at 9:24 AM
To: "Kim, Bo Hyun (CDC/NIOSH/SMRD)" <jn0@cdc.gov>
Cc: Gabriel Walton <gwalton@mines.edu>

Hello Dr. Kim,

I hope you are doing well. I am close to defending my PhD thesis work here at the Colorado School of Mines. I am writing this email to kindly request your consent to use the full content of the conference paper (ARMA 2020): Difficulties in determining the Crack Initiation (CI) thresholds for three different rock types to be included in my PhD thesis as required by the school. The co-author permission can be given by responding to this email.

I am thankful for your time.

--
Sankhaneeel Sinha
PhD Candidate
Geology and Geological Engineering
Colorado School of Mines
Golden, Colorado

Kim, Bo Hyun (CDC/NIOSH/SMRD) <jn0@cdc.gov>  Thu, Jul 9, 2020 at 9:28 AM
To: Sankhaneeel Sinha <sankhaneeelsinha@mymail.mines.edu>
Cc: Gabriel Walton <gwalton@mines.edu>

Hi Sankhaneeel,

As the co-author of the ARMA 2020 paper, I give you my permission for the publication.

Best regards,

Bo-Hyun.

From: Sankhaneeel Sinha <sankhaneeelsinha@mymail.mines.edu>  Thu, July 9, 2020 8:24:59 AM
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Sankhaneel Sinha
PhD Candidate
Geology and Geological Engineering
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---

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Please have this email as a confirmation of my approval to have the aforementioned work (paper) included in your thesis.

All the best,
Deepanshu

with regards,
Deepanshu Shirole, Ph.D.
Berthoud Hall- 126
Underground Construction and Tunneling Programme
Department of Civil and Environmental Engineering
Colorado School of Mines
Golden, Colorado, USA.

Mobile: +1 6465255619
e-mail: dshirole@mymail.mines.edu
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I am thankful for your time.

---

Sankhaneel Sinha
PhD Candidate
Geology and Geological Engineering
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Golden, Colorado

---

Rami Abousleiman <rabousleiman@mymail.mines.edu>  Thu, Jul 9, 2020 at 11:18 AM
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3 messages

Sankhaneel Sinha <sankhaneel.sinha@mymail.mines.edu>  Thu, Jul 9, 2020 at 9:30 AM
To: authors@smenet.org

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Sankhaneel Sinha
PhD Candidate
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Colorado School of Mines
Golden, Colorado

Sankhaneel Sinha <sankhaneel.sinha@mymail.mines.edu>  Fri, Jul 10, 2020 at 10:16 AM
To: cs@smenet.org

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Tara Davis <davis@smenet.org>  Fri, Jul 10, 2020 at 12:13 PM
To: Sankhaneel Sinha <sankhaneel.sinha@mymail.mines.edu>, Authors <authors@smenet.org>

Granted. Please make sure to give proper reference to publishing. Thank you.

Tara Davis
SME
Content Development & Program Director
Permission to use published ISRM paper in thesis
3 messages

Sankhaneel Sinha <sankhaneelsinha@mymail.mines.edu>  Thu, Jul 9, 2020 at 8:42 AM
To: leon.bijnsdorp@taylorandfrancis.com

Dear Mr. Bijnsdorp,

My name is Sankhaneel Sinha, a PhD student in the department of Geology and Geological Engineering at the Colorado School of Mines. I am the first author of the published conference paper titled: *Simulating laboratory-scale damage in granite using Bonded Block Models (SBM)*. This was published at the 14th International conference on Rock Mechanics and Rock Engineering held at Foz do Iguaçu, Brazil in 2019.

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Golden, Colorado

Bijnsdorp, Leon <Leon.Bijnsdorp@taylorandfrancis.com>  Thu, Jul 9, 2020 at 3:31 PM
To: Sankhaneel Sinha <sankhaneelsinha@mymail.mines.edu>

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Léon Bijnsdorp
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Peter Smeallie <peterhsmeallie@gmail.com>
To: sankhaneelsinha@mymail.mines.edu

Sankhaneel

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Peter

Peter Smeallie
Executive Director
American Rock Mechanics Association
600 Woodland Terrace
Alexandria, VA 22302
703-683-1806 (office)
703-801-1086 (cell)
703-997-6112 (fax)
www.armrocks.org

From: sankhaneelsinha@mymail.mines.edu
Sent: Wednesday, July 15, 2020 3:42 PM
To: info@armrocks.org
Subject: New Message (sent by contact form at American Rock Mechanics Association)

Name: Sankhaneel Sinha
Email: sankhaneelsinha@mymail.mines.edu
Topic: Other
Subject: Permission to use published ARMA papers in thesis

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ARMA-2018-265: Application of micromechanical modeling to prediction of in-situ rock behavior.

ARMA-2020-1061: Effect of damping mode in laboratory and field-scale Universal Distinct Element Code (UDEC) models.

ARMA-2020-1074: Difficulties in determining the Crack Initiation (CI) thresholds for three different rock types.

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APPENDIX P

SUPPLEMENTARY ELECTRONIC FILES

The supplementary files show how damage evolves at different stages in the Calibrated and Alternative 1-3 models for the Creighton Mine case study (Chapter 10).

1) **Figure S1.tiff**: This figure shows the plastic shear strain and failed contacts in the calibrated BBM at 6 different model stages. The yielding in the zones and fracturing along the periphery progressed as the pillar was monotonically loaded along the model top and bottom and boundary.

2) **Figure S2.tiff**: This figure shows the plastic shear strain and failed contacts in the Alternative 1 BBM at 6 different model stages.

3) **Figure S3.tiff**: This figure shows the plastic shear strain and failed contacts in the Alternative 2 BBM at 6 different model stages.

4) **Figure S4.tiff**: This figure shows the plastic shear strain and failed contacts in the Alternative 3 BBM at 6 different model stages.

5) **Figure S5.tiff**: This figure shows the plastic shear strain and failed contacts in the Calibrated and Alternative 1-3 BBMs at the 47th stage.