INVERSION OF A LATERAL LOG
USING
NEURAL NETWORKS

by

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Abstract

A non-standard technique for the inversion of a lateral log is developed in this thesis using neural networks. A finite difference method is used to simulate the lateral log which in turn is used as an input to a backpropagation neural network. The neural network reacts to gross and subtle data features in actual logs and produces a response inferred from the "knowledge" stored in the network during a training process. The training process consists of the application of the neural network backpropagation algorithm to a set of specially selected earth models and their lateral responses. The neural network architecture is selected based on a sensitivity analysis of the different parameters important for the neural network and the response to synthetic data.

The neural network response can be used as a "deconvolved" log, or after a post processing scheme, as an initial earth model for a standard inversion package. The "neural deconvolved log" has sharper boundaries than the lateral log and it can be used as a interpretation tool for correlation with other wells or as a bed boundaries detector.

The neural network inversion of lateral logs is tested on synthetic and field data. Field data inverted using the neural network initial earth model give surpris-
ingly good agreement with the actual log data. Test results indicate that the lateral response created with the neural inverted earth model fits the actual data quite well. Comparison with the initial earth model created for the short normal log in the same interval using a different technique, indicates that the initial guess earth model created with the neural network performs quite satisfactorily.
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Chapter 1

INTRODUCTION


There are even a few applications in the well logging arena. J. L. Baldwin et al. (1990) applied a special neural network to the problem of mineral identification. M. D. McCormack (1991) presents a simple application to the lithology identification using two different logs.

In these previous studies the neural network has been used as a tool to do signal processing of different types of information. That processing scheme consists
of the detection of a particular pattern in the data. Most of the applications of neural networks have used a backpropagation technique because of its simplicity and its good behavior.

In this thesis the neural network technique will be used as a method for the automation of the inversion of a lateral electrode well log. By inversion I mean constructing an earth model from the log data; on the other hand, the reverse of inversion - forward modeling - means generating a log from an earth model. Parametric inversion requires a forward model for the earth, using parameters such as mud resistivity, hole diameter, and resistivity and depth of invaded zones. In particular, after the interpreter specifies initial approximations for the various parameters in the assumed earth model, this initial model is turned over to an automatic inversion process, which generates the final earth model. It is the initial model that the neural network will create automatically from the original log data, so that the entire inversion process is performed without the intervention of a log analyst.

The lateral log is an old log introduced in the late 1920's as a tool for the investigation of the surrounding formation. It had some advantages compared with the other developed tool at the same time, the normal, because it was less sensitive to the mud resistivity in the borehole. Unfortunately, the lateral tool response to changes in the resistivity of the surrounding layers is deceiving and the interpretation of the log is extremely difficult. J. T. Dewan (1983) said
The Electrical Survey logs were difficult, sometimes impossible, to interpret.

Added to those difficulties, there is the problem of the interpretation of a log that is not currently in the market and that petrophysicists are not used to the erratic response of the lateral log. For these reasons the detection of bed boundaries for correlation with other logs and the search for hydrocarbon zones is a challenge.

I propose a new technique for the interpretation of the lateral log. The initial interpretation will be performed through a neural network specially trained to recognize the lateral response pattern. The training process will be performed using a finite difference forward model for the response of the lateral tool and a neural network backpropagation algorithm.

In practice, current geophysical inversion methods consist of a form of curve fitting. The inversion consists of minimizing the misfit between field and model data points. This ignores the fact that some parts of the data curve are more important than others. Another problem associated with this inversion is the enormous waste of computer time because this method does not retain experience. Ideally each inversion of the data should be able to use the results of all previous inversions, rather than having to start a new each time. The neural network method is interesting because it provides a solution those difficulties.

First, the lateral tool physical setup and its implementation in a borehole
survey is discussed, presenting the different aspects related to the character of the log response. Special attention is given to the different physical parameters involved in the electrical phenomena, and the problem with the interpretation of the tool is explained. Next, some current uses of the lateral log in exploration and production today are presented.

A review of the neural network principles and the implementation of the backpropagation algorithm is discussed, emphasizing the training process of the network and the elements and parameters necessary for a successful recognition of pattern responses of the lateral log. A careful sensitivity analysis of those parameters will be designed to get a good and fast implementation of the neural network.

Examples of the typical log responses to synthetic earth models usually encountered in geological situations will be discussed and the response of the neural network for different signal to noise ratios is studied carefully. Finally a test of the backpropagation neural network is used in a complex real earth model with the subsequent inversion and a study of the shortcomings and advantages of the neural network technique for real logs is presented.
Chapter 2

THE LATERAL LOG

2.1 Electrical Survey Tool

Before 1955, all resistivity measurements were made with simple electrode devices of the type shown in Figure 2.1 (the Normal) and Figure 2.2 (the Lateral).

For the lateral device a constant survey current $I$ was emitted from electrode $A$ and returned to electrode $B$. The voltage $V$ between electrodes $M$ and $N$ was measured. The ratio $V/I$, multiplied by a constant dependent on the electrode spacing, gave the apparent resistivity at the depth reference, $O$.

Many tools design were made and after a period of experimentation the Electrical Survey (ES) tool configuration settled down in soft rock areas to a short Normal with 16-in. spacing, a long Normal with 64-in. spacing, and a Lateral with 18ft. 8in. spacing. Other spacings continued to be used in hard rock areas. The greater the spacing, the greater the depth of investigation.

All systems can be understood, at least in principle, by superposing the pat-
Figure 2.1: Normal device. (from Schlumberger).

Figure 2.2: Lateral device. (from Schlumberger).
terns of current flow and voltage drop from individual electrodes.

2.2 Resistivity measurement

2.2.1 Homogeneous media

For a constant survey current \( I \) emitted to the earth we get a current density \( J \) equal to

\[
J = \frac{E}{R} = \frac{\nabla \Psi}{R},
\]

where \( E \) is the electric field, \( R \) is the resistivity of a material, and \( \nabla \Psi \) is the gradient of a potential \( \Psi \).

Except at a source or a sink, the current density must be divergence free; that is, the current leaving a volume must equal the current entering. If we consider a single current source of \( I \) amperes at the center of a spherical coordinate system,

\[
\nabla \cdot J = I \delta(0),
\]

where \( \delta(0) = 1 \) at \( r = 0 \) and zero elsewhere. Combining equation 2.1 and equation 2.2 gives us

\[
\nabla \cdot \left( \frac{1}{R} \nabla \Psi \right) = \nabla \left( \frac{1}{R} \right) \cdot \nabla \Psi + \frac{1}{R} \nabla^2 \Psi = -I \delta(0).
\]

If we assume the medium is homogeneous, resistivity \( R \) is constant everywhere
and the first term vanishes. Away from the source, equation 2.3 reduces to Laplace’s equation:

\[ \nabla^2 \psi = 0 \quad (2.4) \]

The problem can be solved for the appropriate boundary conditions. The solution for a single current source at \( r = 0 \) can be found by integrating \( E \) to get the potential

\[ \psi(r) = -\int_0^r E \, dr = -\int_\infty^r R J \, dr. \quad (2.5) \]

A current density \( J \) at any radius \( r \) is the source strength divided by the area of the sphere, \( I/4\pi r^2 \). The other condition necessary for complete determination of \( \psi \) is obtained by defining \( \psi = 0 \) at \( r = \infty \). The result of evaluating equation 2.5 with these boundary conditions is then

\[ \psi(r) = -\int_\infty^r \frac{RI}{4\pi r^2} \, dr = \frac{RI}{4\pi r}. \quad (2.6) \]

This solution obeys Laplace’s equation for \( r > 0 \). Figure 2.3 shows the spatial behavior of \( \psi \) and emphasizes that a voltage \( V \) is determined by taking the difference of the potential \( \psi \) at two different points. For two points distant \( r_M \) and \( r_N \) from the current electrode,
Figure 2.3: Spatial behavior of the potential $\Psi$ for a homogeneous media.

$$V_{MN} = \frac{RI}{4\pi} \left( \frac{1}{r_M} - \frac{1}{r_N} \right).$$ \hspace{1cm} (2.7)

If the current electrode is a sphere of radius $r_e$ rather than a point electrode, the voltage on its surface is found by letting $r_M = r_e$ and $r_N = \infty$.

$$V_{r_e} = \frac{RI}{4\pi r_e},$$ \hspace{1cm} (2.8)

so that the impedance $V/I$ of a single spherical electrode in a homogeneous material is $R/4\pi r_e$. If the electrode is not spherical, the radius $r_e$ is replaced by an appropriate length factor. Anyhow, electrode impedance varies inversely with electrode size.
Multiple electrode devices are used to extend the effective depth of investigation and obtain more representative value of true formation resistivity, $R_t$. Because potentials add, Equation 2.7 shows that the voltage $V_{MN}$ between points $M$ and $N$ in a homogeneous medium can be found by combining the potential generated by the individual current sources. A four electrode system with $+I$ source at point $A$ and a $-I$ source at point $B$ will produce a voltage

$$V_{MN} = \frac{RI}{4\pi} \left[ \left( \frac{1}{r_{AM}} - \frac{1}{r_{AN}} \right) - \left( \frac{1}{r_{BM}} - \frac{1}{r_{BN}} \right) \right],$$

(2.9)

where $r_{AM}$ is the distance between point electrodes $A$ and $M$. The resistivity $R = GV/I$, where

$$G = 4\pi \left[ \frac{1}{r_{AM}} - \frac{1}{r_{AN}} - \frac{1}{r_{BM}} + \frac{1}{r_{BN}} \right]^{-1}$$

(2.10)

is the geometric or device factor.

The formulation simplifies if current electrode $B$ and the voltage electrode $N$ are placed on the surface and themselves separated (so that $1/r_{BN} \simeq 0$), leaving only two electrodes on the sonde with $B$ and $N$ effectively at infinity. This is called the normal device. Here Equation 2.10 gives $G = 4\pi r_{AM}$

The lateral device consists of a current pole at $A$ and a voltage dipole at $MN$. Because $(r_{AM}, r_{AN}) \ll (r_{BM}, r_{BN})$, Equation 2.10 shows that the geometric factor for the lateral is
Figure 2.4: Schematic of the resistivity distribution in an inhomogeneous media. (Reservoir rock penetrated by a borehole).

\[ G = 4\pi \left( \frac{1}{r_{AM}} - \frac{1}{r_{AN}} \right)^{-1} \]  \hspace{1cm} (2.11)

2.2.2 Inhomogeneous Media

The resistivity distribution in a layered rock sequence penetrated by a borehole is shown in Figure 2.4. This earth model presents a three layered inhomogeneous media. One permeable layer with invaded zone and two non permeable shoulder beds cut by a borehole are depicted. A mud cake and an invaded zone are present in the permeable layer. In table 2.1, I present the different zones shown in Figure 2.4 with the notation for their respective resistivities.
Table 2.1: Principal zones in a reservoir rock penetrated by a borehole and resistivity notation.

<table>
<thead>
<tr>
<th>Zone</th>
<th>Resistivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Borehole</td>
<td>$R_m$</td>
</tr>
<tr>
<td>Invaded Zone</td>
<td>$R_{inv}$</td>
</tr>
<tr>
<td>Uninvaded Zone</td>
<td>$R_t$</td>
</tr>
<tr>
<td>Mudcake</td>
<td>$R_{mc}$</td>
</tr>
<tr>
<td>Shoulder Beds</td>
<td>$R_{sh}$</td>
</tr>
</tbody>
</table>

Note: Cylindrical Symmetry. (Non-dipping beds)

To understand sonde response in a well, we must look at the effect that spatial variations in resistivity have on the measurement. Suppose a device is placed in a medium with varying properties; that is, the resistivity changes from place to place. For any current $I$, the potential electrodes at $M$ and $N$ measure some voltage $V$ that implies

$$R_a = G \frac{V}{I}. \quad (2.12)$$

The resulting quantity $R_a$ is called the apparent resistivity, but its exact value is determined by the pattern of current flow. Figure 2.5 shows the current and the potential distribution in a two layer model. Notice the distortion in the current flow and the equipotential lines produced by the more resistive layer.

In most real heterogeneous situations, the value $R_a$ is affected by several zones of differing resistivity, including the mud resistivity $R_m$ and the invaded zone. The goal of resistivity interpretation is to derive from $R_a$ the desired formation resistivity.
Figure 2.5: Current and potential distribution in an inhomogeneous media (a) layer two is five times more resistive than layer one (b) layer two is three times less resistive than layer one.

$R_i$ and the bed boundaries. I will use the neural network technique and an inversion program to try to get a better approximation of those parameters. A more simple earth model without invaded zones will be used, because it is impossible to infer the depth of invasion with only one log.

In an inhomogeneous medium equation 2.3 does not reduce to Laplace's equation. The resistivity is not constant, and equation 2.3 does not simplify, remaining

$$\nabla \cdot \left( \frac{1}{R_i} \nabla \Psi \right) = -I \delta(0), \quad (2.13)$$

where $R_i$ is the position dependent resistivity.
If we assume axial symmetry and employ cylindrical coordinates, equation 2.13 is expressed as

\[ \frac{\partial}{\partial r} \left( \frac{1}{R} \frac{\partial U}{\partial r} \right) + \frac{\partial}{\partial z} \left( \frac{1}{R} \frac{\partial U}{\partial z} \right) + \frac{1}{rrR} \frac{\partial U}{\partial r} = -I\delta(0), \quad (2.14) \]

where \( R = R(r, z) \), \( U = U(r, z) \), \( r \) represents the radial coordinate, and \( z \) is the vertical coordinate.

The problem is difficult to solve analytically in general but a finite difference solution to the problem can be found for the boundary conditions (J.H. Kim, 1986),

\[
\begin{align*}
\left. \frac{\partial U}{\partial r} \right|_{r=0} &= 0 \\
\lim_{r \to \infty} U &= 0 \\
\lim_{z \to \pm \infty} U &= 0
\end{align*}
\quad (2.15)
\]

and for the more general boundary conditions

\[
\begin{align*}
\left. \frac{\partial U}{\partial r} \right|_{r=0} &= 0 \\
\lim_{r \to \infty} \left( \alpha U + \beta \frac{\partial U}{\partial r} \right) &= 0 \\
\lim_{z \to \pm \infty} \left( \alpha U + \beta \frac{\partial U}{\partial r} \right) &= 0
\end{align*}
\quad (2.16)
\]

where \( \alpha \) and \( \beta \) are constant coefficients.

J.H. Kim assumes an exponentially expanding rectangular grid system in cylindrical coordinates for computational efficiency. In Figure 2.6 the principal parameters
in the finite differences numerical modeling program and the exponentially expanded grid is shown. Two additional grid lines at the lateral potential receivers are present to avoid loss of accuracy in the numerical approximation (see detail in J.W. Kim's Ph.D. thesis, 1991).

2.3 Behavior of $R_a$ for simple earth models

In this section, the lateral responses created with the finite difference forward modeling program are used in simple earth models. The shape of the curves will be studied because the neural network uses those shape patterns in the training process. Table 2.2 presents the parameters used in the earth models represented in Figures 2.7 through Figure 2.12. The normal responses are not presented because they are symmetric and the bed boundaries are easy to pick, as opposed to the lateral. Table 2.3 summarizes the principal characteristics of the responses in Figures 2.7 through Figure 2.12.

2.3.1 Response to a thick resistive layer

Figure 2.7 illustrates the response of the lateral device to a thick bed more resistive than the surrounding formations. By thick I mean a layer that is at least two times thicker than the lateral spacing. The case represents a 190 ft. bed, recalling that the usual lateral spacing is 18 ft. 8 in. The curve is unsymmetrical with a long plateau with reading roughly equal to $R_t$; a minimum bed thickness of about
Figure 2.6: An exponentially expanding rectangular grid system in cylindrical coordinates. (from J.H. Kim, 1986)
Table 2.2: Parameters for the finite difference model used in the generation of lateral responses.

<table>
<thead>
<tr>
<th>Figure</th>
<th>$R_t$ (Ω-m.)</th>
<th>Thickness layer 2 (ft.)</th>
<th>Characteristics layer 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.7</td>
<td>1.0</td>
<td>8.0</td>
<td>1.0</td>
</tr>
<tr>
<td>2.8</td>
<td>1.0</td>
<td>8.0</td>
<td>1.0</td>
</tr>
<tr>
<td>2.9</td>
<td>1.0</td>
<td>8.0</td>
<td>1.0</td>
</tr>
<tr>
<td>2.10</td>
<td>8.0</td>
<td>1.0</td>
<td>8.0</td>
</tr>
<tr>
<td>2.11</td>
<td>8.0</td>
<td>1.0</td>
<td>8.0</td>
</tr>
<tr>
<td>2.12</td>
<td>8.0</td>
<td>1.0</td>
<td>8.0</td>
</tr>
</tbody>
</table>

Note: $R_m = 1.0$ Ω-m, borehole diameter $\phi = 8.0$ in.

Table 2.3: Response characteristics for the lateral log for simple earth models.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Earth model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Resistive</td>
</tr>
<tr>
<td></td>
<td>Thick</td>
</tr>
<tr>
<td>Shadow Zone</td>
<td>✓</td>
</tr>
<tr>
<td>Blind Zone</td>
<td>no</td>
</tr>
<tr>
<td>Plateau Zone</td>
<td>✓</td>
</tr>
<tr>
<td>Peak lower boundary</td>
<td>✓</td>
</tr>
<tr>
<td>Peak upper boundary</td>
<td>no</td>
</tr>
<tr>
<td>Anomaly below lower boundary</td>
<td>no</td>
</tr>
</tbody>
</table>
50 ft. is needed to obtain a plateau reading uninfluenced by surrounding formations (Schlumberger, 1987).

2.3.2 Response to a thin resistive layer

Figure 2.8 illustrates the response of the lateral device to a thin bed more resistive than the surrounding formations. By thin, I mean a layer between one and two times thicker than the lateral spacing. Here there is a sharp resistivity peak at the lower boundary and there is a small plateau below the lower boundary. There is also a small plateau below the upper layer. These zones represent the case when the receiver is at the lower boundary and the source is at the upper boundary and are commonly known as the shadow zones.

2.3.3 Response to a very thin resistive layer

Figure 2.9 illustrates the response of the lateral device to a very thin bed more resistive than the surrounding formations. By very thin, I mean a layer thinner than the lateral spacing. The apparent resistivity curve has big notches just beneath the lower boundary of the layer, followed by low readings over the blind zone below the bed. The blind zone corresponds to the receiver and the source located below and above the middle thin layer (the receiver and the source are straddling the layer of interest).
Figure 2.7: Response of the lateral device to a thick resistive layer.
Figure 2.8: Response of the lateral device to a thin resistive layer.
Figure 2.9: Response of the lateral device to a very thin resistive layer.
2.3.4 Response to a thick conductive layer

Figure 2.10 illustrates the response of the lateral device to a thick bed less resistive than the surrounding formations. The curve is again unsymmetrical. Here, the anomaly extends below the upper and lower beds for a distance slightly greater than the AO spacing.

2.3.5 Response to a thin conductive layer

Figure 2.11 illustrates the response of the lateral device to a thin bed less resistive than the surrounding formations. Notice a plateau of length AO below the lower and upper boundaries.

2.3.6 Response to a very thin conductive layer

Figure 2.12 illustrates the response of the lateral device to a very thin bed more conductive than the surrounding formations. The curve is again unsymmetrical. This anomaly, too, extends below the bed for a distance slightly greater than the AO spacing, and there is again a blind zone below the lower boundary.

2.4 Environmental effects

The borehole diameter, the mud resistivity $R_m$, and the invaded zone affect the apparent resistivity reading for the lateral device. The different environmental effects can be summarized in the following way (J.H. Kim, 1986 pages 80-81):
Figure 2.10: Response of the lateral device to a thick conductive layer.
Figure 2.11: Response of the lateral device to a thin conductive layer.
Figure 2.12: Response of the lateral device to a very thin conductive layer.
• **Borehole diameter:** For common borehole sizes, the variations in borehole diameter do not appreciably affect the apparent resistivity if the ratio \( R_t/R_m \) is not too large (or small).

• **Borehole resistivity:** The borehole resistivity affects the apparent resistivity reading in a lateral device when \( R_t/R_m \) is large. Here, the borehole effects are significant, and the apparent resistivity is greater than the true resistivity.

• **Invaded zone:** From analysis of the apparent resistivity curves for the lateral device, it can be concluded that the apparent resistivity curve is not appreciably affected by the invaded zone resistivity, but a large diameter of invasion adversely affects the apparent resistivity reading in resistive beds.

Based on these environmental assumptions, I trained the neural network with the same borehole parameters (mud resistivity and borehole diameter), feeling I will not significantly affect the inversion process.

### 2.5 Use of lateral logs today

Even today, lateral logs are a significant proportion of available logs. In some areas ES logs are the only type of resistivity log measurement available. The Soviet Union is still using the lateral device in the field. This means that the interpretation of the tool is still important for the determination of possible hydrocarbon zones.
Because of the difficulties in the interpretation of the lateral log, many hydrocarbon zones have been bypassed in the past. Since many of those wells are cased today, it is difficult to use new resistivity tools for the evaluation of those zones as potential reservoirs. At this point numerical inversion and forward modeling can be used to help in that formation evaluation and in the correlation with new logs in nearby wells.
Chapter 3

THE NEURAL NETWORKS ALGORITHM

Neural networks are computing circuits or computer simulations modeled after the brain. They are amazing because of their ability to learn from experience, much as people do. For instance, they can be taught by repeated exposure to recognize lateral log responses, feats that are hard for humans. But like humans, neural nets cannot explain how they learn. That makes it difficult to design the best network for tackling a specific problem. The approach used in this thesis, is empirical and it is based on a sensitivity analysis of the different parameters in the network. Many types of neural networks have been developed, but I will use a backpropagation neural network, because it is simple to use and it has proven effective in areas such as speech recognition, handwritten character recognition, military target identification, and other geophysical applications.
3.1 A view of neural networks

3.1.1 Short history

In the 1940’s, psychologists began to improve their understanding of the functioning of the neuron and the pattern of its interconnections. This new knowledge allowed researchers to produce mathematical models to test theories about human learning (Wasserman, 1989). In 1949, Donald Hebb presented the first explicit statement of a learning rule.

For twenty years, the field of neural networks grew fast until 1969 when Marvin Minsky, the father of artificial intelligence, proved that perceptrons, the simple neural network, could not solve many very simple problems. It wasn’t until the 1980’s that new life in this field started again with the use of more complex networks and better mathematical tools.

3.1.2 Biological and artificial neural networks

The brain differs from a serial computer in several fundamental ways. In the brain there is distributed rather than centralized control. Also, there is not a central memory bank. Knowledge appears to be placed in the interneural connections, and learning takes place by modification of connection strength (Rumelhart and McClelland, 1986), and signals in the brain are $10^5$ times slower than signals in a computer circuit. The brain is thought to get its incredible computer power from its massive
parallel structure.

Neural networks are closer to the brain model than conventional computers. We expect the neural networks to get better responses for tasks like pattern recognition, similar to those of the human.

3.2 Artificial neural networks

Figure 3.1 shows a schematic diagram of a backpropagation neural network system. This is a distributed computational system characterized by the following:

- Neurons: computational elements, which put a weighted sum of the inputs through a non linear gate (Figure 3.2).

- Activation Rule: the neuron response to the weighted input sum. Three common types of non linear activation functions are shown in Figure 3.3, the step function, the hyperbolic tangent function, and the sigmoid function. The activation function is characterized by its shape and a threshold value, below which no response occurs, or for the hyperbolic tangent function, below which the response is negative.

- Architecture: this is the topology of what is connected to what. The particular network shown in Figure 3.1 would be said to have four layers, an input layer, two hidden layers, and an output layer. The input layer receives the data while
Figure 3.1: Four layer neural network.
the connections between layers do the processing part of the problem. The output layer creates the result of the problem.

- **Learning Rules**: these govern the way in which network parameters (weights and thresholds) vary during training. In this thesis the learning rules are based on the backpropagation algorithm.

### 3.3 Neural network systems

The neural network system can be divided in two different systems:

- **self-organizing system**
Figure 3.3: Activation functions (a) Step function (b) Hyperbolic tangent function (c) Sigmoid function.
The self-organizing system is an implementation of a statistical clustering algorithm, which will take a set of training vectors and cluster them into a number of classes. The number of classes may be pre-set or set adaptively on the basis of threshold rules for deciding whether or not two pattern vectors can reside in the same class (see Lippman, 1987).

The alternative to a self-organizing net is a supervised net where the user specifies the class of each of the exemplar vectors. The backpropagation algorithm is a supervised method and, it is used in this thesis because it can handle more complex problems than the self-organizing system.

Many variations of the unsupervised and supervised method have been implemented but the most commonly used are the Kohonen network for the unsupervised system and Backpropagation neural network for the supervised system.

### 3.4 Backpropagation algorithm

A four layer neural network is illustrated in Figure 3.1. It has a connected architecture with sigmoidal activation functions for each neuron (see Figure 3.3 (c)).

Figure 3.1 presents an input layer with \( N_1 \) nodes, two hidden layers with \( N_2 \) and \( N_3 \) nodes, respectively, and an output layer with \( N_4 \) nodes. Between layers nodes are related with a connection characterized by an adjustable weight, \( w \).
The general problem is as follows. Given a set of patterns presented at the input nodes, how can the weights be adjusted so each output node gives the correct response for each training pattern, \( p \), \((1 \leq p \leq P)\), with \( P \) equal to the number of training patterns.

Assuming an \( n \) layer network, we number the layers \( 1, 2, \ldots, n-1, n \), starting with the input layer, while \( N_1, N_2, \ldots, N_{n-1}, N_n \), respectively, are the numbers of nodes in layers \( 1 \) to \( n \). \( w_{ij}^{(k)} \) is the weight connecting the output of node \( j \) of layer \( k-1 \) with the input of node \( i \) of layer \( k \) and \( y_{i}^{(k)} \) is the output from node \( i \) of layer \( k \).

Before training, the weights at all levels are set randomly to small numbers (close to zero). When each training pattern is presented, the idea is to adjust each weight such that the difference between the actual output and the desired output is minimized.

Let \( d_{pi} \) \((1 \leq i \leq N_n)\) be the desired output at the \( i^{th} \) node of the output layer upon presentation of the \( p^{th} \) pattern, and \( y_{pi}^{(n)} \) be the actual output of the node. A typical least squares cost function of the system, \( E \), can be expressed as

\[
E = \sum_{p=1}^{P} E_{p}, \tag{3.1}
\]

where

\[
E_{p} = \frac{1}{2} \sum_{i=1}^{N_n} (y_{pi}^{(n)} - d_{pi})^2 \tag{3.2}
\]
The method adjusts the weight $w_{ij}$ by an amount $\delta w_{ij}$, according to

$$
\delta w_{ij} = -\eta \frac{\partial E_p}{\partial w_{ij}} \tag{3.3}
$$

or

$$
\delta w_{ij} = -\eta \sum_{p=1}^{P} \frac{\partial E_p}{\partial w_{ij}} \tag{3.4}
$$

where $\eta$ is a convergence constant determined empirically and frequently called learning rate (see equation 3.11 and discussion).

The first method (see equation 3.3) requires much less storage than the second. However, the results of the second method (see equation 3.4) will not be a function of the order of pattern presentation as may well happen with the first (Raiche, 1991) and this is a desirable feature.

$a_i^{(k)}$ is defined as the total input to node $i$ from all of the nodes in the layer $k - 1$. Since

$$
a_i^{(k)} = \sum_{j=1}^{N_{k-1}} w_{ij}^{(k)} y_j^{(k-1)} \tag{3.5}
$$

and $f(x)$ is the sigmoid function:

$$
f(x) = \frac{1}{1 + e^{-x}}, \tag{3.6}
$$
then the output \( y_{i}^{(k)} \) is

\[
y_{i}^{(k)} = f(a_{i}^{(k)}).
\]  

(3.7)

We can use equation 3.3 or equation 3.4 to calculate the update to \( w_{ij}^{(n)} \) in the output layer (Notice the superscript \( n \) in the weights in equations 3.8 and 3.9) as:

\[
\delta w_{ij}^{(n)} = -\eta \frac{\partial E_{p}}{\partial y_{pi}^{(n)}} \frac{\partial y_{pi}^{(n)}}{\partial w_{ij}^{(n)}}
\]  

(3.8)

\[
\delta w_{ij}^{(n)} = -\eta(y_{pi}^{(n)} - d_{pi})(-y_{i}^{(n-1)})f(a_{i}^{(n)})(1 - f(a_{i}^{(n)}))
\]  

(3.9)

where \( \{(-\eta(y_{pi}^{(n)} - d_{pi})\) \} \) is derived from equation 3.4, and \( \{f(a_{i}^{(n)})(1 - f(a_{i}^{(n)}))\} \) is derived from the derivative of the sigmoid function in equation 3.6.

Computing updates to weights in intermediate layers is more involved, but by working back toward the input, previously computed quantities can be reused; hence, the descriptive name Backpropagation. For example, to compute the update to \( w_{ij}^{(n-1)} \),

\[
\delta w_{ij}^{(n-1)} = -\eta \sum_{i=1}^{N_{n}} \left[ (y_{pi}^{(n)} - d_{pi}) \frac{\partial y_{pi}^{(n)}}{\partial y_{pi}^{(n-1)}} \frac{\partial y_{pi}^{(n-1)}}{\partial w_{ij}^{(n-1)}} \right]
\]  

(3.10)

The procedure is iterated until the system cost function has been reduced to some acceptable tolerance.
3.5 Software implementation

I implement a backpropagation neural network in “ANSI C” programming language. It allows the use of any number of layers, and several nodes per layer. It gives considerable freedom to experiment with different network designs, features essential for the sensitivity analysis of the different parameters. Some of the procedures involving the input of parameters and dynamic allocation of arrays were taken from a public domain program developed in the Center for the Wave Phenomena at Colorado School of Mines.

My implementation of the backpropagation algorithm has a small variation with respect to the standard backpropagation algorithm. It allows the use of a variable \( \eta \) parameter (see equations 3.3 and 3.4). This parameter is function of the error per iteration and has the property of increasing the convergence speed, when correctly chosen. Specifically, I let

\[
\eta_p = 1 - \max_i (\max \{|(y_{pi}^{(n)} - d_{pi}|), \eta_{p-1}\}). \tag{3.11}
\]

Note that a variable \( \eta \) has properties similar to the Marquardt \( \lambda \) parameter in the standard least squares inversion. It adjusts the weight in a different way as the cost function \( E_p \) approaches a minimum.
Chapter 4

NEURAL NETWORK TRAINING PROCESS

In practice, current automated geophysical inversion methods consist of minimizing the misfit between field and model data points. An inversion using a neural network can bring new information to the standard inversion process through its ability to recognize the most important parts of the curve. Experienced interpreters learn to make judgments based upon data features such as relative peak heights, separations and positions of cross-over. Neural network inversion is a general procedure that can quantitatively define and assign importance to both gross and subtle data features based upon the structure of the data learned during the training process.

The interpretation of the lateral tool is complex and the learning process necessary to get part of the knowledge requires a “teacher”. Here the neural network will act as the individual trying to learn the interpretation and I act as the “professor” giving the necessary information to solve the problems that I assign, and hopefully be able to solve new ones. The teaching process can be done in two different ways:

- Randomly generated examples
• "Professor" designed examples.

The first teaching process consists of the generation of random earth models with their respective responses, which are then input to the neural network. This approach has big problems, because it does not allow a careful study of the parameters in the architecture of the network. Even more catastrophic, it is possible to get a non convergent network, because some random earth models generated may be "close" to each other. The second "professor" approach is the one used in this thesis. A set of carefully selected examples are input to the neural network for the learning process. This method fixes the number and categories of examples in the training process. With the number of examples fixed, it is possible to do a careful sensitivity analysis of the parameters used in the neural network. Also, I can study the convergence properties of the network based on the examples presented.

The most important elements in the training process of the neural networks are:

• the earth models used

• the input representation of the geophysical data

• the output representation

• the neural network architectures

Each of these will be studied in turn in the following sections.
4.1 Earth models

A forward modeling program creates lateral tool responses to different earth models. These responses and the earth models create examples that comprise the training set for the neural network. A series of two and three layer earth models are chosen, trying to represent the typical lateral response for common geologic conditions.

Changes in resistivity simulate variations in lithology or presence of hydrocarbon zones. The chosen set of earth models examples have contrasts in resistivity from low resistivity to high resistivity and vice versa. They build what I term the knowledge base for the neural network.

Figure 4.1 and Figure 4.3 show typical examples presented to the neural network during the training process, and Figure 4.2 and Figure 4.4 show the respective network responses after training. A pool of 122 examples shown in the Appendix A (Table A.1 to Table A.6) form the training set for the neural network used during the learning part of the backpropagation algorithm.

The selection of earth models was based on two criteria:

- economy
- neural network efficiency

The convergence speed of the backpropagation algorithm is a function of the
Figure 4.1: Example 1: Earth model and lateral response. High to low resistivity. Boundaries at 106 and 114 ft.

Figure 4.2: Example 1: Neural network and post processing response. High to low resistivity. Boundaries at 106 and 114 ft.
Figure 4.3: Example 2: Earth model and lateral response. Low to high resistivity. Boundaries at 62 and 118 ft.

Figure 4.4: Example 2: Neural network and post processing response. Low to high resistivity. Boundaries at 62 and 118 ft.
number of examples presented during the training process. It is important to use the minimum amount of examples consistent with convergence of the network, while presenting an adequate amount of information for the neural network to respond correctly to inputs not part of the training set.

The order of presentation of the training set is also important. The classification procedure of the neural network depends upon the order of presentation of the examples. If the set of examples is presented sequentially and repeatedly, the neural network will memorize the last example of the set and will not respond consistently to the other examples. Trying to avoid this problem, the set was initially presented in one order and for the next iteration, it was presented in the reverse order. This procedure partially solved the difficulties, but now the network reacted to the first and last examples only.

The problem was solved with the earth models presented to the network in a random sequence for each iteration. The converge speed was improved and the neural response to each of the earth models was more accurate. The final result is a network that will not memorize the last and first examples, but a network that recognizes all the earth models from the training set.

All the earth models used in this thesis represent layers with one or two boundaries in an interval of 100 ft. for reasons explained in the next section. The boundaries between models with similar characteristics (for example low to high resistivity con-
trast) are separated by an interval of 8 ft. (see Figure 4.5). A study of the effects of the separation between boundaries shows that 8 ft. is adequate. Using a separation larger than 8 ft. creates difficulties because the neural network cannot generalize to earth models not part of the training set, and separation smaller than 8 ft. produced a non convergent network.

4.2 Input representation of geophysical data

The design for the input representation of the lateral log data optimizes the feature extraction and reduces the dimension of the input set.

The first representation used as an input was sampled data without any preprocessing scheme. The neural network response was not compatible with the desired output and the network did not converge for some examples. Very resistive beds cause “saturation” to the weights in the network and as a consequence the network did not respond in the expected fashion.

A simple preprocessing scheme allowing the values in the input data to vary between zero and one produced a convergent network. Scaling the curve by the maximum value in the interval creates the necessary characteristics for the input data.
Example ...

Figure 4.5: Separation between earth models examples used in the training process: high to low resistivity contrast.
4.2.1 Input dimensions

The lateral log is a continuous log where the reading represents an average resistivity of the surrounding earth. The process required to analyze the data should be at least compatible with the sensitivity of the tool. The dimension in the input data (number of input nodes) is designed to optimize the computational efficiency of the neural network algorithm, because the CPU time for the process is a function of the number of nodes in the input layer. Obviously the input dimension should be set large enough to have geological significance and small enough to have a fast response.

An inversion of 100 ft. of logging data is large enough to have geological significance and, with an adequate sampling interval, it allows a very fast neural network response. A 100 ft. interval is almost five or six times the typical length for the lateral tool and with this thickness, it is possible to have many thin and thick beds in the interval.

4.2.2 Resistivity ratio

The input data in the training process was selected using a resistivity ratio $R_t/R_m$ of 1 for the conductive beds and 10 for the resistive beds. Even in the best of conditions we cannot expect similar resistivity ratios for all the situations. Nevertheless, J. H. Kim (1986) showed that the shape of the curve does not change dramatically for resistivity ratios different from those used in this thesis.
Another effect created by the preprocessing scheme is produced by scaling of the data to the maximum resistivity value in the interval and setting this value to one. When the interval has more than one big peak, this process gives more importance to the highest peak, while lessening the influence of the other lower peaks (see Figure 4.6).

4.2.3 Sampling rate for the lateral data

The sampling rate is very important in the study of the response of the neural network. The sampling rate of the data should be determined to avoid aliasing, while allowing the network to detect the principal features in the lateral curve. A very coarse sampling can produce data with alias, but the typical sampling wavenumbers for logging data are so small that we generally do not observe aliasing even for very thin layers.

For example a sampling interval of 4 ft. is too coarse for the requirement to detect the different features in the lateral log, and is not good for use in this process. On the other hand, 1 ft. sampling is good for high resolution work for most of applications, but it is not good enough to get a fast response for the network. Another aspect of the problem is that the neural network may be used as an initial guess for a conventional inversion package, so that the response does not have to be perfect. For most of the applications, I considered 2 ft. as a reasonable sampling rate.
Figure 4.6: Neural network response with two peaks of different height.
4.2.4 "Optimal" input parameters

In summary, the parameters for the input data used in this thesis are the following:

- 100 ft. interval
- scaling by the maximum resistivity in the interval
- 2 ft. sampling interval

These parameters produce satisfactory results for synthetic and real data.

4.3 Output representation

The output space (or number of nodes) in the neural network typically corresponds to the input for the standard inversion package. Several aspects should be considered for the design of the output parameters. There are some edge and boundary effects created when the boundary of the earth model is close to the border of the input data file.

The output from the neural network has a value from 0 to 1 (recall the scaling discussion in section 4.2). Since we wish only to determine the boundaries, a post processing scheme is designed to create an initial guess suitable for the inversion package.
Post processing of the data consists initially of an enhancement of the boundaries detected by the neural network. When the neural network response is larger than 0.6 or smaller than 0.4, I consider that the network recognizes a conductive or a resistive layer in a definite way. But when the value is between 0.4 and 0.6 the network reacts to what it thinks is a boundary but does not have a precise answer. For the first case the post processing scheme will provide an answer of 1 or 0 and for the second case the answer will be 0.5. With these values and the log itself, the interpreter can decide if the boundary detected by the network is real or not. When the boundary detected by the post processing scheme is less than one sample in width and the value is 0.5, I consider that it does not represent a true layer. When the width of the 0.5 layer is more than 2 to 3 samples, I consider the boundary well determined. This post processing creates a sharper response and solves some problems created by an imperfect generalization of the neural network.

The next step is to assign a resistivity to the layers. The resistivity assigned to the layer is the one corresponding to the middle of the boundaries determined by the neural network. For the lateral log the resistivity value in the middle of the layer does not always represent the best approximation to the real resistivity, but studies at the Center for Well Logging and Petrophysics at Colorado School of Mines showed that for the convergence of the standard inversion package it is more important to have correct boundaries than correct resistivities. A correction could also be made
which depended on bed thickness. When the bed is thick, the bed boundaries are well determined, but if the bed is thin, shoulder effects could be considered as well.

4.4 Neural network architectures

The neural network has many different parameters, all of them with variable influence on the final response of the net. The effect of each parameter is studied in an empirical way and based on a sensitivity analysis. The sensitivity analysis is performed using the best parameters found for the input layer while the output layer is fixed. Based on geological and physical constraints explained in the previous section, the number of nodes in the input was fixed at 51 nodes, representing 100 ft. of lateral log sampled at 2 ft. space intervals, and 51 nodes at the output response representing the same 100 ft. of earth model.

The supervised training used for the training process was explained in previous sections (4.1 and 4.2) of this chapter. In this section I will concentrate on the effect of the different parameters on the response of the network. Table 4.1 presents the network architectures studied in this thesis with their parameters.

4.4.1 Training efficiencies

The training process used involves the presentation of 122 examples to the network. The efficiency of the different networks was compared based on two aspects:
Table 4.1: Neural Network architectures.

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<th>Network Number</th>
<th>Number of Nodes</th>
<th>Number of Hidden Layer</th>
<th>Max RMS Error</th>
<th>CPU Time (min)</th>
<th>Maximum Initial Weight</th>
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</table>

*Note: 122 examples during training process*
• CPU time

• number of iterations

Figure 4.7 presents the CPU time necessary for the convergence of the network. The CPU times range from one minute to two and a half minutes for those networks that converged. Notice that the networks with 10 or 0 nodes in the hidden layer did not converge, while networks with more than 15 nodes in the hidden layer always converged.

The networks with fastest convergence were networks 12 and 16 with 30 and 40 nodes in the hidden layer, respectively. Nevertheless, the one with least iterations to converge was network 24 (100 nodes in the hidden layer) with 212 iterations (see Figure 4.8). In general the number of nodes in the hidden layer reduces the number of iterations but increases the number of computations (see Figure 4.9). The most efficient network will be the one with fewest iterations for convergence and the smallest amount of CPU time, given that the network offers “good” answers. These criteria are met by network numbers 12 and 16.

4.4.2 Memory requirements and stability

The memory required for the backpropagation algorithm in the implementation provided in this thesis, is a function of the maximum number of nodes in any of the layers in the networks. With the number of nodes in the input and output layers
Figure 4.7: CPU time vs. network number.
Figure 4.8: Number of iterations vs. network number.
Figure 4.9: Number of iterations vs. network number and CPU time.
fixed, the only variable is the number of nodes in the hidden layers. In general the number of nodes in the hidden layer should be reduced to a minimum to get a more efficient network. For the inversion of the lateral log the requirements in memory are so minimal that the program can be run on a small personal computer.

4.4.3 Number of layers

Two different configurations were tested, no hidden layer and only one hidden layer. The convergence time increased considerably when I introduced a hidden layer (see Table 4.1), but I did not get convergence for a network without a hidden layer. Marvin Minsky (1969) explained that a two layer neural network cannot solve many simple problems; such as the XOR problem. I think the lateral inversion is a complex problem for which the solution with two layers is very difficult or impossible because, I think the problem is not linearly separable.

I could use more than one hidden layer in the network, but this is not common practice. "Kolmogorov’s theorem (1957)" shows that all the classification problems solved with networks could be done with a three layer network.

4.4.4 Number of nodes in the hidden layer

Several configurations for the neural network were tested. Networks with 15 to 100 nodes in the hidden layer gave somewhat different results when they were tested on synthetic data (see Figure 4.12 and Figure 4.13) to compare the quality
of response. The selection of the number of nodes was based on the quality of the response more than the CPU time or number of iterations for convergence.

4.4.5 Maximum random weight number

For the first iteration, the weight connections (see Figure 3.1) in the network are set to random numbers. It seems that the convergence speed of the network is function of the maximum value allowed for the initial weights. Figure 4.10 presents the number of iterations as a function of the number of nodes in the hidden layer for different values of initial weight. A sensitivity analysis with different parameters, shows that a maximum random value of 0.1 is necessary to get a small number of iterations for convergence for the different nets presented.

4.4.6 Quality of the response

The most important aspect in the sensitivity analysis is the quality of the response. The training process may be slow, but the ultimate goal is to have a network that works properly for most of the situations. The quality of the response is then essential in the selection of the “best” network.

A study of the maximum root mean square error (RMS) for the networks tested (see Figure 4.11), does not give enough insight into the selection of the network, because the RMS error remains basically constant and is primarily function of the stopping criteria for the backpropagation algorithm. Nevertheless, it is known that
Figure 4.10: Number of iterations vs. number of nodes in the hidden layer for different initial weight values.
a network with too many nodes in the hidden layer will converge very fast, but it will not properly generalize to examples not part of the training set. An example using a 4 layer earth model (see Figure 4.12) is used to compare the response for the networks 8, 12, 16, 20 with 20, 30, 40, and 50 nodes in the hidden layer, respectively. All the networks recognized the first boundary but most of them failed to detect the second and third boundary. The only network that gave an appropriate response to the problem was network 16 with 40 nodes in the hidden layer. I think networks with less than 40 nodes in the hidden layer do not have enough degrees of freedom to solve the problem and with more than 40 nodes, the network will start to memorize the patterns.

4.4.7 Neural network parameters selected

Base on the empirical observations in this chapter, the network selected for the test with synthetic and real data has the following parameters:

- 51 input nodes
- 40 nodes in the hidden layer
- 51 output nodes
- 0.1 maximum value in the initial weights
- sigmoidal transfer function
Figure 4.11: Maximum root means square vs. network number.
Figure 4.12: Four layer earth model and lateral response. Boundaries at 71, 91, 111
Figure 4.13: Earth model and neural networks response for network 8,12,16,20
• learning rate $\eta$ variable per iteration
Chapter 5

NEURAL NETWORK RESPONSE TO SYNTHETIC AND REAL DATA

5.1 Synthetic Data

A set of synthetic lateral responses computed from the earth models described in Table 5.1 are used as test functions for the neural network program. The object is to show the response of the neural network program to examples that are not part of the training set. The synthetic models used have two, three, four and five layers. The goal is to evaluate the "generalization" properties of the neural network. All the examples were designed with mud resistivity $R_m$ equal to 1 Ω-m and borehole diameter of 8 in. and a lateral tool length of 18' 8".

5.1.1 Two layer models

Figure 5.1 shows the lateral and neural network response for an earth model with two layers. The first layer is a conductive bed with resistivity of 4 Ω-m and the second is a resistive bed with resistivity of 20 Ω-m. The post processing scheme
Table 5.1: Parameters for the synthetic models used in the generation of lateral responses.

<table>
<thead>
<tr>
<th>Figure</th>
<th>$R_t$ (Ω-m.)</th>
<th>Layer</th>
<th>Boundaries (ft.)</th>
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</thead>
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<td>3</td>
<td>95 - - -</td>
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<tr>
<td>5.2</td>
<td>20 4</td>
<td>2</td>
<td>95 - - -</td>
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<tr>
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<td>1</td>
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<tr>
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<td>-</td>
<td>71 121 - -</td>
</tr>
<tr>
<td>5.5</td>
<td>30 2 15</td>
<td>-</td>
<td>71 91 - -</td>
</tr>
<tr>
<td>5.6</td>
<td>2 30 5</td>
<td>-</td>
<td>71 91 - -</td>
</tr>
<tr>
<td>5.7</td>
<td>2 30 3 15</td>
<td>2</td>
<td>71 101 121 -</td>
</tr>
<tr>
<td>5.8</td>
<td>20 30 3 15 2</td>
<td></td>
<td>95 115 125 -</td>
</tr>
</tbody>
</table>

Note: $R_m = 1.0$ Ω-m, borehole diameter $d = 8.0$ in, and lateral tool length $l = 18'8''$.

enhances the response even more. Minor “high frequency” response is observed for the first layer. These variations are a function of the examples presented during the training process and the response properties of the neural network. The misfit between the real bed boundary and the boundary detected with the neural network is just one sample (2 ft.).

Figure 5.2 shows a similar earth model but with the resistive layer first. Again, the neural network response compares favorably with the desired earth model. The noise in the zone for the conductive layer is of the same level as in the previous example. For this case the neural bed boundary is exactly in the desired position.

The post processing scheme works well for this example and for the previous one, giving a very good definition for the boundaries in the respective earth models.
Figure 5.1: Example 1: Two layer model. Low to high resistivity
Figure 5.2: Example 2: Two layer model. High to low resistivity
5.1.2 Three layer models

Figure 5.3 and Figure 5.4 show earth models, lateral responses, and neural responses for three layer models. The object is to evaluate the response for earth models with more complex geometries, and compare the responses for thin and thick layers. Another element in the evaluation is the analysis of the response for earth models having different resistivities in the shoulder beds.

Thick middle layer

Figure 5.3 shows the responses for a thick conductive bed (8 Ω-m) followed by a layer with a resistivity of 15 Ω-m. The resistivity of the first layer is twice the resistivity of the last layer.

The neural network detects an event in approximately the correct position, but it has a "noisy" response for the last layer. The response is far from perfect, but the neural network has a sharper response than the lateral device. The post processing scheme brings a response similar to the earth model expected, but with some variation for the lower boundary where 0.5 values are detected. Some of those boundaries with value 0.5 have a small thickness, so they can possibly be considered artifacts of the post processing scheme. Anyway, the interesting aspect is the interpretation of the neural network response, because the neural network log helps in the detection of the boundary.
Figure 5.3: Example 3: Three layer model. Thick conductive middle layer
Figure 5.4 shows a conductive layer of 2 \( \Omega \)-m with a thick conductive bed (30 \( \Omega \)-m) followed by a layer with a resistivity of 5 \( \Omega \)-m. The neural network response is very similar to the real earth model. The pattern learned during the training process was appropriate for this example. Again a small amount of noise is present for the lower layer and the post processing scheme yields a response close to the real one. A peak with value 0.5 is detected in the lower layer, as it is in some other examples. I suspect that training the network with more example will reduce the number of these anomalies.

Thin middle layer

Figure 5.5 and Figure 5.6 present the response to thin conductive and thin resistive middle layers, respectively. (Recall that we are using a lateral tool of length 18’8”.) The response is not as good as the neural network response for thick layers. Nevertheless, the presence of three layers is detected for the conductive middle layer, and the general shape of the desired earth model is obtained. Again, a small “anomaly” is detected around depth 135.

For a thin resistive middle layer (Figure 5.6), the neural network failed to detect the lower boundary in a satisfactory way. It seems that the network has problems with the detection of the shadow zones. Again, more training may solve this problem. If we use more examples with thin beds, it may help the neural network to detect that response pattern.
Figure 5.4: Example 4: Three layer model. Thick resistive middle layer
Figure 5.5: Example 5: Three layer model. Thin conductive middle layer
Figure 5.6: Example 6: Three layer model. Thin resistive middle layer
5.1.3 Four and five layer models

The training process for the networks was performed with one, two, and three layer models. It is interesting to evaluate the network response for earth models having more layers than the ones presented during the learning process (two and three).

The four and five layer models shown in Figure 5.7 and Figure 5.8 present more complex models than the ones used in the training set. The response is very encouraging because the neural network log responses are similar to the earth model expected. These responses suggest that I continue the research with real data.

5.2 Real data

A real lateral log was digitized and used as an input to the neural network code. The well location and the interval of study are the following:

- Well: Wardner number 76
- Field: Stratton
- County: Nueces
- State: Texas
- Interval: 6450-6650 ft.
Figure 5.7: Example 7: Four layer earth model.
Figure 5.8: Example 8: Five layer earth model.
Figure 5.9 shows the lateral log with the neural network response. Two hundred feet of lateral log were used as an input to the neural network. The input dimension for the network is 100 ft. of log. To create a response for the entire log, the neural network code was called three times; a neural response for the interval 6450-6500, another for the interval 6500-6550 and the last one for the interval 6550-6600. Since the network response is considered more reliable at the middle of an interval, the response used for the interval 6500 to 6600 is a combination of the mid interval response from the three neural responses. I consider the response in the middle of the neural output more reliable because most of the training examples are located between the 20 ft. and 80 ft. of the data samples.

Based on the neural response and the post processing scheme, an earth model was proposed for the initial guess to use in the standard inversion package. The post processing output gave the boundaries used for the initial guess model. When the value in the post processing output was 0.5, I looked at the neural network response and decided if the value indicated a bed. The resistivities of the initial earth model were determined using the apparent resistivity values at the middle of each layer. Forward modeling of the initial guess earth model created a lateral log. Figure 5.10 shows the initial log response created using a forward modeling program, compared with the actual log. The initial guess earth model created with the neural network
Figure 5.9: Real log from well G.P. Wardner number 76, neural net response and post processing scheme.
response to the lateral log displayed remarkable similarities to the real log, indicating the promise of this entire neural network approach. The RMS error starts with 1.005 \( \Omega \cdot m \) for the first iteration and ends with 0.26 \( \Omega \cdot m \) for the sixth and last iteration.

The standard inversion packages uses the bed boundaries detected with the neural network, improves the resistivities substantially and somewhat modifies the bed boundaries. As expected, the determination of the boundaries is more important than the determination of the resistivities for the inversion process.

To compare the initial earth model created with the neural network with another technique, a short normal log in the same interval was digitized and processed with an automatic initial guess program discussed by Whitman et al. (1990). This automatic initial guess program creates a simulated laterolog from the short normal log. This algorithm has physical meaning and it is considered a good testing program to compare its initial guess with the “non physical” approach of neural networks.

Figure 5.11 presents a comparison between the neural network initial guess model and the short normal initial guess model. Both responses have reasonable similarities to the real log and provide a good approximation for the initial guess. The problem with the initial guess for the short normal is that it is based on readings close to the borehole, so that it does not evaluate the true resistivity of the formation. Also, sometimes there are electrical surveys without that short normal log. By contrast the lateral tool was the first tool designed for logging and is more frequently found
Figure 5.10: Real log from well G.P. Wardner number 76, lateral response to the neural network initial guess. Real log vs. best fit response
in old logs.

The results with real data are very interesting because they show that the initial guess using neural networks can give an answer that is close to a feasible earth model, indicating its promise as an initial guess for the standard inversion package.

As a final cautionary note, while the neural network initial guess was adequate to obtain a very good fit after processing by the inversion scheme, this is also function of the apparent robustness of the loss surface associated with the inversion scheme. In other words the inversion scheme can tolerate an initial guess that isn’t particularly accurate, as long as the initial guess is not patently poor; e.g. bed boundaries way off or too few of them, or gross errors in resistivities.
Figure 5.11: Real log from well G.P. Wardner number 76. Initial guess from neural network vs. initial guess from short normal.
Chapter 6

CONCLUSIONS

A non-standard technique for the inversion of a lateral log is developed in this thesis using neural networks. A finite difference method is used to simulate the lateral log which in turn is used as an input to a backpropagation neural network. The neural network extracts the information from the log based on the "knowledge" learned during the training process of the neural network. The training process consists of the presentation of a set of earth models with their lateral responses to a neural network. The response to these models includes the most important features in the lateral log. The neural network process gives quantitative importance to the gross and subtle data features in the lateral log data. The neural network then reacts to those features in actual logs and produces a response inferred from the "knowledge" stored in the network.

A sensitivity analysis of the important parameters for the neural network architecture was designed. The neural network architecture is selected based on the efficiency of the backpropagation algorithm and the quality of response to synthetic
data not part of the training set. The result is a neural network with 40 nodes in the hidden layer, 51 nodes in the input and output nodes and a learning rate $\eta$ variable per iteration.

The neural network response can then be used as a "deconvolved" log. This new log gives better definition of the earth layer boundaries than the lateral log and provides a new interpretation tool. In fact, the neural log may offer bed boundary sequences. The neural response is enhanced by a post processing scheme which then provides an initial guess model for a standard inversion package. The standard inversion is then be applied to the data using the neural network earth model to fix the initial misfit between the field and model response data points, in other works as an initial guess. To my knowledge, there are no other lateral initial guess models and its competitor - the short normal initial guess model - is just a shallow device.

Synthetic data was used to evaluate the performance of the network to examples not part of the training set, showing reasonable agreement with the desired output for most of the earth models presented. Finally, in a test using real data, the earth model generated by using the neural network response as an initial guess for an automatic inversion of a normal resistivity log was compared by forward modeling with the original log, giving surprisingly good agreement in the results and showing a promising future for the technique. The main advantages for neural networks are speed and flexibility. Once established and trained, neural networks are easy to
implement and change.

Additional study and experimentation are encouraged. After training with more complex examples, the neural network may yield an even better response for real data. More detailed work with the post processing scheme could help provide a better automation for the initial guess. For instance, a processing with two or more neural networks instead of just one could diminish the detectability problem for the thin layers.

I feel that the strategy for inversion used in this thesis can be applied to other geophysical areas, such as electromagnetics and gravity.
REFERENCES


McCormack, M. D., 1990. Seismic trace editing and first break picking using neural


Appendix A

TRAINING EXAMPLES

A.1 Two layer examples

A.1.1 Low to high resistivity

A.1.2 High to low resistivity
**Table A.1:** Earth model parameters for the examples in the training set. (Two layer model. Low to high resistivity.)

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<th>Boundary</th>
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Note: $R_m = 1.0$ Ω-m, borehole diameter $\phi = 8.0$ in.

**Table A.2:** Earth model parameters for the examples in the training set. (Two layer model. High to low resistivity.)

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<th>$R_t$ (Ω-m.)</th>
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Note: $R_m = 1.0$ Ω-m, borehole diameter $\phi = 8.0$ in.
A.2 Three layer examples

A.2.1 Conductive middle bed

A.2.2 Resistive middle bed
Table A.3: Earth model parameters for the examples in the training set. (Three layer model. Conductive middle layer. Part I)

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Note: $R_m = 1.0$ Ω-m, borehole diameter $\phi = 8.0$ in.
**Table A.4**: Earth model parameters for the examples in the training set. (Three layer model. Conductive middle layer. Part II)

<table>
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*Note: $R_m = 1.0 \, \Omega \cdot m$, borehole diameter $\phi = 8.0 \, \text{in.}$*
Table A.5: Earth model parameters for the examples in the training set. (Three layer model. Resistive middle layer. Part I)

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Note: $R_m = 1.0$ Ω-m, borehole diameter $\phi = 8.0$ in.
Table A.6: Earth model parameters for the examples in the training set. (Three layer model. Resistive middle layer. Part II)

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Note: $R_m = 1.0 \, \Omega\cdot m$, borehole diameter $\phi = 8.0$ in.