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COMPUTATIONAL ANALYSIS OF THE RATLIFF METHOD: CONDENSATION AND GEOMETRIC PROGRAMMING REVEALED

by

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ABSTRACT

Geometric programming, as a segment of mathematical optimization, is a young and growing field of endeavor. The milestones of its development are primarily widely dispersed for numerous reasons. The concepts involved are easily visualized in their simplest form but the transition to higher levels of abstraction or dimensionality tends to cloud continued understanding. An effort to produce an intuitive insight of fundamental concepts and then apply them to a series of problems through an algorithm originally created in 1986 is provided here. Because the computer implementation is the most directly discernible manifestation of the theory in action, a methodical, computationally oriented investigation is an inviting method of approaching the subject.

Richard Ratliff's algorithm for solving a rudimentary category of nonlinear problems appears almost mystical in its apparent success. Looking at its components, defining their role in geometric programming, and gleaning insights and compiling areas for further research is shown to provide synergistic benefit. Enroute to this goal will be a look at the development of geometric programming from its origins to this date. This will be followed by an overview

of important GP concepts and a taxonomy for categorizing GP problems. A restructured, more user-friendly computer program will be utilized to evaluate various permutations of input parameters and their effect on the outcome. Along the way, deeper perspectives on the algorithm and GP in general are derived.

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ACKNOWLEDGEMENTS

It is traditional in academic circles to simply identify in acknowledgements those individuals who assisted in the acquisition of knowledge and the accomplishment of the latest academic achievement. I would like to break from that tradition and speak of people who gave more than time and information; they gave of themselves.

My first and strongest acknowledgment is to Dr. Robert E. D. Woolsey. As an instructor, he has provided me a wealth of information. As a mentor, he has offered an abundance of experience and an appreciation for things that are of most importance. As a professional, he has given me the thirst for precision balanced by the need for responsiveness. And as an academic counselor, he offered me direction in a world where direction is not often conferred but frequently enforced. But it is as a caring human being that shared of himself that Dr. Woolsey tempered all the other elements into something which can be called knowledge. I know that it is now up to me to convert this knowledge into wisdom. But I am already wise enough to recognize the originating force of any of my future successes. Thank-you, Dr. Woolsey. My commitment to you will be evidenced by my sharing of your legacy with the students in my future.

I would like to also thank Professor Bill Astle and Dr. Paul Anderson for the acumen they possessed which aided me so much in the production of this work and the acquisition of my Masters degree. It is unusual to find two finer academics who can relay complicated notions so skillfully. Both endeavored to guide my thoughts in directions which would allow me to discover things on my own. They were, in essence, cultivating on what to them seemed like unprepared soil. For this effort, they will always have my admiration. For their knowledge, they will always have my respect.

It would be a tragic error to fail to mention Mrs. Jean Goldberg, a friend and a listening ear. She eased my mind on numerous occasions by her incisive and uncanny reflections on my troubles and the many similar experiences of her life. Jean, you will always be "Moma Goldberg" in my heart.

Finally, to my wife, Nancy, I apologize for letting my academic life interfere with our family life. You have been so stoically supportive that I wonder if any other man has had such devotion embellished upon him. Thank-you for the time. Thank-you for being there.

Chapter 1

INTRODUCTION

In 1986, Richard Ratliff wrote a thesis entitled "A Generalized Condensation Algorithm for the Solution of Unconstrained Balanced Posynomial Problems Using Geometric Programming." In this document the algorithm developed was structured to set all but one variable to a selected constant, combine or condense terms of similarly signed exponents of the selected variable, derive a simpler problem, calculate estimated answers, and continue to iterate until a selectively accurate set of answers were produced. In this way, the solutions given were thought to converge quadratically to the optimal solution. This was, however, only demonstrated and not proved. The number of iterations as a function of two elements, starting point of the constant and accuracy tolerance, was used as the test of convergence. Although this provided an impressive demonstration of the capability of the algorithm, it did not provide substantial insight into either the strengths of the algorithm or the limitations of the algorithm's implementation in any given computer system.

This author plans to conduct a analysis of the algorithm from a computational perspective. Is there quadratic convergence in the overall solution, in the individual variables, in the Lagrangian multipliers, in the contributional levels of the objective function terms, etc.? If not, what levels of convergence are there? Are there elements of the algorithm which are strongly influenced by the floating point accuracy of the computer and other machine-related characteristics?

In line with answering these and other questions, the author plans to develop or utilize an assortment of computational tools that will further aid in the understanding of the geometric programming realm of mathematical programming and optimization.

Chapter 2
BACKGROUND

Historical Foundation

The area of geometric programming falls under the broad general title of mathematical programming or optimization. Concepts of mathematical programming have been evidenced throughout history but it was not until World War II and the advancement of linear programming that the bulk of mathematical techniques for optimizing of processes and resource management developed. Linear programming (LP) was first used on a grand scale in the optimization of shipping strategies to move material from the United States to Europe in support of the war effort. Linear programming was an adaptable technique for solving a mathematical model allocating scarce or restricted resources. Primarily describing the physical and often the economic aspects of a problem and utilizing linear relationships between material and process components, the techniques were soon rapidly solving substantial problems. But the techniques brought on a greater demand for corresponding technological advancement in computer resources, so the computers got bigger and faster.

Branches of mathematical programming began to appear as the breadth of the concepts crept into numerous other endeavors. A portion focused on integer only solutions while others focused on nonlinear problems. In every case, the problems could be solved in a variety of ways which made use of the tremendous speed and versatility of the computer, but the problems continued to grow, both in size as well as complexity. Strategies to limit the ultimate number of steps required to solve a problem soon became equally if not more important than the knowledge that a particular procedure would accomplish the desired effect in a finite number of steps. In some cases, a finite number of steps constituted years of computer time on the fastest of machines. Obviously, this was not acceptable.

Much nonlinear mathematical programming and optimization development took place in the period following World War II, but it was not until the 1960's that the key insights into solving many nonlinear problems in a realistic amount of time emerged. Many of these insights were in the field of GP, an arena of additive but nonlinear values.

The publication of the book Geometric Programming (Duffin, Peterson, and Zener, 1967) appears to be the watershed point for the field of GP. Combining the work of documents published by the authors over the period 1961 to 1967, the publication provides a broad presentation of what was then known about mathematical optimization utilizing nonlinear characterizations of physical phenomenon. The work was totally within the confines of problems with strictly positive terms and exponents in the objective function. It was Zener, who, in 1962, first coined the name "geometric programming."

Douglass J. Wilde, a professor at the Department of Chemical Engineering at Stanford University published a number of books in the 1960's on the attainment of optima. His initial works looked at techniques of looking for optimum values using many search techniques (Wilde, 1964). In later works, he turned to techniques which accelerated the search process by understanding characteristics of the functions under investigation (Wilde and Beightler, 1967). He was influenced by the work of Zener but continued to focus mainly on linear problems. But his ultimate turn to real world, nonlinear, engineering problems came to a zenith in 1978 when he focused on numerous applications of the techniques of geometric programming. He appears to be one of the first to use monotonicity analysis to determine active/binding constraints. He would then derive bounds and solve a simpler problem, repeating until the desired degree of precision was obtained. He also used orthogonality constraints of dual variables or the exponents of the variables in each term. Looking for balancing in signs, Wilde's concepts brought geometric programming to a new level. But the understanding of how to harness this insight was still missing.

Dr. R. E. D. Woolsey of the Colorado School of Mines was strongly convinced in the late 1960's that geometric programming was much more powerful a tool than many thought. By 1969, Woolsey published a simplification of previously defined algorithmic rules for geometric programming (Woolsey and Swanson, 1969):

WOOLSEY'S 4 RULES FOR GEOMETRIC PROGRAMMING

RULE 1: THE FORM OF THE OPTIMAL SOLUTION OF ANY POSYNOMIAL GP PROBLEM IS:

$$TEC^{*} = \left(\frac{(coeff\ 1\ of\ obj\ function)}{d_1}\right)^{(d_1)} \text{ times } \ldots \left(\frac{(last\ coeff\ of\ obj\ function)}{d_{last}}\right)^{(d_{last})} \text{ times}$$

$$\left\{\left(\frac{(coeff\ 1\ of\ term\ 1\ of\ constraint\ 1)}{d_{last+1}}\right)^{(d_{last+1})} \text{ times } \ldots \left(\frac{(coeff\ of\ last\ term\ of\ constraint\ 1)}{d_{last+M}}\right)^{(d_{last+M})} \text{ times}\right.$$

$$\left.(\Sigma\ of\ d's\ for\ constraint\ 1)^{(\Sigma\ of\ d's\ for\ constraint\ 1)}\right\} \ldots$$

(Where the curly bracket is to be repeated for each constraint and where it is assumed that there are M terms in the constraint.)

RULE 2: THE EXPONENT MATRIX IS CONSTRUCTED AS FOLLOWS:

RULE 2A: The sum of contributions to cost in the objective function is 1:

$d_1 + d_2 + \ldots + d_{last} = 1.$

RULE 2B: For each primal variable the equations in the exponent matrix are:

$$(\text{Power of variable I in term 1}) \cdot d_1 + (\text{Power of variable I in term 2}) \cdot d_2 + \ldots (\text{Power of variable I in last term}) \cdot d_{last+M} = 0$$

RULE 3: AT OPTIMALITY FOR THE OBJECTIVE FUNCTION

$$TEC^* = \left(\frac{(\text{term 1 of obj. func.})}{d_1^*}\right) = \left(\frac{(\text{term 2 of obj. func.})}{d_2^*}\right) = \ldots = \left(\frac{(\text{Last term of obj. func.})}{d_{last}^*}\right)$$

RULE 4: AT OPTIMALITY FOR EACH CONSTRAINT

$d_I = (\text{Ith term of constraint})(\Sigma \text{ of } d_I \text{ for that constraint})$

In the August 1971 issue of Hydrocarbon Processing, Woolsey demonstrated the capabilities of GP to solve realistic problems without using traditional calculus techniques.

This article contained the speculation that the difficult nature of the tableau of coefficients and exponents used by Wilde could be simplified. By using only the sign of the product of coefficients and their individual exponent in each variable of each term of the problem, an easier technique was developed for bounding the problem by the identification of the most restrictive (binding or active) constraints.

In 1987, Frank Grange completed a doctoral dissertation which applied GP to the task of analyzing financial portfolio compilation. Developing a computer routine which reduced the difficulty of even attempting to optimize the nonlinear problem posed by the situation, Grange effectively demonstrated the power of GP in making a nonlinear problem iteratively solvable.

Wall, Greening, and Woolsey published an algorithm based upon a specific application to complex chemical equilibria in Operations Research in 1986. The dominance of the Colorado School of Mines Geometric Programming effort was immediately established.

Richard Ratliff wrote still another algorithm which could solve nonlinear, posynomial, multivariable, unconstrained optimization problems with multiple degrees of difficulty (Ratliff, 1986).

Jim Thome's doctoral dissertation of 1988 involved the use of a transformation and condensation technique first espoused by another graduate student, Doran Greening, and its subsequent computer program. The technique was demonstrated as being able to solve nonlinear, signomial, single variable, unconstrained optimization problems with multiple degrees of difficulty.

C. K. Oatney, a CSM graduate student in 1987, developed a GP-based algorithm centering on reliability in Economics.

While a Ph.D. student in 1988, J. B. Kirk developed a detailed procedure of preprocessing to reduce a given "real world" problem to an easier level where other existing algorithms and schemes could solve for optimality. This characterizes an approach promulgated by his advisor that simpler problems are often at the heart of more difficult ones. The procedure was designed for use by management and other nontechnical individuals who are in need of arriving at some worthwhile solution to a nonlinear optimization problem.

Gysbert Wessels in 1989, then a Ph.D. candidate at the Colorado School of Mines, published a highly informative treatise on the substance of an "advanced sign table." (Wessels, 1989). This advanced sign table combined the sign table notion of Wilde with that of Woolsey. He additionally utilized elementary row and column operations (ERO). The use of ERO allowed for reduction of the "density" of the table and simplified comprehension of the approach required to solve the problem. Here, it appears that the term "sign" now more appropriately inplies the existence of guideposts directing us to the optima. Following this procedure iteratively, insights into which constraints were binding further restricted the problem.

Later in 1989, Jeff Wilkenson, in another doctoral dissertation, wrote on a geometric programming application involving reliability analysis.

In 1990, several more interesting efforts in the expansion of the application of geometric programming Techniques occurred, many at the Colorado School of Mines. David Logan wrote a Master's Thesis concerning the acquisition strategy for the purchasing of the MX missile system (Logan, D., 1990). In this thesis, 20 possible siting location strategies were reviewed, a mathematical model was developed emphasizing the minimization of costs and a second model was developed focusing on the maximization of survivability for the missile systems. A combination of 5 different independent parameters embedded into a nonlinear objective function were analyzed within 25 nonlinear constraints. Optimal values for the 5 parameters were obtained as well as their corresponding objective function values. The outcome of the analysis could serve as an example of how significant an impact of the use of GP techniques could be to multi-billion dollar programs.

Jim Knowles, also a Colorado School of Mines student, wrote on a technique for reducing the feasible region in a GP problem by early identification of the binding constraints within the problem (Knowles, 1990). The Knowles procedure, called the "SNAKE" (**S**imple **N**o-nonsense **A**pproach to all **K**inds of **E**quations) method, lends insight into the effect of the "sign table" along the lines of Wessels' work in 1989.

There are numerous other examples of the application of GP techniques which have not been mentioned here. Suffice it to say, the applicability and utility of GP can be expected to continue to expand. Every individual effort or accomplishment described above was a different way of approaching the solution of nonlinear optimization problems. At the time, some of these approaches were considered radical or at least unusual one way or another. This thesis also falls into this category, but it is the necessity for continued understanding of the "how" and the "how well" of our efforts which form the genesis of this document.

Mathematical Foundation

The notions involved in geometric programming are frequently overwhelming to the average individual.

Buzzwords, euphemisms, and mathematical vernacular abound in any mathematical optimization application. This section will attempt to present, in a straightforward manner, an overview of the most important applicable notions. This is only an overview and it is highly recommended that the interested reader consult the associated references for detailed information on these topics.

Objective Function and Constraints

The objective function is a mathematical expression of the relationship of various structures in a problem to be maximized or minimized. An example should clarify this point. The dimensions of a barge are to be found which minimizes the cost of materials. These dimensions will be L for length, H for height, and W for width. Assume that we are designing a rectangular barge with two ends, two sides, and a bottom. The objective function in this case is the sum of areas of the five surfaces or

$$\text{Min.:}\ 2WH + 2LH + LW.$$

Here we have the desired effect: minimization, the areas of the various surfaces written as the product of the number of these surfaces and their dimensions, and the entirety summed to aggregate their cumulative value.

Notice that the terms of the function have products of variables within them. This is a nonlinear relationship and is characteristic of nonlinear optimization, of which geometric programming is a part. Other nonlinear forms would have the variables raised to various powers both integer and real, or even be the exponents. So one can see that this could get pretty messy.

But let us for now look at the optimal solution to our objective function. Without going into the reasons why, notice that the objective function value (OFV) at optimality is minus infinity. But knowing that our variables in the problem are measures of distance, we can say that each must be non-negative: greater than or equal to zero. This factor, which is quite common in linear programming situations as well, is called a non-negativity constraint. More precisely, there are three non-negativity constraints, as there are three variables of interest. The fact that they limit the range of the OFV makes them a constraint. Non-negativity of the variables is quite often assumed in most problems of interest, so in many cases a problem with just non-negativity constraints is labeled "unconstrained." Such problems will, however, continue to be called constrained problems.

Looking back at the objective function, we see that the trivial answer of zero is the minimum value of the OFV. So it appears that if we want to minimize the total surface area of the sides, ends, and bottom of this barge, we shouldn't build it. Later we will look at an extension of this problem, one that includes a term which is a function of the variables to negative exponents. This will make things more exciting.

Solving Simultaneous Equations

Solving simultaneous equations is, for the most part, a high-school-level endeavor. But the notion of solving these equations with common values for the variables is an integral aspect of understanding some aspects of geometric programming.

It is common knowledge that a system of linear equations with n unknowns can be solved directly if there are n independent (or nonredundant) equations. Additionally, most students of linear algebra know that if there are more unknowns than equations, there is substantial flexibility in the solution, i.e., an infinite number of solutions. From linear programming, add the slack variables or subtract the surplus variables to the equations in order to create the structure of equalities rather than inequalities within the constraint equations. Once again the problem becomes one of solving for the variables of interest. In this case, what eventually happens to the slack/surplus variables gives us insight into the sensitivity of the solution to values in the problem formulation. Now, to make matter worse, we add the nonlinear aspect to the problem, and we see that we do not have a clear approach to solving a system of inequalities of a nonlinear form. As a matter of fact, we do not have a straight-forward way of acquiring the optimum solution to many unconstrained, nonlinear problems. Some methods incorporate a methodical search which could consume extensive amounts of times on the fastest of machines, while others require mathematical insight to direct the search process. But in both cases, a search process of one type or another is required except in so-called "uninteresting" or trivial cases. It is in the latter vein that the initiative of geometric programming efforts is derived.

Geometric Mean Inequality

The fundamental mathematical notion of geometric programming derives its source from a relationship between an arithmetic mean and a geometric mean. This relationship is defined as follows:

The Arithmetic Mean is ≥ The Geometric Mean

or

$$(A+B)/2 \geq A^{\frac{1}{2}}B^{\frac{1}{2}}$$

or more generally

$$\frac{\sum_{i=1}^{n} x_i c_i}{n} \geq \left(\prod_{i=1}^{n} x_i c_i\right)^{\frac{1}{n}}$$

with n the number of terms, x_i a variable, and c_i the constant associated with the variable. It is obvious here that the inequality itself contains all the elements of a nonlinear expression, specifically: constants, exponents, products of variables, and fractional elements. It is this construction which makes it an ideal starting point for the theory. The inequality itself is an extension of Cauchy's Inequality (Bartle and Sherbert, 1982) as it applies to means. It can be shown that the equality relationship only occurs when the variables are in "some sense" equal. Further explanation of this "sense" will be given later. Suffice it now to say that it is this effort to manipulate values of the variables so as to obtain an equality which provides the basic motivation for geometric programming.

Let us look at a rudimentary example of a nonlinear function to be minimized (Duffin, et al., 1967):
We are given a function of t in two terms:

$$g(t) = 4t + \frac{1}{t}.$$

Rewrite the function using the Geometric Mean Inequality:

$$\frac{(4t) + \frac{1}{t}}{2} \geq \left(4t\frac{1}{t}\right)^{\frac{1}{2}}.$$

Multiply through by 2:

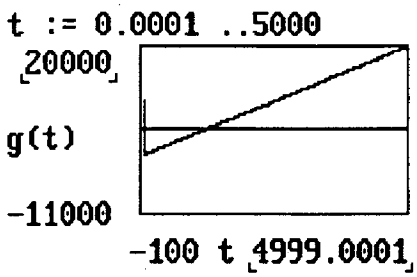
$$g(t) \geq \left(8t\frac{2}{t}\right)^{\frac{1}{2}}.$$

Simplify product within parenthesis:

$$g(t) \geq 16^{\frac{1}{2}}$$
$$g(t) \geq 4$$

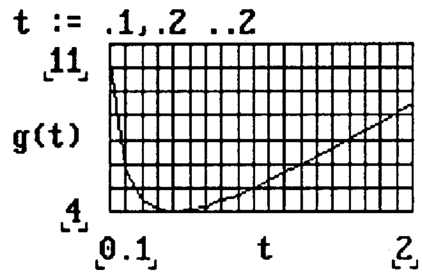
The quantity 4 can now be described as the lower bound for the function with positive t, but note that if we set g(t) equal to 4, the value of t can be derived algebraically as 0.5. 4 can now be referred to as the greatest lower bound or infimum of g(t). So, if we were minimizing this function with respect to the variable t and t was non-negative, the OFV would be optimal at 4 with t being 0.5 at optimality.

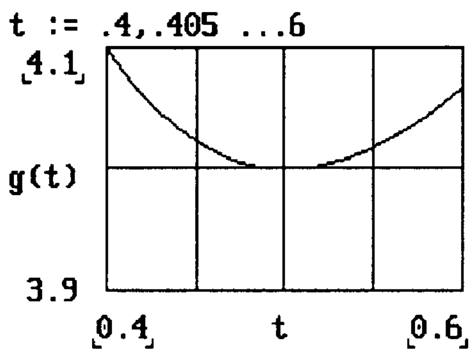
Using the MathCAD software, the function is graphically displayed in this way:



This is a graph of the function over an interval of .0001 to 50000. As you can see, the minimum appears to be down near the t=0 region.

Now the plot to the right is over .1 to 2.0. It looks as if the region is correct.





This plot clearly identifies the apparent minimum of the function at t=.5 as was derived in the text. We also see that it occurs where the function value is 4.

Fig. 1. Graphical View of the Function g(t)

The Geometric Mean Inequality allowed us to see a lower bound to the function. Combining this with our knowledge of the lower bound on the variable, we were able to algebraically derive the minimum which we confirmed intuitively by the graph.

A similar function yields a slightly different result:

$$g(t) = 4t + \frac{1}{t^2}.$$

Here, the impact of the first term is increased with the increased magnitude of the exponent in the second term. Now the contribution of the first term is two-thirds while the second term is reduced to one-third. This is still analogous to what was demonstrated in the original function but now consists of a "weighted" (0.667 and 0.333) rather than symmetric values (0.5 and 0.5) of contribution to the value of the function. The optimal value can be derived as 4.7622 at a variable value of 0.7937.

Convexity

Entire books can be written on the nuances of convexity. It is a subject which can easily overpower the mathematically naive. But taken in a larger perspective, it is a convenient as well as powerful concept.

The physical depiction of convexity in reality is a shape in which the central regions are "greater" in a sense than the edges. A hill would be convex as its center is higher than its periphery. A deflated ball stuck to a wall would also be convex as its center would stick out from the wall farther than the rest. We are all familiar with convex lenses which are thicker in the middle than on the edges. But the mathematical notion of convexity, by necessity, must be more precise. Convexity is defined as a condition of an object in which a line segment connecting any two points within the object must lie totally within the object. This would obviously eliminate any hills which have multiple peaks, or objects which have "folds" of some kind, from being considered convex. Mathematicians considers shapes, sets, or regions of space in any dimension which fulfill this condition as being convex. Most often, the points in this convex set or space are denoted in an ordered set or vector of n components, each component representing a value particular to a dimension. This can be related to our nonlinear problem thusly: The set of all possible points which can be defined by the objective function at a fixed value defines a space. The space can be further restricted by intersecting this space with the points which can fulfill the mathematical relationships in the set of inequality or equality constraints. If this final intersection meets the above definition, we can say it is convex. But so what?

It was noted earlier that a hill with multiple peaks could not be convex because it did not meet the condition of convexity, i.e., a line segment from one peak to the other would not necessarily be wholly contained in the hill. Using the mathematical definition, a nonempty set S in E_n (n-dimensional space) is said to be convex if the line segment joining any two points of the set also belongs to the set. In other words, if $\mathbf{x}_1$ and $\mathbf{x}_2$ are in S, then $\lambda x_1 + (1-\lambda)x_2$ must also belong to S for each $\lambda \in [0,1]$ (Bazaraa and Shetty, 1979). So for the points along the line between the two peaks, at least one point is not a convex combination of points, hence the two peaks do not represent a single convex set. It has been mathematically proven that if a function is not convex then the existence of a global minimum or maximum is not guaranteed (Bazarra and Shetty, 1979). In this case, global refers to existing anywhere. For this reason, it is desirable to have a region of interest which is convex. Otherwise, we would not know if the results of our efforts were globally optimal.

But one additional aspect should be considered. Just as the region around either peak of our two-peak hill can be considered convex, the issue of convexity is one which is relative to the space of interest. In the case of a nonlinear problem the objective function could define a nonconvex space, but if the constraints pare down the region to one that is convex, then global optimality is obtainable. This is a common strategy in solving real-world nonlinear problems.

Duality

Duality in GP is functionally identical to duality in linear programming and promotes the notion that there exists a "mirror image" of any problem. This image is not directly "reflective" as in LP but behaves at optimality in a similar fashion (they have points in common, hence they touch). In Woolsey's four rules of geometric programming, the weights or d's are the dual variables. Hence, the functional depiction of the d's, when maximized or minimized, provide an inverse functional representation of the original problem at optimality. The key element here is that the dual of a GP problem is significantly different than the original by the very nature of nonlinearity. But because these dual conditions exist, we are not required to work within a single set of conditions. We are able to choose the set which provides the easiest road to an optimal solution. If the answer is obvious in the original or "primal" problem, there is no need to go to the dual and vice versa. However, when the road get rough, sometimes the dual/primal switch is a necessity.

Optimality and Kuhn-Tucker Conditions

The definition of optimality in the dictionary is:

> "The point of best or most favorable condition, degree, or amount for a particular situation."

This applies equally to optimality in GP. But the quantification of optimality in GP is often the minimally precise answer that fulfills the requirements. Because of the nonlinear nature of the subject, exquisite precision is often paid for at an exquisite price (time, equipment, manipulation, etc.). Wilde points out that there are many reasons why a truly optimal answer is unnecessary or even unreasonable (Wilde, 1978). Woolsey adds an additional perspective by espousing that a good (feasible but not necessarily optimal) solution immediately is often "better" than an optimal solution paid for with lots of time and money (Woolsey, 1988). Geometric programming drives these points home daily for it is replete with situations where only exhaustive efforts are the only sure means of obtaining optimality.

But Kuhn and Tucker, a pair of mathematicians, did develop a set of six conditions which provide for optimality when met. These can be generalized into three statements given below:

1. A solution must exist, (i.e., an infimum or supremum).
2. Solution must be primal feasible.
3. Solution must be dual feasible.

A more specific outline of the Kuhn-Tucker conditions are given in the following program and subsequent theorem (Phillips, Ravindran, and Solberg, 1976).

Program

$$\text{Minimize} f(x)$$

$$\text{Subject to } h_j(x) = 0 \qquad j = 1, 2, \ldots, m$$

$$g_j(x) \geq 0 \qquad j = m+1, \ldots, p$$

Theorem

If x^* is a solution to the program above, and the functions $f(x)$, $h_j(x)$, and $g_j(x)$ are once differentiable, then there exists a set of vectors μ^* and λ^* such that x^*, μ^*, and λ^* satisfy the following relations:

$$h_j(x) = 0 \qquad j = 1, \ldots, m \tag{1}$$

$$g_j(x) \geq 0 \tag{2}$$

$$\mu_j[g_j(x)] = 0 \tag{3}$$

$$\mu_j \geq 0 \tag{4}$$

$$\frac{\partial f(x)}{\partial x_k} + \sum_{j=1}^{m} \lambda_j \left[\partial h_j(x) \frac{}{\partial x_k} \right] - \sum_{j=m+1}^{p} \mu_j \left[\frac{\partial g_j(x)}{\partial} x_k \right] = 0 \tag{5}$$

$$k = 1, 2, \ldots, N$$

Equations 1 and 2 specify primal feasibility. Equations 3 are complementary slackness conditions, analogous to those in linear programming; equation 4 contains nonnegative dual variables corresponding to the Lagrangian multipliers or weights previously mentioned.

Degrees of Difficulty

One way of categorizing geometric programming problems is referred to as the degree of difficulty (DD). This is defined as the number of terms in the problem minus the number of variables of interest minus 1. Outwardly, this value tells us the relationships between the number of variables and the number of terms within the problem. Just as some of the characteristics of nonlinear programming included the possibility of products, ratios, and exponents of variables, the notion that the relationship of the terms and the variables they contain somehow reveals aspects to the problem is intuitively attractive. The details of the origins of the concept of degrees of difficulty will not be addressed here, but let it be explained without proof that they are analogous to the number of free variables in a set of simultaneous equations with more unknowns than equations. For n equations, m variables can only be determined if the m - n extra variables are fixed.

Correspondingly, in a nonlinear problem, a similar number of variables must be fixed to allow for a solution to be derived.

Posynomials and Signomials

So far, the discussion of the objective function and constraint inequalities has focused on positive coefficients of terms. Posynomials are polynomials with positive coefficients and real exponents. Note that the difference between polynomials and posynomials is that polynomials have integer rather than real exponents.

Signomials are defined as posynomials with at least one negative coefficient.

Negative Variables

Negative variables in posynomial objective functions wreck havoc on most geometric programming problems. If the constraint of non-negativity is lifted, the problem is apt to tend toward negative infinity unless at least one other constraint precludes such an event. For the most part, non-negativity of variables is assumed in GP.

Negative Objective Function Values

OFVs are automatically positive when all terms and variables are positive, but the presence of negative coefficients or variables which can be negative immediately creates the possibility of a negative OFV.

Convergence

Probably the least understood of all the characteristics of geometric and other nonlinear mathematics, the subject of convergence of algorithms is often looked upon as a ancillary effect of these procedures. However, it should rightfully be considered one of the paramount characteristics as it is justifiably the very reason the algorithms and heuristics can be deemed more or less valuable tools for mathematical optimization. The faster the convergence, the quicker and more efficient the technique. It has long been known that geometric programming techniques converge to a solution. But how fast? Ratliff spoke of quadratic convergence in conjunction with his technique. What is quadratic convergence?

A definition for quadratic convergence is given as follows:

If $\exists$ constants $p > 1, c \geq 0, \hat{k} \geq 0 \ni \{x_k\}$ converge to x_* and for $\forall k \geq \hat{k}$

$$|x_{k+1} - x_*| \leq c|x_k - x_*|^p$$

then $\{x_k\}$ is said to converge to x_* with q-order at least p.

If $p = 2$ or 3, convergence is q-quadratic or q-cubic respectively (Dennis and Schnabel, 1983, page 20).

It is convergence which is the consummate issue in all nonlinear iterative techniques, and it is a key element of analysis for this thesis.

Taxonomy for Geometric Programming

Problems in this field come in so many forms that it is often difficult to distinguish one from another. A taxonomy of these problems and a categorical designation may serve to improve on this situation.

The taxonomy will consist of the following:

Number of Variables - An appropriate starting label which identifies the number of variables in the problem.

Constrained/Unconstrained - If there are any constraints other than non-negativity of variables, C will represent constrained, U is unconstrained.

Positive/negative coefficients - P represents all positive coefficients, N represents any negative coefficients.

Degree of Difficulty - A non-negative integer representing the number of terms minus the number of variables of interest minus 1.

Balanced/unbalanced objective function - B will signify problems where all variables appearing in the objective function have at least one positive and negative exponent. Any failure of a variable to meet the above criteria will be designated by N.

The problems will be categorized using the letter codes in the order they appear above. Each code will consist of five elements.

Examples:

$$70.00H + 2333.33L^{-1} + 3333.33H^{-1}$$

This is a **2UP0N** categorization: **2**-variable **U**nconstrained **P**ositive **0**-dd u**N**balanced problem.

$$40H^{-1}L^{-1}W^{-1} + 10LW + 20HL + 40HW + 10L$$

This is a **3UP1B** categorization: **3**-variable **U**nconstrained **P**ositive **1**-dd **B**alanced problem.

In general the code for the defined taxonomy is: (number of variables / **C**onstrained, **U**nconstrained / **P**osynomial or **S**ignomial / degree of difficulty / **B**alanced or u**N**balanced in the objective function).

Computer Implementation

Ratliff's implementation was developed in 1986. It was written in Microsoft FORTRAN as well as a VAX VMS implementation in FORTRAN 77 with the assistance of Mike Tapia (an experienced programmer). It contains 692 lines of code and comments with four subroutines.

In 1989, the code was translated into Apple BASIC by Robert E. D. Woolsey. It was his code which was originally modified to run in DOS under the QuickBASIC programming language. The program was further modified to allow for easier input of problems and parameter values as well as recording of problems and corresponding program run outputs. The design of the outputs was tailored to meet the needs of display and utility requirements but the outputs were specifically arranged to be imported directly into Lotus 1-2-3 spreadsheets.

The program was later converted to C. The C language was deemed desirable because of its speed, portability from one system to another, and the flexibility to control elemental structures within the computer system. Some exquisite efficiency and exploratory work was thought to be possible in this language, and it was hoped that the language itself would help to provide further insights into the method and geometric programming in general. Coincidentally, the first translation of the FORTRAN code to the C language by this author occurred exactly four years to the day after the publication of the original, February 19, 1990.

Chapter 3

METHODOLOGY

Analysis of Algorithm

The initial activity will be a dissection of the algorithm. It is hoped that this will identify programming elements and their role in the accomplishment of the task of finding near optimal solutions. In this way, programmatic aspects of the algorithm may provide insight or at least provoke inquisitive thoughts on the real nature of the mathematical procedure. Serendipity is not always a player in studies such as these but it is often unwise to refuse to consider the possibility of finding something not considered before.

Basic Programming

The programming done by Ratliff was exemplary in accomplishing the application of his ideas, but it provided somewhat of a "black box" appearance to the user in that the intimate details of what, how, and how well things were being done within the program were hidden and only the results were depicted.

Rewriting the code in BASIC was the first approach as it was desired to work in a language easily accessible to those interested in conducting further analysis on commonly available personal computers. Additionally, the desire to conduct extensive runs of numerous problem sets and even more numerous parametric modifications of these runs prompted provision of the ability to add additional capabilities to the current software.

In an effort to look at optimizing the code and possibly find improvements in the technique via programming means and machine flexibility, the FORTRAN program was rewritten in the C programming language. The language allows for extensive control of values used in the algorithm. This particular line of effort did not meet with much success due to several other considerations, but can be looked upon in the future to provide even further insights.

Design of Experiment

As the number of desired factors to be considered is eight, with many factors coming in three levels, it was thought that an incomplete factorial experimental design could be accomplished. The first two factors are for credibility only. The results of these will not be used in the basic experiment as there are uncontrolled elements participating.

The first factor will be evaluated using a comparison of a FORTRAN run on the PC and that of the published problems. This will provide some confidence in the comparability of the output for each problem based upon the use of a mainframe or microcomputer.

The second factor will look to compare the Microsoft QuickBASIC runs with the FORTRAN runs in order to see if any inconsistencies developed during the translation and subsequent modification of the FORTRAN code to QuickBASIC. Of secondary importance will be the comparison of the C programming output to the FORTRAN output to observe any inconsistencies there.

For a common measurement of performance, the number of iterations conducted to achieve the desired precision will be recorded. As a secondary measure, the difference between the test answers and the results with the greatest precision will be examined.

The test vehicle for this experiment will be a PC-compatible, Advanced Logic Research 25 Megahertz 386 Computer with 64k of 25 Nanosecond Cache RAM, 2 Megabytes of 65 Nanosecond RAM, and a 150 Megabyte ESDI hard disk. The computer will be running Microsoft FORTRAN, Microsoft C Version 5.1, and Microsoft QuickBASIC 4.5.

The primary factors for this experiment are outlined below.

Convergence Condition

A tolerance condition which, when met, will terminate the program run. This can be looked upon as overall function precision. Five levels will be used within this factor: 1.0×10^{-2}, 1.0×10^{-4}, 1.0×10^{-7}, 1.0×10^{-8}, and 1.0×10^{-11}. Note: the machine precision of the test computer is approximately 1.401298E-45, but the software is precise to only 17 digits in double precision mode.

Number of Terms

The four problems utilized contain three different numbers of terms: 4, 5, and 7.

Number of Variables

3 levels - 1, 2, 3

Starting Vector

The starting values for each variable will include the original values used in the Ratliff runs as well as the three levels: 0.002, 1.0, and 999999. This wide spectrum of values is intentionally weighted toward the low side because of the anticipated effect of using values around 1.0.

Collective Variation

Start all variables at 0.002, 1, or 999999.

Individual Variation

Start with all columns set to one except one and conduct runs with that column set to 0.002, 1, or 999999 iteratively.

Iteration Analysis

Output from the above will be used to input into a Minitab Statistical Software package and multiple linear regression will be used to attempt to find strong relationships between the factors and the response variable.

Chapter 4

ANALYSIS OF ALGORITHM

It should be pointed at the onset of this analysis that the FORTRAN code created by Ratliff and Tapia is a very good example of straight-forward programming. It was consistently demonstrated that superfluous code was restricted except in instances where clarity of purpose was intended. The internal documentation was very precise and comprehensive.

A pseudocode listing of the program (FORTRAN version) similar to one in the original Ratliff document is presented here. This portrays the original intent of the program.

Put equation in unconstrained, balanced posynomial form.	⇐⇐⇐⇐⇐⇐	⇑

⇓ ⇑

Choose initial values for each variable in the problem. Label these XOLD(1), XOLD(2), ... , XOLD(NVAR)		⇑

⇓ ⇑

Treat all variables except one as constants using the values of XOLD(I); this results in a simplified problem in only one variable with revised coefficients.

⇓

Solve the simplified problem using the single variable algorithm. Obtain a solution for the variable which is presently active X(J). Using the new value of X(J), calculate a value for the simplified objective function. Label this FNCVAL.

⇓

Compare the values of FNCVAL on successive iterations.

⇓

Is difference in FNCVAL(K) and FNCVAL(K+1) negligible?

YES => STOP. Use current values as solution.

⇓

Return to step 3 treating the presently active variable as a constant (X(J)). The next variable in the equation (X(J+1)) now becomes active.

⇒⇒⇒⇒⇒⇒ ⇑

Fig. 2. Pseudocode Listing of Program (FORTRAN version)

Pseudocode can be written to different levels of detail which varies from individual to individual. A more detailed pseudocode outline is given here.

```
Start with the first variable and iterate through to the
last.
If the exponent is negative, then increment NNEG.
    Make pwneg(NNEG) equal to the exponent.
    Make ngcoef(NNEG) equal to the coefficent of var.
        Take all (except k=var) xold^exp and mult by ngcoef.
Condense positive terms (analogous to condensing negative
terms shown next).
Condense neg terms.
    If nneg >1 then
        sum the negcoef * xold(var)^pwneg - val of function
        For each neg exp:
            Calc wgt by dividing ngcoef by (sum above *
xold^pwneg)
            Add each weight to total weight.
            Add wgtneg * pwneg to a temporary location
ptemp.
            Add (ngcoef/wgtneg)^wgtneg to ctemp.
        Wgtneg(1)= 1 - total weight.
        Condcoef(2) = ctemp* (ngcoef(1)/
wgtneg(1))^wgtneg(1)
        Condpow(2) = ptemp + pwneg(1)*wgtneg(1)
    If nneg =1 then
        Cndpow(2)=pwneg(1)
        cdcof(2)=ngcoeg(1)
        wgtneg(1)=1
Calc new deltas
    dlta(2)=1/(1-cndpow(2)/cndpow(1))
    dlta(1)=1-dlta(2)
Eval funct -
cdcof(1)/dlta(1)^dlta(1)*cdcof(2)/dlta(2)^dlta(2)
New var value is funvl*dltal/cdcoefl^(1/cndpowl)
Check for var conv by if newvar-oldvar / oldvar <= tol
            then set temp as product of all xold^powers
            value of orig funct is sum of all coef*temp
            if not last var then goto next variable at top
of loop
            Otherwise chk for prob conv
            old fval - nefval / newfval
```

```
                if &lt;= tol print results
                otherwise start over
Set xold to xnew and increment iteration
Goto condense pos terms procedure
```

It was found that the plan for the algorithm and the execution therein were well orchestrated. The program did what it was supposed to do. If there is a criticism of the program it would reside in the meticulous and recurring demands on the operator. Specifically speaking, the input mechanism provided only for interactive input of all values prior to each run. Compounding the pressure to repeatedly input the same parameters in order to perturb them was the strict floating point versus integer requirement. This generated inordinate retyping or corrupted output and limits the utility of the code as it is currently written. The QuickBASIC version utilized for a predominate portion of this document made several important improvements in this area but was first an effort to accomplish the task. It should not, for any reason, be considered excellent code on the order of the FORTRAN algorithm.

One aspect of the code should be brought out strongly: the use of the variable and term iteration as a measure of performance. As is usually the case in programs such as these, there is a main loop which comprises the bulk of the programming effort, calling subroutines to support it. In this case, the main loop calls subroutines which aggregate positive and negative exponent portions of the objective function in what is referred to as condensation. This condensation results in an artificial or pseudo problem that is, by its very nature, a 2-term 1-variable balanced 0-dd posynomial. Other subroutines are used to straightforwardly solve for the optimum solution to this pseudo problem.

The iterative aspect of the code lies in the 1st check to see if the variable has converged. This is done by checking if the difference in the previous and current value of the variable, divided by the previous value, is within the tolerance established initially as an input. If this is not true then the current value is retained as the previous value and more condensing is done. If within tolerance, there is a check to see if the overall function value has converged in similar fashion: a positive difference in the previous and current values, only this time divided by the current value of the function.

The tolerance check is not, therefore, a check on the absolute variation of the values but rather a check on the ratio of the variation to one of the values. A closer look at the general output of the algorithm, reveals a situation where values of any specific variable tend to jump around the ultimate value (call it the settle value), while the value of the objective function is monotonically convergent to the settle value. This is understandable, as the objective function value is by default zero. The nature of these two tests of convergence therefore lends itself to a division by the respective previous or current values: the current value of the objective function is not likely to be zero (although it could be very close), nor is the previous value of the variable. It is important to recognize that zero is not allowed as a starting point of any variable but is the starting point for the value of the objective function by default.

One additional phenomenon of interest within the algorithm is the pseudo problem itself. As was mentioned previously, it is a two-term, one-variable problem. One term has a negative exponent and the other has a positive exponent. The values of the coefficients and the exponents are allowed to change during the program run and they, too, seem to converge to some values. It was not clear if the settle values of these items have some direct significance, but further analysis seems to be in order.

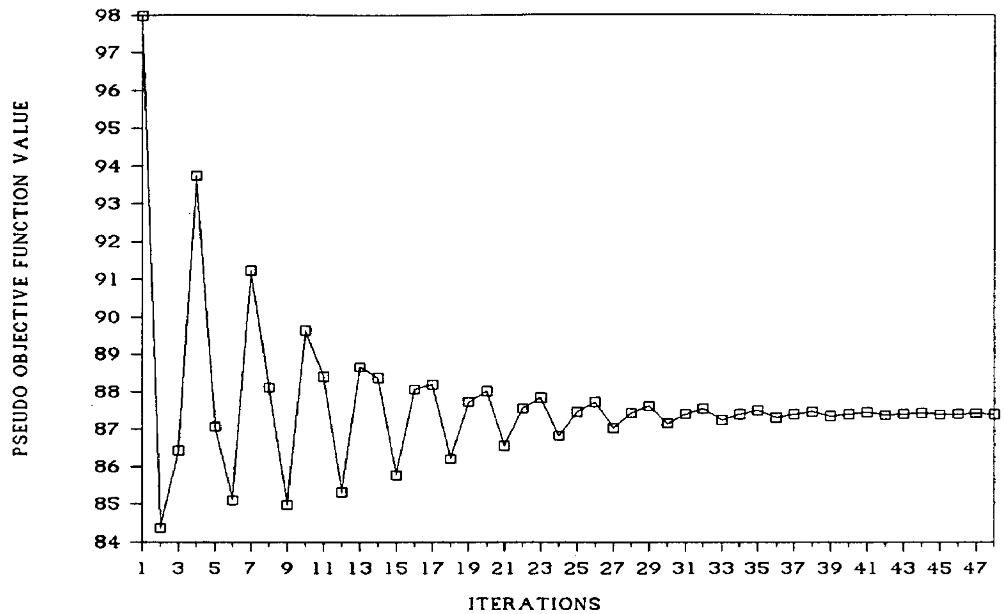


Fig. 3. Convergence of Pseudo Coefficients

Here you can see that the values do tend to converge quickly. In problems where only one variable has a negative exponent, the coefficient of that variable became the coefficient of the negative exponent term of the pseudo problem. A look to see if the ratios of the values could be associated with their analogous positive and negative elements in the original function did not provide anything substantial.

Chapter 5
ANALYSIS OF RESULTS

General

The four problems used for the computational analysis were ones used by Ratliff in his original work. Because he had already run his algorithm on this data and published the results, it was thought that continuity in their use would facilitate further analysis. The four problems include: the Gravelbox, Cofferdam, Shrink Stope, and the Spaceshuttle Problem.

The four problems are broken down into four subdivisions of inspection:

1. Machine comparison will be a comparison of the mainframe-generated published results with that of his Microsoft FORTRAN algorithm running on an IBM AT compatible microcomputer. This will provide confidence that the mainframe solution is not significantly different than the PC solution.
2. The second subdivision will be that of comparing the Microsoft FORTRAN solution to the highly modified QuickBASIC programming output. The large number of changes in the software after first making the initial

translation from FORTRAN to BASIC is thought to have a possibly pronounced effect on the output. Here we will see if this effect appears.

3. This section will look at the variation of the convergence criteria or tolerance condition in the output. Values of 10^{-2}, 10^{-4}, 10^{-7}, 10^{-8}, and 10^{-11} not used in the documented runs will be used to provide a spectrum of possible results.

4. The last section will look at the changes that result from adjusting the starting point values (starting vector), first uniformly through all variables, and then by permuting one element at a time, leaving others set to 1.0. The values used will be 0.002, 1.0, and 999999. All other parameters will be the same as those published by Ratliff.

Problem 1 - Gravelbox Problem

This problem was originally presented by Wilde (1978) as a variation of one proposed by Duffin, Peterson, and Zener (1967). Per the title, it involved an open topped box for transporting a pile of material such as gravel. The dimensions of (H)eight, (L)ength, and (W)idth were to be optimized in the design with regard to the transportation costs of material within the box and given costs of each side. This problem falls into a **3UP1B** categorization and reads as follows:

Minimize: $\text{COST} = 40H^{-1}L^{-1}W^{-1} + 10LW + 20HL + 40HW + 10L$

= construction cost of box of height H, length L, and width W

The first division of the analysis, machine comparison, displays the results of the published results of Ratliff and that of the PC FORTRAN runs conducted on the test computer. The output is shown to the level of precision presented by Ratliff or generated by the PC runs. In each case, the optimal values of all variables involved are displayed. This is followed by the optimal cost or value as generated by the output using the original input parameters. The weights or contributions of each term are then displayed. Finally, the starting values of each variable and the convergence condition (tolerance) is shown.

Table 1 - Gravelbox Machine Comparison

	Ratliff	PC FORTRAN
OPTIMAL H	0.5952	0.59505022
OPTIMAL L	1.2942	1.29346134
OPTIMAL W	1.1884	1.18939703
OPTIMAL COST	115.72	115.71696088
WGT 1	0.3776	0.3776
WGT 2	0.1329	0.1329
WGT 3	0.1331	0.1330
WGT 4	0.2445	0.2446
WGT 5	0.1118	0.1118
INIT VAL OF H	1.0	1.0
INIT VAL OF L	1.0	1.0
INIT VAL OF W	1.0	1.0
CONV COND	1E-7	1E-7
ITER	43	48

The Gravelbox Problem values unfortunately appeared to differ somewhat between the mainframe run and the PC

FORTRAN run. The resulting values of H, L, and W all differed in the third or fourth significant digit. Although the OFV differed in the fifth digit, it was assumed to be a function of round-off in the mainframe output. Two of the five weights also differed in the fourth digit and the number of iterations conducted were 43 and 48.

A summary of the other runs for this problem is given below in a format similar (less weights) to that used in the machine comparison display above:

Table 2 - Gravel Box Result Summary

INIT VAL\RUN	1	2	3	4	5
H	1.0	1.0	1.0	1.0	1.0
L	1.0	1.0	1.0	1.0	1.0
W	1.0	1.0	1.0	1.0	1.0
COV COND	1E-7	.01	1E-4	1E-8	1E-11
ITER	48	6	27	54	72

INIT VAL\RUN	6	7	8	9	10
H	.002	999999	.002	999999	1.0
L	.002	999999	1.0	1.0	.002
W	.002	999999	1.0	1.0	1.0
COV COND	1E-7	1E-7	1E-7	1E-7	1E-7
ITER	120	150	48	48	81

INIT VAL\RUN	11	12	13		
H	1.0	1.0	1.0		
L	999999	1.0	1.0		
W	1.0	.002	999999		
COV COND	1E-7	1E-7	1E-7		
ITER	66	66	147		

In terms of the number of iterations required given a convergence condition, runs 1 through 5 show an apparent correlation with a consistent increase in the number of required iterations as the convergence condition becomes smaller. Recognizing the nonlinearity of this relationship, an adjustment was in order. The following plot shows the apparent linearity of the iterations to the **logarithm** of the convergence condition.

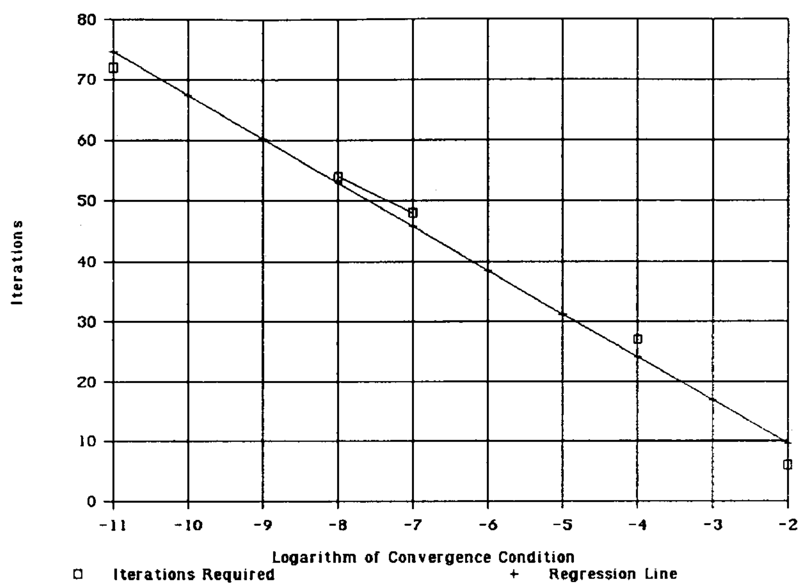


Fig. 4. Iterations vs Logarithm of Convergence Condition

A linear regression run on the above provided a good (R-square > 98% fit) linear relationship with a regression equation of ITER = -4.80487 -7.21951 * Log(CONVERGENCE CONDITION). At this point, it is surmised that with all other things being equal, the number of iterations required is approximately a linear function of the log of the convergence condition.

Runs 1, 6, and 7 have uniform starting values (all starting variables are the same). Here again we see consistency in the relative magnitudes of the uniform starting points and the number of iterations. As the starting point increases, so do the number of iterations.

But does the consistency waver with starting values around the final value of each variable? How about the geometric or arithmetic mean of the final values? So far, we can't say.

Runs 9, 11, and 13 have starting values of one in two variables and 999999 in the third. Here the number of iterations increase by 38% and then by 123% over the previous run when the L and W variables respectively were changed to 999999. It is not known if this is a function of the magnitudes of the coefficients of the terms with each of the respective variables, the exponents of the respective variables, or the order in which the variables are evaluated.

Problem 2 - Cofferdam Problem

This problem, proposed by Wilde (1978) deals with a structure commonly used when constructing in rivers and lakes. The cofferdam under design here is a multifaceted structure which surrounds construction in water. It is taller than the water when the water is at its expected peak and the overall internal dimensions are a function of the dimension of the construction within it. The design is to minimize the cost of the dam and the problem reads:

$$\text{Minimize: COST} = 25100X + 341X^2 + 1.34X^3 + 50000X^{-1}$$

$$= \text{construction cost} + \text{expected cost of flooding}$$

The machine comparison appeared to be an close match, differing only in the least significant digit of the mainframe run, presumably due to round-off.

Table 3 - Cofferdam Machine Comparison

	Ratliff	PC FORTRAN
OPTIMAL X	1.3854	1.38535185
OPTIMAL COST	71522.26	71522.25541960
WGT 1	0.4862	0.4862
WGT 2	0.0092	0.0092
WGT 3	0.0000	0.0000
WGT 4	0.5046	0.5046
INIT VAL OF X	10000	10000
CONV COND	1E-8	1E-8
ITER	6	6

The output to the general runs are depicted below.

Table 4 - Cofferdam Result Summary

INIT VAL\RUN	1	2	3	4	5
X	10000.0	10000.0	10000.0	10000.0	10000.0
COV COND	1E-8	.01	1E-4	1E-7	1E-11
ITER	6	3	4	5	7

INIT VAL\RUN	6	7			
X	.002	999999			
COV COND	1E-8	1E-8			
ITER	5	6			

In terms of the number of iterations required given a convergence condition, runs 1 through 5 follow the same patterns as the gravelbox problem. The apparent consistent increase in the number of required iterations as the convergence condition becomes smaller is maintained.

A linear regression run on the logarithm of the convergence condition again provided a good (R-square > 98% fit) linear relationship with a regression equation of ITER = 2.138211 -0.44715 * Log(CONVERGENCE CONDITION).

As there is only one variable, the uniform and individual variations of start points are common and require only three runs. The number of iterations are 5, 6, and 6 with increasing start points, and nothing conclusive can be derived from such a small variation.

Problem 3 - Space Shuttle Problem

This problem was first proposed by Nicholson in an issue of Optimization in Industry. It is concerned with a single stage rocket powered launch vehicle. Although in a form with 13 dd and 14 constraints, Ratliff and Woolsey showed that it could be approximated by the following 5dd problem in 1 variable. The problem reads:

$$\text{Minimize: } COST = 11.861X^{0.479} + 441.119X^{-0.146}$$
$$+ 3.218X^{0.648} + 1467706X^{0.568}$$
$$+ 1040.0X + 0.078X^{0.736}$$
$$+ 23.688^{-0.229}$$

Table 5 - Space Shuttle Result Summary

	Ratliff	PC FORTRAN
OPTIMAL X	0.00000237	0.00000237
OPTIMAL COST	4319.55	4319.54457299
WGT 1	0.0000	0.0000
WGT 2	0.6767	0.6767
WGT 3	0.0000	0.0000
WGT 4	0.2169	0.2169
WGT 5	0.0000	0.0000
WGT 6	0.0000	0.0000
WGT 7	0.1065	0.1065
INIT VAL OF X	10000	10000
CONV COND	1E-8	1E-8
ITER	6	6

The machine comparison appeared to be a close match, differing only in the least significant digit of the optimal cost on the mainframe run, presumably due to round-off.

One unique feature of this problem was the fact that since the algorithm attempts to find a variable value within tolerance before checking for convergence of the objective function and because this problem has only one variable, in some cases all intermediate OFVs were nearly identical until an appropriate solution was reached. By that time, the OFV was already within tolerance. This may represent a small problem with the algorithm or an insightful slant on the need for two tolerance conditions: one for the variables and one for the OFV. Because the aspect of variable convergence and OFV convergence effectively use different procedures, a certain relationship between these tolerance conditions may accelerate convergence even more. This will be left for further analysis in a subsequent thesis or paper.

Table 6 - Space Shuttle Result Summary

INIT VAL\RUN	1	2	3	4	5
X	999999	999999	999999	999999	999999
COV COND	1E-11	0.01	1E-4	1E-7	1E-8
ITER	7	3	4	5	5

INIT VAL\RUN	6	7			
X	0.002	1.0			
COV COND	1E-11	1E-11			
ITER	6	6			

In terms of the number of iterations required given a convergence condition, runs 1 through 5 follow the same patterns as the gravelbox problem. The apparent correlation of a consistent increase in the number of required iterations as the convergence condition becomes smaller is maintained.

The now common linear regression run on the logarithm of the convergence condition again provided a good (R-square > 98%) linear relationship with a regression equation of ITER = 2.138211 -0.44715 * Log(CONVERGENCE CONDITION).

Nothing conclusive can be derived from the small variation in the number of iterations with respect to the start points used.

Problem 4 - Shrink Stope Problem

A shrink stope is a process of mining involving raise stations and blasting. Danny L. Taylor first proposed the problem by looking at the shrink stope as an Economic Order Quantity-type problem. The variables H and L refer to heights and lengths. The problem falls into the **2UPON** categorization and reads:

Minimize: $$\text{COST} = 70.0040HL + 2333.33L^{-1} + 3333.33H^{-1} + 83333.33H^{-1}L^{-1}$$

= cost of remaining broken ore

+ vertical development cost

+ horizontal development cost

+ cost of raising stations

Tbl 7 - Shrink Stope Machine Comparison

	Ratliff	PC FORTRAN
OPTIMAL H	7.357	7.35741510
OPTIMAL L	5.150	5.15003361
OPTIMAL COST	5758	5757.78812925
WGT 1	0.4607	0.4607
WGT 2	0.0787	0.0787
WGT 3	0.0787	0.0787
WGT 4	0.3820	0.3820
INIT VAL OF H	10000.0	10000.0
INIT VAL OF L	0.002	0.002
CONV COND	1E-11	1E-11
ITER	133	138

The machine comparison appeared to be a close match, differing only in the least significant digit of the optimal cost in the mainframe run, presumably due to round-off and the number of iterations. With respect to the iterations, this was analogous to deviation in the gravelbox problem, and it is suspected that a typographical error in converting a 3 to an 8 occurred in the mainframe transcriptions.

Tbl 8 - Shrink Stope Result Summary

INIT VAL\RUN	1	2	3	4	5
H	10000.0	10000.0	10000.0	10000.0	10000.0
L	0.002	0.002	0.002	0.002	0.002
COV COND	1E-11	.01	1E-4	1E-7	1E-8
ITER	138	22	48	86	100

INIT VAL\RUN	6	7	8	9	10
H	0.002	999999	0.002	999999	1.0
L	0.002	999999	1.0	1.0	0.002
COV COND	1E-11	1E-11	1E-11	1E-11	1E-11
ITER	138	138	128	128	138

INIT VAL\RUN	11				
H	1.0				
L	999999				
COV COND	1E-11				
ITER	138				

In terms of the number of iterations required given a convergence condition, runs 1 through 5 follow the same patterns as the gravelbox problem. The apparent correlation of a consistent increase in the number of required iterations as the convergence condition becomes smaller is maintained.

A linear regression run on the logarithm of the convergence condition again provided a near perfect (R-square > 99.99%) linear relationship with a regression equation of ITER = -3.72357 -12.8943 * Log(CONVERGENCE CONDITION).

This particular example did present an anomalous result in that for all starting point combinations of 0.002, 1.0, and 999999, the number of iterations conducted was either 128 or 138 when a convergence condition of 1E-11 was used. It was not clear why. Later runs (not depicted above) had 78 and 86 iterations under a convergence condition of 10E-4 and 90 and 100 iterations under 10E-8.

Convergence Within the Procedure

To check for convergence within the procedure, the various state variables were analyzed at each successive iteration. As was mentioned in the analysis of the algorithm, the procedure utilizes a number of recursive actions within the iterations themselves. For the sake of clarity, we will use the final values of the state variables at each iteration to define the progress accomplished to that point within the algorithm. Despite this semantic graininess, the analysis should bear considerable information.

The gravelbox problem will be used, as the number of iterations are generally small enough to be manageably analyzed while large enough to provide insight.

Overall Function Value

A plot of the function value versus iterations is displayed as Figure 5.

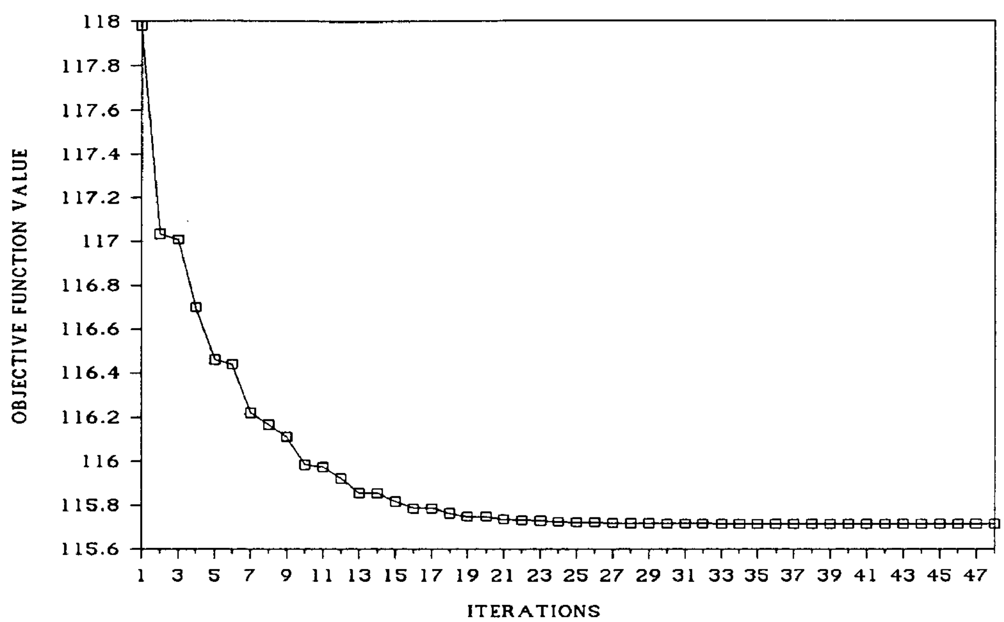


Fig. 5. Function Value vs. Iteration

The plot looks rather erratic, but can be explained by the fact that the problem has three variables. These three variables are sequentially manipulated until convergence, then the value is passed to the next iteration. The initial and every third value plotted can be interpreted as the output of the first variable convergence. The other corresponding values are equivalently distributed.

It appears that the algorithm's overall function value converges to the ultimate value. Checking these values with the condition of quadratic convergence defined earlier shows that this convergence is at a quadratic rate when function values are taken from sequential evaluations of the same variable.

Pseudo Function Value

The plot of the pseudo function value per iteration is shown as Figure 6.

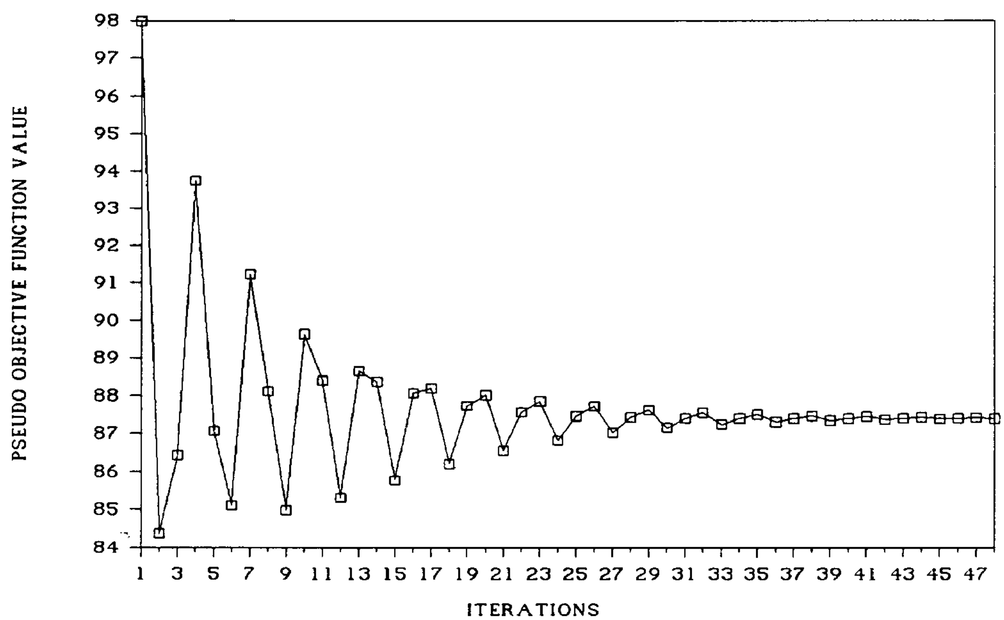


Fig. 6. Pseudo OFV vs. Iteration

It appears that one variable converges monotonically, while the others fluctuate but ultimately converge to the terminal value. Checking these values with the condition of quadratic convergence defined earlier shows that they both individually and collectively converge at a quadratic rate.

As would be suspected, each subsequent value is strongly influenced by the prior one. Hence, all calculations are dominated by the influence of the initial value of the first variable. Additionally, it would seem reasonable that the start point values for each variable would impact the speed of convergence within the variable in the iteration loop.

The shape of this plot is strongly reminiscent of a dampened harmonic wave and this may be insightful. It would be very plausible that the pseudo function value should follow both the greater influence of the general convergence of the objective function value as well as the pull of the individual variables as they move to their settle values. If the parameters of this dampening could be derived, some initial pseudofunction values themselves would be enough to evaluate the values of the objective function and variables at optimality. This predictive use of the pseudofunction could speed the optimization process even further.

Pseudoconstants

The plots of the two pseudo constants provide an entertaining display in Figures 7 and 8.

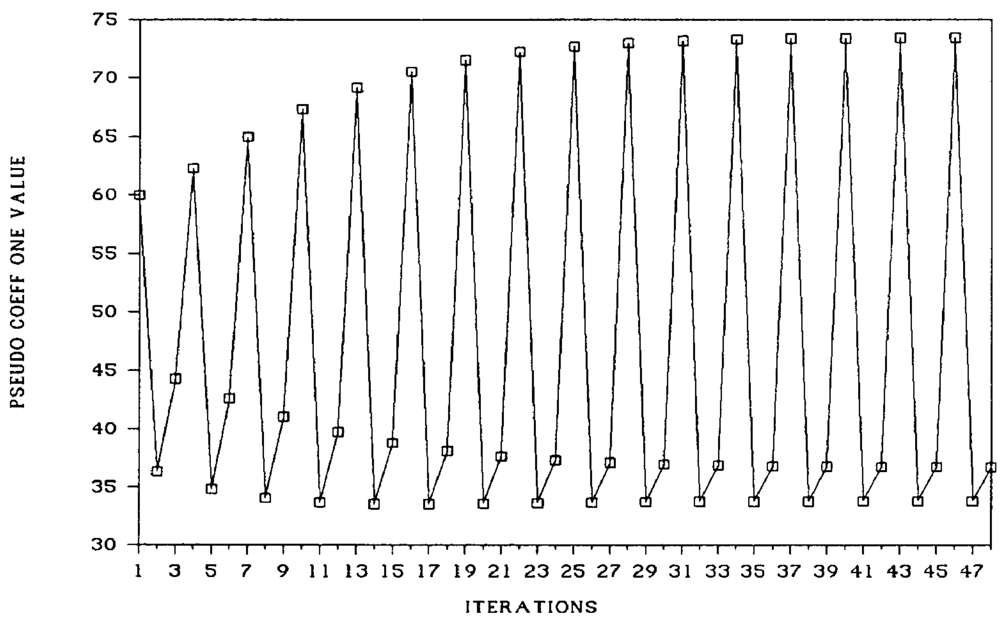


Fig. 7. Pseudoconstant One vs. Iteration

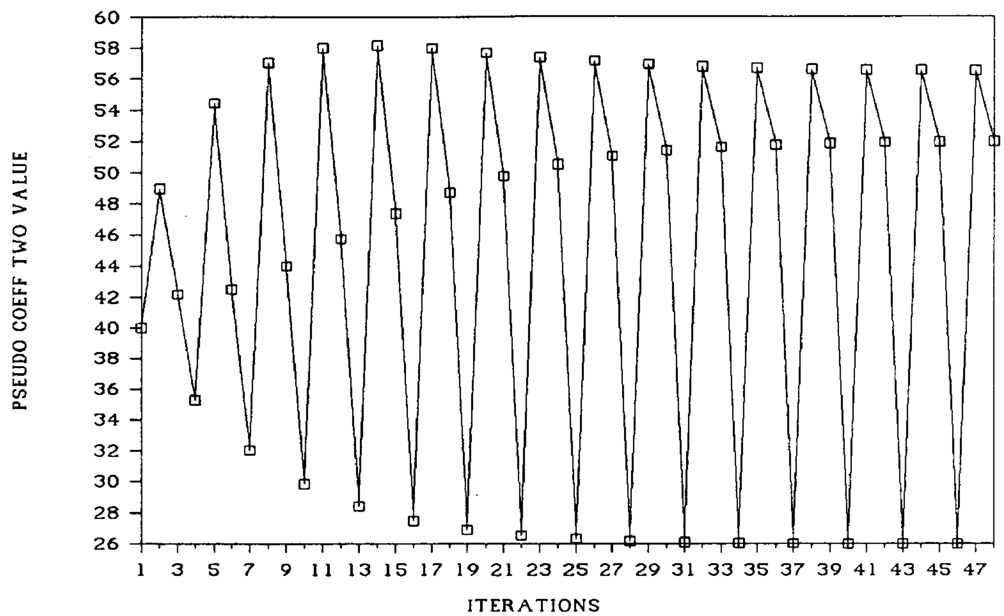


Fig. 8. Pseudoconstant Two vs. Iteration

Because they represent a linear and proportional element in the pseudo function, it is not surprising to see the "reversed" action at every inflection point. It is, however, interesting to note the variation of convergence of the values corresponding to each variable. Collectively, these coefficients do not converge but individually they do converge at a nonlinear rate. It is not apparent what the significance of the terminal values is.

Convergence in the Variables

Figure 9 shows the variation of the variables at each iteration.

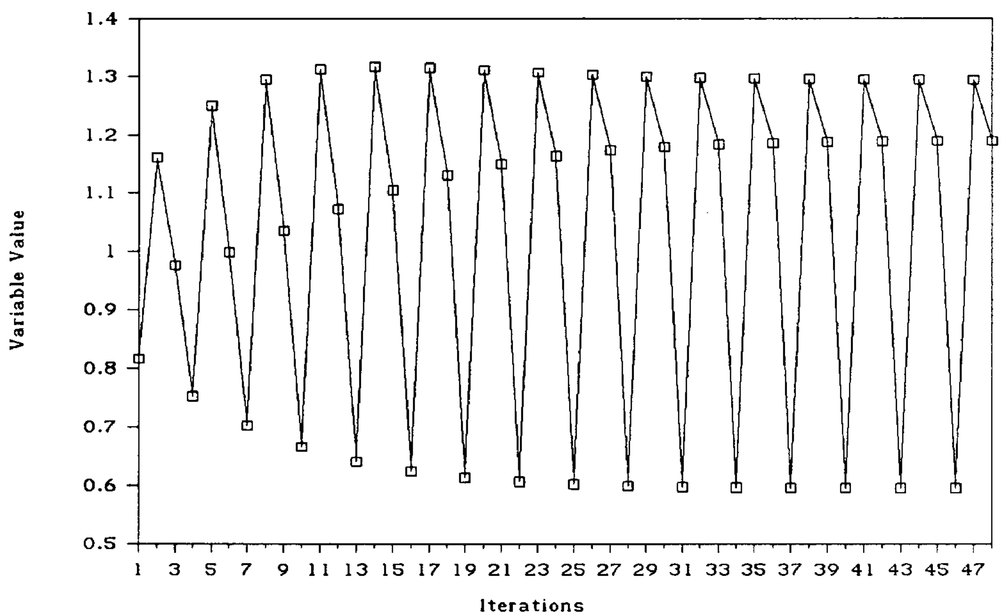


Fig. 9. Variable Values vs. Iteration

The plot shows remarkable similarity to the plot of the second pseudocoefficient as well as the inversion of the plot of the first pseudocoefficient. This may be due to the strong algorithmic association of the variable values to the pseudo objective function.

Pseudoweights

The contribution of each term in the pseudo objective function is ideally one half. The runs conducted on the Gravelbox problem showed the intermediate pseudoweights to converge quickly to 0.5, but other problems exhibited different weights as was anticipated. In each case, the values settled quickly. No check for the degree of convergence was conducted.

Term Weights - Lagrangian Multipliers

Because the algorithm looks for pseudocoefficients which make the pseudo objective function balance at an estimated current variable value, the weights or contributions of the terms in the original function change at each iteration. To see if these values have a characteristic convergence pattern, they are plotted here against iterations. Due to the number of variables, there are three separate plots displayed, Figure 10 being for the first variable of the gravelbox problem H.

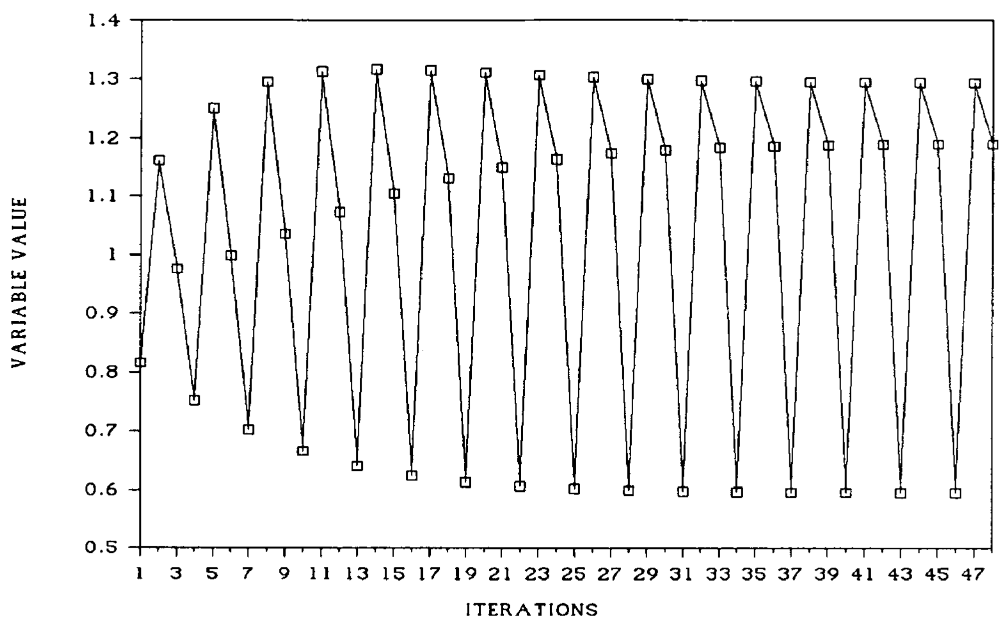


Fig. 10. 1st Variable Term Weights vs. Iteration

There does appear to be a pattern of convergence. Some terms appear to initially overshoot their final values but align quickly soon after.

The second and third variable, L and W, plots are given in Figures 11 and 12.

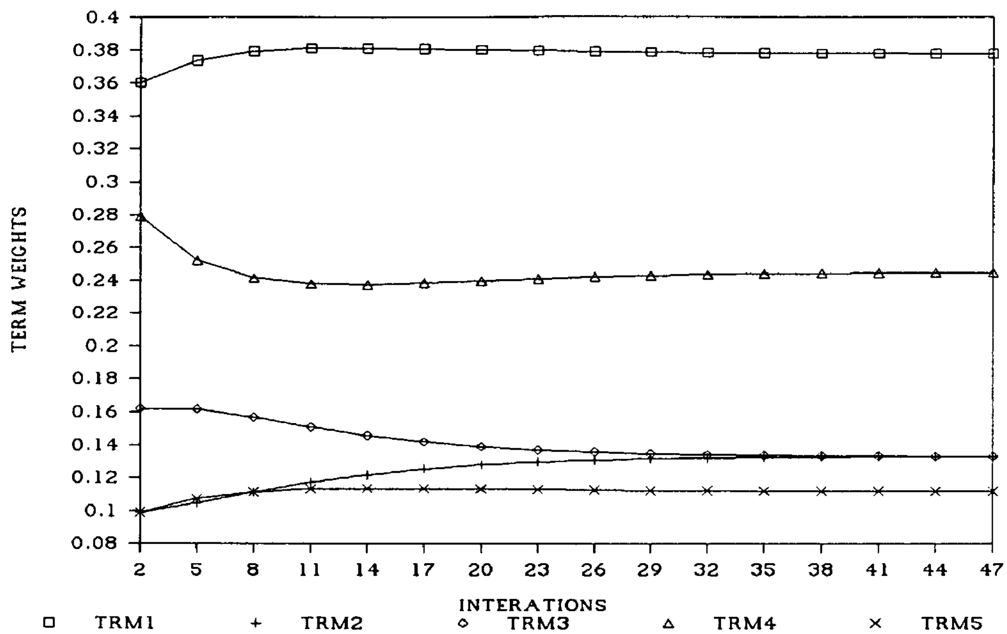


Fig. 11. 2nd Var. Term Weights vs. Iteration

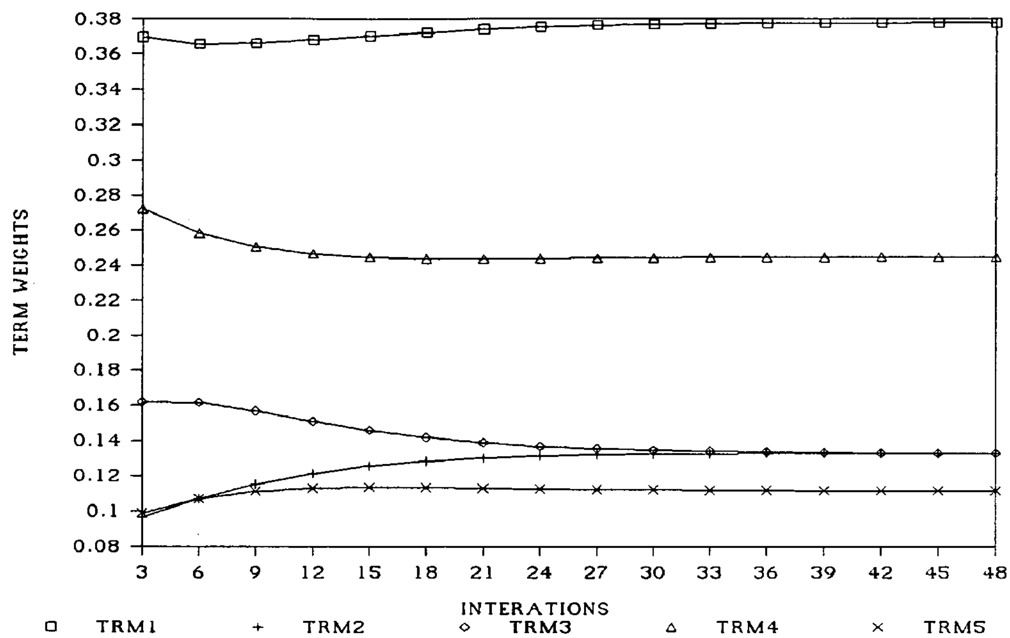


Fig. 12. 3rd Var. Term Weights vs. Iteration

They all seem to follow similar patterns. One item to note is that in comparing a single term weight over the three variables, there is an ultimate relation between their final values. The term weights all tend to increase from the first to third variable plots. Looking at the initial values, no such pattern exists, the term 1 weights actually crossing each other in value during the iterations. It is unclear exactly what this structure denotes, but the overall convergence does seem to occur, at least within a variable under consideration.

Iteration Analysis

The specific analysis on the impact of the various inputs to the algorithm and their effect on the number of iterations conducted is done with slightly different inputs. Because we are interested in the identified factors of the starting values of the variables, the tolerance condition, the number of terms within the problem, and the degree of difficulty, these were structured within an experimental design to properly test for their impact.

The starting values used were all possible combinations of three levels: 0.002, 1.0, and 999999. The tolerance condition was also evaluated at three levels: 1E-4, 1E-8, and 1E-11. The number of terms and the degree of difficulty used were those applicable to each problem. The response variable, the number of iterations required, was examined as a function of these factors. The data was entered into a Minitab matrix and regression was conducted. The results of the Minitab regression on the following data follows.

Table 9. Experimental Runs Input

ROW	VAR 1	VAR 2	VAR 3	LOG(TOL)	NTERMS	DD	ITER
1	0	0	0	-7	5	1	120
2	0	0	1	-7	5	1	81
3	0	0	999999	-7	5	1	144
4	0	1	0	-7	5	1	66
5	0	1	1	-7	5	1	48
6	0	1	999999	-7	5	1	147
7	0	999999	0	-7	5	1	69

8	0	999999	1	-7	5	1	66
9	0	999999	999999	-7	5	1	150
10	1	0	0	-7	5	1	120
11	1	0	1	-7	5	1	81
12	1	0	999999	-7	5	1	120
13	1	1	0	-7	5	1	66
14	1	1	1	-7	5	1	48
15	1	1	999999	-7	5	1	147
16	1	999999	0	-7	5	1	69
17	1	999999	1	-7	5	1	66
18	1	999999	999999	-7	5	1	150
19	999999	0	0	-7	5	1	120
20	999999	0	1	-7	5	1	81
21	999999	0	999999	-7	5	1	144
22	999999	1	0	-7	5	1	66
23	999999	1	1	-7	5	1	48
24	999999	1	999999	-7	5	1	147
25	999999	999999	0	-7	5	1	69
26	999999	999999	1	-7	5	1	66
27	999999	999999	999999	-7	5	1	150
28	0	0	0	-8	5	1	123
29	0	0	1	-8	5	1	87
30	0	0	999999	-8	5	1	150
31	0	1	0	-8	5	1	72
32	0	1	1	-8	5	1	54
33	0	1	999999	-8	5	1	153
34	0	999999	0	-8	5	1	75
35	0	999999	1	-8	5	1	69
36	0	999999	999999	-8	5	1	156
37	1	0	0	-8	5	1	126
38	1	0	1	-8	5	1	87
39	1	0	999999	-8	5	1	150
40	1	1	0	-8	5	1	72
41	1	1	1	-8	5	1	54
42	1	1	999999	-8	5	1	153
43	1	999999	0	-8	5	1	75
44	1	999999	1	-8	5	1	69
45	1	999999	999999	-8	5	1	156
46	999999	0	0	-8	5	1	120
47	999999	0	1	-8	5	1	87
48	999999	0	999999	-8	5	1	150
49	999999	1	0	-8	5	1	72
50	999999	1	1	-8	5	1	54
51	999999	1	999999	-8	5	1	153
52	999999	999999	0	-8	5	1	75
53	999999	999999	1	-8	5	1	69
54	999999	999999	999999	-8	5	1	156
55	0	0	0	-11	5	1	141
56	0	0	1	-11	5	1	102

57	0	0	999999	-11	5	1	171
58	0	1	0	-11	5	1	87
59	0	1	1	-11	5	1	72
60	0	1	999999	-11	5	1	174
61	0	999999	0	-11	5	1	90
62	0	999999	1	-11	5	1	87
63	0	999999	999999	-11	5	1	177
64	1	0	0	-11	5	1	141
65	1	0	1	-11	5	1	102
66	1	0	999999	-11	5	1	171
67	1	1	0	-11	5	1	87
68	1	1	1	-11	5	1	72
69	1	1	999999	-11	5	1	174
70	1	999999	0	-11	5	1	90
71	1	999999	1	-11	5	1	87
72	1	999999	999999	-11	5	1	177
73	999999	0	0	-11	5	1	141
74	999999	0	1	-11	5	1	102
75	999999	0	999999	-11	5	1	171
76	999999	1	0	-11	5	1	87
77	999999	1	1	-11	5	1	72
78	999999	1	999999	-11	5	1	174
79	999999	999999	0	-11	5	1	90
80	999999	999999	1	-11	5	1	87
81	999999	999999	999999	-11	5	1	177
82	0	*	*	-7	4	2	4
83	1	*	*	-7	4	2	4
84	999999	*	*	-7	4	2	5
85	0	*	*	-8	4	2	5
86	1	*	*	-8	4	2	4
87	999999	*	*	-8	4	2	6
88	0	*	*	-11	4	2	6
89	1	*	*	-11	4	2	6
90	999999	*	*	-11	4	2	7
91	0	*	*	-7	7	5	4
92	1	*	*	-7	7	5	4
93	999999	*	*	-7	7	5	5
94	0	*	*	-8	7	5	5
95	1	*	*	-8	7	5	5
96	999999	*	*	-8	7	5	5
97	0	*	*	-11	7	5	6
98	1	*	*	-11	7	5	6
99	999999	*	*	-11	7	5	7
100	0	0	*	-7	4	1	86
101	0	1	*	-7	4	1	78
102	0	999999	*	-7	4	1	86
103	1	0	*	-7	4	1	86
104	1	1	*	-7	4	1	78
105	1	999999	*	-7	4	1	86

106	999999	0	*	-7	4	1	86
107	999999	1	*	-7	4	1	78
108	999999	999999	*	-7	4	1	86
109	0	0	*	-8	4	1	100
110	0	1	*	-8	4	1	90
111	0	999999	*	-8	4	1	100
112	1	0	*	-8	4	1	100
113	1	1	*	-8	4	1	90
114	1	999999	*	-8	4	1	100
115	999999	0	*	-8	4	1	100
116	999999	1	*	-8	4	1	90
117	999999	999999	*	-8	4	1	100
118	0	0	*	-11	4	1	138
119	0	1	*	-11	4	1	128
120	0	999999	*	-11	4	1	138
121	1	0	*	-11	4	1	138
122	1	1	*	-11	4	1	128
123	1	999999	*	-11	4	1	138
124	999999	0	*	-11	4	1	138
125	999999	1	*	-11	4	1	128
126	999999	999999	*	-11	4	1	138

First a comprehensive regression was run, effectively telling the software to evaluate all of the factors together.

```
MTB > regr c7 6 c1-c6
*   NTERMS is (essentially) constant
*   NTERMS has been removed from the equation
*       DD is (essentially) constant
*       DD has been removed from the equation

The regression equation is
ITER = 33.3 +0.000000 VAR 1 -0.000006 VAR 2 +0.000073 VAR 3
- 6.04 LOG(TOL)
81 cases used 45 cases contain missing values
Predictor         Coef         Stdev    t-ratio        p
Constant         33.33         11.02       3.02    0.003
VAR 1       0.00000028    0.00000438       0.06    0.950
VAR 2      -0.00000589    0.00000438      -1.34    0.183
VAR 3       0.00007328    0.00000438      16.74    0.000
LOG(TOL)        -6.043         1.214      -4.98    0.000
s = 18.58        R-sq = 80.1%    R-sq(adj) = 79.1%

Analysis of Variance
```

```
SOURCE          DF          SS            MS          F         p
Regression       4      105823         26456      76.66     0.000
Error           76       26228           345
Total           80      132052
SOURCE          DF      SEQ SS
VAR 1            1           1
VAR 2            1         624
VAR 3            1       96653
LOG(TOL)         1        8544
Unusual Observations
Obs.    VAR 1     ITER       Fit Stdev.Fit  Residual St.Resid
  1         0   120.00     75.63      3.84     44.37     2.44R
 10         1   120.00     75.63      3.84     44.37     2.44R
 19    999999   120.00     75.91      4.60     44.09     2.45R
 28         0   123.00     81.68      3.36     41.32     2.26R
 37         1   126.00     81.68      3.36     44.32     2.43R
 46    999999   120.00     81.95      4.21     38.05     2.10R
 55         0   141.00     99.80      4.32     41.20     2.28R
 64         1   141.00     99.80      4.32     41.20     2.28R
 73    999999   141.00    100.08      5.01     40.92     2.29R
R denotes an obs. with a large st. resid.
```

As is evident here, the degree of difficulty and number of terms, at least in our problem set, have such little impact that they have been removed from the analysis. Now we shall conduct a stepwise regression, forcing variable one and two into the calculations (the forcing was shown to be necessary by other runs not shown here).

```
MTB > stepwise c7 c3,c4,c1,c2;
SUBC> force c1,c2.
 STEPWISE REGRESSION OF ITER ON  4 PREDICTORS, WITH N = 81
 N(CASES WITH MISSING OBS.) =  45 N(ALL CASES) =  126
    STEP          1          2          3
CONSTANT     110.13      85.70      33.33
VAR 1       0.00000    0.00000    0.00000
T-RATIO        0.03       0.06       0.06
VAR 2      -0.00001   -0.00001   -0.00001
T-RATIO       -0.61      -1.18      -1.34
VAR 3                  0.00007    0.00007
T-RATIO                  14.63      16.74
LOG(TOL)                             -6.0
T-RATIO                             -4.98
```

S	41.0	21.3	18.6
R-SQ	0.47	73.67	80.14

We see above that variable 1 and 2 have little impact, while in combination with 3 and 4 the equation should be able to explain 80% of the variation in the response variable, iterations, when the input variables are similar to those used. But we also see that variable 3 seems to account for almost three-fourths of our prediction equation value. This has an unusual consequence: that the last variable has the strongest influence on the number of iterations required. Could this be from some intrinsic characteristic of the algorithm? It is unclear at this point.

This apparent influence of the third variable prompts the question whether the order of the variables has any effect? Through other runs not presented in the thesis, the order of the variables did show an effect, although no conclusive pattern was discerned as it both decreased and increased the number of iterations required. This facet of the algorithm's nature will require further study but will not be done in this document.

As the relationships of iterations to convergence condition seem to be strong, a plot of their values can possibly provide additional insight.

```
MTB > plot c7 c4

ITER
    -
 180+   6
    -   3
    -                                         9           3
    -   9                                                 5
    -   3                                     *
 120+                                         2           4
    -   3
    -   3                                     9
    -   6                                     3           +
    -   3                                     9           9
  60+                                         3
    -                                                     3
    -
    -
    -   6                                     *
   0+                                         5           6
      +---------+---------+---------+---------+---------+--
----LOG(TOL)
 -11.20    -10.40     -9.60     -8.80     -8.00     -7.20
```

It was observed in the above plot that the values seem to occupy three diagonal bands. The significance of this is yet unknown, although the gaps between the bands may tell us a relationship in a supremum and infimum sense of the convergence condition and the number of iterations.

The strong impact of variable 3 was intriguing as well, and it was plotted again iterations.

```
MTB > plot c7 c3
ITER
     -
     -                                                    3
     -                                                    6
     -                                                    3
  150+                                                    +
     - 3                                                  2
     - *
     - 5                                                  *
     -
  100+ 3
     - +
     - 6
     - +
     -
   50+ 6
     -
     -
       +---------+---------+---------+---------+---------+--
----VAR 3
       0    200000    400000    600000    800000   1000000
    N* = 45
```

Once again we see what can be described as three bands on the left and what may be a convergence of the three bands to two or even one on the right. The significance is still unknown.

Further Insights

It is surmised that some of the characteristic algorithmic responses of the Ratliff method may be elements of design for future programming efforts. Specifically, the notion that the method divides the problem into as many parts as there are variables, iterates within one variable to achieve a local convergence, and then passes the interim values to other portions of the algorithm is a revelation.

Its significance becomes obvious when one considers that if there were a way to concurrently iterate to various local convergence points, the process would be dramatically quickened. Until recently this would have only been a nice idea, but technology has provided the tool to do such a thing: parallel processing (Woolsey, 1988). Having as many processing units as there are variables is a potential reality.

Programming, implementing Ratliff's procedure (or at least the notional kernel as such), and running on one of these machines, could revolutionize optimization as we know it. The possibilities of utilizing such optimizing potential are mindboggling. Real-time optimization would become a reality and the effects would be felt worldwide. Traffic light control, antiballistic missile management, environmental administration in buildings and homes, and nuclear power plant regulation are only a few applications which come to mind. It is this aspect which is really exciting.

Chapter 6
CONCLUSIONS

The purpose of this thesis was to study a geometric programming algorithm from a computational point of view in order to discover how, why, how well, and when the technique works. Serendipity was also hoped for as the concepts of nonlinear optimization are not straightforward. An effort in one direction was sure to spawn insight in another or at least further questions in that direction. Attacking this subject in this way was different, but, from the history of GP one can recognize that each significant accomplishment was done from a different perspective, technique, or application. In this vein, this author believes that the purpose was accomplished.

The computer code was found to be an excellent manifestation of the algorithm. It seems to have run equally well on mainframes as well as microcomputers. It was, however, difficult to run repetitively, hence a variation of the program was developed for ease of analysis. This variation was run and found to have comparable success in solving the problems.

The magnitude of the convergence condition had a strong influence on the number of iterations; the smaller the tolerance, the greater the iterations. Specifically, the iterations were a linear function of the log of the tolerance.

The algorithm also revealed that the utilization of one convergence condition may affect the number of required iterations. It is believed that there is some relationship between these values which may accelerate the convergence process.

The degrees of difficulty and the number of terms seemed to have an insignificant effect on the number of iterations, but the starting value of the third variable appeared to have a tremendous effect on the iterations. This is an enigma and is subject to further research.

The aggregation of numbers of iterations in three apparent bands in the plots of iterations to some of the predictor variables is also interesting. The number of bands may have something to do with the number of variables, but it cannot be evaluated at this time.

The algorithm generates an iteratively changing pseudo objective function whose value also converged. The coefficients of the two terms in the objective function converged as well. The significance of these values is also unknown.

Finally, the potential for utilizing parallel processing to speed up the optimization characteristic of Ratliff's method adds the final crescendo to the conclusions gleaned from this effort.

Chapter 7

AREAS FOR FURTHER RESEARCH

This area of nonlinear programming is replete with potential gold mines of information. An obvious choice for further research from this effort on unconstrained problems is to move on to constrained problems. If similar condensation techniques were to be applied to constraints and iteratively put into the objective function as a new variable value for the program, the technique may constructively converge on constrained problems. The concept is intuitively exciting.

The future analysis need mentioned in Chapter 6 of the relationship between a tolerance value for variables and that of one for the objective function is another possibility. Although the acceleration of the algorithm is not believed to be on the order-of-magnitude level, the study of this aspect may provide serendipitous results.

Also, work focusing on the moving of some or all the terms of the objective function to the constraints might be beneficial. Here, some types of matrix operations may prove to provide the insight to solve these problems expeditiously.

As the Geometric Mean Inequality is the source of methods which appear to converge quadratically, research into developments of a realm of mathematics which derives from other inequalities and even other types of means may be worthwhile.

Conducting continued computational analysis on isolating possible starting vector values based upon relationships of coefficients and exponents may prove desirable.

Any work in implementing the concepts of Ratliff's method in an parallel processing environment is sure to provide tremendous rewards, both academic and economic.

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APPENDICES

APP. A. ORIGINAL ALGORITHM (FORTRAN)

```
C*************************************************************
***
C
C program: MULTICON
C
C PURPOSE      : This program
C                solves multivariable posynomial problems
C                by using condensation
C
C AUTHOR: Richard Ratliff
C                Department of Mineral Economics
C                Colorado School of Mines
C                (303) 273-3672
C
C WRITTEN      : 19 February 1986
C
C SYSTEM       : IBM PC operating under MS-DOS
C                Written in MS-FORTRAN
C                Minimum configuration of 256K + 1 disk
drive
C
C INPUTS       : NTRMS      - number of terms in the
objective
C                    function (I3)
C                NVARS      - number of variables considered
(I3)
C                TOL        - convergence condition (F14.10)
C                XOLD(I)    - initial value of the Ith
variable
C                    (F15.4)
C                COEFF(I)   - coefficient of the Ith term
(F20.8)
C                POWER(I,J)- power in the Ith term of the
Jth
C                    variable (F20.8)
C                (inputs are interactive from the terminal,
C                    logic unit 5)
C
C OUTPUTS      : XNEW(I)    - optimal value of the Ith
variable
C                    (F30.8)
C                FNCVAL     - value of the dual function
```

```
(F30.8)
C                WEIGHT    - contribution to FNCVAL of each
term
                     (F7.4)
C                ITER      - number of iterations (I10)
C                (outputs are interactive to the terminal,
C                     logic unit 6)
C
C SUBROUTINES  : CONDNS    - condenses terms passed to it
C                OUTRES    - print the results to the screen
C                RDDATA    - read in the data
C                SENSE     - does limited sensitivity
analysis
C                     of function value to changes in the
C                     variables
C
C VARIABLES    : COEFF(I)  - coefficient of the Ith term
C                CONCOF(I) - coefficient of the Ith
condensed
C                     term
C                CONEG(I)  - coefficients of the terms w/
C                     negative powers
C                CONPOW(I) - power of the Ith condensed term
C                COPOS(I)  - coefficients of the terms w/
C                     positive powers
C                DELTA(I)  - contribution to FNCVAL of the
Ith
C                     condensed term
C                FNCVAL    - value of the dual function
C                I         - loop counter
C                ITER      - number of iterations through
the
C                     algorithm
C                K         - loop counter
C                NNEG      - number of terms with a negative
C                     exponent
C                NPOS      - number of terms with a positive
C                     exponent
C                NVARS     - number of variables considered
C                NTRMS     - number of terms in the
objective
C                     function
C                POWER(I,J)- power in the Ith term of the
Jth
C                     variable
C                POWNEG(I) - powers of the variables w/
C                     negative powers
C                POWPOS(I) - powers of the variables w/
C                     positive powers
```

```
C                TEMP      - partial value of the term
C                TOL       - Convergence condition
C                VALHAT    - value of the function
C                VAR       - loop counter
C                WGTNEG(I) - contribution to DELTA(2) of Ith
C                     term w/ neg power
C                WGTPOS(I) - contribution to DELTA(1) of Ith
C                     term w/ pos power
C                XHAT(I)   - outer loop value of variable
C                XNEW      - current value of the variable
C                XOLD      - previous value of the variable
C
C*****************************************************************
*
C
C
C
C------------------
C DECLARATIONS
C------------------
C
      INTEGER  ITER, NNEG, NPOS, NTRMS,NVARS,VAR
      DOUBLE PRECISION COEFF(50), CONCOF(2), CONEG(50),
     +        CONPOW(2),DELTA(2), FNCVAL, POWER(50,50),
     +        POWNEG(50), POWPOS(50), TOL,WGTNEG(50),
     +        WGTPOS(50), XNEW(50), XOLD(50), XHAT(50),
      +        VALUE, TEMP, VALHAT, COPOS(50)
      COMMON /PARAMS/ COEFF, CONEG, COPOS, DELTA, FNCVAL,
     +        ITER, NNEG, NPOS, NTRMS, POWER, POWNEG,
POWPOS,
     +        TOL, WGTNEG,WGTPOS, XNEW, XOLD, NVARS, VAR,
XHAT
C----------------
C INITIALIZE
C----------------
C
      ITER = 0
      NNEG = 0
      NPOS = 0
      VALHAT = 0!
C
C------------------
C PRINT HEADING
C------------------
C
      WRITE(0,5)
    5 FORMAT(/,' GENERALIZED MULTI- VARIABLE POSYNOMIAL
     +        CONDENSATION PROGRAM')
C
```

```
C-----------------------
C GET THE PARAMETERS
C-----------------------
C
      CALL RDDATA
C
C-----------------------------------
C MAIN LOOP; CONDENSE IF REQUIRED
C-----------------------------------
C
   10 DO 100 VAR=1,NVARS
C
       NPOS = 0
       NNEG = 0
C
        DO 18 I= 1,NTRMS
C
           IF (POWER(I,VAR). GT .0.0) THEN
             NPOS = NPOS + 1
             POWPOS(NPOS) = power(I, VAR)
             COPOS(NPOS) = COEFF(I)
C
               DO 14 K= 1,NVARS
C
                 IF (VAR.NE.K) THEN
                   COPOS(NPOS) =
    +
XOLD(K)**POWER(I,K)*COPOS(NPOS)
                 END IF
14         CONTINUE
           END IF
C
           IF (POWER(I,VAR). LT .0.0) THEN
             NNEG = NNEG + 1
             POWNEG(NNEG) = power(I, VAR)
             CONEG(NNEG) = COEFF(I)
C
               DO 15 K= 1,NVARS
C
                 IF (VAR.NE.K) THEN
                   CONEG(NNEG) =
    +
XOLD(K)**POWER(I,K)*CONEG(NNEG)
                 END IF
15          CONTINUE
           END IF
C
C
18    CONTINUE
```

```
C
C
   20 IF (NPOS .GT. 1) THEN
         CALL CONDNS(COPOS, CONCOF(1), CONPOW(1), NPOS,
POWPOS,
    +         WGTPOS, XOLD, VAR)
      ELSE
         CONPOW(1) = POWPOS(1)
         CONCOF(1) = COPOS(1)
         WGTPOS(1) = 1!
      END IF
C
C
      IF (NNEG .GT. 1) THEN
         CALL CONDNS(CONEG, CONCOF(2), CONPOW(2), NNEG,
POWNEG,
    +         WGTNEG, XOLD, VAR)
      ELSE
         CONPOW(2) = POWNEG(1)
         CONCOF(2) = CONEG(1)
         WGTNEG(1) = 1!
      END IF
C
C
C------------------------------
C CALCULATE THE NEW DELTAS
C------------------------------
C
      DELTA(2) = (1.0 - CONPOW(2)/CONPOW(1)) ** (-1)
      DELTA(1) = 1! - DELTA(2)
C
C-----------------------------------------
C CALCULATE THE NEW VALUE OF THE VARIABLE
C-----------------------------------------
C
      FNCVAL = (CONCOF(1)/DELTA(1)) ** DELTA(1) *
(CONCOF(2)/
    +         DELTA(2))** DELTA(2)
      XNEW(VAR) = (FNCVAL * DELTA(1)/CONCOF(1)) **
    +         (1.0/CONPOW(1))
C
C-------------------------------------
C CHECK & SEE IF IT HAS CONVERGED YET
C-------------------------------------
C
      IF ((DABS(XNEW(VAR)-XOLD(VAR))/XOLD(VAR)) .GT. TOL)
THEN
         XOLD(VAR) = XNEW(VAR)
         ITER = ITER + 1
```

```
          GOTO 20
       END IF
C
       value = 0!
C
       DO 62 I = 1,NTRMS
C
         TEMP = 1!
C
         DO 61 J = 1,NVARS
C
           TEMP= XOLD(J) ** POWER(I,J) * TEMP
C
61     CONTINUE
C
         value = COEFF(I) * TEMP + value
C
62 CONTINUE
C
100 CONTINUE
C
       IF ((DABS(VALHAT - VALUE) / VALUE). GT .TOL) THEN
          VALHAT = value
       ELSE
          CALL OUTRES
          GOTO 200
       END IF
C
       GOTO 10
C
200 CALL SENSE(COEFF, power, NTRMS, NVARS, XNEW)
       STOP
       END
C
```

```
C=============================================================
=
C
      SUBROUTINE CONDNS(COEF, CONCOF, CONPOW, NTRMS, PWER,
WGT,
     +                   XOLD, VAR)
C
C PURPOSE      : This subroutine condenses a posynomial
C                      function w/ 1 variable
C
C AUTHOR: Richard Ratliff
C                Department of Mineral Economics
C                Colorado School of Mines
C                Golden, CO  80401
C                (303) 273-3672
C
C WRITTEN      : 19 February 1986
C
C SYSTEM       : IBM PC operating under MS-DOS
C                Written in MS-FORTRAN
C                Minimum configuration of 256K + 1 disk
drive
C
C INPUTS       : COEF(I)    - coefficient of the Ith term in
the
C                     function
C                NTRMS      - number of terms in the function
C                POWER(I)   - power of the variable in the
Ith
C                     term
C                XOLD       - current value of the variable
C                (inputs are passed to the subroutine
through
C                     the param list)
C
C OUTPUTS      : CONCOF     - coefficient of the condensed
term
C                CONPOW     - power of the variable in the
C                     condensed term
C                WGT(I)     - contribution of term I towards
C                     condensed term
C                (outputs are returned to the main program
with
C                     the param list)
C
C SUBROUTINES: None
C
C VARIABLES    : COEF(I)    - coefficient of the Ith term in
the
```

```
C                    function
C               CONCOF     - coefficient of the condensed
term
C               CONPOW     - power of the variable in the
Ith
C                    term
C               CTEMP      - temporary variable used in
C                    calculating CONCOF
C               I          - loop variable
C               NTRMS      - number of terms in the function
C               PTEMP      - temporary variable used in
C                    calculating CONPOW
C               POWER(I)   - power of the variable in the
Ith
C                    term
C               SUM        - current value of the function
C               VAR        - loop counter
C               WGT(I)     - contribution of term I towards
C                    condensed term
C               WGTSUM     - sum of the weights of terms 2
to
C                    NTRMS
C               XOLD       - current value of the variable
C
C============================================================
=
C
C
C
C------------------
C DECLARATIONS
C------------------
C
      INTEGER  I, NTRMS, VAR
      DOUBLE PRECISION COEF(50), CONCOF, CONPOW, CTEMP,
PTEMP,
   +          SUM, WGT(50), WGTSUM, XOLD(50), PWER(50)
C
C
C--------------------------------
C INITIALIZE LOCAL VARIABLES
C--------------------------------
C
      CTEMP = 1!
      PTEMP = 0!
      Sum = 0!
      WGTSUM = 0!
C
C-----------------------------------------
```

```
C CALCULATE THE VALUE OF THE FUNCTION
C------------------------------------------
C
      DO 105 I=1,NTRMS
         SUM = SUM + COEF(I)*(XOLD(VAR) ** PWER(I))
105 CONTINUE
C
C-------------------------------------------------------
C CALCULATE THE CONTRIBUTIONS TO THE CONDENSED TERM
C-------------------------------------------------------
C
      DO 110 I=2,NTRMS
         WGT(I) = COEF(I)/SUM * (XOLD(VAR) ** PWER(I))
         WGTSUM = WGTSUM + WGT(I)
         PTEMP = PTEMP + PWER(I) * WGT(I)
         CTEMP = CTEMP * (COEF(I)/WGT(I)) ** WGT(I)
110 CONTINUE
C
C--------------------------------------------------------
C CALCULATE COEFFICIENT, POWER OF THE CONDENSED TERM
C--------------------------------------------------------
C
      WGT(1) = 1! - WGTSUM
      CONCOF = CTEMP * (COEF(1)/WGT(1)) ** WGT(1)
      CONPOW = PTEMP + PWER(1) * WGT(1)
C
C
      RETURN
      END
C
```

```
C==========================================================
=
C
      SUBROUTINE OUTRES
C
C PURPOSE      : This subroutine outputs the solution to the
C                   user's terminal
C
C AUTHOR: Richard Ratliff
C               Department of Mineral Economics
C               Colorado School of Mines
C               Golden, CO  80401
C               (303) 273-3672
C
C WRITTEN      : 19 February 1986
C
C SYSTEM       : IBM PC operating under MS-DOS
C                Written in MS-FORTRAN
C                Minimum configuration of 256K + 1 disk
drive
C
C INPUTS       : PARAMS     - COMMON block of variables
C                (inputs are from the COMMON block)
C
C OUTPUTS      : XNEW       - optimal value of the variable
C                     (F30.8)
C                FNCVAL     - value of the function (F30.8)
C                WEIGHT     - contribution to FNCVAL of each
C                     term (F7.4)
C                ITER       - number of iterations (I10)
C                (outputs are interactive to the terminal,
logic
C                     unit 6)
C
C SUBROUTINES: None
C
C VARIABLES    : COEFF(I)   - coefficient of the Ith term
C                CONEG(I)   - coefficients of the terms w/
C                     negative powers
C                COPOS(I)   - coefficients of the terms w/
C                     positive powers
C                DELTA(I)   - contribution to FNCVAL of the
Ith
C                     condensed term
C                FNCVAL     - value of the function at
optimality
C                I          - loop variable
C                ITER       - number of iterations through
the
```

```
C                        algorithm
C                  K           - loop counter
C                  NEGCNT      - # of terms w/ neg powers that
have
C                         been evaluated
C                  NNEG        - number of terms with a negative
C                        exponent
C                  NPOS        - number of terms with a positive
C                        exponent
C                  NTRMS       - number of terms in the
objective
C                        function
C                  POSCNT      - # of terms w/ pos powers that
have
C                        been evaluated
C                  POWER(I,J)- power in the Ith term of the
Jth
C                        variable
C                  POWNEG(I) - powers of the variables w/
C                        negative powers
C                  POWPOS(I) - powers of the variables w/
C                        positive powers
C                  TOL         - convergence condition
C                  WEIGHT      - contribution towards FNCVAL of
C                        each term
C                  WGTNEG(I) - contribution to DELTA(2) of Ith
C                        term w/ neg power
C                  WGTPOS(I) - contribution to DELTA(1) of Ith
C                        term w/ pos power
C                  XNEW        - current value of the variable
C                  XOLD        - previous value of the variable
C
C===========================================================
=
C
C
C
C------------------
C DECLARATIONS
C------------------
C
      INTEGER  I, ITER, NEGCNT, NNEG, NPOS, NTRMS, POSCNT,
    +          NVARS, VAR
      DOUBLE PRECISION COEFF(50), CONEG(50), COPOS(50),
    +          DELTA(2), POWER(50,50), POWNEG(50),
POWPOS(50),
    +          TOL, WEIGHT, WGTNEG(50), WGTPOS(50),
XNEW(50),
    +          XOLD(50), TEMP, XHAT(50), FNCVAL
```

```
C
      COMMON /PARAMS/ COEFF, CONEG, COPOS, DELTA, FNCVAL,
    +         ITER, NNEG, NPOS, NTRMS, POWER, POWNEG,
POWPOS,
    +         TOL, WGTNEG, WGTPOS, XNEW, XOLD, NVARS, VAR,
    +         XHAT
C
C----------------
C INITIALIZE
C----------------
C
      NEGCNT = 1
      POSCNT = 1
      FNCVAL = 0!
C
      DO 202 I = 1,NTRMS
C
       TEMP = 1!
C
        DO 201 J = 1,NVARS
C
          TEMP = (XOLD(J)**POWER(I,J)) *TEMP
C
201   CONTINUE
        FNCVAL = (COEFF(I) * TEMP) + FNCVAL
C
202 CONTINUE
C
C-------------------------------------------------------------
--
C PRINT OPTIMAL VALUES FOR THE VARIABLE AND THE FUNCTION
C-------------------------------------------------------------
--
C
      DO 207 K= 1,NVARS
C
      WRITE(0,204)
  204 FORMAT('  ')
      WRITE(0,205) K, XOLD(K)
  205 FORMAT(' The optimal value of variable',I3,' is',
F30.8)
C
207 CONTINUE
C
      WRITE(0,208)
  208 FORMAT('  ')
      WRITE(0,210) FNCVAL
  210 FORMAT(' The value of the function at optimality is',
   +         F30.8)
```

```
C
C---------------------------------------------------------------
---
C PRINT OUT THE CONTRIBUTIONS OF EACH TERM TO THE FINAL
VALUE
C---------------------------------------------------------------
---
C
      DO 220 I=1,NTRMS
C
       TEMP = 1!
C
        DO 212 J= 1,NVARS
C
          TEMP = (XOLD(J)**POWER(I,J)) *TEMP
C
212   CONTINUE
C
         WEIGHT = (COEFF(I) * TEMP) / FNCVAL
C
         WRITE(0,215) I, WEIGHT
  215    FORMAT(' Weight', I3, ' =', F7.4)
C
220 CONTINUE
C
C----------------------------------------
C PRINT OUT THE NUMBER OF ITERATIONS
C----------------------------------------
C
      WRITE(0,225) ITER
  225 FORMAT(' Number of iterations is', I10)
C
C
      RETURN
      END
C
C
```

```
C==============================================================
=
C
      SUBROUTINE RDDATA
C
C PURPOSE      : This subroutine reads in the data for the
C                     program
C
C AUTHOR: Richard Ratliff
C                Department of Mineral Economics
C                Colorado School of Mines
C                Golden, CO  80401
C                (303) 273-3672
C
C WRITTEN      : 19 February 1986
C
C SYSTEM       : IBM PC operating under MS-DOS
C                Written in MS-FORTRAN
C                Minimum configuration of 256K + 1 disk
drive
C
C INPUTS       : NTRMS     - number of terms in the
objective
C                     function (I3)
C                NVARS     - number of variables considered
(I3)
C                TOL       - convergence condition (F14.10)
C                XOLD      - initial value for the variable
C                     (F15.4)
C                COEFF(I)  - coefficient of the Ith term
(F20.8)
C                POWER(I,J)- power in the Ith term of the
Jth
C                     variable (F20.8)
C                (inputs are interactive from the terminal,
C                     logic unit 6)
C
C OUTPUTS      : PARAMS    - COMMON block of variables
C                (outputs are to the COMMON block)
C
C SUBROUTINES: None
C
C VARIABLES    : COEFF(I)  - coefficient of the Ith term
C                CONEG(I)  - coefficients of the terms w/
C                     negative powers
C                COPOS(I)  - coefficients of teh terms w/
C                     positive powers
C                DELTA(I)  - contribution to FNCVAL of the
Ith
```

```
C                    condensed term
C               FNCVAL     - value of the function at
optimality
C               I          - loop variable
C               ITER       - number of iterations through
the
C                    algorithm
C               NNEG       - number of terms with a negative
C                    exponent
C               NPOS       - number of terms with a positive
C                    exponent
C               NTRMS      - number of terms in the
objective
C                    function
C               NVARS      - number of variables considered
C               POWER(I,J)- power in the Ith term of the
Jth
C                    variable
C               POWNEG(I) - powers of the variables w/
C                    negative powers
C               POWPOS(I) - powers of the variables w/
C                    positive powers
C               TOL        - convergence condition
C               WGTNEG(I) - contribution to DELTA(2) of Ith
C                    term w/ neg power
C               WGTPOS(I) - contribution to DELTA(1) of Ith
C                    term w/ pos power
C               XNEW       - current value of the variable
C               XOLD       - initial value for the variable
C
C==========================================================
===
C
C
C
C------------------
C DECLARATIONS
C------------------
C
      INTEGER  ITER, NNEG, NPOS, NTRMS, NVARS, VAR
      DOUBLE PRECISION COEFF(50), CONEG(50), COPOS(50),
    +          DELTA(2), XNEW(50), POWNEG(50), POWPOS(50),
    +          TOL, WGTNEG(50), WGTPOS(50), XOLD(50),
    +          POWER(50,50),XHAT(50), FNCVAL
C
      COMMON /PARAMS/ COEFF, CONEG, COPOS, DELTA, FNCVAL,
    +          ITER, NNEG, NPOS, NTRMS, POWER, POWNEG,
POWPOS,
    +          TOL, WGTNEG, WGTPOS, XNEW, XOLD, NVARS, VAR,
```

```
     +          XHAT
C
C-----------------------------
C GET THE NUMBER OF TERMS
C-----------------------------
C
      WRITE(0,305)
  305 FORMAT(/,' Enter the number of terms:  [no decimals]
')
      READ(0,310) NTRMS
310 FORMAT (I3)
C
C-----------------------------------
C GET THE CONVERGENCE CONDITION
C-----------------------------------
C
      WRITE(0,315)
  315 FORMAT(' Enter the convergence condition:  [decimal
     +          required] ')
      READ(0,320) TOL
320 FORMAT (F14.10)
C
      WRITE(0,321)
  321 FORMAT(' Enter the # of variables considered:  [no
     +          decimals] ')
      READ(0,322) NVARS
322 FORMAT (I3)
C
C------------------------------------------------------
C GET AN INITIAL ESTIMATE OF THE VARIABLE'S VALUE
C------------------------------------------------------
C
      DO 331 I = 1,NVARS
C
      WRITE(0,325)I
  325 FORMAT(' Enter an initial value for variable',I3,
     +':   [decimal required]')
      READ(0,330) XOLD(I)
330 FORMAT (F15.4)
C
      XHAT(I) = XOLD(I)
C
331 CONTINUE
C----------------------------------------------
C GET THE TERMS OF THE OBJECTIVE FUNCTION
C----------------------------------------------
C
      DO 360 I=1,NTRMS
C
```

```
          WRITE(0,335) I
  335     FORMAT(/,' Enter the data for term', I3)
C
          WRITE(0,340)
C 340     FORMAT(' Coefficient:  [decimal required] ')
          READ(0,345) COEFF(I)
345     FORMAT (F20.8)
C
        DO 356 J = 1,NVARS
C
          WRITE(0,350)J
  350     FORMAT(' Enter the power for variable',I3,
     + ':   [decimal required]')
          READ(0,355) POWER(I,J)
355     FORMAT (F20.8)
C
356   CONTINUE
C
360 CONTINUE
C
C
      RETURN
      END
C
```

```
C============================================================
===
C
C
      SUBROUTINE SENSE(COEFF, power, NTRMS, NVARS, XNEW)
C
C PURPOSE      : This subroutine performs a limited
C                sensitivity analysis.
C
C AUTHOR: Richard Ratliff
C                Department of Mineral Economics
C                Colorado School of Mines
C                Golden, CO  80401
C                (303) 273-3672
C
C WRITTEN      : 19 February 1986
C
C SYSTEM       : IBM PC operating under MS-DOS
C                Written in MS-FORTRAN
C                Minimum configuration of 256K + 1 disk
drive
C
C INPUTS       : X(I)       - new value for the Ith variable
C                     (F25.10)
C                (inputs are passed to the subroutine
through
C                     the param list)
C
C OUTPUTS      : VALUE      - new function value (F30.8)
C
C SUBROUTINES: None
C
C VARIABLES    : COEFF(I)   - coefficient of the Ith term
C                VALUE      - value of the function
C                I          - loop counter
C                J          - loop counter
C                K          - loop counter
C                NVARS      - number of variables considered
C                NTRMS      - number of terms in the
objective
C                     function
C                POWER(I,J)- power in the Ith term of the
Jth
C                     variable
C                TEMP       - partial value of the term
C                VAR        - loop counter
C                X(I)       - value of the Ith variable
C
      INTEGER NTRMS,NVARS,I,J,K
```

```
C
      DOUBLE PRECISION
COEFF(50),POWER(50,50),VALUE,TEMP,X(50)
C
C
  10  WRITE(0,20)
  20  FORMAT(/,' Sensitivity analysis?  [0 = no, 1 = yes]
')
C
      READ(0,40) I
40  FORMAT (I3)
C
      IF (I.NE.1) GOTO 400
C
      DO 100 J = 1,NVARS
C
        WRITE(0,60) J
  60    FORMAT(' Enter new value for variable ',I3,
     +    ':  [decimal required] ')
C
        READ(0,65)X(J)
65    FORMAT (F25.10)
C
  100 CONTINUE
C
      value = 0!
C
      DO 220 J = 1,NTRMS
C
        TEMP = 1!
C
         DO 210 K= 1,NVARS
C
          TEMP= TEMP * X(K) ** POWER(J,K)
C
210    CONTINUE
C
         value = value + COEFF(J) * TEMP
C
  220 CONTINUE
C
       WRITE(0,230)VALUE
  230  FORMAT(' The new function value is ',F30.8)
C
      GOTO 10
C
400 RETURN
C
      END
```

APP. B. APPLE BASIC ALGORITHM

The original Apple BASIC program written by Woolsey. This copy does not contain all line numbers as it was to be immediately translated into QuickBASIC which does not require them except as GOTO objects.

```
REM MULTICON
HOME
PRINT "FNLMULTICON"
DIM POWER(20, 20), PLSPOW(20)
DIM PSCOF(20), COFF(20), XOLD(20)
DIM NGCOEF(20), DLTA(2), CNDPOW(2)
DIM CDCOF(20), XNW(20), WTPLS(20)
DIM PWNEG(20), WGTNEG(20)
PRINT "NON-CONSTRAINED POSYNOMIALS"
PRINT "           MULTICON"
PP = 0
INPUT "SPPRESS PRINTS (Y/N)?"; A$
IF A$ = "Y" THEN PP = 1
VH = 0!: ITER = 0
INPUT "DO GRAVEL BOX (Y/N)"; A$
IF A$ = "Y" THEN GOTO 2000
150 GOSUB 1320
155 INPUT "WHAT IS THE CONVERGENCE CONDITION"; TL
GOSUB 640
GOSUB 1520
INPUT "TRY ANOTHER (Y/N)"; A$
IF A$ = "Y" THEN 150
STOP
REM *** CONDENSE POS1+ ***
180 CTEMP = 1
PTEMP = 0
SUM = 0
WSUM = 0
FOR I = 1 TO NPLS
        SUM = SUM + PSCOF(I) * XOLD(VAR) ^ PLSPOW(I)
NEXT I
FOR I = 2 TO NPLS
        WTPLS(I) = PSCOF(I) / SUM * (XOLD(VAR) ^ PLSPOW(I))
        WSUM = WSUM + WTPLS(I)
        PTERM = PTERM + WTPLS(I) * PLSPOW(I)
        CTEMP = CTEMP * (PSCOF(I) / WTPLS(I)) ^ WTPLS(I)
NEXT I
```

```
WTPLS(1) = 1 - WSUM
CDCOF(1) = CTEMP * (PSCOF(1) / WTPLS(1)) ^ WTPLS(1)
CNDPOW(1) = PTEMP + PLSPOW(1) * WTPLS(1)
RETURN
REM *** CONDENSE NEG1 ***
360 CTEMP = 1
PTEMP = 0
SUM = 0
WSUM = 0
FOR I = 1 TO NNEG
        SUM = SUM + NGCOEF(I) * XOLD(VAR) ^ PWNEG(I)
NEXT I
FOR I = 2 TO NNEG
        WGTNEG(I) = NGCOEF(I) / SUM * (XOLD(VAR) ^
PWNEG(I))
        WSUM = WSUM + WGTNEG(I)
        PTEMP = PTEMP + WGTNEG(I) * PWNEG(I)
        CTEMP = CTEMP * (NGCOEF(I) / WGTNEG(I)) ^ WGTNEG(I)
NEXT I
WGTNEG(1) = 1 - WSUM
CDCOF(2) = CTEMP * (NGCOEFF(1) / WGTNEG(1)) ^ WGTNEG(1)
CNDPOW(2) = PTEMP + PWNEG(1) * WGTNEG(1)
RETURN
REM  *** POS1 ***
540 CNDPOW(1) = PLSPOW(1)
CDCOF(1) = PSCOF(1)
WTPLS(1) = 1
RETURN
REM *** NEG 1 ***
590 CNDPOW(2) = PWNEG(1)
CDCOF(2) = NGCOEF(1)
WGTNEG(1) = 1
RETURN
REM  *** MAIN LOOP ***
640 FOR VAR = 1 TO NVARS
NPLS = 0
NNEG = 0
FOR I = 1 TO NTERMS
        IF (POWER(I, VAR) <= 0) THEN 750
        NPLS = NPLS + 1
        PLSPOW(NPLS) = POWER(I, VAR)
        PSCOF(NPLS) = COFF(I)
        FOR K = 1 TO NVARS
                IF (VAR <> K) THEN PSCOF(NPLS) = XOLD(K) ^
POWER(I, K) * PSCOF(NPLS)
        NEXT K
750     IF (POWER(I, VAR) >= 0) THEN 815
        NNEG = NNEG + 1
        PWNEG(NNEG) = POWER(I, VAR)
```

```
        NGCOEF(NNEG) = COFF(I)
        FOR K = 1 TO NVARS
                IF (VAR <> K) THEN NGCOEF(NNEG) = XOLD(K) ^
POWER(I, K) * NGCOEF(NNEG)
        NEXT K
815 NEXT I
REM  *** CONDENSE POSITIVE TERMS ***
830 IF (NPLS > 1) THEN GOSUB 180
IF (NPLS = 1) THEN GOSUB 540
REM   *** CONDENSE NEGATIVE TERMS ***
IF (NNEG > 1) THEN GOSUB 360
IF (NNEG = 1) THEN GOSUB 590
REM *** CALCULATE NEW DELTAS ***
DLTA(2) = 1 / (1 - CNDPOW(2) / CNDPOW(1))
DLTA(1) = 1 - DLTA(2)
REM *** CALC NEW VALUE OF VAR ***
PRINT "PROBLEM FOR VBL "; VAR
PRINT "Y="; CDCOF(1); "*X^"; CNDPOW(1); " + "; CDCOF(2);
"*X^"; CNDPOW(2)
FUNVL = (CDCOF(1) / DLTA(1)) ^ DLTA(1) * (CDCOF(2) /
DLTA(2)) ^ DLTA(2)
REM X NEW (VAR)= (FUNVL*DLTA(1)/CDCOF(1))^(1/CNDPOW(1))
XNW(VAR) = (FUNVL * DLTA(1) / CDCOF(1)) ^ (1 / CNDPOW(1))
PRINT "XNW("; VAR; ")="; XNW(VAR)
REM *** CHK FOR VAR CONVERGENCE ***
XH = ABS(XNW(VAR) - XOLD(VAR)) / XOLD(VAR)
IF (ABS(XNW(VAR) - XOLD(VAR)) / XOLD(VAR) <= TL) THEN 1000
PRINT "TL,XH"; TL, XH
XOLD(VAR) = XNW(VAR)
ITER = ITER + 1
PRINT "ITER"; ITER
GOTO 830
1000 VLUE = 0
FOR I = 1 TO NTERMS
        TEMP = 1
        FOR J = 1 TO NVARS
                TEMP = XOLD(J) ^ POWER(I, J) * TEMP
        NEXT J
        VLUE = COEF(I) * TEMP + VLUE
NEXT I
NEXT VAR
REM *** CHK FOR PROB CONVERG ***
IF VLUE = 0 THEN PRINT "***** WARNING *******:  VLUE=0":
STOP
IF ((ABS(VH - VLUE) / VLUE) <= TL) THEN RETURN
VH = VLUE
GOTO 640
REM *** READ IN PROB ***
1320 INPUT "HOW MANY TERMS (<=20)"; NTERMS
```

```
IF NTERMS > 20 THEN 1320
INPUT "WHAT IS THE CONVERGENCE CONDITION"; TL
1340 INPUT "HOW MANY VARIABLES (<=20)"; NVARS
IF NVARS > 20 THEN 1340
REM *** GET INIT EST OF VAR ***
FOR I = 1 TO NVARS
        PRINT "INITIAL VALUE FOR VAR "; I;
        INPUT "IS "; XOLD(I)
        XNW(I) = XOLD(I)
NEXT I
PRINT
REM *** GET TERMS ***
FOR I = 1 TO NTERMS
        PRINT "PLEASE ENTER DATA FOR TERM ", I
        INPUT "COEFFICIENT:"; COEFF(I)
        FOR J = 1 TO NVARS
                PRINT "ENTER POWER FOR VAR"; J;
                INPUT "IS "; POWER(I, J)
        NEXT J
NEXT I
RETURN
REM *** PRINT RESULTS ***
1520 REM
FUNVL = 0
FOR I = 1 TO NTERMS
        TEMP = 1
        FOR J = 1 TO NVARS
                TEMP = (XOLD(J) ^ POWER(I, J)) * TEMP
        NEXT J
        FUNVL = COFF(I) * TEMP + FUNVL
NEXT I
FOR I = 1 TO NVARS
        PRINT "VARIABLE", I, " = ", XOLD(I)
NEXT I
PRINT "FUNCTION =", FUNVL
REM *** PRINT CONTRIB TO ANS ***
FOR I = 1 TO NTERMS
        TEMP = 1
        FOR J = 1 TO NVARS
                TEMP = (XOLD(J) ^ POWER(I, J)) * TEMP
        NEXT J
        WEIGHT = COFF(I) * TEMP / FUNVL
        PRINT "WEIGHT ", I, " = ", WEIGHT
NEXT I
REM *** # OF ITERATIONS ***
PRINT "# OF ITERATIONS = ,ITER"
RETURN
2000 REM GRAVEL BOX PROBLEM INPUT
NTERMS = 5: NVARS = 3
```

```
INPUT "INPUT CONV. COND "; TL
PRINT "ENTER STARTING POINTS"
INPUT "STRTNG PNTS"; SP
FOR I = 1 TO NVARS
        XOLD(I) = SP: NEXT I
COFF(1) = 40: COFF(2) = 10
COFF(3) = 20: COFF(4) = 40: COFF(5) = 10
POWER(1, 1) = -1: POWER(1, 2) = -1: POWER(1, 3) = -1
POWER(2, 1) = 0: POWER(2, 2) = 1
POWER(3, 1) = 1: POWER(3, 2) = 1: POWER(3, 3) = 0
POWER(4, 1) = 1: POWER(4, 2) = 1: POWER(4, 3) = 1
POWER(5, 1) = 0: POWER(5, 2) = 1: POWER(5, 3) = 0
GOTO 155
```

APP. C. ORIGINAL ALGORITHM (QUICK BASIC)

```
REM MULTICON
CLS
PRINT "FNLMULTICON"
DIM POWER(20, 20), PLSPOW(20), PSCOF(20), COFF(20),
XOLD(20)
DIM NGCOEF(20), DLTA(2), CNDPOW(2), CDCOF(20), XNW(20),
DIM WTPLS(20), PWNEG(20), WGTNEG(20)
PRINT "NON-CONSTRAINED POSYNOMIALS"
PRINT "          MULTICON"
VH = 0: ITER = 0
INPUT "DO GRAVEL BOX (Y/N)"; A$
IF A$ = "Y" THEN GOTO 2000
150 GOSUB 1320
155 INPUT "WHAT IS THE CONVERGENCE CONDITION"; TL
GOSUB 640
GOSUB 1520
INPUT "TRY ANOTHER (Y/N)"; A$
IF A$ = "Y" THEN 150
STOP
REM *** CONDENSE POS1+ ***
180 CTEMP = 1
PTEMP = 0
SUM = 0
WSUM = 0
FOR I = 1 TO NPLS
        SUM = SUM + PSCOF(I) * XOLD(VAR) ^ PLSPOW(I)
NEXT I
FOR I = 2 TO NPLS
        WTPLS(I) = PSCOF(I) / SUM * (XOLD(VAR) ^ PLSPOW(I))
        WSUM = WSUM + WTPLS(I)
        PTERM = PTERM + WTPLS(I) * PLSPOW(I)
        CTEMP = CTEMP * (PSCOF(I) / WTPLS(I)) ^ WTPLS(I)
NEXT I
WTPLS(1) = 1 - WSUM
CDCOF(1) = CTEMP * (PSCOF(1) / WTPLS(1)) ^ WTPLS(1)
CNDPOW(1) = PTEMP + PLSPOW(1) * WTPLS(1)
RETURN
REM *** CONDENSE NEG1 ***
360 CTEMP = 1
PTEMP = 0
SUM = 0
WSUM = 0
FOR I = 1 TO NNEG
        SUM = SUM + NGCOEF(I) * XOLD(VAR) ^ PWNEG(I)
NEXT I
```

```
FOR I = 2 TO NNEG
        WGTNEG(I) = NGCOEF(I) / SUM * (XOLD(VAR) ^
PWNEG(I))
        WSUM = WSUM + WGTNEG(I)
        PTEMP = PTEMP + WGTNEG(I) * PWNEG(I)
        CTEMP = CTEMP * (NGCOEF(I) / WGTNEG(I)) ^ WGTNEG(I)
NEXT I
WGTNEG(1) = 1 - WSUM
CDCOF(2) = CTEMP * (NGCOEFF(1) / WGTNEG(1)) ^ WGTNEG(1)
CNDPOW(2) = PTEMP + PWNEG(1) * WGTNEG(1)
RETURN
REM  *** POS1 ***
540 CNDPOW(1) = PLSPOW(1)
CDCOF(1) = PSCOF(1)
WTPLS(1) = 1
RETURN
REM *** NEG 1 ***
590 CNDPOW(2) = PWNEG(1)
CDCOF(2) = NGCOEF(1)
WGTNEG(1) = 1
RETURN
REM  *** MAIN LOOP ***
640 FOR VAR = 1 TO NVARS
NPLS = 0
NNEG = 0
FOR I = 1 TO NTERMS
        IF (POWER(I, VAR) <= 0) THEN 750
        NPLS = NPLS + 1
        PLSPOW(NPLS) = POWER(I, VAR)
        PSCOF(NPLS) = COFF(I)
        FOR K = 1 TO NVARS
                IF (VAR <> K)
               THEN PSCOF(NPLS) = XOLD(K) ^ POWER(I, K) *
               PSCOF(NPLS)
        NEXT K
750     IF (POWER(I, VAR) >= 0) THEN 815
        NNEG = NNEG + 1
        PWNEG(NNEG) = POWER(I, VAR)
        NGCOEF(NNEG) = COFF(I)
        FOR K = 1 TO NVARS
                IF (VAR <> K)
               THEN NGCOEF(NNEG) = XOLD(K) ^ POWER(I, K) *
               NGCOEF(NNEG)
        NEXT K
815 NEXT I
REM  *** CONDENSE POSITIVE TERMS ***
830 IF (NPLS > 1) THEN GOSUB 180
IF (NPLS = 1) THEN GOSUB 540
REM   *** CONDENSE NEGATIVE TERMS ***
```

```
IF (NNEG > 1) THEN GOSUB 360
IF (NNEG = 1) THEN GOSUB 590
REM *** CALCULATE NEW DELTAS ***
DLTA(2) = 1 / (1 - CNDPOW(2) / CNDPOW(1))
DLTA(1) = 1 - DLTA(2)
REM *** CALC NEW VALUE OF VAR ***
PRINT "PROBLEM FOR VBL "; VAR
PRINT "Y="; CDCOF(1); "*X^"; CNDPOW(1); " + "; CDCOF(2);
"*X^";
PRINT CNDPOW(2)
FUNVL = (CDCOF(1) / DLTA(1)) ^ DLTA(1) * (CDCOF(2) /
    DLTA(2)) ^ DLTA(2)
REM X NEW (VAR)= (FUNVL*DLTA(1)/CDCOF(1))^(1/CNDPOW(1))
XNW(VAR) = (FUNVL * DLTA(1) / CDCOF(1)) ^ (1 / CNDPOW(1))
PRINT "XNW("; VAR; ")="; XNW(VAR)
REM *** CHK FOR VAR CONVERGENCE ***
XH = ABS(XNW(VAR) - XOLD(VAR)) / XOLD(VAR)
IF (ABS(XNW(VAR) - XOLD(VAR)) / XOLD(VAR) <= TL) THEN 1000
PRINT "TL,XH"; TL, XH
XOLD(VAR) = XNW(VAR)
ITER = ITER + 1
PRINT "ITER"; ITER
GOTO 830
1000 VLUE = 0
FOR I = 1 TO NTERMS
        TEMP = 1
        FOR J = 1 TO NVARS
                TEMP = XOLD(J) ^ POWER(I, J) * TEMP
        NEXT J
        VLUE = COEF(I) * TEMP + VLUE
NEXT I
NEXT VAR
REM *** CHK FOR PROB CONVERG ***
IF ((ABS(VH - VLUE) / VLUE) <= TL) THEN RETURN
VH = VLUE
GOTO 640
REM *** READ IN PROB ***
1320 INPUT "HOW MANY TERMS (<=20)"; NTERMS
IF NTERMS > 20 THEN 1320
INPUT "WHAT IS THE CONVERGENCE CONDITION"; TL
1340 INPUT "HOW MANY VARIABLES (<=20)"; NVARS
IF NVARS > 20 THEN 1340
REM *** GET INIT EST OF VAR ***
FOR I = 1 TO NVARS
        PRINT "INITIAL VALUE FOR VAR "; I;
        INPUT "IS "; XOLD(I)
        XNW(I) = XOLD(I)
NEXT I
PRINT
```

```
REM *** GET TERMS ***
FOR I = 1 TO NTERMS
        PRINT "PLEASE ENTER DATA FOR TERM ", I
        INPUT "COEFFICIENT:"; COEFF(I)
        FOR J = 1 TO NVARS
                PRINT "ENTER POWER FOR VAR"; J;
                INPUT "IS "; POWER(I, J)
        NEXT J
NEXT I
RETURN
REM *** PRINT RESULTS ***
1520 REM
FUNVL = 0
FOR I = 1 TO NTERMS
        TEMP = 1
        FOR J = 1 TO NVARS
                TEMP = (XOLD(J) ^ POWER(I, J)) * TEMP
        NEXT J
        FUNVL = COFF(I) * TEMP + FUNVL
NEXT I
FOR I = 1 TO NVARS
        PRINT "VARIABLE", I, " = ", XOLD(I)
NEXT I
PRINT "FUNCTION =", FUNVL
REM *** PRINT CONTRIB TO ANS ***
FOR I = 1 TO NTERMS
        TEMP = 1
        FOR J = 1 TO NVARS
                TEMP = (XOLD(J) ^ POWER(I, J)) * TEMP
        NEXT J
        WEIGHT = COFF(I) * TEMP / FUNVL
        PRINT "WEIGHT ", I, " = ", WEIGHT
NEXT I
REM *** # OF ITERATIONS ***
PRINT "# OF ITERATIONS = ,ITER"
RETURN
2000 REM GRAVEL BOX PROBLEM INPUT
NTERMS = 5: NVARS = 3
INPUT "INPUT CONV. COND "; TL
PRINT "ENTER STARTING POINTS"
INPUT "STARTING PNTS"; SP
FOR I = 1 TO NVARS
        XOLD(I) = SP: NEXT I
COFF(1) = 40: COFF(2) = 10
COFF(3) = 20: COFF(4) = 40: COFF(5) = 10
POWER(1, 1) = -1: POWER(1, 2) = -1: POWER(1, 3) = -1
POWER(2, 1) = 0: POWER(2, 2) = 1
POWER(3, 1) = 1: POWER(3, 2) = 1: POWER(3, 3) = 0
```

```
POWER(4, 1) = 1: POWER(4, 2) = 1: POWER(4, 3) = 1
POWER(5, 1) = 0: POWER(5, 2) = 1: POWER(5, 3) = 0
GOTO 155
```

APP. D. EFFICIENCY ALGORITHM (QUICK BASIC)

```
10 CLS : CLEAR ALL
DEFDBL A-Z
DIM POWER(20, 20), PLSPOW(20), PSCOF(20), COEFF(20),
XOLD(20)
DIM NGCOEF(20), DLTA(2), CNDPOW(2), CDCOF(20), XNW(20),
WTPLS(20)
DIM PWNEG(20), WGTNEG(20), VAR$(20)
ZZ$ = "NON-CONSTRAINED POSYNOMIALS": PRINT ZZ$
ZZ$ = "            MULTICON": PRINT ZZ$
PP = 0
VH = 0: ITER = 0
INPUT "DO AN EXISTING PROBLEM (Y/N)"; A$
IF LEFT$(UCASE$(A$), 1) = "Y" THEN GOTO 2000
150 GOSUB 1310
155 INPUT "WHAT IS THE CONVERGENCE CONDITION"; TL
175 CLS : PRINT "CURRENT STARTING VECTOR": PRINT
PRINT "VAR NUM", "VAR", "VALUE"
FOR I = 1 TO NVARS
        PRINT I, VAR$(I), XOLD(I)
NEXT I
INPUT "IF CORRECT, HIT <ENTER>, OTHERWISE ENTER NUMBER"; AA
IF AA <= NVARS AND AA > 0 THEN
        PRINT "ENTER NEW VALUE FOR "; VAR$(AA);
        INPUT " :", XOLD(AA)
        GOTO 175
END IF
GOSUB 640
GOSUB 1520
CLOSE #2
INPUT "RERUN WITH DIFFERENT CONVERGENCE CONDITION"; A$
IF LEFT$(UCASE$(A$), 1) = "Y" THEN REDO = 1: VH = 0: ERASE
PLSPOW, PSCOF, NGCOEF, DLTA, CNDPOW, CDCOF, XNW, WTPLS,
PWNEG, WGTNEG: GOTO 2000
INPUT "TRY ANOTHER PROBLEM (Y/N)"; A$
IF A$ = "Y" THEN 10
SYSTEM
REM *** CONDENSE POS1+ ***
180 CTEMP = 1
PTEMP = 0
SUM = 0
WSUM = 0
FOR I = 1 TO NPLS
        SUM = SUM + PSCOF(I) * XOLD(VAR) ^ PLSPOW(I)
NEXT I
FOR I = 2 TO NPLS
```

```
        WTPLS(I) = PSCOF(I) / SUM * (XOLD(VAR) ^ PLSPOW(I))
        WSUM = WSUM + WTPLS(I)
        PTEMP = PTEMP + WTPLS(I) * PLSPOW(I)
        CTEMP = CTEMP * (PSCOF(I) / WTPLS(I)) ^ WTPLS(I)
NEXT I
WTPLS(1) = 1 - WSUM
CDCOF(1) = CTEMP * (PSCOF(1) / WTPLS(1)) ^ WTPLS(1)
CNDPOW(1) = PTEMP + PLSPOW(1) * WTPLS(1)
RETURN
REM *** CONDENSE NEG1 ***
360 CTEMP = 1
PTEMP = 0
SUM = 0
WSUM = 0
FOR I = 1 TO NNEG
        SUM = SUM + NGCOEF(I) * XOLD(VAR) ^ PWNEG(I)
NEXT I
FOR I = 2 TO NNEG
        WGTNEG(I) = NGCOEF(I) / SUM * (XOLD(VAR) ^
PWNEG(I))
        WSUM = WSUM + WGTNEG(I)
        PTEMP = PTEMP + WGTNEG(I) * PWNEG(I)
        CTEMP = CTEMP * (NGCOEF(I) / WGTNEG(I)) ^ WGTNEG(I)
NEXT I
WGTNEG(1) = 1 - WSUM
CDCOF(2) = CTEMP * (NGCOEF(1) / WGTNEG(1)) ^ WGTNEG(1)
CNDPOW(2) = PTEMP + PWNEG(1) * WGTNEG(1)
RETURN
REM  *** POS1 ***
540 CNDPOW(1) = PLSPOW(1)
CDCOF(1) = PSCOF(1)
WTPLS(1) = 1
RETURN
REM *** NEG 1 ***
590 CNDPOW(2) = PWNEG(1)
CDCOF(2) = NGCOEF(1)
WGTNEG(1) = 1
RETURN
REM  *** MAIN LOOP ***
640 IF PP > 0 THEN 649
CLS : FILES "*.TRD"
INPUT "NAME OF RESULTANT DATA FILE (.TRD ADDED
AUTOMATICALLY)"; RNM$
RNM$ = RNM$ + ".TRD"
IF RNM$ = ".TRD" THEN RNM$ = LEFT$(NM$, LEN(NM$) - 4) +
".TRD"
OPEN RNM$ FOR OUTPUT AS #2
WRITE #2, "FILE: " + RNM$
WRITE #2, TITLE$
```

```
WRITE #2, DUMY$, DUMY$, TIME$, DATE$
WRITE #2, DUMY$, DUMY$, DUMY$, "TOLERANCE", " VALUE=",
STR$(TL)
WRITE #2, "******* LIST OF TERMS ******"
FOR I = 1 TO NTERMS
        Z$ = STR$(COEFF(I)) + " "
        FOR J = 1 TO NVARS
                IF POWER(I, J) = 1 THEN Z$ = Z$ + VAR$(J) +
" ": GOTO 602
                IF POWER(I, J) = 0 THEN Z$ = Z$ ELSE Z$ =
Z$ + VAR$(J) + "^" + STR$(POWER(I, J)) + " "
602     NEXT J
        WRITE #2, DUMY$, DUMY$, "TERM" + STR$(I), Z$
        Z$ = ""
NEXT I
WRITE #2, "** VARIABLE START POINTS **"
FOR I = 1 TO NVARS
        WRITE #2, DUMY$, DUMY$, VAR$(I), XOLD(I)
NEXT I
WRITE #2, "IT", "VAR", "CDCOF(1)", "CDCOF(2)", "XNW(VAR)",
"FUNVL", "VLUE"
649 VLUE = 0
650 'ZZZ$ = "PROBLEM EXECUTED: " + TITLE$
FOR VAR = 1 TO NVARS
NPLS = 0
NNEG = 0
FOR I = 1 TO NTERMS
        IF (POWER(I, VAR) <= 0) THEN 750
        NPLS = NPLS + 1
        PLSPOW(NPLS) = POWER(I, VAR)
        PSCOF(NPLS) = COEFF(I)
        FOR K = 1 TO NVARS
                IF (VAR <> K) THEN PSCOF(NPLS) = XOLD(K) ^
POWER(I, K) * PSCOF(NPLS)
        NEXT K
750     IF (POWER(I, VAR) >= 0) THEN 815
        NNEG = NNEG + 1
        PWNEG(NNEG) = POWER(I, VAR)
        NGCOEF(NNEG) = COEFF(I)
        FOR K = 1 TO NVARS
                IF (VAR <> K) THEN NGCOEF(NNEG) = XOLD(K) ^
POWER(I, K) * NGCOEF(NNEG)
        NEXT K
815 NEXT I
REM  *** CONDENSE POSITIVE TERMS ***
830 IF (NPLS > 1) THEN GOSUB 180
IF (NPLS = 1) THEN GOSUB 540
REM   *** CONDENSE NEGATIVE TERMS ***
IF (NNEG > 1) THEN GOSUB 360
```

```
IF (NNEG = 1) THEN GOSUB 590
REM *** CALCULATE NEW DELTAS ***
DLTA(2) = 1 / (1 - CNDPOW(2) / CNDPOW(1))
DLTA(1) = 1 - DLTA(2)
REM *** CALC NEW VALUE OF VAR ***
IF PP = 1 THEN 920
PRINT "PROBLEM FOR VBL "; VAR$(VAR)
ZZ$ = "Y=" + STR$(CDCOF(1)) + "*X^" + STR$(CNDPOW(1)) + " +
" + STR$(CDCOF(2)) + "*X^" + STR$(CNDPOW(2))
PRINT ZZ$
920 FUNVL = (CDCOF(1) / DLTA(1)) ^ DLTA(1) * (CDCOF(2) /
DLTA(2)) ^ DLTA(2)
XNW(VAR) = (FUNVL * DLTA(1) / CDCOF(1)) ^ (1 / CNDPOW(1))
IF PP = 1 THEN GOTO 955
PRINT "XNW("; VAR; ")="; XNW(VAR)
PRINT "TEC*= "; FUNVL
REM *** CHK FOR VAR CONVERGENCE ***
955 XH = ABS(XNW(VAR) - XOLD(VAR)) / XOLD(VAR)
IF (ABS(XNW(VAR) - XOLD(VAR)) / XOLD(VAR) <= TL) THEN 1000
XOLD(VAR) = XNW(VAR)
ITER = ITER + 1
GOSUB 1269 'CALC CURRENT OFV
WRITE #2, ITER, VAR$(VAR), CDCOF(1), CDCOF(2), XNW(VAR),
FUNVL, VLUE
GOTO 830
1000 VLUE = 0
FOR I = 1 TO NTERMS
        TEMP = 1
        FOR J = 1 TO NVARS
                TEMP = XOLD(J) ^ POWER(I, J) * TEMP
        NEXT J
        VLUE = COEFF(I) * TEMP + VLUE
NEXT I
NEXT VAR
REM *** CHK FOR PROB CONVERG ***
PRINT " OLD TTEC= "; VH; " NEW TTEC= "; VLUE
IF ((ABS(VH - VLUE) / VLUE) <= TL) THEN RETURN
VH = VLUE
GOTO 650
REM *** CALC CURRENT FUNCTION VALUE ***
1269 VLUE = 0
FOR I = 1 TO NTERMS
        TEMP = 1
        FOR J = 1 TO NVARS
                TEMP = XOLD(J) ^ POWER(I, J) * TEMP
        NEXT J
        VLUE = COEFF(I) * TEMP + VLUE
```

```
NEXT I
RETURN
REM *** READ IN PROB ***
1310 CLS
FILES "*.TDA"
PRINT "ABOVE FILES CURRENTLY EXIST.  IF YOU CHOOSE TO
OVERWRITE"
PRINT "A CURRENT FILE, USE IT'S NAME, OTHERWISE USE
ANOTHER"
INPUT "NAME OF FILE TO CREATE (.TDA IS AUTOMATICALLY
APPENDED):"; NM$
NM$ = NM$ + ".TDA"
OPEN NM$ FOR OUTPUT AS #1
INPUT "PROBLEM TITLE"; TITLE$
DDMMYY$ = DATE$
HHMM$ = TIME$
1320 INPUT "HOW MANY TERMS (<=20)"; NTERMS
IF NTERMS > 20 THEN 1320
1340 INPUT "HOW MANY VARIABLES (<=20)"; NVARS
IF NVARS > 20 THEN 1340
WRITE #1, TITLE$, DATE$, TIME$, NTERMS, NVARS
REM *** GET INIT EST OF VAR ***
FOR I = 1 TO NVARS
        PRINT "IDENTIFICATION OF VARIABLE "; I;
        INPUT "( E.G. 'X', 'Y', 'MOL')"; VAR$(I)
        WRITE #1, VAR$(I)
        PRINT "INITIAL VALUE FOR VAR "; VAR$(I);
        INPUT "' IS "; XOLD(I)
        WRITE #1, XOLD(I)
        XNW(I) = XOLD(I)
NEXT I
PRINT
REM *** GET TERMS ***
FOR I = 1 TO NTERMS
        PRINT "PLEASE ENTER DATA FOR TERM ", I
        INPUT "COEFFICIENT:"; COEFF(I)
        WRITE #1, COEFF(I)
        FOR J = 1 TO NVARS
                PRINT "POWER FOR VAR '"; VAR$(J);
                INPUT "' :"; POWER(I, J)
                WRITE #1, POWER(I, J)
        NEXT J
NEXT I
CLOSE #1
RETURN
REM *** PRINT RESULTS ***
1520 REM
FUNVL = 0
FOR I = 1 TO NTERMS
```

```
        TEMP = 1
        FOR J = 1 TO NVARS
                TEMP = (XOLD(J) ^ POWER(I, J)) * TEMP
        NEXT J
        FUNVL = COEFF(I) * TEMP + FUNVL
NEXT I
WRITE #2, ""
WRITE #2, DUMY$, DUMY$, DUMY$, DUMY$, DUMY$, "*** OBJ FUNC
VALUE ***"
WRITE #2, DUMY$, DUMY$, DUMY$, DUMY$, DUMY$, DUMY$, FUNVL
WRITE #2, ""
WRITE #2, DUMY$, DUMY$, DUMY$, DUMY$, DUMY$, "*** VARIABLE
VALS ***"
FOR I = 1 TO NVARS
        PRINT VAR$(I); " = "; XOLD(I)
        WRITE #2, DUMY$, DUMY$, DUMY$, DUMY$, DUMY$,
VAR$(I) + " =", XOLD(I)
NEXT I
PRINT "FUNCTION =", FUNVL
REM *** PRINT CONTRIB TO ANS ***
WRITE #2, ""
WRITE #2, DUMY$, DUMY$, DUMY$, DUMY$, DUMY$, "*** TERM
WEIGHTS ***"
FOR I = 1 TO NTERMS
        TEMP = 1
        FOR J = 1 TO NVARS
                TEMP = (XOLD(J) ^ POWER(I, J)) * TEMP
        NEXT J
        WEIGHT = COEFF(I) * TEMP / FUNVL
        PRINT "WEIGHT "; I; " = "; WEIGHT
        WRITE #2, DUMY$, DUMY$, DUMY$, DUMY$, DUMY$, "WGT "
+ STR$(I), WEIGHT
NEXT I
REM *** # OF ITERATIONS ***
PRINT "# OF ITERATIONS = "; ITER
RETURN
2000 REM INPUT PROBLEM FROM EXISTING FILE
CLS : IF REDO = 1 THEN GOTO 2200
FILES "*.TDA"
INPUT "NAME OF EXISTING FILE TO USE (DO NOT INCLUDE .TDA
EXTENTION"; NM$
NM$ = NM$ + ".TDA"
IF NM$ = ".TDA" THEN NM$ = "DEFAULT.TDA"
2200 OPEN NM$ FOR INPUT AS #1
DO WHILE NOT EOF(1)
        INPUT #1, TITLE$, DDMMYY$, HHMM$, NTERMS, NVARS
        FOR I = 1 TO NVARS
                INPUT #1, VAR$(I), XOLD(I)
                XNW(I) = XOLD(I)
```

```
        NEXT I
        FOR I = 1 TO NTERMS
                INPUT #1, COEFF(I)
                FOR J = 1 TO NVARS
                        INPUT #1, POWER(I, J)
                NEXT J
        NEXT I
LOOP
CLOSE #1
IF REDO = 1 THEN ITER = 0: FUNVL = 0
GOTO 155
```

APP. E. C PROGRAM OF ORIGINAL ALGORITHM

```
/* MULTICON PROGRAM
  ORIGINALLY WRITTEN IN MS-FORTRAN
  BY RICHARD RATLIFF
  DATE STARTED: 19 FEB 90
  DATE UPDATED: 19 FEB 90          */
#include "c:\c\include\stdio.h"
#include "c:\c\include\math.h"
main()
{
  int i,j,k,iter=0,nneg=0,npls=0,nterms,nvars,var;
  float coeff[50], concof[2],ngcoef[50],conpow[2];
  float delta[2],fncval,power[50][50],powneg[50];
  float plspow[50],tol,wgtneg[50],wgtpos[50];
  float xnew[50],xold[50],xhat[50],value,temp,valhat=0.0;
  float pscof[50],sp,weight,negcnt,poscnt,x[50];
  float pwer[50],wgt[50],ctemp,ptemp,sum,wsum;

  printf("\n Generalized Multi-variable Posynomial");
  printf(" Condensation Program \n");
  /* this section replaces the rddata function */
  nterms=5,tol=.0001,nvars=3,sp=1.0;
  for(i=1;i<=nvars;i++)
    xold[i]=sp;
  coeff[1]= 40, coeff[2]= 10, coeff[3]= 20;
  coeff[4]= 40, coeff[5]= 10;
  power[1][1]=-1, power[1][2]=-1, power[1][3]=-1;
  power[2][1]= 0, power[2][2]= 1, power[2][3]= 1;
  power[3][1]= 1, power[3][2]= 1, power[1][3]= 0;
  power[4][1]= 1, power[4][2]= 0, power[2][3]= 1;
  power[5][1]= 0, power[5][2]= 1, power[1][3]= 0;
  /* this is the end of the section which replaces the
  rddata function                                          */
c10:     for(var=1;var<=nvars;var++)
  {
  npls=0, nneg=0;
    for(i=1;i<=nterms;i++)
    {
    if(power[i][var]>0.0, npls+=npls+1,0);
    plspow[npls]=power[i][var];
    pscof[npls]=coeff[i];
      for(k=1;k<=nvars;k++){
        if(var!=k)
          pscof[npls]=pow(xold[k],power[i][k])*pscof[npls];
        }
    if(power[i][var]<0.0){
```

```
      nneg=+1, powneg[nneg]=power[i][var];
      ngcoef[nneg]=coeff[i];
      for(k=1;k<=nvars;k++){
         if(var!=k)

ngcoef[nneg]=pow(xold[k],power[i][k])*ngcoef[nneg];
         }
    }
c20:    if(npls>1){
    ctemp=1.0;
    ptemp=0.0, sum=0.0;
    wsum=0.0;
  for(i=1;i<=nterms;i++)
    sum=sum+coeff[i]*pow(xold[var],pwer[i]);
  for(i=2;i<=nterms;i++) {
    wgt[i]=coeff[i]/sum*pow(xold[var],pwer[i]);
    wsum=wsum+wgt[i];
    ptemp=ptemp+pwer[i]*wgt[i];
    ctemp=ctemp*pow(coeff[i]/wgt[i],wgt[i]);
  }
  wgt[1]=1.0-wsum;
  concof[1]=ctemp*pow(coeff[1]/wgt[1],wgt[1]);
  conpow[1]=ptemp+pwer[1]*wgt[1];
}
  else {
    conpow[1]=plspow[1];
    concof[1]=pscof[1];
    wgtpos[1]=1.0;
  }
  if(nneg>1){
  float pwer[50], wgt[50];
  ctemp=1.0;
  ptemp=0.0, sum=0.0;
  wsum=0.0;
  for(i=1;i<=nterms;i++)
    sum=sum+coeff[i]*pow(xold[var],pwer[i]);
  for(i=2;i<=nterms;i++) {
    wgt[i]=coeff[i]/sum*pow(xold[var],pwer[i]);
    wsum=wsum+wgt[i];
    ptemp=ptemp+pwer[i]*wgt[i];
    ctemp=ctemp*pow(coeff[i]/wgt[i],wgt[i]);
  }
  wgt[1]=1.0-wsum;
  concof[2]=ctemp*pow(coeff[1]/wgt[1],wgt[1]);
  conpow[2]=ptemp+pwer[1]*wgt[1];
}
  else {
    conpow[2]=powneg[1];
    concof[2]=ngcoef[1];
```

```
    wgtneg[1]=1.0;
  }
  delta[2]=pow(1.0-conpow[2]/conpow[1],-1);
  delta[1]=1.0-delta[2];
  fncval=pow(concof[1]/delta[1],delta[1])*pow(concof[2]/delt
a[2],delta[2]);
  xnew[var]=pow(fncval*delta[1]/concof[1],1.0/conpow[1]);
/* Check & see if it has converged yet */
  if(abs(xnew[var]-xold[var])/xold[var]>tol) {
    xold[var]=xnew[var];
    iter=iter+1;
    goto c20;
    /* goto 20 which is if(npls>1) */
  }
  value=0.0;
  for(i=1;i<=nterms;i++) {
    temp=1.0;
    for(j=1;j<=nvars;j++) {
      temp=pow(xold[j],power[i][j])*temp;
    }
  value=coeff[i]*temp+value;
  }
  }
  if(abs(valhat-value)/value>tol) {
    valhat=value;
    }
  else {
negcnt=1.0;
poscnt=1;
fncval=0.0;
for(i=1;i<=nterms;++i){
  temp=1.0;
  for(j=1;j<=nvars;++i){
    temp=pow(xold[j],power[i][j])*temp;
  }
  fncval=(coeff[i]*temp)+fncval;
}
for(k=1;k<=nvars;++i){
  printf("  ");
  printf(" The optimal value of variable %d is
%d",k,xold[k]);
}
  printf(" ");
  printf(" The value of the function at optimality is
%d",fncval);
```

```
  for(i=1;i<=nterms;++i){
    temp=1.0;
    for(j=1;j<=nvars;++i){
      temp=pow(xold[j],power[i][j])*temp;
    }
    weight=(coeff[i]*temp)/fncval;
    printf(" Weight %d = %d",i,weight);
  }
    printf(" Number of iterations is %d",iter);
    goto c200;
  }
  /* goto 10 */
  goto c10;
c200:   ;
  /* call sense */
printf("\n Sensitivity analysis? [ 0= no, 1 = yes] ");
scanf("%i",i);
if(i!=1)
  return;
for(j=1;j<=nvars;++j){
  printf(" Enter new value for variable %i:  [decimal
required] ",j);
  scanf("%d",x[j]);
}
value=0.0;
for(j=1;j<=nterms;++j){
  temp=1.0;
  for(k=1;k<=nvars;++k){
    temp=temp*pow(x[k],power[j][k]);
  }
  value=value+coeff[j]*temp;
}
printf(" The new function value is %d",value);
goto c200;
}

/*
printf("\n Enter the number of terms: [no decimals] ");
scanf("%i";nterms);
printf(" Enter the convergence condition:  [decimal
required]");
scanf("%d";tol);
printf(" Enter the # of variables considered:  [no
decimals] ");
scanf("%i";nvars);
for(i=1;i<=nvars;++i){
  printf(" Enter an initial value for variable %i";i);
  scanf("%d";xold[i]);
  xhat[i]=xold[i];
```

```
}
/* Get the terms of the objective function */
for(i=1;i<=nterms;++i){
  printf("\n Enter the data for term %i",i);
  printf(" Coefficient:  [decimal required] ");
  scanf("%d",coeff[i]);
  for(j=1;j<=nvars;++i){
     printf(" Enter the power for variable %i:  [decimal
required]",j);
     scanf("%d",power[i][j]);
  }
}
return;
}
```

APP. F. COMPUTATION

This appendix presents a compilation of the results of the computer runs as described in the mainbody of the thesis. Each of the four problems are presented in turn with the four divisions of analysis.

The first division, machine comparison, displays the results of the published results of Ratliff and that of the PC FORTRAN runs conducted on the test computer. The output is shown to the level of precision presented by Ratliff or generated by the PC runs. In each case, the optimal values of all variables involved are displayed. This is followed by the optimal cost or value as generated by the output using the original input parameters. The weights or contributions of each term are then displayed. Finally, the starting values of each variable and the convergence condition (tolerance) is shown.

In all subsequent sections for the same problem, the format of the output is the same:

1- Filename where the output is stored.

2- Short Identification of the problem used.

3- Time and date of the run which generated the file.

4- Tolerance value used. May be displayed in scientific notation format with E (D is used for double precision) denoting previous value should be multiplied by 10 raised to the power of the subsequent value. For example, 9.999999999999999D-12 is read 9.999999999999999 times 10 to the -12th power and is equal to .000000000009999999999999999 which is the closest the computer can express 1E-11.

5- List of terms in problem. Each term is presented with the included variables shown raised ('^') to the respective exponent value.

6- Starting values. The initial value of each variable is presented. Collectively, these may be referred to as the starting vector.

7- Table of iteration outputs. Here the information generated during the run is displayed. Refer to the program and pseudoprogram listings for further information. Please note that all displays are limited to the decimal values displayed by Lotus 1-2-3 as the software was used to format the output. The actual file may contain more significant digits. An example line of the display follows:

IT	VAR	CDCOF(1)	CDCOF(2)	XNW(VAR)	FUNVL	VLUE
1	H	60	40	0.816496	97.97958	115.716960883

IT- Iteration identification.

VAR - Variable under scrutiny.

CDCOF(1) - Coefficient of 1st term of pseudo problem. This represents the coefficients generated by the condensation of all locations in the original function where the variable under scrutiny appears with positive exponents.

CDCOF(2) - Coefficient of 2nd term of pseudo problem. This represents the coefficients generated by the condensation of all locations in the original function where the variable under scrutiny appears with negative exponents.

XNW(VAR) - The latest estimated value of the variable under scrutiny.

FUNVL - The value of the pseudo problem when the value XNW(VAR) is used in both terms.

VLUE - The value of the last objective function value calculated

8 - The value of the final objective function as evaluated with the final values of the individual variables.

9 - The final value of each variable.

10 - The weights (proportional contributions) of each term to the overall objective function value. The iterative weights of each term grouped by variable are displayed for the Gravelbox problem only and are read as the current term weights of term 1 through 5 respectively.

Problem 1 - Gravelbox Problem

Machine Comparison

	Ratliff	PC FORTRAN
OPTIMAL H	.5952	.59505022
OPTIMAL L	1.2942	1.29346134
OPTIMAL W	1.1884	1.18939703
OPTIMAL COST	115.72	115.71696088
WGT 1	0.3776	0.3776
WGT 2	0.1329	0.1329
WGT 3	0.1331	0.1330
WGT 4	0.2445	0.2446
WGT 5	0.1118	0.1118
INIT VAL OF H	1.0	1.0
INIT VAL OF L	1.0	1.0
INIT VAL OF W	1.0	1.0
CONV COND	1E-7	1E-7
ITER	43	48

Language Comparison

```
FILE: GRAV7E.TRD
GRAVELBOX PROBLEM
       13:04:08 03-17-1990
                TOLERANCE VALUE=   .0000001
******* LIST OF TERMS ******
       TERM 1    40 H^-1 L^-1 W^-1
       TERM 2    10 L W
       TERM 3    20 H L
       TERM 4    40 H W
       TERM 5    10 L
** VARIABLE START POINTS **
       H               1
       L               1
       W               1
IT VAR CDCOF(1) CDCOF(2) XNW(VAR) FUNVL    VLUE
 1 H         60       40 0.816496 97.97958 117.979589711
 2 L   36.32993 48.98979 1.161236 84.37525 117.035117835
 3 W   44.27222 42.18762 0.976173 86.43471 117.009983769
 4 H   62.27164 35.28682 0.752767 93.75220 116.700237975
 5 L   34.81709 54.43421 1.250373 87.06873 116.462010356
 6 W   42.61445 42.49708 0.998621 85.11145 116.440010794
 7 H   64.95234 32.03458 0.702282 91.22985 116.220094279
 8 L   34.03187 57.03569 1.294584 88.11428   116.1668948
 9 W   41.03716 43.99643 1.035428 84.98208   6.111224563
10 H   67.30882 29.84073 0.665838 89.63358 115.983924589
11 L   33.67104 58.01912 1.312675 88.39829 115.975412552
12 W   39.76028 45.76505 1.072858 85.31427 115.921615148
13 H   69.16782 28.40275 0.640808 88.64664 115.856534233
14 L   33.54474 58.18212 1.316990 88.35620 115.856058438
15 W   38.80223 47.39682 1.105213 85.76952 115.818195322
16 H   70.54835 27.48091 0.624125 88.06209 115.787555759
17 L   33.53465 57.98845 1.314994 88.19575  115.78745433
18 W   38.11497 48.73758 1.130795 86.20050  15.764885041
19 H   71.53171 26.89998 0.613234 87.73144 115.751290045
20 L   33.57264 57.68317 1.310785 88.01311 115.750837895
21 W   37.63724 49.76242 1.149851 86.55450 115.738751404
22 H   72.20978 26.53911 0.606240 87.55303  115.73299165
23 L   33.62333 57.38164 1.306369 87.84901  15.732491381
24 W   37.31332 50.50667 1.163435 86.82331 115.726503534
25 H   72.66482 26.31791 0.601815 87.46168 115.724156694
26 L   33.67067 57.12865 1.302570 87.71682 115.723784568
27 W   37.09833 51.02643 1.172790 87.01714 115.72099408
28 H   72.96301 26.18415 0.599056 87.41795 115.720071015
29 L   33.70903 56.93400 1.299608 87.61713  115.71984411
30 W   36.95835 51.37826 1.179053 87.15173  15.718608079
```

31	H	73.15430	26.10440	0.597361	87.39908	115.718257061
32	L	33.73775	56.79231	1.297438	87.54530	115.71813474
33	W	36.86882	51.61029	1.183146	87.24244	15.717610852
34	H	73.27462	26.05762	0.596335	87.39250	115.717481795
35	L	33.75817	56.69318	1.295913	87.49533	115.717421297
36	W	36.81255	51.75989	1.185765	87.30209	115.717207894
37	H	73.34888	26.03066	0.595725	87.39153	115.717162075
38	L	33.77215	56.62593	1.294876	87.46152	115.717134047
39	W	36.77776	51.85442	1.187408	87.34048	115.71705028
40	H	73.39387	26.01545	0.595368	87.39279	15.717034612
41	L	33.78145	56.58143	1.294189	87.43919	115.717022295
42	W	36.75663	51.91302	1.188421	87.36470	115.71699057
43	H	73.42062	26.00709	0.595164	87.39467	115.71698543
44	L	33.78749	56.55262	1.293744	87.42474	15.716980256
45	W	36.74401	51.94870	1.189033	87.37972	115.716968662
46	H	73.43621	26.00264	0.595050	87.39647	115.716967055
47	L	33.79133	56.53433	1.293461	87.41558	115.716964967
48	W	36.73662	51.97002	1.189397	87.38885	115.716960883

*** OBJ FUNC VALUE ***
115.716960883
*** VARIABLE VALS ***
H = 0.5950502213
L = 1.2934613354
W = 1.1893970268
*** TERM WEIGHTS ***
WGT 1 0.3775974489
WGT 2 0.1329484507
WGT 3 0.1330270771
WGT 4 0.2446489982
WGT 5 0.1117780251

*** INTERIM DELTAS FOR VAR H ***

1	0.41524	0.08476	0.138413	0.276826	0.08476
4	0.40168	0.097135	0.14981	0.25187	0.099506
7	0.392487	0.107438	0.151113	0.241375	0.107587
10	0.386405	0.115572	0.148639	0.237767	0.111618
13	0.382571	0.121557	0.145209	0.237361	0.113302
16	0.380274	0.125709	0.141979	0.238296	0.113742
19	0.378965	0.128464	0.139333	0.239632	0.113605
22	0.378254	0.130232	0.137325	0.240929	0.113259

25	0.377889	0.131336	0.135874	0.242015	0.112887
28	0.377713	0.132012	0.134862	0.242851	0.112562
31	0.377637	0.132417	0.134177	0.243461	0.112308
34	0.377612	0.132656	0.133724	0.243888	0.112121
37	0.377608	0.132794	0.13343	0.244178	0.11199
40	0.377614	0.132871	0.133244	0.24437	0.1119
43	0.377623	0.132914	0.133127	0.244495	0.111841
46	0.37763	0.132937	0.133056	0.244574	0.111802

*** INTERIM DELTAS FOR VAR L ***

2	0.36047	0.099221	0.162027	0.27906	0.099221
5	0.373807	0.104805	0.161639	0.252385	0.107363
8	0.379257	0.111288	0.156527	0.241485	0.111442
11	0.381108	0.117196	0.150727	0.237784	0.113186
14	0.381319	0.121957	0.145687	0.237362	0.113675
17	0.380852	0.125519	0.141764	0.238296	0.11357
20	0.380183	0.128054	0.138888	0.239633	0.113242
23	0.379535	0.129793	0.136863	0.24093	0.112878
26	0.378992	0.130955	0.135479	0.242016	0.112559
29	0.378574	0.131712	0.134556	0.242851	0.112306
32	0.37827	0.132196	0.133953	0.243461	0.112121
35	0.378056	0.1325	0.133567	0.243888	0.111989
38	0.377911	0.132687	0.133323	0.244178	0.1119
41	0.377815	0.132801	0.133173	0.244371	0.111841
44	0.377752	0.132868	0.133082	0.244495	0.111802

47	0.377713	0.132908	0.133027	0.244574	0.111778

*** INTERIM DELTAS FOR VAR W ***

3	0.369348	0.096878	0.162062	0.27247	0.099242
6	0.365473	0.107236	0.16167	0.258238	0.107383
9	0.365951	0.115445	0.156602	0.250506	0.111495
12	0.367983	0.121488	0.150797	0.246494	0.113238
15	0.370277	0.125676	0.145735	0.244601	0.113712
18	0.372309	0.128449	0.141791	0.243859	0.113592
21	0.373922	0.130225	0.138902	0.243697	0.113254
24	0.375123	0.131334	0.13687	0.243789	0.112884
27	0.375978	0.132011	0.135482	0.243967	0.112561
30	0.376567	0.132417	0.134557	0.244151	0.112308
33	0.376963	0.132656	0.133954	0.244307	0.112121
36	0.377222	0.132793	0.133567	0.244428	0.11199
39	0.377388	0.132871	0.133324	0.244517	0.1119
42	0.377493	0.132914	0.133173	0.244579	0.111841
45	0.377558	0.132937	0.133082	0.244621	0.111802
48	0.377597	0.132948	0.133027	0.244649	0.111778

Convergence

```
FILE: GRAV2E.TRD
GRAVELBOX PROBLEM
       16:40:30 03-17-1990
                TOLERANCE VALUE=    .01
******* LIST OF TERMS ******
       TERM 1    40 H^-1 L^-1 W^-1
       TERM 2    10 L W
       TERM 3    20 H L
```

```
       TERM 4    40 H W
       TERM 5    10 L
** VARIABLE START POINTS **
       H                1
       L                1
       W                1
IT VAR CDCOF(1) CDCOF(2) XNW(VAR) FUNVL    VLUE
 1 H         60       40 0.816496 97.97958 117.979589711
 2 L   36.32993 48.98979 1.161236 84.37525 117.035117835
 3 W   44.27222 42.18762 0.976173 86.43471 117.009983769
 4 H   62.27164 35.28682 0.752767 93.75220 116.700237975
 5 L   34.81709 54.43421 1.250373 87.06873 116.462010356
 6 W   42.61445 42.49708 0.998621 85.11145 116.440010794
                                  *** OBJ FUNC VALUE ***
                                           116.440010794
                                  *** VARIABLE VALS ***
                                  H =       0.7527679323
                                  L =       1.2503735011
                                  W =       0.9986219426
                                  *** TERM WEIGHTS ***
                                  WGT  1    0.3654734043
                                  WGT  2    0.1072355117
                                  WGT  3     0.161669699
                                  WGT  4    0.2582378925
                                  WGT  5    0.1073834924
```

```
FILE: GRAV4E.TRD
GRAVELBOX PROBLEM
       16:40:53 03-17-1990
                TOLERANCE VALUE=   .0001
******* LIST OF TERMS ******
       TERM 1    40 H^-1 L^-1 W^-1
       TERM 2    10 L W
       TERM 3    20 H L
       TERM 4    40 H W
       TERM 5    10 L
** VARIABLE START POINTS **
       H               1
       L               1
       W               1
```

IT	VAR	CDCOF(1)	CDCOF(2)	XNW(VAR)	FUNVL	VLUE
1	H	60	40	0.816496	97.97958	117.979589711
2	L	36.32993	48.98979	1.161236	84.37525	117.035117835
3	W	44.27222	42.18762	0.976173	86.43471	117.009983769
4	H	62.27164	35.28682	0.752767	93.75220	116.700237975
5	L	34.81709	54.43421	1.250373	87.06873	116.462010356
6	W	42.61445	42.49708	0.998621	85.11145	116.440010794
7	H	64.95234	32.03458	0.702282	91.22985	116.220094279
8	L	34.03187	57.03569	1.294584	88.11428	116.1668948
9	W	41.03716	43.99643	1.035428	84.98208	6.111224563
10	H	67.30882	29.84073	0.665838	89.63358	115.983924589
11	L	33.67104	58.01912	1.312675	88.39829	115.975412552
12	W	39.76028	45.76505	1.072858	85.31427	115.921615148
13	H	69.16782	28.40275	0.640808	88.64664	115.856534233
14	L	33.54474	58.18212	1.316990	88.35620	115.856058438
15	W	38.80223	47.39682	1.105213	85.76952	115.818195322
16	H	70.54835	27.48091	0.624125	88.06209	115.787555759
17	L	33.53465	57.98845	1.314994	88.19575	115.78745433
18	W	38.11497	48.73758	1.130795	86.20050	15.764885041
19	H	71.53171	26.89998	0.613234	87.73144	115.751290045
20	L	33.57264	57.68317	1.310785	88.01311	115.750837895
21	W	37.63724	49.76242	1.149851	86.55450	115.738751404
22	H	72.20978	26.53911	0.606240	87.55303	115.73299165
23	L	33.62333	57.38164	1.306369	87.84901	15.732491381
24	W	37.31332	50.50667	1.163435	86.82331	115.726503534
25	H	72.66482	26.31791	0.601815	87.46168	115.724156694
26	L	33.67067	57.12865	1.302570	87.71682	115.723784568
27	W	37.09833	51.02643	1.172790	87.01714	115.720994086

```
*** OBJ FUNC VALUE ***
         115.720994086
*** VARIABLE VALS ***
H =        0.6018158823
L =        1.3025700206
W =        1.1727904526
```

```
*** TERM WEIGHTS ***
WGT  1     0.3759782226
WGT  2      0.132010764
WGT  3     0.1354823008
WGT  4     0.2439674586
WGT  5      0.112561254
```

FILE: GRAV8E.TRD
GRAVELBOX PROBLEM
16:41:18 03-17-1990
TOLERANCE VALUE= .00000001

******* LIST OF TERMS ******

TERM 1	40	H^-1 L^-1 W^-1
TERM 2	10	L W
TERM 3	20	H L
TERM 4	40	H W
TERM 5	10	L

** VARIABLE START POINTS **

H	1
L	1
W	1

IT	VAR	CDCOF(1)	CDCOF(2)	XNW(VAR)	FUNVL	VLUE
1	H	60	40	0.816496	97.97958	117.979589711
2	L	36.32993	48.98979	1.161236	84.37525	117.035117835
3	W	44.27222	42.18762	0.976173	86.43471	117.009983769
4	H	62.27164	35.28682	0.752767	93.75220	116.700237975
5	L	34.81709	54.43421	1.250373	87.06873	116.462010356
6	W	42.61445	42.49708	0.998621	85.11145	116.440010794
7	H	64.95234	32.03458	0.702282	91.22985	116.220094279
8	L	34.03187	57.03569	1.294584	88.11428	116.1668948
9	W	41.03716	43.99643	1.035428	84.98208	6.111224563
10	H	67.30882	29.84073	0.665838	89.63358	115.983924589
11	L	33.67104	58.01912	1.312675	88.39829	115.975412552
12	W	39.76028	45.76505	1.072858	85.31427	115.921615148
13	H	69.16782	28.40275	0.640808	88.64664	115.856534233
14	L	33.54474	58.18212	1.316990	88.35620	115.856058438
15	W	38.80223	47.39682	1.105213	85.76952	115.818195322
16	H	70.54835	27.48091	0.624125	88.06209	115.787555759
17	L	33.53465	57.98845	1.314994	88.19575	115.78745433
18	W	38.11497	48.73758	1.130795	86.20050	15.764885041
19	H	71.53171	26.89998	0.613234	87.73144	115.751290045
20	L	33.57264	57.68317	1.310785	88.01311	115.750837895
21	W	37.63724	49.76242	1.149851	86.55450	115.738751404
22	H	72.20978	26.53911	0.606240	87.55303	115.73299165
23	L	33.62333	57.38164	1.306369	87.84901	15.732491381
24	W	37.31332	50.50667	1.163435	86.82331	115.726503534
25	H	72.66482	26.31791	0.601815	87.46168	115.724156694
26	L	33.67067	57.12865	1.302570	87.71682	115.723784568
27	W	37.09833	51.02643	1.172790	87.01714	115.720994086
28	H	72.96301	26.18415	0.599056	87.41795	115.720071015
29	L	33.70903	56.93400	1.299608	87.61713	115.71984411
30	W	36.95835	51.37826	1.179053	87.15173	15.718608079
31	H	73.15430	26.10440	0.597361	87.39908	115.718257061
32	L	33.73775	56.79231	1.297438	87.54530	115.71813474
33	W	36.86882	51.61029	1.183146	87.24244	15.717610852
34	H	73.27462	26.05762	0.596335	87.39250	115.717481795

35	L	33.75817	56.69318	1.295913	87.49533	115.717421297
36	W	36.81255	51.75989	1.185765	87.30209	115.717207894
37	H	73.34888	26.03066	0.595725	87.39153	115.717162075
38	L	33.77215	56.62593	1.294876	87.46152	115.717134047
39	W	36.77776	51.85442	1.187408	87.34048	115.71705028
40	H	73.39387	26.01545	0.595368	87.39279	115.717034612
41	L	33.78145	56.58143	1.294189	87.43919	115.717022295
42	W	36.75663	51.91302	1.188421	87.36470	115.71699057
43	H	73.42062	26.00709	0.595164	87.39467	115.71698543
44	L	33.78749	56.55262	1.293744	87.42474	15.716980256
45	W	36.74401	51.94870	1.189033	87.37972	115.716968662
46	H	73.43621	26.00264	0.595050	87.39647	115.716967055
47	L	33.79133	56.53433	1.293461	87.41558	115.716964967
48	W	36.73662	51.97002	1.189397	87.38885	115.716960883
49	H	73.44510	26.00037	0.594988	87.39795	115.716960409
50	L	33.79373	56.52293	1.293285	87.40986	115.716959598
51	W	36.73238	51.98251	1.189608	87.39431	115.716958215
52	H	73.45004	25.99929	0.594955	87.39907	115.716958085
53	L	33.79520	56.51596	1.293177	87.40637	115.716957781
54	W	36.73000	51.98968	1.189729	87.39751	115.716957332

*** OBJ FUNC VALUE ***
115.716957332

*** VARIABLE VALS ***

H =	0.5949558872
L =	1.2931771371
W =	1.18972914

*** TERM WEIGHTS ***

WGT	1	0.3776348815
WGT	2	0.1329563582
WGT	3	0.1329767683
WGT	4	0.2446785233
WGT	5	0.1117534687

```
FILE: GRAV11E.TRD
GRAVELBOX PROBLEM
       16:42:01 03-17-1990
                TOLERANCE VALUE=   9.999999999999999D-12
******* LIST OF TERMS ******
       TERM 1    40 H^-1 L^-1 W^-1
       TERM 2    10 L W
       TERM 3    20 H L
       TERM 4    40 H W
       TERM 5    10 L
** VARIABLE START POINTS **
       H                1
       L                1
       W                1
IT VAR CDCOF(1) CDCOF(2) XNW(VAR) FUNVL    VLUE
 1 H         60       40 0.816496 97.97958 117.979589711
 2 L   36.32993 48.98979 1.161236 84.37525 117.035117835
 3 W   44.27222 42.18762 0.976173 86.43471 117.009983769
 4 H   62.27164 35.28682 0.752767 93.75220 116.700237975
 5 L   34.81709 54.43421 1.250373 87.06873 116.462010356
 6 W   42.61445 42.49708 0.998621 85.11145 116.440010794
 7 H   64.95234 32.03458 0.702282 91.22985 116.220094279
 8 L   34.03187 57.03569 1.294584 88.11428   116.1668948
 9 W   41.03716 43.99643 1.035428 84.98208   6.111224563
10 H   67.30882 29.84073 0.665838 89.63358 115.983924589
11 L   33.67104 58.01912 1.312675 88.39829 115.975412552
12 W   39.76028 45.76505 1.072858 85.31427 115.921615148
13 H   69.16782 28.40275 0.640808 88.64664 115.856534233
14 L   33.54474 58.18212 1.316990 88.35620 115.856058438
15 W   38.80223 47.39682 1.105213 85.76952 115.818195322
16 H   70.54835 27.48091 0.624125 88.06209 115.787555759
17 L   33.53465 57.98845 1.314994 88.19575  115.78745433
18 W   38.11497 48.73758 1.130795 86.20050  15.764885041
19 H   71.53171 26.89998 0.613234 87.73144 115.751290045
20 L   33.57264 57.68317 1.310785 88.01311 115.750837895
21 W   37.63724 49.76242 1.149851 86.55450 115.738751404
22 H   72.20978 26.53911 0.606240 87.55303  115.73299165
23 L   33.62333 57.38164 1.306369 87.84901  15.732491381
24 W   37.31332 50.50667 1.163435 86.82331 115.726503534
25 H   72.66482 26.31791 0.601815 87.46168 115.724156694
26 L   33.67067 57.12865 1.302570 87.71682 115.723784568
27 W   37.09833 51.02643 1.172790 87.01714 115.720994086
28 H   72.96301 26.18415 0.599056 87.41795 115.720071015
29 L   33.70903 56.93400 1.299608 87.61713  115.71984411
30 W   36.95835 51.37826 1.179053 87.15173  15.718608079
31 H   73.15430 26.10440 0.597361 87.39908 115.718257061
32 L   33.73775 56.79231 1.297438 87.54530  115.71813474
33 W   36.86882 51.61029 1.183146 87.24244  15.717610852
34 H   73.27462 26.05762 0.596335 87.39250 115.717481795
```

35	L	33.75817	56.69318	1.295913	87.49533	115.717421297
36	W	36.81255	51.75989	1.185765	87.30209	115.717207894
37	H	73.34888	26.03066	0.595725	87.39153	115.717162075
38	L	33.77215	56.62593	1.294876	87.46152	115.717134047
39	W	36.77776	51.85442	1.187408	87.34048	115.71705028
40	H	73.39387	26.01545	0.595368	87.39279	15.717034612
41	L	33.78145	56.58143	1.294189	87.43919	115.717022295
42	W	36.75663	51.91302	1.188421	87.36470	115.71699057
43	H	73.42062	26.00709	0.595164	87.39467	115.71698543
44	L	33.78749	56.55262	1.293744	87.42474	15.716980256
45	W	36.74401	51.94870	1.189033	87.37972	115.716968662
46	H	73.43621	26.00264	0.595050	87.39647	115.716967055
47	L	33.79133	56.53433	1.293461	87.41558	115.716964967
48	W	36.73662	51.97002	1.189397	87.38885	115.716960883
49	H	73.44510	26.00037	0.594988	87.39795	115.716960409
50	L	33.79373	56.52293	1.293285	87.40986	115.716959598
51	W	36.73238	51.98251	1.189608	87.39431	115.716958215
52	H	73.45004	25.99929	0.594955	87.39907	115.716958085
53	L	33.79520	56.51596	1.293177	87.40637	115.716957781
54	W	36.73000	51.98968	1.189729	87.39751	115.716957332
55	H	73.45270	25.99883	0.594939	87.39988	115.7169573
56	L	33.79608	56.51176	1.293112	87.40426	15.71695719
57	W	36.72871	51.99370	1.189796	87.39935	15.716957052
58	H	73.45408	25.99867	0.594932	87.40043	115.716957046
59	L	33.79660	56.50928	1.293073	87.40302	115.716957007
60	W	36.72803	51.99588	1.189831	87.40038	115.716956967
61	H	73.45475	25.99866	0.594929	87.40081	115.716956966
62	L	33.79691	56.50785	1.293051	87.40230	115.716956953
63	W	36.72769	51.99703	1.189850	87.40094	115.716956943
64	H	73.45505	25.99870	0.594928	87.40105	115.716956943
65	L	33.79708	56.50704	1.293039	87.40190	115.716956939
66	W	36.72754	51.99760	1.189859	87.40123	115.716956936
67	H	73.45516	25.99875	0.594928	87.40121	115.716956936
68	L	33.79717	56.50659	1.293032	87.40167	115.716956935
69	W	36.72748	51.99786	1.189863	87.40138	115.716956934
70	H	73.45518	25.99880	0.594929	87.40131	115.716956934
71	L	33.79722	56.50636	1.293028	87.40156	115.716956934
72	W	36.72746	51.99796	1.189865	87.40145	115.716956934

*** OBJ FUNC VALUE ***
115.716956934

*** VARIABLE VALS ***
H = 0.5949294658
L = 1.2930286586
W = 1.1898650349

*** TERM WEIGHTS ***
WGT 1 0.377651883
WGT 2 0.132956278

WGT	3	0.1329555961
WGT	4	0.244695605
WGT	5	0.1117406379

Starting Vector

```
FILE: GRAV2S.TRD
GRAVELBOX PROBLEM
       16:47:50 03-17-1990
                TOLERANCE VALUE=    .0000001
******* LIST OF TERMS ******
       TERM 1    40 H^-1 L^-1 W^-1
       TERM 2    10 L W
       TERM 3    20 H L
       TERM 4    40 H W
       TERM 5    10 L
** VARIABLE START POINTS **
       H           0.002
       L           0.002
       W           0.002
```

IT	VAR	CDCOF(1)	CDCOF(2)	XNW(VAR)	FUNVL	VLUE
1	H	0.12	10000000	9128.709	2190.890	2190.91027
2	L	182584.2	2.190890	0.003464	1264.945	95.2425172
3	W	365148.4	1.264945	0.001861	1359.254	1991.726927
4	H	0.143729	6204117.	6570.024	1888.613	1888.647937
5	L	131410.5	3.271084	0.004989	1311.266	800.4008031
6	W	262801.0	1.220287	0.002154	1132.594	1788.2280387
7	H	0.185978	3720588.	4472.753	1663.668	1663.7180897
8	L	89465.08	4.150185	0.006810	1218.682	1604.2073427
9	W	178910.1	1.313040	0.002709	969.3633	1578.7043909
10	H	0.244581	2167861.	2977.169	1456.323	1456.3919631
11	L	59553.42	4.959465	0.009125	1086.928	1409.5434799
12	W	119086.8	1.472286	0.003516	837.4485	1380.9123104
13	H	0.323158	1246613.	1964.076	1269.414	1269.5058276
14	L	39291.56	5.792118	0.012141	954.1098	1230.3472612
15	W	78563.17	1.677384	0.004620	726.0320	1203.0865251
16	H	0.427655	712991.1	1291.204	1104.381	1104.5037443
17	L	25834.13	6.704375	0.016109	832.3501	1071.0002961
18	W	51648.32	1.923015	0.006101	630.3032	1046.4775923
19	H	0.566265	406925.1	847.7100	960.0572	960.21934492
20	L	16964.26	7.733024	0.021350	724.3895	931.29436405
21	W	33908.61	2.210067	0.008073	547.5045	909.69804866
22	H	0.749938	232062.5	556.2751	834.3445	834.5597339
23	L	11135.58	8.906818	0.028281	629.8654	09.50313508
24	W	22251.29	2.542526	0.010689	475.7078	790.63834098
25	H	0.993210	132312.2	364.9885	725.0212	725.30710877
26	L	7309.878	10.25240	0.037450	547.5173	703.57832902
27	W	14599.91	2.926328	0.014157	413.3964	687.15107383
28	H	1.315309	75442.48	239.4936	630.0166	630.39644712
29	L	4800.014	11.79722	0.049575	475.9278	611.55301223
30	W	9580.241	3.368972	0.018752	359.3080	597.26494179

31	H	1.741615	43025.99	157.1771	547.4842	547.98928315
32	L	3153.729	13.57094	0.065598	413.7588	531.6577922
33	W	6287.740	3.879518	0.024839	312.3677	19.23490519
34	H	2.305544	24548.53	103.1872	475.8055	476.47778067
35	L	2073.993	15.60602	0.086744	359.8154	462.33998265
36	W	4128.357	4.468805	0.032900	271.6529	451.53911648
37	H	3.050926	14015.55	67.77809	413.5719	414.46790935
38	L	1365.890	17.93756	0.114597	313.0543	402.25264878
39	W	2712.269	5.149877	0.043574	236.3713	392.86085754
40	H	4.034920	8010.401	44.55635	359.5627	360.75861125
41	L	901.5627	20.60242	0.151168	272.5757	350.23647611
42	W	1783.765	5.938669	0.057699	205.8464	342.06848968
43	H	5.331368	4585.884	29.32865	312.7237	314.3226865
44	L	597.1501	23.63698	0.198954	237.6117	05.30222899
45	W	1175.135	6.855095	0.076377	179.5067	298.19781153
46	H	7.034175	2632.346	19.34482	272.1498	274.2913502
47	L	397.6603	27.07275	0.260921	207.5163	66.61637216
48	W	776.4023	7.924740	0.101029	156.8794	260.4383044
49	H	9.259620	1517.402	12.80129	237.0702	39.94305012
50	L	267.0361	30.92837	0.340324	181.7580	233.49044475
51	W	515.4549	9.181479	0.133463	137.5883	228.12353365
52	H	12.14501	880.6544	8.515373	206.8386	210.69615237
53	L	181.6421	35.19616	0.440189	159.9137	205.37329189
54	W	345.0168	10.67128	0.175868	121.3552	200.72466076
55	H	15.83852	516.6926	5.711612	180.9270	186.10312065
56	L	125.9909	39.82107	0.562194	141.6629	181.84263239
57	W	234.0864	12.45702	0.230684	108.0003	177.84312333
58	H	20.47128	308.4282	3.881543	158.9204	165.83924715
59	L	89.93771	44.67211	0.704769	126.7707	162.58731057
60	W	162.3094	14.62204	0.300145	97.43298	159.19257584
61	H	26.10122	189.0951	2.691595	140.5078	149.67091817
62	L	66.83336	49.51285	0.860720	115.0497	147.36458187
63	W	116.2710	17.26584	0.385352	89.61066	144.55211629
64	H	32.62851	120.5978	1.922520	125.4580	137.38202945
65	L	52.30394	53.99216	1.016010	106.2826	135.91662842
66	W	87.06093	20.47815	0.484991	84.44754	133.67367758
67	H	39.71985	81.17608	1.429585	113.5658	128.65353869
68	L	43.44162	57.69205	1.152404	100.1246	27.858113673
69	W	68.70748	24.27978	0.594456	81.68727	126.160543759
70	H	46.82636	58.38948	1.116662	104.5785	122.953129186
71	L	38.27782	60.25838	1.254686	96.05332	122.60564278
72	W	57.21337	28.54978	0.706403	80.83141	21.399508064
73	H	53.34984	45.13072	0.919749	98.13698	119.546992687
74	L	35.45902	61.56556	1.317666	93.44633	119.434893262
75	W	49.96664	33.00539	0.812741	81.21991	118.635047003
76	H	58.86298	37.35098	0.796580	93.77826	117.664151555
77	L	34.05903	61.78424	1.346860	91.74555	117.642121314
78	W	45.33184	37.28270	0.906884	82.22149	117.147777813
79	H	63.21259	32.74803	0.719765	90.99645	116.679531518

80	L	33.46414	61.27977	1.353220	90.56877	116.678526481
81	W	42.32281	41.06770	0.985060	83.38107	116.39331262
82	H	66.46684	30.00739	0.671910	89.31958	16.181838712
83	L	33.28882	60.43456	1.347390	89.70608	116.181002473
84	W	40.35033	44.18296	1.046414	84.44637	116.026806979
85	H	68.80439	28.37021	0.642130	88.36279	115.935990105
86	L	33.30675	59.52959	1.336904	89.05588	115.933271906
87	W	39.05425	46.59469	1.092280	85.31638	115.85476009
88	H	70.42929	27.39212	0.623643	87.84549	15.817273721
89	L	33.39566	58.72050	1.326019	88.56659	115.814314065
90	W	38.20591	48.36976	1.125179	85.97700	115.776455779
91	H	71.52754	26.80948	0.612220	87.58120	115.76148982
92	L	33.49619	58.06718	1.316641	88.20497	15.759268489
93	W	37.65522	49.62319	1.147967	86.45397	115.741888499
94	H	72.25155	26.46443	0.605212	87.45504	115.736092704
95	L	33.58392	57.57348	1.309319	87.94415	115.734724988
96	W	37.30168	50.47853	1.163293	86.78558	115.72709347
97	H	72.71811	26.26184	0.600954	87.40050	15.724914962
98	L	33.65201	57.21757	1.303944	87.76073	115.724172568
99	W	37.07761	51.04573	1.173339	87.00928	115.720955416
101	H	73.01249	26.14429	0.598397	87.38100	115.72016138
102	L	33.70135	56.97000	1.300167	87.63483	115.71979271
103	W	36.93758	51.41272	1.179780	87.15645	15.718487017
104	H	73.19456	26.07711	0.596884	87.37740	115.718206961
105	L	33.73549	56.80265	1.297599	87.55033	115.718035832
106	W	36.85137	51.64507	1.183825	87.25118	115.71752473
107	H	73.30500	26.03943	0.596003	87.38011	15.717429461
108	L	33.75832	56.69221	1.295899	87.49478	115.717354225
109	W	36.79914	51.78926	1.186317	87.31095	115.717161103
110	H	73.37069	26.01882	0.595501	87.38465	115.717129995
111	L	33.77319	56.62085	1.294798	87.45895	115.717098403
112	W	36.76802	51.87705	1.187825	87.34796	115.717027977
113	H	73.40897	26.00790	0.595220	87.38909	115.717018295
114	L	33.78266	56.57562	1.294099	87.43627	115.71700556
115	W	36.74982	51.92950	1.188719	87.37048	15.716980807
116	H	73.43077	26.00236	0.595069	87.39276	115.716977965
117	L	33.78857	56.54747	1.293664	87.42216	115.716973021
118	W	36.73940	51.96021	1.189239	87.38392	115.716964659
119	H	73.44287	25.99973	0.594989	87.39554	115.716963886
120	L	33.79219	56.53025	1.293398	87.41353	115.716962035
121	W	36.73357	51.97782	1.189535	87.39179	15.716959332

*** OBJ FUNC VALUE ***
115.716959332
*** VARIABLE VALS ***
H = 0.5949899349
L = 1.2933982601
W = 1.1895355395

*** TERM WEIGHTS ***

WGT	1	0.3776101546
WGT	2	0.1329574512
WGT	3	0.1330071151
WGT	4	0.2446527034
WGT	5	0.1117725757

```
FILE: GRAV9S.TRD
GRAVELBOX PROBLEM
       16:33:30 03-17-1990
                TOLERANCE VALUE=   .0000001
******* LIST OF TERMS ******
       TERM 1    40 H^-1 L^-1 W^-1
       TERM 2    10 L W
       TERM 3    20 H L
       TERM 4    40 H W
       TERM 5    10 L
** VARIABLE START POINTS **
       H          999999
       L          999999
       W          999999
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IT	VAR	CDCOF(1)	CDCOF(2)	XNW(VAR)	FUNVL	VLUE
1	H	59999940	4.0E-11	0.000000	0.097979	9999990000000
2	L	10000000	48989.77	0.069992	1399853.	1399853.8873
3	W	0.699926	7.0E+11	999999.4	1399853.	1399853.8873
4	H	39999980	0.000571	0.000003	302.3873	700229.64833
5	L	10000004	10.58245	0.001028	20574.21	20725.408688
6	W	0.010438	1.0E+10	992731.2	20724.84	20724.85727
7	H	39709251	0.039168	0.000031	2494.269	706.6089355
8	L	9927322.	1.282940	0.000359	7137.552	8384.6877831
9	W	0.004851	3.5E+09	854579.1	8291.416	8291.4203786
10	H	34183166	0.130202	0.000061	4219.358	7291.4910676
11	L	8545801.	0.758409	0.000297	5091.645	7201.3249961
12	W	0.005447	2.2E+09	631950.8	6885.368	6885.371925
13	H	25278032	0.212471	0.000091	4635.027	517.6329045
14	L	6319518.	0.690395	0.000330	4177.541	6495.0556051
15	W	0.006972	1.3E+09	435103.5	6067.524	6067.5275332
16	H	17404140	0.278138	0.000126	4400.343	5838.4808729
17	L	4351045.	0.727215	0.000408	3557.611	5757.7829445
18	W	0.009144	7.7E+08	290918.4	5320.834	5320.8386202
19	H	11636739	0.336320	0.000170	3956.603	5145.9481605
20	L	2909194.	0.808774	0.000527	3067.821	5046.1235465
21	W	0.012072	4.5E+08	192256.6	4642.160	4642.165627
22	H	7690265.	0.394594	0.000226	3483.985	497.6888034
23	L	1922576.	0.918488	0.000691	2657.716	4399.7089497
24	W	0.015972	2.6E+08	126471.2	4040.156	4040.1632839
25	H	5058849.	0.457586	0.000300	3042.933	3917.0921403
26	L	1264722.	1.051616	0.000911	2306.515	3827.9821097
27	W	0.021148	1.5E+08	83045.50	3512.626	3512.6354623
28	H	3321820.	0.528217	0.000398	2649.258	3406.5320526
29	L	830465.0	1.207884	0.001206	2003.103	3327.7333453
30	W	0.028010	83174390	54491.93	3052.722	3052.7350589
31	H	2179677.	0.608661	0.000528	2303.636	2960.8284818
32	L	544929.3	1.389108	0.001596	1740.075	2891.8938127
33	W	0.037103	47410042	35746.03	2652.604	2652.6201761
34	H	1429841.	0.700864	0.000700	2002.124	2572.8638142

35	L	357470.3	1.598302	0.002114	1511.748	2512.8104599
36	W	0.049149	27019505	23446.45	2304.783	2304.8046839
37	H	937858.3	0.806813	0.000927	1739.743	2235.5422154
38	L	234474.6	1.839351	0.002800	1313.440	2183.3118848
39	W	0.065108	15397759	15378.35	2002.523	2002.5514962
40	H	615134.2	0.928678	0.001228	2002.551	1942.3852612
41	L	153793.5	2.116909	0.003710	1141.169	1896.9885714
42	W	0.086248	8774656.	10086.44	1739.890	1739.9275793
43	H	403457.9	1.068906	0.001627	1313.406	1687.6573705
44	L	100874.5	2.436413	0.004914	991.5080	1648.2109431
45	W	0.114253	5000396.	6615.585	1511.701	1511.7512661
46	H	264623.5	1.230288	0.002156	1141.162	1466.3391572
47	L	66165.90	2.804157	0.006510	861.4862	1432.0675095
48	W	0.151348	2849618.	4339.145	1313.446	1313.5118842
49	H	173565.9	1.416027	0.002856	991.5122	1274.0578005
50	L	43401.51	3.227395	0.008623	748.5288	1244.2846299
51	W	0.200484	1623988.	2846.102	1141.201	1141.2881255
52	H	113844.2	1.629805	0.003783	861.4964	1107.0107127
53	L	28471.09	3.714472	0.011422	650.4002	1081.147803
54	W	0.265567	925552.8	1866.865	991.5580	91.67315271
55	H	74674.85	1.875860	0.005012	748.5441	961.89387753
56	L	18678.75	4.274977	0.015128	565.1593	939.43028101
57	W	0.351764	527538.2	1224.618	861.5554	861.70825913
58	H	48985.03	2.159067	0.006638	650.4214	835.83785545
59	L	12256.31	4.919917	0.020035	491.1214	816.33012002
60	W	0.465913	300718.1	803.3912	748.6219	748.82491537
61	H	32136.04	2.485043	0.008793	565.1883	726.351717
62	L	8044.087	5.661899	0.026530	426.8246	9.41530745
63	W	0.617050	171453.5	527.1241	650.5245	650.79447693
64	H	21085.49	2.860252	0.011646	491.1612	631.27434952
65	L	5281.474	6.515335	0.035122	371.0017	616.57621776
66	W	0.817105	97781.99	345.9317	565.3253	565.68474726
67	H	13837.97	3.292143	0.015424	426.8797	548.73241374
68	L	3469.625	7.496635	0.046482	322.5555	535.98462609
69	W	1.081796	55791.10	227.0961	491.3435	491.82267016
70	H	9084.774	3.789294	0.020423	371.0789	477.10427729
71	L	2281.369	8.624385	0.061484	280.5381	466.05860758
72	W	1.431770	31854.56	149.1588	427.1226	427.76256049
73	H	5967.584	4.361588	0.027034	322.6648	414.9893807
74	L	1502.129	9.919456	0.081262	244.1336	05.43279445
75	W	1.894016	18207.34	98.04635	371.4028	372.25944152
76	H	3923.479	5.020399	0.035771	280.6950	361.18265464
77	L	991.1789	11.40499	0.107268	212.6442	352.93362761
78	W	2.503531	10424.49	64.52841	323.0978	324.24722622
79	H	2583.282	5.778797	0.047296	244.3625	314.65378412
80	L	656.2301	13.10618	0.141322	185.4795	307.55940409
81	W	3.305097	5984.352	42.55165	281.2747	282.8216271
82	H	1704.892	6.651713	0.062462	212.9831	74.53126813
83	L	436.7657	15.04961	0.185625	162.1499	268.46503513

84	W	4.354751	3449.875	28.14621	245.1395	247.22770414
85	H	1129.561	7.655995	0.082327	185.9883	240.09122319
86	L	293.1087	17.26211	0.242679	142.2628	234.95132742
87	W	5.719900	2002.079	18.70882	214.0252	216.85158036
88	H	753.2064	8.810100	0.108151	162.9211	210.75037264
89	L	199.2512	19.76879	0.314984	125.5222	206.45786617
90	W	7.475916	1174.186	12.53245	187.3832	191.21440593
91	H	507.5980	10.13290	0.141288	143.4356	186.06081889
92	L	138.1503	22.59001	0.404373	111.7285	182.55633153
93	W	9.695274	700.1170	8.497775	164.7765	169.96293499
94	H	347.9984	11.64052	0.182893	127.2931	165.69956363
95	L	98.63562	25.73695	0.510812	100.7686	162.93607281
96	W	12.42385	428.1547	5.870460	145.8675	152.84411553
97	H	245.0346	13.33908	0.233318	114.3422	149.43746199
98	L	73.37097	29.20374	0.630895	92.57877	147.36626793
99	W	15.64169	271.7400	4.168069	130.3913	139.64425725
101	H	179.3406	15.21135	0.291235	104.4608	137.06589753
102	L	57.50540	32.95190	0.756982	87.06118	135.61680494
103	W	19.21925	181.4385	3.072532	118.1035	130.08259757
104	H	138.0409	17.19798	0.352967	97.44796	128.27632764
105	L	47.78467	36.88320	0.878557	83.96313	27.343314214
106	W	22.90427	128.9897	2.373117	108.7090	123.696702212
107	H	112.4958	19.18539	0.412968	92.91452	122.549289895
108	L	41.99055	40.81535	0.985907	82.79756	121.998500217
109	W	26.37781	98.24419	1.929896	101.8129	119.814961657
110	H	96.91400	21.02277	0.465748	90.27515	119.161213868
111	L	38.61394	44.50145	1.073532	82.90661	118.860492356
112	W	29.36527	80.00059	1.650553	96.93790	117.673150289
113	H	87.49278	22.57435	0.507950	88.88403	117.338581708
114	L	36.66454	47.70993	1.140725	83.64839	117.184380791
115	W	31.72528	69.03307	1.475114	93.59687	116.592783126
116	H	81.81910	23.77129	0.539012	88.20309	116.437367222
117	L	35.53140	50.30777	1.189902	84.55780	116.362040466
118	W	33.45953	62.36623	1.365258	91.36181	116.088295554
119	H	78.40837	24.62259	0.560383	87.87759	116.021857322
120	L	34.86026	52.28288	1.224656	85.38371	115.986470591
121	W	34.66192	58.28541	1.296742	89.89515	115.867284776
122	H	76.36282	25.18790	0.574321	87.71360	115.840814139
123	L	34.45384	53.70953	1.248552	86.03488	115.824749864
124	W	35.45838	55.78252	1.254265	88.94848	115.775414871
125	H	75.14168	25.54250	0.583030	87.61979	115.765491142
126	L	34.20326	54.69896	1.264607	86.50741	115.75843002
127	W	35.96729	54.25165	1.228153	88.34670	15.738876824
128	H	74.41828	25.75441	0.588282	87.55796	115.735355933
129	L	34.04718	55.36322	1.275175	86.83229	115.732348837
130	W	36.28306	53.32168	1.212271	87.96986	115.724897379
131	H	73.99438	25.87557	0.591351	87.51335	115.7237129
132	L	33.94974	55.79746	1.282002	87.04733	5.722472057
133	W	36.47408	52.76250	1.202736	87.73742	115.719736577

134	H	73.74952	25.94182	0.593090	87.48021	115.719359551
134	L	33.88916	56.07493	1.286334	87.18561	115.718863511
135	W	36.58694	52.43066	1.197097	87.59630	115.717896243
136	H	73.61060	25.97624	0.594043	87.45575	115.717783485
137	L	33.85184	56.24868	1.289035	87.27248	115.717591448
138	W	36.65208	52.23684	1.193820	87.51204	115.717262652
139	H	73.53354	25.99296	0.594545	87.43809	115.717231408
140	L	33.82911	56.35542	1.290691	87.32592	115.717159461
141	W	36.68874	52.12574	1.191954	87.46263	115.717052452
142	H	73.49202	26.00026	0.594796	87.42566	115.71704464
143	L	33.81548	56.41979	1.291688	87.35817	15.717018589
144	W	36.70876	52.06349	1.190917	87.43424	115.71698547
145	H	73.47048	26.00281	0.594913	87.41714	15.716983798
146	L	33.80744	56.45787	1.292278	87.37726	115.7169747
147	W	36.71931	52.02956	1.190358	87.41830	15.71696506
148	H	73.45990	26.00315	0.594960	87.41143	15.716964789
149	L	33.80278	56.47995	1.292620	87.38832	115.716961735
150	W	36.72460	52.01172	1.190068	87.40960	15.716959144

*** OBJ FUNC VALUE ***

115.716959144

*** VARIABLE VALS ***

H =	0.5949601228
L =	1.2926201291
W =	1.190068707

*** TERM WEIGHTS ***

WGT	1	0.3776871164
WGT	2	0.1329370195
WGT	3	0.1329204356
WGT	4	0.2447500969
WGT	5	0.1117053316

```
FILE: GRAVW2.TRD
GRAVELBOX PROBLEM
       16:45:20 03-17-1990
                TOLERANCE VALUE=   .0000001
******* LIST OF TERMS ******
       TERM 1    40 H^-1 L^-1 W^-1
       TERM 2    10 L W
       TERM 3    20 H L
       TERM 4    40 H W
       TERM 5    10 L
** VARIABLE START POINTS **
       H               1
       L               1
       W           0.002
```

IT	VAR	CDCOF(1)	CDCOF(2)	XNW(VAR)	FUNVL	VLUE
1	H	20.08	20000	31.55972	1267.438	1277.4583614
2	L	641.2144	633.7191	0.994138	1274.911	1277.4363294
3	W	1272.330	1.274911	0.031654	80.55081	717.98670826
4	H	21.14895	1271.080	7.752504	327.9148	338.17088906
5	L	165.3666	162.9963	0.992807	328.3544	338.17059439
6	W	320.0282	5.197003	0.127433	81.56440	245.42734222
7	H	24.95347	316.1642	3.559515	177.6445	188.8377645
8	L	82.46463	88.18339	1.034092	170.5521	88.69618573
9	W	152.7215	10.86699	0.266750	81.47698	165.43529776
10	H	31.35186	145.0092	2.150632	134.8526	147.95204999
11	L	55.68015	69.72509	1.119036	124.6162	147.56353729
12	W	97.21567	16.62070	0.413481	80.39385	139.71696021
13	H	38.92001	86.44882	1.490366	116.0101	131.82754314
14	L	43.94214	64.90981	1.215386	106.8134	131.46300591
15	W	71.76852	22.08271	0.554701	79.62018	128.00149415
16	H	46.49578	59.33163	1.129630	105.0461	23.941738104
17	L	38.13961	63.83582	1.293731	98.68483	123.749128623
18	W	58.12252	27.37030	0.686226	79.77044	121.936519724
19	H	53.32368	45.05555	0.919208	98.03118	119.846430373
20	L	35.24643	63.41300	1.341317	94.55332	119.784737921
21	W	50.18151	32.44250	0.804054	80.69731	118.769496068
22	H	58.98851	37.08883	0.792935	93.54817	117.746261601
23	L	33.89925	62.73889	1.360421	92.23451	117.737038082
24	W	45.32163	37.08077	0.904527	81.98929	117.168028744
25	H	63.38950	32.50611	0.716100	90.78648	116.696073483
26	L	33.36727	61.75392	1.360416	90.78678	116.696073482
27	W	42.24817	41.05955	0.985832	83.29925	116.387310472
28	H	66.64163	29.82530	0.668990	89.16517	116.180771655
29	L	33.23812	60.65089	1.350828	89.79803	116.178525735
30	W	40.26788	44.26291	1.048432	84.43634	116.018456891
31	H	68.95388	28.24353	0.640000	88.26101	115.931834994
32	L	33.28433	59.61276	1.338288	89.08806	115.927960007
33	W	38.98289	46.70142	1.094530	85.33596	115.848946679
34	H	70.54700	27.30752	0.622159	87.78299	115.81386196

35	L	33.38850	58.73948	1.326375	88.57140	15.810321528
36	W	38.15014	48.47206	1.127191	86.00503	115.773141052
37	H	71.61517	26.75443	0.611216	87.54480	115.759357558
38	L	33.49625	58.05865	1.316544	88.19857	115.756916406
39	W	37.61411	49.70834	1.149580	86.48087	115.740189834
40	H	72.31408	26.42928	0.604548	87.43476	115.734929091
41	L	33.58677	57.55585	1.309063	87.93441	115.733501417
42	W	37.27257	50.54382	1.164499	86.80780	115.726284333
43	H	72.76124	26.23977	0.600523	87.38966	115.724334255
44	L	33.65546	57.19929	1.303669	87.75121	115.723586356
45	W	37.05763	51.09312	1.174200	87.02621	115.720591351
46	H	73.04142	26.13063	0.598122	87.37549	115.719890425
47	L	33.70446	56.95438	1.299929	87.62687	115.719528812
48	W	36.92421	51.44577	1.180372	87.16868	115.718330881
49	H	73.21351	26.06878	0.596712	87.37477	115.718087254
50	L	33.73797	56.79053	1.297413	87.54421	115.717922959
51	W	36.84261	51.66741	1.184221	87.25967	115.717460556
52	H	73.31715	26.03444	0.595897	87.37898	115.717378992
53	L	33.76016	56.68334	1.295762	87.49032	115.717308057
54	W	36.79351	51.80397	1.186576	87.31667	115.717135743
55	H	73.37833	26.01588	0.595436	87.38426	115.717109589
56	L	33.77449	56.61463	1.294702	87.45583	115.717080277
57	W	36.76447	51.88653	1.187990	87.35173	115.717018326
58	H	73.41368	26.00620	0.595182	87.38904	115.71701036
59	L	33.78355	56.57139	1.294034	87.43415	15.716998717
60	W	36.74763	51.93549	1.188823	87.37291	115.716977269
61	H	73.43363	26.00140	0.595046	87.39284	115.716974994
62	L	33.78916	56.54466	1.293621	87.42075	115.716970538
63	W	36.73807	51.96393	1.189303	87.38545	115.716963411
64	H	73.44457	25.99920	0.594976	87.39566	115.716962815
65	L	33.79257	56.52843	1.293370	87.41262	115.71696117
66	W	36.73278	51.98008	1.189574	87.39274	115.71695891

*** OBJ FUNC VALUE ***

115.71695891

*** VARIABLE VALS ***

H =	0.5949769802
L =	1.2933701425
W =	1.1895743631

*** TERM WEIGHTS ***

WGT 1	0.3776142627
WGT 2	0.1329589006
WGT 3	0.1330013282
WGT 4	0.2446553622
WGT 5	0.1117701463

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FILE: GRAVW9.TRD
GRAVELBOX PROBLEM
        16:35:04 03-17-1990
                 TOLERANCE VALUE=    .0000001
******* LIST OF TERMS ******
        TERM 1    40 H^-1 L^-1 W^-1
        TERM 2    10 L W
        TERM 3    20 H L
        TERM 4    40 H W
        TERM 5    10 L
** VARIABLE START POINTS **
        H                 1
        L                 1
        W            999999
IT  VAR CDCOF(1) CDCOF(2) XNW(VAR) FUNVL    VLUE
  1 H   39999980  0.00004 0.000001 80.00002        10000080
  2 L   10000000 40.00001 0.002000 40000.00        40.00499
  3 W   0.020040  2.0E+10 999000.9 40039.96     40039.98503
  4 H   39960039 0.020019 0.000022 1788.854     1768.896716
  5 L   9990019. 1.788854 0.000423 8454.748    9349.1754179
  6 W   0.005126  4.2E+09 907590.7 9306.288    9306.2929409
  7 H   36303631 0.104151 0.000053 3888.998    7729.5618925
  8 L   9075917. 0.822833 0.000301 5465.518    7410.0177775
  9 W   0.005153  2.5E+09 693737.5 7150.332    7150.3353397
 10 H   27749502 0.191493 0.000083 4610.357    6699.2043548
 11 L   6937385. 0.694089 0.000316 4388.697    6693.8757811
 12 W   0.006485  1.5E+09 484467.5 6284.432    6284.4360339
 13 H   19378703 0.261027 0.000116 4498.162    6030.5735564
 14 L   4844685. 0.711401 0.000383 3712.959    5962.0408333
 15 W   0.008474  9.0E+08 325779.5 5521.552    5521.5558748
 16 H   13031181 0.320414 0.000156 4086.746    5335.1347709
 17 L   3257805. 0.783019 0.000490 3194.322     5237.695308
 18 W   0.011174  5.2E+08 215782.3 4822.659     822.6646559
 19 H   8631293. 0.378112 0.000209 3613.085     4670.977535
 20 L   2157833. 0.885669 0.000640 2764.870     571.4133792
 21 W   0.014778  3.0E+08 142073.4 4199.308    4199.3145422
 22 H   5682938. 0.439460 0.000278 3160.650    4070.8637398
 23 L   1420744. 1.012449 0.000844 2398.693    3979.0188903
 24 W   0.019564  1.7E+08 93323.12 3651.728    3651.7371339
 25 H   3732924. 0.507740 0.000368 2753.440    3541.2521552
 26 L   933241.2 1.162182 0.001115 2082.879    3459.5994418
 27 W   0.025911 97190401 61244.17 3173.865    3173.8768951
 28 H   2449767. 0.585268 0.000488 2394.803    3078.2619513
 29 L   612451.8 1.336226 0.001477 1809.280    3006.6826647
 30 W   0.034322 55403963 40177.56 2757.955    2757.9698691
 31 H   1607102. 0.674019 0.000647 2081.555    2675.0249514
 32 L   401785.6 1.537312 0.001956 1571.839    2612.6172936
 33 W   0.045465 31576347 26353.71 2396.349    2396.3687906
 34 H   1054148. 0.775951 0.000857 1808.831    2324.3473364
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35	L	263547.1	1.769098	0.002590	1365.636	2270.0521054
36	W	0.060227	17994793	17285.31	2082.089	2082.1157521
37	H	691412.8	0.893173	0.001136	1571.688	2019.5562468
38	L	172863.2	2.036026	0.003431	1186.514	1972.3587804
39	W	0.079782	10254639	11337.21	1809.023	1809.0579124
40	H	453488.4	1.028047	0.001505	1365.588	1754.7096121
41	L	113382.1	2.343312	0.004546	1030.902	1713.6961845
42	W	0.105687	5843776.	7435.930	1571.767	1571.8130186
43	H	297437.3	1.183264	0.001994	1186.502	1524.5957687
44	L	74369.34	2.697003	0.006022	895.7105	1488.9616576
45	W	0.140002	3330221.	4877.185	1365.632	1365.6927997
46	H	195087.5	1.361905	0.002642	1030.904	1324.6704037
47	L	48781.90	3.104073	0.007976	778.2611	1293.7129079
48	W	0.185455	1897862.	3198.984	1186.540	1186.6204567
49	H	127959.5	1.567513	0.003500	895.7193	1150.9803631
50	L	31999.91	3.572552	0.010566	676.2288	1124.0879723
51	W	0.245661	1081622.	2098.309	1030.947	1031.0535789
52	H	83932.57	1.804162	0.004636	778.2750	1000.0902033
53	L	20993.18	4.111667	0.013994	587.5950	976.73158375
54	W	0.325401	616477.9	1376.413	895.7741	895.91540797
55	H	55056.83	2.076543	0.006141	676.2482	869.01598577
56	L	13774.26	4.732013	0.018534	510.6074	848.72988874
57	W	0.431003	351403.5	902.9484	778.3469	778.53462259
58	H	36118.30	2.390056	0.008134	587.6216	755.16705233
59	L	9039.646	5.445736	0.024544	443.7455	737.55340496
60	W	0.570831	200339.6	592.4198	676.3435	676.59300485
61	H	23697.28	2.750918	0.010774	510.6439	656.2953602
62	L	5934.414	6.266727	0.032496	385.6908	41.00752588
63	W	0.755933	114245.4	388.7564	587.7482	588.08018103
64	H	15550.90	3.166291	0.014269	443.7959	570.45166721
65	L	3897.850	7.210822	0.043011	335.3010	557.18978961
66	W	1.000875	65175.23	255.1827	510.8122	511.25468071
67	H	10208.16	3.644424	0.018894	385.7610	495.94786965
68	L	2562.205	8.295989	0.056901	291.5889	484.45320644
69	W	1.324808	37204.22	167.5790	444.0201	444.61068283
70	H	6704.298	4.194818	0.025013	335.4001	431.32490824
71	L	1686.290	9.542461	0.075225	253.7034	421.37507464
72	W	1.752805	21257.68	110.1263	386.0600	386.84997127
73	H	4406.557	4.828417	0.033101	291.7306	375.32581334
74	L	1111.925	10.97276	0.099339	220.9153	366.7308744
75	W	2.317466	12164.29	72.44972	335.7996	36.85879038
76	H	2899.975	5.557800	0.043777	253.9093	326.87363631
77	L	735.3728	12.61156	0.130957	192.6053	319.4729975
78	W	3.060689	6977.100	47.74499	292.2652	93.68945037
79	H	1912.419	6.397369	0.057837	221.2189	285.05418012
80	L	488.6067	14.48514	0.172179	168.2562	278.71420743
81	W	4.035294	4016.696	31.54982	254.6256	256.54660849
82	H	1265.436	7.363449	0.076281	193.0593	249.10350718
83	L	327.0238	16.62044	0.225440	147.4487	243.71570975

84	W	5.305672	2325.990	20.93793	222.1795	224.77793035
85	H	842.0260	8.474120	0.100319	168.9429	218.39989345
86	L	221.3856	19.04327	0.293289	129.8600	213.87921831
87	W	6.945664	1359.500	13.99048	194.3464	197.86778054
88	H	565.4851	9.748350	0.131297	148.4930	192.45854565
89	L	152.5307	21.77570	0.377839	115.2643	188.74069527
90	W	9.030276	806.3018	9.449271	170.6590	175.42964598
91	H	385.5276	11.20351	0.170470	131.4422	170.92370193
92	L	107.9021	24.83203	0.479723	103.5264	167.95930725
93	W	11.61605	489.1245	6.489037	150.7540	157.18690982
94	H	269.1559	12.84956	0.218495	117.6186	153.54536313
95	L	79.26028	28.21223	0.596610	94.57503	151.2880295
96	W	14.70592	306.8505	4.567907	134.3505	42.92380921
97	H	194.6485	14.67748	0.274599	106.9009	140.11963984
98	L	61.17107	31.88910	0.722018	88.33325	138.50712705
99	W	18.20417	201.7490	3.329049	121.2052	132.39071251
100	H	147.6023	16.64146	0.335775	99.12254	130.37906561
101	L	50.00600	35.78413	0.845929	84.60311	129.31566615
102	W	21.89032	140.8239	2.536367	111.0438	25.183959879
103	H	118.3733	18.64290	0.396853	93.95365	123.868831933
104	L	43.30074	39.73908	0.957990	82.96341	123.226039435
105	W	25.45403	105.2128	2.033087	103.5005	120.684106638
106	H	100.4833	20.53726	0.452089	90.85488	119.911583267
107	L	39.37266	43.51906	1.051338	82.78795	119.553455392
108	W	28.59695	84.15759	1.715484	98.11525	118.134613385
109	H	89.64613	22.17843	0.497392	89.17871	117.727633925
110	L	37.10270	46.87848	1.124045	83.41027	117.541071002
111	W	31.13617	71.54450	1.515847	94.39539	116.817702173
112	H	83.11481	23.47580	0.531460	88.34448	116.623765427
113	L	35.78768	49.65162	1.177876	84.30698	116.531504708
114	W	33.03719	63.89826	1.390730	91.89165	116.190327272
115	H	79.18675	24.41839	0.555305	87.94574	116.105606724
116	L	35.01342	51.79463	1.216256	85.17058	116.06182135
117	W	34.37479	59.22466	1.312596	90.24047	15.910925601
118	H	76.82898	25.05553	0.571069	87.74944	115.876543856
119	L	34.54736	53.36290	1.242831	85.87310	115.85648315
120	W	35.27111	56.35837	1.264065	89.17000	15.793194887
121	H	75.41925	25.46115	0.581029	87.64157	115.780095725
122	L	34.26123	54.46185	1.260795	86.39284	115.77119966
123	W	35.84912	54.60311	1.234154	88.48669	15.745825998
124	H	74.58208	25.70667	0.587091	87.57298	115.741108631
125	L	34.08336	55.20582	1.272685	86.75483	115.737286814
126	W	36.21051	53.53442	1.215903	88.05700	115.727514247
127	H	74.08986	25.84875	0.590663	87.52441	115.72590363
128	L	33.97231	55.69553	1.280405	86.99669	15.724312739
129	W	36.43061	52.88980	1.204904	87.79084	115.720688057
130	H	73.80431	25.92743	0.592705	87.48843	115.720167222
131	L	33.90315	56.01034	1.285328	87.15337	115.719525616
132	W	36.56150	52.50576	1.198371	87.62852	115.718230389

133	H	73.64142	25.96895	0.593835	87.46178	115.718071743
134	L	33.86042	56.20854	1.288412	87.25240	115.717821119
135	W	36.63754	52.28039	1.194555	87.53114	115.717375864
136	H	73.55046	25.98954	0.594438	87.44240	115.717330877
137	L	33.83431	56.33094	1.290312	87.31366	115.717236114
138	W	36.68064	52.15050	1.192369	87.47374	115.717089366
139	H	73.50101	25.99886	0.594744	87.42866	115.717077749
140	L	33.81858	56.40514	1.291462	87.35083	115.717043104
141	W	36.70440	52.07722	1.191145	87.44057	115.716997008
142	H	73.47505	26.00240	0.594890	87.41918	115.716994392
143	L	33.80925	56.44927	1.292145	87.37295	115.716982167
144	W	36.71706	52.03694	1.190479	87.42182	115.7169685
145	H	73.46209	26.00319	0.594951	87.41278	5.716968033
146	L	33.80382	56.47501	1.292543	87.38585	115.716963882
147	W	36.72350	52.01553	1.190130	87.41150	115.716960117

*** OBJ FUNC VALUE ***
115.716960117
*** VARIABLE VALS ***
H = 0.5949516712
L = 1.2925436607
W = 1.1901302309
*** TERM WEIGHTS ***
WGT 1 0.3776952972
WGT 2 0.1329360263
WGT 3 0.1329106831
WGT 4 0.2447592709
WGT 5 0.1116987224

FILE: GRAVL9.TRD
GRAVELBOX PROBLEM
16:36:58 03-17-1990
TOLERANCE VALUE= .0000001
******* LIST OF TERMS ******
TERM 1 40 H^-1 L^-1 W^-1
TERM 2 10 L W
TERM 3 20 H L
TERM 4 40 H W
TERM 5 10 L
** VARIABLE START POINTS **
H 1
L 999999
W 1

IT	VAR	CDCOF(1)	CDCOF(2)	XNW(VAR)	FUNVL	VLUE
1	H	20000020	0.00004	0.000001	56.56859	20000036.569
2	L	20.00002	28284271	1189.206	47568.31	47568.318293
3	W	11892.06	23784.15	1.414214	33635.85	45527.953067
4	H	23840.69	0.023784	0.000998	47.62480	28757.615809
5	L	24.16212	28317.85	34.23439	1654.351	1654.4077528
6	W	342.3839	1169.803	1.848416	1265.736	1608.7639686
7	H	758.6245	0.632117	0.028865	43.79678	018.93495388
8	L	29.06148	749.6781	5.079003	295.2067	297.34101916
9	W	51.94467	272.8325	2.291804	238.0940	291.81631565
10	H	193.2522	3.436401	0.133348	51.53997	218.73085227
11	L	35.58502	130.8858	1.917841	136.4928	148.71727511
12	W	24.51237	156.4075	2.526015	123.8373	148.13056419
13	H	139.3974	8.256788	0.243376	67.85205	135.47546065
14	L	40.12768	65.06475	1.273358	102.1938	26.784777576
15	W	22.46863	129.0716	2.396774	107.7045	126.636202584
16	H	121.3381	13.10635	0.328656	79.75716	123.010290354
17	L	40.54087	50.77974	1.119176	90.74481	122.253439649
18	W	24.33803	108.7474	2.113813	102.8921	121.440404185
19	H	106.9360	16.90808	0.397635	85.04316	119.892249563
20	L	39.09084	47.58916	1.103358	86.26241	119.883509481
21	W	26.93899	91.17134	1.839663	99.11740	118.925671993
22	H	95.65371	19.70629	0.453891	86.83272	118.164386127
23	L	37.47445	47.90379	1.130621	84.73886	118.139140802
24	W	29.46185	77.94551	1.626542	95.84194	117.411739544
25	H	87.67414	21.75089	0.498084	87.33822	117.03448717
26	L	36.22711	49.37324	1.167424	84.58487	16.991087145
27	W	31.59762	68.79045	1.475492	93.24408	116.547852204
28	H	82.36818	23.22170	0.530966	87.46953	116.369044971
29	L	35.37425	51.05705	1.201390	84.99659	116.334087623
30	W	33.25257	62.70590	1.373224	91.32651	116.098385263
31	H	78.95679	24.24566	0.554143	87.50680	116.018506374
32	L	34.81511	52.56492	1.228751	85.55826	115.996811949
33	W	34.45326	58.74535	1.305784	89.97708	115.882702574
34	H	76.80642	24.93011	0.569722	87.51669	115.849063232

35	L	34.45229	53.76814	1.249261	86.07986	115.837268525
36	W	35.28151	56.20089	1.262112	89.05846	115.785734669
37	H	75.46972	25.36930	0.579786	87.51261	115.772318439
38	L	34.21684	54.66307	1.263941	86.49620	115.766415862
39	W	35.83086	54.58395	1.234252	88.44864	115.744380072
40	H	74.64891	25.64065	0.586074	87.49964	115.739289009
41	L	34.06400	55.29722	1.274100	86.80196	115.736507511
42	W	36.18398	53.56775	1.216727	88.05213	115.727504627
43	H	74.15113	25.80255	0.589891	87.48231	115.725660822
44	L	33.96511	55.73064	1.280944	87.01489	115.724412103
45	W	36.40512	52.93672	1.205861	87.79904	115.720878788
46	H	73.85334	25.89597	0.592149	87.46437	115.720240934
47	L	33.90159	56.01852	1.285451	87.15772	115.719703366
48	W	36.54048	52.55005	1.199221	87.64026	115.718367565
49	H	73.67789	25.94805	0.593449	87.44822	115.718157154
50	L	33.86120	56.20523	1.288359	87.25083	115.717934381
51	W	36.62157	52.31654	1.195228	87.54231	115.717447495
52	H	73.57634	25.97597	0.594178	87.43493	115.717381636
53	L	33.83585	56.32383	1.290201	87.31013	115.717292565
54	W	36.66914	52.17778	1.192868	87.48291	115.717121609
55	H	73.51875	25.99022	0.594574	87.42468	115.717102232
56	L	33.82016	56.39772	1.291347	87.34713	115.717067833
57	W	36.69643	52.09680	1.191498	87.44751	115.717010154
58	H	73.48689	25.99701	0.594780	87.41715	115.717004883
59	L	33.81059	56.44293	1.292047	87.36977	115.716992048
60	W	36.71169	52.05049	1.190721	87.42681	115.716973434
61	H	73.46980	25.99988	0.594882	87.41181	115.716972148
62	L	33.80486	56.47009	1.292467	87.38338	115.716967525
63	W	36.71997	52.02464	1.190291	87.41495	115.716961822
64	H	73.46101	26.00081	0.594928	87.40815	115.716961557
65	L	33.80149	56.48610	1.292715	87.39140	115.716959953
66	W	36.72430	52.01062	1.190061	87.40833	115.716958314

*** OBJ FUNC VALUE ***
115.716958314

*** VARIABLE VALS ***

H =	0.5949288871
L =	1.2927152133
W =	1.1900610167

*** TERM WEIGHTS ***

WGT	1	0.3776816077
WGT	2	0.1329459401
WGT	3	0.1329232352
WGT	4	0.2447356676
WGT	5	0.1117135493

FILE: GRAVL2.TRD
GRAVELBOX PROBLEM
16:46:08 03-17-1990
TOLERANCE VALUE= .0000001
******* LIST OF TERMS ******
TERM 1 40 H^-1 L^-1 W^-1
TERM 2 10 L W
TERM 3 20 H L
TERM 4 40 H W
TERM 5 10 L
** VARIABLE START POINTS **
H 1
L 0.002
W 1

IT	VAR	CDCOF(1)	CDCOF(2)	XNW(VAR)	FUNVL	VLUE
1	H	40.04	20000	22.34950	1789.748	1789.7885857
2	L	466.9901	1.789748	0.061907	57.82023	951.80055023
3	W	894.5993	28.91011	0.179767	321.6393	349.9304371
4	H	8.428837	3594.241	20.64998	348.1107	48.84112197
5	L	424.7973	10.77530	0.159266	135.3118	283.79947854
6	W	827.5920	12.16231	0.121227	200.6533	268.02291142
7	H	8.034413	2071.743	16.05798	258.0329	259.81866179
8	L	332.3719	20.54797	0.248640	165.2824	243.14897418
9	W	644.8056	10.01836	0.124647	160.7469	243.08674613
10	H	9.958719	1290.635	11.38413	226.7428	229.53918563
11	L	238.9292	28.18875	0.343481	164.1355	220.89574529
12	W	458.8002	10.22954	0.149319	137.0156	218.65525159
13	H	12.84240	779.9023	7.792859	200.1581	204.10588126
14	L	167.3503	34.37531	0.453220	151.6934	198.23843684
15	W	316.2466	11.32538	0.189240	119.6932	194.86316476
16	H	16.63403	466.3761	5.295042	176.1557	181.54568328
17	L	117.7932	39.91874	0.582141	137.1445	177.22599915
18	W	217.6231	12.97664	0.244190	106.2829	173.75360616
19	H	21.41043	281.3863	3.625256	155.2366	162.47961657
20	L	84.94703	45.18482	0.729326	123.9083	159.31842158
21	W	152.3035	15.12861	0.315169	96.00293	156.17614017
22	H	27.19332	174.0176	2.529679	137.5807	147.17267584
23	L	63.74528	50.17067	0.887157	113.1042	144.9954102
24	W	110.0587	17.82352	0.402424	88.58069	42.33677467
25	H	33.84014	112.0403	1.819580	123.1496	135.59142052
26	L	50.41584	54.62661	1.040923	104.9580	134.24778501
27	W	83.19243	21.11884	0.503840	83.83145	132.12154289
28	H	40.97208	76.26902	1.364363	111.8016	27.455451132
29	L	42.32567	58.18844	1.172509	99.25452	126.751392581
30	W	66.29963	25.00423	0.614117	81.43147	125.151157802
31	H	48.01487	55.55106	1.075618	103.2913	122.217061805
32	L	37.65354	60.55507	1.268155	95.50105	121.92328521
33	W	55.70629	29.32441	0.725542	80.83450	20.797079737
34	H	54.38478	43.47354	0.894074	97.24812	119.130670609

35	L	35.13691	61.66284	1.324737	93.09441	119.041962854
36	W	49.01037	33.77195	0.830107	81.36770	118.303381563
37	H	59.69904	36.37440	0.780574	93.19908	117.44320353
38	L	33.91255	61.73217	1.349197	91.50947	17.427888129
39	W	44.71494	37.98133	0.921634	82.42167	116.976633899
40	H	63.84932	32.16812	0.709797	90.64023	116.566882949
41	L	33.41230	61.14579	1.352789	90.39959	116.566563504
42	W	41.91980	41.65769	0.996868	83.57708	116.309114241
43	H	66.93053	29.66142	0.665708	89.11239	116.125812854
44	L	33.28284	60.27513	1.345732	89.57964	116.12458785
45	W	40.08565	44.64957	1.055392	84.61222	15.986859905
46	H	69.13037	28.16352	0.638276	88.24862	115.908718915
47	L	33.31946	59.37953	1.334963	88.96053	115.905847355
48	W	38.88070	46.94416	1.098812	85.44523	115.836393292
49	H	70.65178	27.26885	0.621257	87.78594	115.804332206
50	L	33.41328	58.59551	1.324258	88.49562	115.801463663
51	W	38.09289	48.62006	1.129758	86.07157	115.768264188
52	H	71.67551	26.73631	0.610752	87.55202	115.755532008
53	L	33.51264	57.97073	1.315225	88.15333	115.753467127
54	W	37.58235	49.79600	1.151079	86.52055	115.738345819
55	H	72.34767	26.42132	0.604317	87.44190	115.733440068
56	L	33.59713	57.50289	1.308258	87.90751	115.732200443
57	W	37.25528	50.59425	1.165350	86.83094	115.725608339
58	H	72.77921	26.23672	0.600414	87.39537	115.723773848
59	L	33.66179	57.16789	1.303189	87.73537	115.723112507
60	W	37.04846	51.12124	1.174669	87.03939	115.720351973
61	H	73.05055	26.12985	0.598076	87.37963	115.719686949
62	L	33.70822	56.93609	1.299648	87.61768	115.719362699
63	W	36.91954	51.46089	1.180620	87.17597	115.718249342
64	H	73.21780	26.06895	0.596696	87.37760	115.718016169
65	L	33.74013	56.78008	1.297252	87.53897	115.717867166
66	W	36.84038	51.67517	1.184346	87.26358	115.717433989
67	H	73.31892	26.03493	0.595895	87.38085	115.717355191
68	L	33.76138	56.67752	1.295672	87.48740	115.717290227
69	W	36.79255	51.80771	1.186635	87.31868	115.717127533
70	H	73.37886	26.01641	0.595440	87.38546	115.717101998
71	L	33.77515	56.61148	1.294653	87.45425	115.717074918
72	W	36.76414	51.88815	1.188014	87.35270	115.717015953
73	H	73.41366	26.00665	0.595187	87.38979	115.717008078
74	L	33.78389	56.56975	1.294008	87.43333	115.716997233
75	W	36.74758	51.93605	1.188830	87.37332	115.716976643
76	H	73.43340	26.00175	0.595051	87.39330	115.716974359
77	L	33.78933	56.54386	1.293608	87.42035	115.716970174
78	W	36.73814	51.96400	1.189303	87.38560	115.716963268
79	H	73.44431	25.99946	0.594981	87.39594	115.716962657
80	L	33.79265	56.52807	1.293364	87.41244	115.7169611
81	W	36.73288	51.97996	1.189571	87.39276	115.716958886

*** OBJ FUNC VALUE ***
115.716958886

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*** VARIABLE VALS ***
H =        0.5949810282
L =        1.2933644838
W =         1.189571213
*** TERM WEIGHTS ***
WGT  1     0.3776143457
WGT  2     0.1329579668
WGT  3     0.1330016512
WGT  4     0.2446563789
WGT  5     0.1117696573
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FILE: GRAVH2.TRD
GRAVELBOX PROBLEM
       16:47:03 03-17-1990
                TOLERANCE VALUE=   .0000001
******* LIST OF TERMS ******
       TERM 1    40 H^-1 L^-1 W^-1
       TERM 2    10 L W
       TERM 3    20 H L
       TERM 4    40 H W
       TERM 5    10 L
** VARIABLE START POINTS **
       H             0.002
       L                 1
       W                 1
```

IT	VAR	CDCOF(1)	CDCOF(2)	XNW(VAR)	FUNVL	VLUE
1	H	60	40	0.816496	97.97958	117.979589711
2	L	36.32993	48.98979	1.161236	84.37525	117.035117835
3	W	44.27222	42.18762	0.976173	86.43471	117.009983769
4	H	62.27164	35.28682	0.752767	93.75220	116.700237975
5	L	34.81709	54.43421	1.250373	87.06873	116.462010356
6	W	42.61445	42.49708	0.998621	85.11145	116.440010794
7	H	64.95234	32.03458	0.702282	91.22985	116.220094279
8	L	34.03187	57.03569	1.294584	88.11428	116.1668948
9	W	41.03716	43.99643	1.035428	84.98208	6.111224563
10	H	67.30882	29.84073	0.665838	89.63358	115.983924589
11	L	33.67104	58.01912	1.312675	88.39829	115.975412552
12	W	39.76028	45.76505	1.072858	85.31427	115.921615148
13	H	69.16782	28.40275	0.640808	88.64664	115.856534233
14	L	33.54474	58.18212	1.316990	88.35620	115.856058438
15	W	38.80223	47.39682	1.105213	85.76952	115.818195322
16	H	70.54835	27.48091	0.624125	88.06209	115.787555759
17	L	33.53465	57.98845	1.314994	88.19575	115.78745433
18	W	38.11497	48.73758	1.130795	86.20050	15.764885041
19	H	71.53171	26.89998	0.613234	87.73144	115.751290045
20	L	33.57264	57.68317	1.310785	88.01311	115.750837895
21	W	37.63724	49.76242	1.149851	86.55450	115.738751404
22	H	72.20978	26.53911	0.606240	87.55303	115.73299165
23	L	33.62333	57.38164	1.306369	87.84901	15.732491381
24	W	37.31332	50.50667	1.163435	86.82331	115.726503534
25	H	72.66482	26.31791	0.601815	87.46168	115.724156694
26	L	33.67067	57.12865	1.302570	87.71682	115.723784568
27	W	37.09833	51.02643	1.172790	87.01714	115.720994086
28	H	72.96301	26.18415	0.599056	87.41795	115.720071015
29	L	33.70903	56.93400	1.299608	87.61713	115.71984411
30	W	36.95835	51.37826	1.179053	87.15173	15.718608079
31	H	73.15430	26.10440	0.597361	87.39908	115.718257061
32	L	33.73775	56.79231	1.297438	87.54530	115.71813474
33	W	36.86882	51.61029	1.183146	87.24244	15.717610852
34	H	73.27462	26.05762	0.596335	87.39250	115.717481795

35	L	33.75817	56.69318	1.295913	87.49533	115.717421297
36	W	36.81255	51.75989	1.185765	87.30209	115.717207894
37	H	73.34888	26.03066	0.595725	87.39153	115.717162075
38	L	33.77215	56.62593	1.294876	87.46152	115.717134047
39	W	36.77776	51.85442	1.187408	87.34048	1115.71705028
40	H	73.39387	26.01545	0.595368	87.39279	115.717034612
41	L	33.78145	56.58143	1.294189	87.43919	115.717022295
42	W	36.75663	51.91302	1.188421	87.36470	115.71699057
43	H	73.42062	26.00709	0.595164	87.39467	115.71698543
44	L	33.78749	56.55262	1.293744	87.42474	15.716980256
45	W	36.74401	51.94870	1.189033	87.37972	115.716968662
46	H	73.43621	26.00264	0.595050	87.39647	115.716967055
47	L	33.79133	56.53433	1.293461	87.41558	115.716964967
48	W	36.73662	51.97002	1.189397	87.38885	115.716960883

*** OBJ FUNC VALUE ***
115.716960883
*** VARIABLE VALS ***
H = 0.5950502213
L = 1.2934613354
W = 1.1893970268
*** TERM WEIGHTS ***
WGT 1 0.3775974489
WGT 2 0.1329484507
WGT 3 0.1330270771
WGT 4 0.2446489982
WGT 5 0.1117780251

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FILE: GRAVH9.TRD
GRAVELBOX PROBLEM
       16:38:28 03-17-1990
                TOLERANCE VALUE=   .0000001
******* LIST OF TERMS ******
       TERM 1    40 H^-1 L^-1 W^-1
       TERM 2    10 L W
       TERM 3    20 H L
       TERM 4    40 H W
       TERM 5    10 L
** VARIABLE START POINTS **
       H          999999
       L               1
       W               1
```

IT	VAR	CDCOF(1)	CDCOF(2)	XNW(VAR)	FUNVL	VLUE
1	H	60	40	0.816496	97.97958	117.979589711
2	L	36.32993	48.98979	1.161236	84.37525	117.035117835
3	W	44.27222	42.18762	0.976173	86.43471	117.009983769
4	H	62.27164	35.28682	0.752767	93.75220	116.700237975
5	L	34.81709	54.43421	1.250373	87.06873	116.462010356
6	W	42.61445	42.49708	0.998621	85.11145	116.440010794
7	H	64.95234	32.03458	0.702282	91.22985	116.220094279
8	L	34.03187	57.03569	1.294584	88.11428	116.1668948
9	W	41.03716	43.99643	1.035428	84.98208	6.111224563
10	H	67.30882	29.84073	0.665838	89.63358	115.983924589
11	L	33.67104	58.01912	1.312675	88.39829	115.975412552
12	W	39.76028	45.76505	1.072858	85.31427	115.921615148
13	H	69.16782	28.40275	0.640808	88.64664	115.856534233
14	L	33.54474	58.18212	1.316990	88.35620	115.856058438
15	W	38.80223	47.39682	1.105213	85.76952	115.818195322
16	H	70.54835	27.48091	0.624125	88.06209	115.787555759
17	L	33.53465	57.98845	1.314994	88.19575	115.78745433
18	W	38.11497	48.73758	1.130795	86.20050	15.764885041
19	H	71.53171	26.89998	0.613234	87.73144	115.751290045
20	L	33.57264	57.68317	1.310785	88.01311	115.750837895
21	W	37.63724	49.76242	1.149851	86.55450	115.738751404
22	H	72.20978	26.53911	0.606240	87.55303	115.73299165
23	L	33.62333	57.38164	1.306369	87.84901	15.732491381
24	W	37.31332	50.50667	1.163435	86.82331	115.726503534
25	H	72.66482	26.31791	0.601815	87.46168	115.724156694
26	L	33.67067	57.12865	1.302570	87.71682	115.723784568
27	W	37.09833	51.02643	1.172790	87.01714	115.720994086
28	H	72.96301	26.18415	0.599056	87.41795	115.720071015
29	L	33.70903	56.93400	1.299608	87.61713	115.71984411
30	W	36.95835	51.37826	1.179053	87.15173	15.718608079
31	H	73.15430	26.10440	0.597361	87.39908	115.718257061
32	L	33.73775	56.79231	1.297438	87.54530	115.71813474
33	W	36.86882	51.61029	1.183146	87.24244	15.717610852
34	H	73.27462	26.05762	0.596335	87.39250	115.717481795

35	L	33.75817	56.69318	1.295913	87.49533	115.717421297
36	W	36.81255	51.75989	1.185765	87.30209	115.717207894
37	H	73.34888	26.03066	0.595725	87.39153	115.717162075
38	L	33.77215	56.62593	1.294876	87.46152	115.717134047
39	W	36.77776	51.85442	1.187408	87.34048	1115.71705028
40	H	73.39387	26.01545	0.595368	87.39279	115.717034612
41	L	33.78145	56.58143	1.294189	87.43919	115.717022295
42	W	36.75663	51.91302	1.188421	87.36470	115.71699057
43	H	73.42062	26.00709	0.595164	87.39467	115.71698543
44	L	33.78749	56.55262	1.293744	87.42474	15.716980256
45	W	36.74401	51.94870	1.189033	87.37972	115.716968662
46	H	73.43621	26.00264	0.595050	87.39647	115.716967055
47	L	33.79133	56.53433	1.293461	87.41558	115.716964967
48	W	36.73662	51.97002	1.189397	87.38885	115.716960883

*** OBJ FUNC VALUE ***

115.716960883

*** VARIABLE VALS ***

H =	0.5950502213
L =	1.2934613354
W =	1.1893970268

*** TERM WEIGHTS ***

WGT	1	0.3775974489
WGT	2	0.1329484507
WGT	3	0.1330270771
WGT	4	0.2446489982
WGT	5	0.1117780251

Problem 2 - Cofferdam Problem

Machine Comparison

	Ratliff	PC FORTRAN
OPTIMAL X	1.3854	1.38535185
OPTIMAL COST	71522.26	71522.25541960
WGT 1	0.4862	.4862
WGT 2	0.0092	.0092
WGT 3	0.0000	.0000
WGT 4	0.5046	.5046
INIT VAL OF X	10000	10000
CONV COND	1E-8	1E-8
ITER	6	6

Language Comparison

```
FILE: COFF8E.TRD
COFFERDAM PROBLEM
       19:20:09 03-18-1990
                TOLERANCE VALUE=   .00000001
******* LIST OF TERMS ******
       TERM 1    25100 X
```

```
       TERM 2    341 X^ 2
       TERM 3    1.34 X^ 3
       TERM 4    50000 X^-1
** VARIABLE START POINTS **
       X           10000
IT VAR CDCOF(1)  CDCOF(2) XNW(VAR) FUNVL     VLUE
 1 X   1.733027     50000 10.06905 6634.951   309149.27949
 2 X   21271.17     50000 1.411019 66815.17   71539.068977
 3 X   25416.86     50000 1.385122 71522.14   71522.256781
 4 X   25419.81     50000 1.385353 71522.25    71522.25542
 5 X   25419.79     50000 1.385351 71522.25    71522.25542
 6 X   25419.79     50000 1.385351 71522.25    71522.25542
                                   *** OBJ FUNC VALUE ***
                                               71522.25542
                                   *** VARIABLE VALS ***
                                   X =        1.3853518542
                                   *** TERM WEIGHTS ***
                                   WGT  1     0.4861749862
                                   WGT  2     0.0091502584
                                   WGT  3     0.0000498131
                                   WGT  4     0.5046249423
```

Convergence

```
FILE: COFF2E.TRD
COFFERDAM PROBLEM
       19:20:34 03-18-1990
                TOLERANCE VALUE=   .01
******* LIST OF TERMS ******
       TERM 1    25100 X
       TERM 2    341 X^ 2
       TERM 3    1.34 X^ 3
       TERM 4    50000 X^-1
** VARIABLE START POINTS **
       X           10000
IT VAR CDCOF(1)  CDCOF(2) XNW(VAR) FUNVL     VLUE
 1 X   1.733027     50000 10.06905 6634.951    71522.25542
 2 X   21271.17     50000 1.411019 66815.17    71522.25542
 3 X   25416.86     50000 1.385122 71522.14   71522.256424
                                   *** OBJ FUNC VALUE ***
                                              71522.256424
                                   *** VARIABLE VALS ***
                                   X =        1.3851229222
```

```
                                            *** TERM WEIGHTS ***
                                            WGT  1      0.486094638
                                            WGT  2     0.0091472343
                                            WGT  3     0.0000497884
                                            WGT  4     0.5047083392
FILE: COFF4E.TRD
COFFERDAM PROBLEM
       19:20:53 03-18-1990
                TOLERANCE VALUE=   .0001
******* LIST OF TERMS ******
       TERM 1    25100 X
       TERM 2    341 X^ 2
       TERM 3    1.34 X^ 3
       TERM 4    50000 X^-1
** VARIABLE START POINTS **
       X            10000
IT VAR CDCOF(1) CDCOF(2) XNW(VAR) FUNVL     VLUE
 1 X    1.733027    50000 10.06905 6634.951   71522.256424
 2 X    21271.17    50000 1.411019 66815.17   71522.256424
 3 X    25416.86    50000 1.385122 71522.14   71522.256424
 4 X    25419.81    50000 1.385353 71522.25    71522.25542
                                            *** OBJ FUNC VALUE ***
                                                   71522.25542
                                            *** VARIABLE VALS ***
                                            X =       1.3853539166
                                            *** TERM WEIGHTS ***
                                            WGT  1       0.48617571
                                            WGT  2     0.0091502856
                                            WGT  3     0.0000498134
                                            WGT  4      0.504624191
FILE: COFF7E.TRD
COFFERDAM PROBLEM
       19:21:15 03-18-1990
                TOLERANCE VALUE=   .0000001
******* LIST OF TERMS ******
       TERM 1    25100 X
       TERM 2    341 X^ 2
       TERM 3    1.34 X^ 3
       TERM 4    50000 X^-1
** VARIABLE START POINTS **
       X            10000
IT VAR CDCOF(1) CDCOF(2) XNW(VAR) FUNVL     VLUE
 1 X    1.733027    50000 10.06905 6634.951    71522.25542
 2 X    21271.17    50000 1.411019 66815.17    71522.25542
 3 X    25416.86    50000 1.385122 71522.14    71522.25542
 4 X    25419.81    50000 1.385353 71522.25    71522.25542
 5 X    25419.79    50000 1.385351 71522.25    71522.25542
                                            *** OBJ FUNC VALUE ***
                                                   71522.25542
```

```
                                       *** VARIABLE VALS ***
                                       X =        1.3853518355
                                       *** TERM WEIGHTS ***
                                       WGT  1     0.4861749796
                                       WGT  2     0.0091502581
                                       WGT  3     0.0000498131
                                       WGT  4     0.5046249491
FILE: COFF11E.TRD
COFFERDAM PROBLEM
       19:21:36 03-18-1990
                TOLERANCE VALUE=   9.999999999999999D-12
******* LIST OF TERMS ******
       TERM 1    25100 X
       TERM 2    341 X^ 2
       TERM 3    1.34 X^ 3
       TERM 4    50000 X^-1
** VARIABLE START POINTS **
       X            10000
IT VAR CDCOF(1) CDCOF(2) XNW(VAR) FUNVL    VLUE
 1 X   1.733027    50000 10.06905 6634.951  309149.27949
 2 X   21271.17    50000 1.411019 66815.17   71539.06898
 3 X   25416.86    50000 1.385122 71522.14   71522.25678
 4 X   25419.81    50000 1.385353 71522.25   71522.25542
 5 X   25419.79    50000 1.385351 71522.25   71522.25542
 6 X   25419.79    50000 1.385351 71522.25   71522.25542
 7 X   25419.79    50000 1.385351 71522.25   71522.25542
                                       *** OBJ FUNC VALUE ***
                                                71522.25542
                                       *** VARIABLE VALS ***
                                       X =        1.3853518541
                                       *** TERM WEIGHTS ***
                                       WGT  1     0.4861749861
                                       WGT  2     0.0091502584
                                       WGT  3     0.0000498131
                                       WGT  4     0.5046249423
```

Starting Vector

```
FILE: COFF2S.TRD
COFFERDAM PROBLEM
       19:23:00 03-18-1990
                TOLERANCE VALUE=   .00000001
******* LIST OF TERMS ******
       TERM 1    25100 X
       TERM 2    341 X^ 2
       TERM 3    1.34 X^ 3
```

```
       TERM 4     50000 X^-1
** VARIABLE START POINTS **
       X            0.002
IT VAR CDCOF(1) CDCOF(2) XNW(VAR) FUNVL     VLUE
 1 X    25104.92    50000 1.411229 70859.23   71534.84988
 2 X    25416.84    50000 1.385121 71522.14   71522.25644
 3 X    25419.81    50000 1.385353 71522.25   71522.25542
 4 X    25419.79    50000 1.385351 71522.25   71522.25542
 5 X    25419.79    50000 1.385351 71522.25   71522.25542
                                   *** OBJ FUNC VALUE ***
                                              71522.25542
                                   *** VARIABLE VALS ***
                                   X =       1.3853518542
                                   *** TERM WEIGHTS ***
                                   WGT  1    0.4861749862
                                   WGT  2    0.0091502584
                                   WGT  3    0.0000498131
                                   WGT  4    0.5046249423
FILE: COFF9S.TRD
COFFERDAM PROBLEM
       19:23:24 03-18-1990
                TOLERANCE VALUE=    .00000001
******* LIST OF TERMS ******
       TERM 1     25100 X
       TERM 2     341 X^ 2
       TERM 3     1.34 X^ 3
       TERM 4     50000 X^-1
** VARIABLE START POINTS **
       X          999999
IT VAR CDCOF(1) CDCOF(2) XNW(VAR) FUNVL     VLUE
 1 X    1.345061    50000 10.55239 6317.815 309149.279485
 2 X    20983.25    50000 1.415293 66452.41   71539.06898
 3 X    25416.36    50000 1.385085 71522.10   71522.25678
 4 X    25419.82    50000 1.385354 71522.25   71522.25542
 5 X    25419.79    50000 1.385351 71522.25   71522.25542
 6 X    25419.79    50000 1.385351 71522.25   71522.25542
                                   *** OBJ FUNC VALUE ***
                                              71522.25542
                                   *** VARIABLE VALS ***
                                   X =       1.3853518543
                                   *** TERM WEIGHTS ***
                                   WGT  1    0.4861749862
                                   WGT  2    0.0091502584
                                   WGT  3    0.0000498131
                                   WGT  4    0.5046249423
```

Problem 3 - Space Shuttle Problem

Machine Comparison

	Ratliff	PC FORTRAN
OPTIMAL X	.00000237	0.00000237
OPTIMAL COST	4319.55	4319.54457299
WGT 1	0.0000	0.0000
WGT 2	0.6767	0.6767
WGT 3	0.0000	0.0000
WGT 4	0.2169	0.2169
WGT 5	0.0000	0.0000
WGT 6	0.0000	0.0000
WGT 7	0.1065	0.1065
INIT VAL OF X	10000	10000
CONV COND	1E-8	1E-8
ITER	6	6

Language Comparison

```
FILE: SPACE.TRD

SPACE SHUTTLE DESIGN PROBLEM

       05:50:45 03-21-1990

                TOLERANCE VALUE=   9.999999999999999D-12

******* LIST OF TERMS ******

       TERM 1    11.861 X^ .479

       TERM 2    441.119 X^-.146

       TERM 3    3.218 X^ .648

       TERM 4    1467706 X^ .5679999999999999

       TERM 5    1040 X

       TERM 6    .078 X^ .736

       TERM 7    23.683 X^-.229

** VARIABLE START POINTS **

       X          999999

IT VAR CDCOF(1) CDCOF(2) XNW(VAR) FUNVL    VLUE

 1 X   513642.5 457.3537 0.000026 2642.991 6011.4814048

 2 X   1467771. 450.6376 0.000002 4312.153 4319.5174865

 3 X   1467727. 441.0185 0.000002 4319.446 4319.4475000

 4 X   1467728. 441.1071 0.000002 4319.447 4319.4474969

 5 X   1467728. 441.1065 0.000002 4319.447 4319.4474969

 6 X   1467728. 441.1065 0.000002 4319.447 4319.4474969

 7 X   1467728. 441.1065 0.000002 4319.447 4319.4474969
```

```
*** OBJ FUNC VALUE ***

          4319.4474969

*** VARIABLE VALS ***

X =       0.0000023709

*** TERM WEIGHTS ***

WGT  1    0.0000055498

WGT  2     0.676691201

WGT  3    0.0000001687

WGT  4    0.2168505023

WGT  5    0.0000005708

WGT  6    0.0000000013

WGT  7    0.1064520062
```

Convergence

```
FILE: SPAC2E.TRD
SPACE SHUTTLE DESIGN PROBLEM
       19:26:04 03-18-1990
                TOLERANCE VALUE=   .01
******* LIST OF TERMS ******
       TERM 1    11.861 X^ .479
       TERM 2    441.119 X^-.146
       TERM 3    3.218 X^ .648
       TERM 4    1467706 X^ .5679999999999999
       TERM 5    1040 X
       TERM 6    .078 X^ .736
       TERM 7    23.683 X^-.229
** VARIABLE START POINTS **
       X          999999
IT VAR CDCOF(1) CDCOF(2) XNW(VAR) FUNVL    VLUE
 1 X   513642.5 457.3537 0.000026 2642.991 4320.11389883
 2 X   1467771. 450.6376 0.000002 4312.153  4319.4475305
 3 X   1467727. 441.0185 0.000002 4319.446  4319.4475004
                                  *** OBJ FUNC VALUE ***
                                           4319.4475004
```

```
                                  *** VARIABLE VALS ***
                                  X =        0.0000023712
                                  *** TERM WEIGHTS ***
                                  WGT  1     0.0000055501
                                  WGT  2     0.6766778937
                                  WGT  3     0.0000001687
                                  WGT  4     0.2168670925
                                  WGT  5     0.0000005709
                                  WGT  6     0.0000000013
                                  WGT  7     0.1064487227
FILE: SPAC4E.TRD
SPACE SHUTTLE DESIGN PROBLEM
       19:26:18 03-18-1990
                TOLERANCE VALUE=   .0001
******* LIST OF TERMS ******
       TERM 1    11.861 X^ .479
       TERM 2    441.119 X^-.146
       TERM 3    3.218 X^ .648
       TERM 4    1467706 X^ .5679999999999999
       TERM 5    1040 X
       TERM 6    .078 X^ .736
       TERM 7    23.683 X^-.229
** VARIABLE START POINTS **
       X          999999
IT VAR CDCOF(1) CDCOF(2) XNW(VAR) FUNVL    VLUE
 1 X   513642.5 457.3537 0.000026 2642.991  4319.4475004
 2 X   1467771. 450.6376 0.000002 4312.153  4319.4475004
 3 X   1467727. 441.0185 0.000002 4319.446  4319.4475004
 4 X   1467728. 441.1071 0.000002 4319.447  4319.4474969
                                  *** OBJ FUNC VALUE ***
                                             4319.4474969
                                  *** VARIABLE VALS ***
                                  X =        0.0000023709
                                  *** TERM WEIGHTS ***
                                  WGT  1     0.0000055498
                                  WGT  2     0.6766912954
                                  WGT  3     0.0000001687
                                  WGT  4     0.2168503846
                                  WGT  5     0.0000005708
                                  WGT  6     0.0000000013
                                  WGT  7     0.1064520295
FILE: SPAC8E.TRD
SPACE SHUTTLE DESIGN PROBLEM
       19:27:00 03-18-1990
                TOLERANCE VALUE=   .00000001
******* LIST OF TERMS ******
       TERM 1    11.861 X^ .479
       TERM 2    441.119 X^-.146
       TERM 3    3.218 X^ .648
```

```
       TERM 4    1467706 X^ .5679999999999999
       TERM 5    1040 X
       TERM 6    .078 X^ .736
       TERM 7    23.683 X^-.229
** VARIABLE START POINTS **
       X          999999
IT VAR CDCOF(1) CDCOF(2) XNW(VAR) FUNVL    VLUE
 1 X   513642.5 457.3537 0.000026 2642.991  4319.4474969
 2 X   1467771. 450.6376 0.000002 4312.153  4319.4474969
 3 X   1467727. 441.0185 0.000002 4319.446  4319.4474969
 4 X   1467728. 441.1071 0.000002 4319.447  4319.4474969
 5 X   1467728. 441.1065 0.000002 4319.447  4319.4474969
                                  *** OBJ FUNC VALUE ***
                                            4319.4474969
                                  *** VARIABLE VALS ***
                                  X =       0.0000023709
                                  *** TERM WEIGHTS ***
                                  WGT  1    0.0000055498
                                  WGT  2    0.6766912003
                                  WGT  3    0.0000001687
                                  WGT  4    0.2168505031
                                  WGT  5    0.0000005708
                                  WGT  6    0.0000000013
                                  WGT  7     0.106452006
```

Starting Vector

```
FILE: SPAC2S.TRD
SPACE SHUTTLE DESIGN PROBLEM
       19:27:51 03-18-1990
                TOLERANCE VALUE=   9.999999999999999D-12
******* LIST OF TERMS ******
       TERM 1    11.861 X^ .479
       TERM 2    441.119 X^-.146
       TERM 3    3.218 X^ .648
       TERM 4    1467706 X^ .5679999999999999
       TERM 5    1040 X
       TERM 6    .078 X^ .736
       TERM 7    23.683 X^-.229
** VARIABLE START POINTS **
       X           0.002
IT VAR CDCOF(1) CDCOF(2) XNW(VAR) FUNVL    VLUE
 1 X   1467979. 460.7555 0.000002 4272.187  4319.4474969
 2 X   1467727. 440.8337 0.000002 4319.442  4319.4474969
 3 X   1467728. 441.1084 0.000002 4319.447  4319.4474969
```

```
 4 X    1467728. 441.1065 0.000002 4319.447  4319.4474969
 5 X    1467728. 441.1065 0.000002 4319.447  4319.4474969
 6 X    1467728. 441.1065 0.000002 4319.447  4319.4474969
                                   *** OBJ FUNC VALUE ***
                                             4319.4474969
                                   *** VARIABLE VALS ***
                                   X =       0.0000023709
                                   *** TERM WEIGHTS ***
                                   WGT  1    0.0000055498
                                   WGT  2     0.676691201
                                   WGT  3    0.0000001687
                                   WGT  4    0.2168505023
                                   WGT  5    0.0000005708
                                   WGT  6    0.0000000013
                                   WGT  7    0.1064520062
```

Problem 4 - Shrink Stope Problem

Machine Comparison

	Ratliff	PC FORTRAN
OPTIMAL H	7.357	7.35741510
OPTIMAL L	5.150	5.15003361
OPTIMAL COST	5758	5757.78812925
WGT 1	0.4607	0.4607
WGT 2	0.0787	0.0787
WGT 3	0.0787	0.0787
WGT 4	0.3820	0.3820
INIT VAL OF H	10000.0	10000.0
INIT VAL OF L	.002	.002
CONV COND	1E-11	1E-11
ITER	133	138

Language Comparison

```
FILE: STOPE.TRD
SHRINK STOPE MINING PROBLEM
        16:43:56 03-17-1990
                 TOLERANCE VALUE=   9.99999999999999D-12
******* LIST OF TERMS ******
        TERM 1    70 H L
        TERM 2    2333.33 L^-1
        TERM 3    3333.33 H^-1
        TERM 4    83333.33 H^-1 L^-1
** VARIABLE START POINTS **
        H           10000
        L           0.002
IT  VAR CDCOF(1) CDCOF(2) XNW(VAR) FUNVL    VLUE
  1 H       0.14 41669998 17252.32 4830.652  5757.7882236
  2 L   1207663. 2338.160 0.044001 106277.1   1171495.652
  3 H   3.080082 1897222. 784.8342 4834.707 106277.371573
  4 L   54938.39 2439.509 0.210723 23153.63  57863.521208
  5 H   14.75065 398795.9 164.4257 4850.773   23157.88564
  6 L   11509.79 2840.144 0.496748 11434.94  15923.712681
  7 H   34.77237 171091.0 70.14494 4878.213 11455.2180115
  8 L   4910.146 3521.346 0.846851 8316.327  9575.4216211
  9 H   59.27958 101737.0 41.42733 4911.590  8363.8479528
 10 L   2899.913 4344.884 1.224042 7099.236  7666.8918822
 11 H   85.68298 71413.74 28.86978 4947.299  7179.6982732
 12 L   2020.885 5219.853 1.607157 6495.760  6853.5482533
 13 H   112.5010 55184.71 22.14783 4983.306  6611.2210105
 14 L   1550.348 6095.925 1.982920 6148.433  6435.1433953
 15 H   138.8044 45358.89 18.07713 5018.371  6298.9366797
 16 L   1265.399 6943.204 2.342427 5928.213   6195.085971
 17 H   163.9699 38908.95 15.40432 5051.692  6112.6079134
 18 L   1078.302 7743.065 2.679699 5779.055  6047.8085754
 19 H   187.5789 34431.34 13.54830 5082.753  5995.4442931
 20 L   948.3812 8484.161 2.990976 5673.171  5953.4966108
 21 H   209.3683 31194.91 12.20636 5111.253  5919.2047876
 22 L   854.4455 9160.368 3.274268 5595.368   5891.376205
 23 H   229.1987 28784.30 11.20654 5137.052   5868.449579
 24 L   784.4581 9769.460 3.528989 5536.689  5849.6793287
 25 H   247.0292 26947.26 10.44439 5160.140  5834.1344833
 26 L   731.1073 10312.09 3.755630 5491.538  5821.3299942
 27 H   262.8941 25522.23 9.853009 5180.596  5810.6884044
 28 L   689.7106 10790.98 3.955460 5456.246   5801.885364
 29 H   276.8822 24401.25 9.387685 5198.566   5794.551986
 30 L   657.1379 11210.20 4.130270 5428.315  5788.4673142
 31 H   289.1189 23509.56 9.017456 5214.235  5783.3904539
 32 L   631.2219 11574.66 4.282162 5405.990  5779.1690254
```

33	H	299.7513	22793.90	8.720245	5227.811	5775.6431049
34	L	610.4171	11889.63	4.413375	5388.000	5772.7067708
35	H	308.9362	22215.32	8.479922	5239.511	5770.2525354
36	L	593.5945	12160.46	4.526162	5373.410	5768.2064058
37	H	316.8313	21744.80	8.284450	5249.547	5766.495465
38	L	579.9115	12392.33	4.622700	5361.513	5765.0678706
39	H	323.5890	21360.31	8.124692	5258.122	5763.8738106
40	L	568.7284	12590.12	4.705032	5351.771	5762.8769021
41	H	329.3522	21044.86	7.993605	5265.424	5762.0429363
42	L	559.5524	12758.32	4.775034	5343.764	5761.3463588
43	H	334.2524	20785.21	7.885694	5271.624	5760.7635816
44	L	551.9985	12900.98	4.834399	5337.163	5760.2766509
45	H	338.4079	20570.90	7.796621	5276.877	5759.869252
46	L	545.7634	13021.72	4.884633	5331.708	5759.5287714
47	H	341.9243	20393.63	7.722934	5281.317	5759.243898
48	L	540.6053	13123.70	4.927061	5327.192	5759.0057723
49	H	344.8943	20246.72	7.661862	5285.065	5758.8065377
50	L	536.3303	13209.71	4.962841	5323.445	5758.6399735
51	H	347.3989	20124.78	7.611169	5288.224	5758.5006147
52	L	532.7818	13282.15	4.992976	5320.334	5758.3840948
53	H	349.5083	20023.44	7.569036	5290.882	5758.2866084
54	L	529.8325	13343.09	5.018327	5317.746	5758.2050915
55	H	351.2829	19939.12	7.533981	5293.118	5758.1368918
56	L	527.3786	13394.32	5.039635	5315.592	5758.0798603
57	H	352.7744	19868.91	7.504789	5294.996	5758.0321468
58	L	525.3352	13437.34	5.057530	5313.798	5757.9922446
59	H	354.0271	19810.40	7.480462	5296.573	5757.9588624
60	L	523.6323	13473.46	5.072550	5312.302	5757.9309442
61	H	355.0785	19761.62	7.460176	5297.896	5757.9075882
62	L	522.2123	13503.75	5.085149	5311.055	5757.8880545
63	H	355.9604	19720.91	7.443251	5299.006	5757.8717131
64	L	521.0276	13529.15	5.095712	5310.014	5757.8580456
65	H	356.6998	19686.94	7.429125	5299.936	5757.846612
66	L	520.0387	13550.44	5.104566	5309.145	5757.837049
67	H	357.3196	19658.58	7.417330	5300.715	5757.8290492
68	L	519.2131	13568.27	5.111984	5308.419	5757.822358
69	H	357.8389	19634.89	7.407479	5301.369	5757.8167606
70	L	518.5235	13583.22	5.118198	5307.812	5757.8120789
71	H	358.2738	19615.09	7.399249	5301.915	5757.8081625
72	L	517.9474	13595.73	5.123402	5307.306	5757.8048866
73	H	358.6381	19598.56	7.392372	5302.373	5757.8021464
74	L	517.4661	13606.20	5.127759	5306.883	5757.7998543
75	H	358.9431	19584.74	7.386625	5302.757	5757.7979369
76	L	517.0638	13614.98	5.131406	5306.529	5757.7963332
77	H	359.1984	19573.19	7.381822	5303.078	5757.7949916
78	L	516.7275	13622.32	5.134459	5306.233	5757.7938695
79	H	359.4121	19563.53	7.377806	5303.347	5757.7929308
80	L	516.4464	13628.46	5.137015	5305.986	5757.7921456
81	H	359.5910	19555.46	7.374448	5303.571	5757.7914888

82	L	516.2114	13633.60	5.139153	5305.779	5757.7909395
83	H	359.7407	19548.70	7.371641	5303.760	5757.7904799
84	L	516.0149	13637.91	5.140943	5305.606	5757.7900955
85	H	359.8660	19543.06	7.369294	5303.917	5757.789774
86	L	515.8506	13641.51	5.142440	5305.462	5757.789505
87	H	359.9708	19538.34	7.367331	5304.049	5757.78928
88	L	515.7132	13644.52	5.143693	5305.341	5757.7890918
89	H	360.0585	19534.39	7.365690	5304.159	5757.7889344
90	L	515.5983	13647.04	5.144741	5305.240	5757.7888027
91	H	360.1319	19531.09	7.364317	5304.251	5757.7886926
92	L	515.5022	13649.15	5.145618	5305.155	5757.7886005
93	H	360.1933	19528.33	7.363169	5304.328	5757.7885234
94	L	515.4218	13650.91	5.146352	5305.085	5757.7884589
95	H	360.2446	19526.02	7.362209	5304.393	5757.788405
96	L	515.3546	13652.39	5.146966	5305.026	5757.7883599
97	H	360.2876	19524.09	7.361406	5304.447	5757.7883222
98	L	515.2984	13653.63	5.147479	5304.976	5757.7882906
99	H	360.3235	19522.48	7.360734	5304.492	5757.7882642
100	L	515.2514	13654.66	5.147909	5304.935	5757.7882421
101	H	360.3536	19521.13	7.360172	5304.530	5757.7882236
102	L	515.2120	13655.52	5.148268	5304.900	5757.7882082
103	H	360.3788	19520.00	7.359702	5304.562	5757.7881953
104	L	515.1792	13656.25	5.148569	5304.871	5757.7881844
105	H	360.3998	19519.05	7.359309	5304.588	5757.7881754
106	L	515.1516	13656.85	5.148820	5304.847	5757.7881678
107	H	360.4174	19518.26	7.358981	5304.610	5757.7881615
108	L	515.1286	13657.36	5.149031	5304.827	5757.7881562
109	H	360.4321	19517.60	7.358706	5304.629	5757.7881518
110	L	515.1094	13657.78	5.149207	5304.810	5757.7881481
111	H	360.4445	19517.05	7.358476	5304.644	5757.788145
112	L	515.0933	13658.13	5.149354	5304.796	5757.7881424
113	H	360.4548	19516.58	7.358283	5304.657	5757.7881402
114	L	515.0798	13658.43	5.149477	5304.784	5757.7881384
115	H	360.4634	19516.20	7.358122	5304.668	5757.7881369
116	L	515.0685	13658.68	5.149580	5304.774	5757.7881356
117	H	360.4706	19515.87	7.357988	5304.677	5757.7881346
118	L	515.0591	13658.88	5.149666	5304.766	5757.7881337
119	H	360.4766	19515.60	7.357875	5304.685	5757.7881329
120	L	515.0512	13659.06	5.149738	5304.759	5757.7881323
121	H	360.4817	19515.37	7.357781	5304.691	5757.7881318
122	L	515.0446	13659.20	5.149799	5304.753	5757.7881314
123	H	360.4859	19515.19	7.357702	5304.696	5757.788131
124	L	515.0391	13659.32	5.149849	5304.748	5757.7881307
125	H	360.4894	19515.03	7.357636	5304.701	5757.7881304
126	L	515.0345	13659.43	5.149891	5304.744	5757.7881302
127	H	360.4924	19514.89	7.357581	5304.704	5757.7881301
128	L	515.0307	13659.51	5.149927	5304.741	5757.7881299
129	H	360.4948	19514.78	7.357535	5304.707	5757.7881298
130	L	515.0274	13659.58	5.149956	5304.738	5757.7881297

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131 H   360.4969 19514.69 7.357496 5304.710  5757.7881296
132 L   515.0247 13659.64 5.149981 5304.735  5757.7881295
133 H   360.4986 19514.61 7.357464 5304.712  5757.7881295
134 L   515.0225 13659.69 5.150001 5304.733  5757.7881294
135 H   360.5001 19514.55 7.357437 5304.714  5757.7881294
136 L   515.0206 13659.73 5.150019 5304.732  5757.7881293
137 H   360.5013 19514.49 7.357415 5304.716  5757.7881293
138 L   515.0190 13659.77 5.150033 5304.730  5757.7881293
                                   *** OBJ FUNC VALUE ***
                                             5757.7881292
                                   *** VARIABLE VALS ***
                                   H =       7.3574151046
                                   L =       5.1500336137
                                   *** TERM WEIGHTS ***
                                   WGT  1    0.4606570088
                                   WGT  2    0.0786883468
                                   WGT  3    0.0786859824
                                   WGT  4     0.381968662
```

Convergence

```
FILE: STOP2E.TRD
SHRINK STOPE MINING PROBLEM
         20:46:22 03-18-1990
                  TOLERANCE VALUE=   .01
******* LIST OF TERMS ******
         TERM 1    70 H L
         TERM 2    2333.33 L^-1
         TERM 3    3333.33 H^-1
         TERM 4    83333.33 H^-1 L^-1
 ** VARIABLE START POINTS **
         H             10000
         L             0.002
 IT  VAR CDCOF(1) CDCOF(2) XNW(VAR) FUNVL    VLUE
   1 H       0.14 41669998 17252.32 4830.652   1171495.652
   2 L   1207663. 2338.160 0.044001 106277.1 106277.371573
   3 H   3.080082 1897222. 784.8342 4834.707  57863.521208
   4 L   54938.39 2439.509 0.210723 23153.63   23157.88564
   5 H   14.75065 398795.9 164.4257 4850.773  15923.712681
   6 L   11509.79 2840.144 0.496748 11434.94 11455.2180115
   7 H   34.77237 171091.0 70.14494 4878.213  9575.4216211
   8 L   4910.146 3521.346 0.846851 8316.327  8363.8479528
   9 H   59.27958 101737.0 41.42733 4911.590  7666.8918822
  10 L   2899.913 4344.884 1.224042 7099.236  7179.6982732
  11 H   85.68298 71413.74 28.86978 4947.299  6853.5482533
  12 L   2020.885 5219.853 1.607157 6495.760  6611.2210105
```

```
  13 H    112.5010 55184.71 22.14783 4983.306   6435.1433953
  14 L    1550.348 6095.925 1.982920 6148.433   6298.9366797
  15 H    138.8044 45358.89 18.07713 5018.371    6195.085971
  16 L    1265.399 6943.204 2.342427 5928.213   6112.6079134
  17 H    163.9699 38908.95 15.40432 5051.692   6047.8085754
  18 L    1078.302 7743.065 2.679699 5779.055   5995.4442931
  19 H    187.5789 34431.34 13.54830 5082.753   5953.4966108
  20 L    948.3812 8484.161 2.990976 5673.171   5919.2047876
  21 H    209.3683 31194.91 12.20636 5111.253   5891.376205
  22 L    854.4455 9160.368 3.274268 5595.368    5868.449579
                                    *** OBJ FUNC VALUE ***
                                                 5868.449579
                                    *** VARIABLE VALS ***
                                    H =       12.2063651418
                                    L =        3.2742684534
                                    *** TERM WEIGHTS ***
                                    WGT  1     0.4767330969
                                    WGT  2     0.1214334975
                                    WGT  3     0.0465338061
                                    WGT  4     0.3552995995
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Starting Vector

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FILE: STOP2S.TRD
SHRINK STOPE MINING PROBLEM
        19:36:30 03-18-1990
                 TOLERANCE VALUE=   9.999999999999999D-12
******* LIST OF TERMS ******
        TERM 1    70 H L
        TERM 2    2333.33 L^-1
        TERM 3    3333.33 H^-1
        TERM 4    83333.33 H^-1 L^-1
** VARIABLE START POINTS **
        H           0.002
        L           0.002
IT  VAR CDCOF(1) CDCOF(2) XNW(VAR) FUNVL    VLUE
  1 H       0.14 41669998 17252.32 4830.652   1171495.652
  2 L   1207663. 2338.160 0.044001 106277.1 106277.371573
  3 H   3.080082 1897222. 784.8342 4834.707  57863.521208
  4 L   54938.39 2439.509 0.210723 23153.63   23157.88564
  5 H   14.75065 398795.9 164.4257 4850.773  15923.712681
  6 L   11509.79 2840.144 0.496748 11434.94 11455.2180115
  7 H   34.77237 171091.0 70.14494 4878.213  9575.4216211
  8 L   4910.146 3521.346 0.846851 8316.327  8363.8479528
  9 H   59.27958 101737.0 41.42733 4911.590  7666.8918822
 10 L   2899.913 4344.884 1.224042 7099.236  7179.6982732
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11	H	85.68298	71413.74	28.86978	4947.299	6853.5482533
12	L	2020.885	5219.853	1.607157	6495.760	6611.2210105
13	H	112.5010	55184.71	22.14783	4983.306	6435.1433953
14	L	1550.348	6095.925	1.982920	6148.433	6298.9366797
15	H	138.8044	45358.89	18.07713	5018.371	6195.085971
16	L	1265.399	6943.204	2.342427	5928.213	6112.6079134
17	H	163.9699	38908.95	15.40432	5051.692	6047.8085754
18	L	1078.302	7743.065	2.679699	5779.055	5995.4442931
19	H	187.5789	34431.34	13.54830	5082.753	5953.4966108
20	L	948.3812	8484.161	2.990976	5673.171	5919.2047876
21	H	209.3683	31194.91	12.20636	5111.253	5891.376205
22	L	854.4455	9160.368	3.274268	5595.368	5868.449579
23	H	229.1987	28784.30	11.20654	5137.052	5849.6793287
24	L	784.4581	9769.460	3.528989	5536.689	5834.1344833
25	H	247.0292	26947.26	10.44439	5160.140	5821.3299942
26	L	731.1073	10312.09	3.755630	5491.538	5810.6884044
27	H	262.8941	25522.23	9.853009	5180.596	5801.885364
28	L	689.7106	10790.98	3.955460	5456.246	5794.551986
29	H	276.8822	24401.25	9.387685	5198.566	5788.4673142
30	L	657.1379	11210.20	4.130270	5428.315	5783.3904539
31	H	289.1189	23509.56	9.017456	5214.235	5779.1690254
32	L	631.2219	11574.66	4.282162	5405.990	5775.6431049
33	H	299.7513	22793.90	8.720245	5227.811	5772.7067708
34	L	610.4171	11889.63	4.413375	5388.000	5770.2525354
35	H	308.9362	22215.32	8.479922	5239.511	5768.2064058
36	L	593.5945	12160.46	4.526162	5373.410	5766.495465
37	H	316.8313	21744.80	8.284450	5249.547	5765.0678706
38	L	579.9115	12392.33	4.622700	5361.513	5763.8738106
39	H	323.5890	21360.31	8.124692	5258.122	5762.8769021
40	L	568.7284	12590.12	4.705032	5351.771	5762.0429363
41	H	329.3522	21044.86	7.993605	5265.424	5761.3463588
42	L	559.5524	12758.32	4.775034	5343.764	5760.7635816
43	H	334.2524	20785.21	7.885694	5271.624	5760.2766509
44	L	551.9985	12900.98	4.834399	5337.163	5759.869252
45	H	338.4079	20570.90	7.796621	5276.877	5759.5287714
46	L	545.7634	13021.72	4.884633	5331.708	5759.243898
47	H	341.9243	20393.63	7.722934	5281.317	5759.0057723
48	L	540.6053	13123.70	4.927061	5327.192	5758.8065377
49	H	344.8943	20246.72	7.661862	5285.065	5758.6399735
50	L	536.3303	13209.71	4.962841	5323.445	5758.5006147
51	H	347.3989	20124.78	7.611169	5288.224	5758.3840948
52	L	532.7818	13282.15	4.992976	5320.334	5758.2866084
53	H	349.5083	20023.44	7.569036	5290.882	5758.2050915
54	L	529.8325	13343.09	5.018327	5317.746	5758.1368918
55	H	351.2829	19939.12	7.533981	5293.118	5758.0798603
56	L	527.3786	13394.32	5.039635	5315.592	5758.0321468
57	H	352.7744	19868.91	7.504789	5294.996	5757.9922446
58	L	525.3352	13437.34	5.057530	5313.798	5757.9588624
59	H	354.0271	19810.40	7.480462	5296.573	5757.9309442

60	L	523.6323	13473.46	5.072550	5312.302	5757.9075882
61	H	355.0785	19761.62	7.460176	5297.896	5757.8880545
62	L	522.2123	13503.75	5.085149	5311.055	5757.8717131
63	H	355.9604	19720.91	7.443251	5299.006	5757.8580456
64	L	521.0276	13529.15	5.095712	5310.014	5757.846612
65	H	356.6998	19686.94	7.429125	5299.936	5757.837049
66	L	520.0387	13550.44	5.104566	5309.145	5757.8290492
67	H	357.3196	19658.58	7.417330	5300.715	5757.822358
68	L	519.2131	13568.27	5.111984	5308.419	5757.8167606
69	H	357.8389	19634.89	7.407479	5301.369	5757.8120789
70	L	518.5235	13583.22	5.118198	5307.812	5757.8081625
71	H	358.2738	19615.09	7.399249	5301.915	5757.8048866
72	L	517.9474	13595.73	5.123402	5307.306	5757.8021464
73	H	358.6381	19598.56	7.392372	5302.373	5757.7998543
74	L	517.4661	13606.20	5.127759	5306.883	5757.7979369
75	H	358.9431	19584.74	7.386625	5302.757	5757.7963332
76	L	517.0638	13614.98	5.131406	5306.529	5757.7949916
77	H	359.1984	19573.19	7.381822	5303.078	5757.7938695
78	L	516.7275	13622.32	5.134459	5306.233	5757.7929308
79	H	359.4121	19563.53	7.377806	5303.347	5757.7921456
80	L	516.4464	13628.46	5.137015	5305.986	5757.7914888
81	H	359.5910	19555.46	7.374448	5303.571	5757.7909395
82	L	516.2114	13633.60	5.139153	5305.779	5757.7904799
83	H	359.7407	19548.70	7.371641	5303.760	5757.7900955
84	L	516.0149	13637.91	5.140943	5305.606	5757.789774
85	H	359.8660	19543.06	7.369294	5303.917	5757.789505
86	L	515.8506	13641.51	5.142440	5305.462	5757.78928
87	H	359.9708	19538.34	7.367331	5304.049	5757.7890918
88	L	515.7132	13644.52	5.143693	5305.341	5757.7889344
89	H	360.0585	19534.39	7.365690	5304.159	5757.7888027
90	L	515.5983	13647.04	5.144741	5305.240	5757.7886926
91	H	360.1319	19531.09	7.364317	5304.251	5757.7886005
92	L	515.5022	13649.15	5.145618	5305.155	5757.7885234
93	H	360.1933	19528.33	7.363169	5304.328	5757.7884589
94	L	515.4218	13650.91	5.146352	5305.085	5757.788405
95	H	360.2446	19526.02	7.362209	5304.393	5757.7883599
96	L	515.3546	13652.39	5.146966	5305.026	5757.7883222
97	H	360.2876	19524.09	7.361406	5304.447	5757.7882906
98	L	515.2984	13653.63	5.147479	5304.976	5757.7882642
99	H	360.3235	19522.48	7.360734	5304.492	5757.7882421
100	L	515.2514	13654.66	5.147909	5304.935	5757.7882236
101	H	360.3536	19521.13	7.360172	5304.530	5757.7882082
102	L	515.2120	13655.52	5.148268	5304.900	5757.7881953
103	H	360.3788	19520.00	7.359702	5304.562	5757.7881844
104	L	515.1792	13656.25	5.148569	5304.871	5757.7881754
105	H	360.3998	19519.05	7.359309	5304.588	5757.7881678
106	L	515.1516	13656.85	5.148820	5304.847	5757.7881615
107	H	360.4174	19518.26	7.358981	5304.610	5757.7881562
108	L	515.1286	13657.36	5.149031	5304.827	5757.7881518

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109 H   360.4321 19517.60 7.358706 5304.629  5757.7881481
110 L   515.1094 13657.78 5.149207 5304.810   5757.788145
111 H   360.4445 19517.05 7.358476 5304.644  5757.7881424
112 L   515.0933 13658.13 5.149354 5304.796  5757.7881402
113 H   360.4548 19516.58 7.358283 5304.657  5757.7881384
114 L   515.0798 13658.43 5.149477 5304.784  5757.7881369
115 H   360.4634 19516.20 7.358122 5304.668  5757.7881356
116 L   515.0685 13658.68 5.149580 5304.774  5757.7881346
117 H   360.4706 19515.87 7.357988 5304.677  5757.7881337
118 L   515.0591 13658.88 5.149666 5304.766  5757.7881329
119 H   360.4766 19515.60 7.357875 5304.685  5757.7881323
120 L   515.0512 13659.06 5.149738 5304.759  5757.7881318
121 H   360.4817 19515.37 7.357781 5304.691  5757.7881314
122 L   515.0446 13659.20 5.149799 5304.753   5757.788131
123 H   360.4859 19515.19 7.357702 5304.696  5757.7881307
124 L   515.0391 13659.32 5.149849 5304.748  5757.7881304
125 H   360.4894 19515.03 7.357636 5304.701  5757.7881302
126 L   515.0345 13659.43 5.149891 5304.744  5757.7881301
127 H   360.4924 19514.89 7.357581 5304.704  5757.7881299
128 L   515.0307 13659.51 5.149927 5304.741  5757.7881298
129 H   360.4948 19514.78 7.357535 5304.707  5757.7881297
130 L   515.0274 13659.58 5.149956 5304.738  5757.7881296
131 H   360.4969 19514.69 7.357496 5304.710  5757.7881295
132 L   515.0247 13659.64 5.149981 5304.735  5757.7881295
133 H   360.4986 19514.61 7.357464 5304.712  5757.7881294
134 L   515.0225 13659.69 5.150001 5304.733  5757.7881294
135 H   360.5001 19514.55 7.357437 5304.714  5757.7881293
136 L   515.0206 13659.73 5.150019 5304.732  5757.7881293
137 H   360.5013 19514.49 7.357415 5304.716  5757.7881293
138 L   515.0190 13659.77 5.150033 5304.730  5757.7881292
                                     *** OBJ FUNC VALUE ***
                                               5757.7881292
                                     *** VARIABLE VALS ***
                                     H =         7.3574151046
                                     L =         5.1500336137
                                     *** TERM WEIGHTS ***
                                     WGT  1      0.4606570088
                                     WGT  2      0.0786883468
                                     WGT  3      0.0786859824
                                     WGT  4       0.381968662
FILE: STOP9S.TRD
SHRINK STOPE MINING PROBLEM
        19:37:26 03-18-1990
                 TOLERANCE VALUE=   9.999999999999999D-12
******* LIST OF TERMS ******
        TERM 1    70 H L
        TERM 2    2333.33 L^-1
        TERM 3    3333.33 H^-1
        TERM 4    83333.33 H^-1 L^-1
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** VARIABLE START POINTS **

		H	999999			
		L	999999			
IT	VAR	CDCOF(1)	CDCOF(2)	XNW(VAR)	FUNVL	VLUE
1	H	69999930	3333.413	0.006900	966102.8	966102.8954
2	L	0.483051	12078329	5000.420	4830.925	487870.296
3	H	350029.4	3349.995	0.097829	68486.40	68486.872042
4	L	6.848064	854155.4	353.1705	4837.070	38909.922749
5	H	24721.94	3569.287	0.379970	18787.20	18793.806932
6	L	26.59791	221648.7	91.28699	4856.086	13628.696735
7	H	6390.089	4246.201	0.815167	10417.98	10443.5468992
8	L	57.06172	104561.8	42.80693	4885.274	8974.4097788
9	H	2996.485	5280.055	1.327434	7955.276	8009.7850227
10	L	92.92042	65111.05	26.47108	4919.409	7430.5159853
11	H	1852.976	6481.418	1.870252	6931.064	7019.2108736
12	L	130.9176	46890.60	18.92534	4955.323	6737.6124647
13	H	1324.774	7736.595	2.416595	6402.887	6526.1789581
14	L	169.1617	36817.09	14.75277	4991.209	6370.5589279
15	H	1032.694	8981.984	2.949173	6091.188	6249.3504883
16	L	206.4421	30589.83	12.17277	5025.944	6156.2041681
17	H	852.0939	10179.21	3.456314	5890.209	6081.8943585
18	L	241.9420	26443.78	10.45456	5058.799	6023.2170815
19	H	731.8198	11304.32	3.930250	5752.470	5975.6584117
20	L	275.1175	23536.38	9.249340	5089.311	5937.4330823
21	H	647.4538	12342.98	4.366220	5653.851	5906.1217964
22	L	305.6354	21419.25	8.371442	5117.218	5880.654611
23	H	586.0009	13287.80	4.761869	5580.920	5859.6452199
24	L	333.3308	19833.45	7.713677	5142.413	5842.417933
25	H	539.9574	14136.65	5.116741	5525.645	5828.1379883
26	L	358.1719	18619.73	7.210094	5164.907	5816.3627012
27	H	504.7066	14891.19	5.431819	5482.950	5806.5704959
28	L	380.2273	17675.02	6.818021	5184.796	5798.4639297
29	H	477.2615	15555.83	5.709111	5449.478	5791.7079905
30	L	399.6377	16929.87	6.508691	5202.237	5786.0994042
31	H	455.6083	16136.72	5.951299	5422.923	5781.4185031
32	L	416.5909	16335.87	6.262045	5217.423	5777.5248102
33	H	438.3432	16641.01	6.161447	5401.657	5774.2720475
34	L	431.3013	15858.28	6.063700	5230.564	5771.5624387
35	H	424.4590	17076.31	6.342772	5384.494	5769.2974422
36	L	443.9940	15471.64	5.903093	5241.876	5767.4087002
37	H	413.2165	17450.22	6.498477	5370.556	5765.829249
38	L	454.8934	15156.84	5.772308	5251.571	5764.5111723
39	H	404.0616	17770.07	6.631637	5359.180	5763.4086653
40	L	464.2146	14899.35	5.665318	5259.847	5762.4880923
41	H	396.5723	18042.71	6.745120	5349.855	5761.7179653
42	L	472.1584	14687.93	5.577461	5266.890	5761.0746571
43	H	390.4222	18274.41	6.841549	5342.186	5760.5364394
44	L	478.9084	14513.80	5.505089	5272.868	5760.0867122
45	H	385.3562	18470.83	6.923282	5335.860	5759.710438

46	L	484.6297	14370.00	5.445321	5277.929	759.3959551
47	H	381.1724	18636.98	6.992413	5330.631	5759.1328333
48	L	489.4689	14251.00	5.395854	5282.206	5758.9128819
49	H	377.7098	18777.28	7.050781	5326.298	5758.7288539
50	L	493.5547	14152.35	5.354841	5285.814	5758.5749984
51	H	374.8389	18895.56	7.099988	5322.704	5758.4462731
52	L	496.9992	14070.43	5.320787	5288.854	5758.3386418
53	H	372.4551	18995.17	7.141420	5319.717	5758.2485924
54	L	499.8994	14002.34	5.292477	5291.413	5758.173293
55	H	370.4734	19078.94	7.176268	5317.233	5758.1102953
56	L	502.3387	13945.67	5.268918	5293.564	5758.0576133
57	H	368.8242	19149.35	7.205552	5315.165	5758.0135389
58	L	504.3886	13898.48	5.249296	5295.370	5757.9766796
59	H	367.4507	19208.47	7.230142	5313.442	5757.9458433
60	L	506.1099	13859.15	5.232941	5296.887	5757.920054
61	H	366.3058	19258.08	7.250778	5312.005	5757.8984792
62	L	507.5545	13826.34	5.219301	5298.160	5757.8804349
63	H	365.3511	19299.70	7.268087	5310.807	5757.8653397
64	L	508.7661	13798.97	5.207920	5299.227	5757.8527144
65	H	364.5544	19334.59	7.282598	5309.807	5757.8421527
66	L	509.7818	13776.13	5.198421	5300.121	5757.8333189
67	H	363.8894	19363.83	7.294759	5308.972	5757.825929
68	L	510.6331	13757.05	5.190488	5300.871	5757.8197481
69	H	363.3342	19388.33	7.304948	5308.275	5757.8145775
70	L	511.3463	13741.12	5.183862	5301.499	5757.8102527
71	H	362.8704	19408.85	7.313482	5307.692	5757.8066349
72	L	511.9437	13727.81	5.178327	5302.024	5757.8036089
73	H	362.4829	19426.04	7.320628	5307.205	5757.8010776
74	L	512.4440	13716.68	5.173701	5302.465	5757.7989602
75	H	362.1591	19440.42	7.326612	5306.799	5757.7971891
76	L	512.8628	13707.39	5.169836	5302.833	5757.7957076
77	H	361.8885	19452.47	7.331621	5306.459	5757.7944684
78	L	513.2134	13699.61	5.166604	5303.142	5757.7934318
79	H	361.6623	19462.55	7.335813	5306.174	5757.7925647
80	L	513.5069	13693.12	5.163903	5303.400	5757.7918394
81	H	361.4732	19470.99	7.339322	5305.937	5757.7912327
82	L	513.7525	13687.69	5.161644	5303.616	5757.7907252
83	H	361.3151	19478.05	7.342259	5305.738	5757.7903007
84	L	513.9581	13683.15	5.159756	5303.797	5757.7899456
85	H	361.1829	19483.96	7.344716	5305.572	5757.7896486
86	L	514.1301	13679.35	5.158177	5303.948	5757.7894001
87	H	361.0724	19488.90	7.346772	5305.433	5757.7891923
88	L	514.2740	13676.17	5.156856	5304.075	5757.7890184
89	H	360.9799	19493.04	7.348492	5305.317	5757.788873
90	L	514.3944	13673.52	5.155752	5304.181	5757.7887514
91	H	360.9026	19496.50	7.349932	5305.220	5757.7886496
92	L	514.4952	13671.30	5.154829	5304.270	5757.7885645
93	H	360.8380	19499.40	7.351136	5305.139	5757.7884933
94	L	514.5795	13669.44	5.154056	5304.344	5757.7884338

95	H	360.7839	19501.82	7.352143	5305.071	5757.788384
96	L	514.6500	13667.89	5.153410	5304.406	5757.7883423
97	H	360.7387	19503.84	7.352986	5305.014	5757.7883074
98	L	514.7090	13666.59	5.152870	5304.458	5757.7882783
99	H	360.7009	19505.54	7.353691	5304.966	5757.7882539
100	L	514.7584	13665.50	5.152418	5304.501	5757.7882335
101	H	360.6692	19506.96	7.354281	5304.927	5757.7882164
102	L	514.7997	13664.59	5.152040	5304.537	5757.7882022
103	H	360.6428	19508.15	7.354774	5304.893	5757.7881902
104	L	514.8342	13663.83	5.151724	5304.568	5757.7881802
105	H	360.6207	19509.14	7.355187	5304.866	5757.7881719
106	L	514.8631	13663.20	5.151459	5304.593	5757.7881649
107	H	360.6021	19509.97	7.355533	5304.842	5757.788159
108	L	514.8873	13662.66	5.151238	5304.615	5757.7881542
109	H	360.5867	19510.66	7.355821	5304.823	5757.7881501
110	L	514.9075	13662.22	5.151053	5304.632	5757.7881466
111	H	360.5737	19511.24	7.356063	5304.807	5757.7881438
112	L	514.9244	13661.85	5.150899	5304.647	5757.7881414
113	H	360.5629	19511.73	7.356265	5304.793	5757.7881394
114	L	514.9385	13661.54	5.150769	5304.660	5757.7881377
115	H	360.5538	19512.14	7.356434	5304.782	5757.7881363
116	L	514.9504	13661.28	5.150661	5304.670	5757.7881351
117	H	360.5462	19512.48	7.356576	5304.772	5757.7881342
118	L	514.9603	13661.06	5.150570	5304.679	5757.7881333
119	H	360.5399	19512.76	7.356694	5304.764	5757.7881326
120	L	514.9686	13660.88	5.150495	5304.686	5757.7881321
121	H	360.5346	19513.00	7.356793	5304.757	5757.7881316
122	L	514.9755	13660.72	5.150431	5304.692	5757.7881312
123	H	360.5302	19513.20	7.356876	5304.752	5757.7881309
124	L	514.9813	13660.60	5.150378	5304.697	5757.7881306
125	H	360.5265	19513.36	7.356945	5304.747	5757.7881303
126	L	514.9861	13660.49	5.150334	5304.701	5757.7881301
127	H	360.5234	19513.50	7.357003	5304.743	5757.78813
128	L	514.9902	13660.40	5.150297	5304.705	5757.7881298
129	H	360.5208	19513.62	7.357051	5304.740	5757.7881297
130	L	514.9936	13660.33	5.150266	5304.708	5757.7881296
131	H	360.5186	19513.72	7.357092	5304.737	5757.7881296
132	L	514.9964	13660.26	5.150240	5304.711	5757.7881295
133	H	360.5168	19513.80	7.357126	5304.735	5757.7881294
134	L	514.9988	13660.21	5.150218	5304.713	5757.7881294
135	H	360.5152	19513.87	7.357154	5304.733	5757.7881293
136	L	515.0008	13660.17	5.150200	5304.714	5757.7881293
137	H	360.5140	19513.92	7.357178	5304.731	5757.7881293
138	L	515.0024	13660.13	5.150185	5304.716	5757.7881293

*** OBJ FUNC VALUE ***
5757.7881293

*** VARIABLE VALS ***
H = 7.357178294
L = 5.1501852234

```
                                            *** TERM WEIGHTS ***
                                            WGT  1     0.4606557424
                                            WGT  2     0.0786860304
                                            WGT  3     0.0786885151
                                            WGT  4      0.381969712
FILE: STOPH2.TRD
SHRINK STOPE MINING PROBLEM
        19:41:00 03-18-1990
                 TOLERANCE VALUE=   9.999999999999999D-12
******* LIST OF TERMS ******
        TERM 1    70 H L
        TERM 2    2333.33 L^-1
        TERM 3    3333.33 H^-1
        TERM 4    83333.33 H^-1 L^-1
** VARIABLE START POINTS **
        H            0.002
        L                1
```

IT	VAR	CDCOF(1)	CDCOF(2)	XNW(VAR)	FUNVL	VLUE
1	H	70	86666.66	35.18657	4926.120	7259.4506644
2	L	2463.060	4701.657	1.381617	6806.016	6900.7496537
3	H	96.71325	63649.08	25.65388	4962.140	6650.979606
4	L	1795.771	5581.701	1.763021	6331.970	6461.904747
5	H	123.4115	50600.65	20.24884	4997.881	6321.3645523
6	L	1417.418	6448.791	2.132996	6046.697	6211.3162883
7	H	149.3097	42402.00	16.85191	5032.308	6126.2296482
8	L	1179.633	7278.367	2.483953	5860.309	6058.1111515
9	H	173.8767	36882.00	14.56419	5064.749	6004.1108986
10	L	1019.493	8055.123	2.810889	5731.369	5960.2408049
11	H	196.7622	32979.93	12.94654	5094.784	5924.8879593
12	L	906.2582	8770.052	3.110821	5638.415	5895.884277
13	H	217.7575	30121.53	11.76120	5122.183	5872.2521356
14	L	823.2846	9418.768	3.382377	5569.318	5852.736044
15	H	236.7664	27970.83	10.86907	5146.864	5836.7136584
16	L	760.8353	10000.34	3.625451	5516.742	5823.4230447
17	H	253.7815	26318.97	10.18367	5168.857	5812.4542049
18	L	712.8570	10516.36	3.840887	5476.007	5803.3282772
19	H	268.8621	25029.70	9.648572	5188.271	5795.7687454
20	L	675.4000	10970.18	4.030198	5443.992	5789.4666862
21	H	282.1138	24010.55	9.225477	5205.271	5784.2326566
22	L	645.7834	11366.28	4.195327	5418.545	5779.863432
23	H	293.6728	23196.69	8.887531	5220.054	5776.2278828
24	L	622.1272	11709.76	4.338448	5398.133	5773.1903488
25	H	303.6913	22541.42	8.615380	5232.833	5770.6594674
26	L	603.0766	12005.95	4.461820	5381.639	5768.5436817
27	H	312.3274	22010.31	8.394755	5243.824	5766.7790787
28	L	587.6328	12260.16	4.567673	5368.230	5765.3033549
29	H	319.7371	21577.48	8.214928	5253.236	5764.0716942
30	L	575.0450	12477.46	4.658136	5357.276	5763.0414349
31	H	326.0695	21223.17	8.067705	5261.266	5762.1811118

32	L	564.7393	12662.57	4.735185	5348.291	5761.4613744
33	H	331.4630	20932.07	7.946732	5268.095	5760.8601187
34	L	556.2712	12819.81	4.800623	5340.897	5760.3570789
35	H	336.0436	20692.18	7.847030	5273.889	5759.936725
36	L	549.2921	12953.05	4.856064	5334.796	5759.5850256
37	H	339.9245	20494.00	7.764654	5278.792	5759.2910707
38	L	543.5258	13065.72	4.902940	5329.749	5759.0451243
39	H	343.2058	20329.93	7.696452	5282.935	5758.8395246
40	L	538.7516	13160.82	4.942506	5325.567	5758.6675046
41	H	345.9754	20193.86	7.639889	5286.429	5758.5236852
42	L	534.7922	13240.99	4.975854	5322.097	5758.4033571
43	H	348.3098	20080.87	7.592912	5289.372	5758.3027453
44	L	531.5039	13308.47	5.003926	5319.213	55758.218569
45	H	350.2748	19986.91	7.553851	5291.848	5758.1481799
46	L	528.7695	13365.23	5.027533	5316.813	5758.0892905
47	H	351.9273	19908.71	7.521338	5293.929	5758.0400434
48	L	526.4937	13412.91	5.047368	5314.815	5757.998843
49	H	353.3158	19843.58	7.494255	5295.677	5757.9643869
50	L	524.5978	13452.95	5.064022	5313.150	5757.9355612
51	H	354.4815	19789.28	7.471679	5297.145	5757.9114532
52	L	523.0175	13486.55	5.077996	5311.762	5757.8912851
53	H	355.4597	19744.00	7.452849	5298.376	5757.8744172
54	L	521.6994	13514.73	5.089716	5310.604	5757.8603061
55	H	356.2801	19706.21	7.437137	5299.408	5757.8485039
56	L	520.5996	13538.35	5.099540	5309.637	5757.8386308
57	H	356.9678	19674.66	7.424020	5300.273	5757.8303728
58	L	519.6814	13558.15	5.107774	5308.830	5757.8234648
59	H	357.5441	19648.32	7.413067	5300.998	5757.8176868
60	L	518.9147	13574.74	5.114671	5308.156	5757.8128533
61	H	358.0270	19626.32	7.403918	5301.605	5757.8088104
62	L	518.2742	13588.63	5.120448	5307.593	5757.8054285
63	H	358.4314	19607.94	7.396274	5302.114	5757.8025997
64	L	517.7391	13600.26	5.125286	5307.123	5757.8002334
65	H	358.7700	19592.58	7.389886	5302.539	5757.7982542
66	L	517.2920	13610.00	5.129336	5306.730	5757.7965985
67	H	359.0535	19579.74	7.384547	5302.896	5757.7952136
68	L	516.9183	13618.15	5.132727	5306.401	5757.7940551
69	H	359.2909	19569.01	7.380084	5303.194	5757.7930861
70	L	516.6059	13624.97	5.135565	5306.126	5757.7922755
71	H	359.4895	19560.04	7.376353	5303.444	5757.7915975
72	L	516.3447	13630.69	5.137940	5305.897	5757.7910303
73	H	359.6558	19552.53	7.373234	5303.653	5757.7905559
74	L	516.1264	13635.47	5.139927	5305.704	5757.7901591
75	H	359.7949	19546.26	7.370626	5303.828	5757.7898272
76	L	515.9438	13639.46	5.141590	5305.544	5757.7895495
77	H	359.9113	19541.02	7.368445	5303.974	5757.7893172
78	L	515.7911	13642.81	5.142982	5305.410	5757.789123
79	H	360.0087	19536.63	7.366621	5304.096	5757.7889605
80	L	515.6635	13645.61	5.144146	5305.297	5757.7888245

81	H	360.0902	19532.97	7.365096	5304.199	5757.7887108
82	L	515.5567	13647.95	5.145121	5305.203	5757.7886157
83	H	360.1584	19529.90	7.363820	5304.285	5757.7885361
84	L	515.4674	13649.91	5.145936	5305.125	5757.7884696
85	H	360.2155	19527.33	7.362754	5304.356	5757.7884139
86	L	515.3927	13651.55	5.146618	5305.059	5757.7883673
87	H	360.2632	19525.19	7.361861	5304.416	5757.7883284
88	L	515.3303	13652.92	5.147188	5305.004	5757.7882958
89	H	360.3031	19523.39	7.361115	5304.467	5757.7882686
90	L	515.2781	13654.07	5.147665	5304.958	5757.7882458
91	H	360.3365	19521.89	7.360491	5304.508	5757.7882267
92	L	515.2344	13655.03	5.148064	5304.920	5757.7882107
93	H	360.3645	19520.64	7.359969	5304.544	5757.7881974
94	L	515.1978	13655.84	5.148398	5304.888	5757.7881862
95	H	360.3879	19519.59	7.359532	5304.573	5757.7881769
96	L	515.1673	13656.51	5.148678	5304.861	5757.7881691
97	H	360.4074	19518.71	7.359167	5304.598	5757.7881626
98	L	515.1417	13657.07	5.148911	5304.838	5757.7881571
99	H	360.4238	19517.97	7.358862	5304.618	5757.7881525
100	L	515.1203	13657.54	5.149107	5304.820	5757.7881487
101	H	360.4375	19517.36	7.358606	5304.635	5757.7881455
102	L	515.1024	13657.93	5.149270	5304.804	5757.7881428
103	H	360.4489	19516.85	7.358392	5304.650	5757.7881406
104	L	515.0875	13658.26	5.149407	5304.791	5757.7881387
105	H	360.4585	19516.42	7.358214	5304.662	5757.7881371
106	L	515.0749	13658.54	5.149522	5304.780	5757.7881358
107	H	360.4665	19516.06	7.358064	5304.672	5757.7881347
108	L	515.0645	13658.77	5.149617	5304.770	5757.7881338
109	H	360.4732	19515.75	7.357939	5304.680	5757.7881331
110	L	515.0557	13658.96	5.149697	5304.763	5757.7881324
111	H	360.4788	19515.50	7.357834	5304.687	5757.7881319
112	L	515.0484	13659.12	5.149764	5304.756	5757.7881314
113	H	360.4835	19515.29	7.357747	5304.693	5757.7881311
114	L	515.0423	13659.25	5.149820	5304.751	5757.7881307
115	H	360.4874	19515.12	7.357674	5304.698	5757.7881305
116	L	515.0371	13659.37	5.149867	5304.746	5757.7881303
117	H	360.4907	19514.97	7.357612	5304.702	5757.7881301
118	L	515.0329	13659.46	5.149907	5304.743	5757.7881299
119	H	360.4934	19514.85	7.357561	5304.706	5757.7881298
120	L	515.0293	13659.54	5.149939	5304.739	5757.7881297
121	H	360.4957	19514.74	7.357518	5304.709	5757.7881296
122	L	515.0263	13659.61	5.149967	5304.737	5757.7881295
123	H	360.4977	19514.66	7.357482	5304.711	5757.7881295
124	L	515.0238	13659.66	5.149990	5304.735	5757.7881294
125	H	360.4993	19514.59	7.357452	5304.713	5757.7881294
126	L	515.0217	13659.71	5.150009	5304.733	5757.7881293
127	H	360.5006	19514.52	7.357427	5304.715	5757.7881293
128	L	515.0199	13659.75	5.150025	5304.731	5757.7881293

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                                              *** OBJ FUNC VALUE ***
                                                        5757.7881293
                                              *** VARIABLE VALS ***
                                              H =       7.3574279129
                                              L =       5.1500254139
                                              *** TERM WEIGHTS ***
                                              WGT  1    0.4606570773
                                              WGT  2    0.0786884721
                                              WGT  3    0.0786858454
                                              WGT  4    0.3819686052
FILE: STOPH9.TRD
SHRINK STOPE MINING PROBLEM
        19:39:40 03-18-1990
                 TOLERANCE VALUE=   9.999999999999999D-12
******* LIST OF TERMS ******
        TERM 1    70 H L
        TERM 2    2333.33 L^-1
        TERM 3    3333.33 H^-1
        TERM 4    83333.33 H^-1 L^-1
** VARIABLE START POINTS **
        H           999999
        L                1
```

IT	VAR	CDCOF(1)	CDCOF(2)	XNW(VAR)	FUNVL	VLUE
1	H	70	86666.66	35.18657	4926.120	7259.4506644
2	L	2463.060	4701.657	1.381617	6806.016	6900.7496537
3	H	96.71325	63649.08	25.65388	4962.140	6650.979606
4	L	1795.771	5581.701	1.763021	6331.970	6461.904747
5	H	123.4115	50600.65	20.24884	4997.881	6321.3645523
6	L	1417.418	6448.791	2.132996	6046.697	6211.3162883
7	H	149.3097	42402.00	16.85191	5032.308	6126.2296482
8	L	1179.633	7278.367	2.483953	5860.309	6058.1111515
9	H	173.8767	36882.00	14.56419	5064.749	6004.1108986
10	L	1019.493	8055.123	2.810889	5731.369	5960.2408049
11	H	196.7622	32979.93	12.94654	5094.784	5924.8879593
12	L	906.2582	8770.052	3.110821	5638.415	5895.884277
13	H	217.7575	30121.53	11.76120	5122.183	5872.2521356
14	L	823.2846	9418.768	3.382377	5569.318	5852.736044
15	H	236.7664	27970.83	10.86907	5146.864	5836.7136584
16	L	760.8353	10000.34	3.625451	5516.742	5823.4230447
17	H	253.7815	26318.97	10.18367	5168.857	5812.4542049
18	L	712.8570	10516.36	3.840887	5476.007	5803.3282772
19	H	268.8621	25029.70	9.648572	5188.271	5795.7687454
20	L	675.4000	10970.18	4.030198	5443.992	5789.4666862
21	H	282.1138	24010.55	9.225477	5205.271	5784.2326566
22	L	645.7834	11366.28	4.195327	5418.545	5779.863432
23	H	293.6728	23196.69	8.887531	5220.054	5776.2278828
24	L	622.1272	11709.76	4.338448	5398.133	5773.1903488
25	H	303.6913	22541.42	8.615380	5232.833	5770.6594674
26	L	603.0766	12005.95	4.461820	5381.639	5768.5436817

27	H	312.3274	22010.31	8.394755	5243.824	5766.7790787
28	L	587.6328	12260.16	4.567673	5368.230	5765.3033549
29	H	319.7371	21577.48	8.214928	5253.236	5764.0716942
30	L	575.0450	12477.46	4.658136	5357.276	5763.0414349
31	H	326.0695	21223.17	8.067705	5261.266	5762.1811118
32	L	564.7393	12662.57	4.735185	5348.291	5761.4613744
33	H	331.4630	20932.07	7.946732	5268.095	5760.8601187
34	L	556.2712	12819.81	4.800623	5340.897	5760.3570789
35	H	336.0436	20692.18	7.847030	5273.889	5759.936725
36	L	549.2921	12953.05	4.856064	5334.796	5759.5850256
37	H	339.9245	20494.00	7.764654	5278.792	5759.2910707
38	L	543.5258	13065.72	4.902940	5329.749	5759.0451243
39	H	343.2058	20329.93	7.696452	5282.935	5758.8395246
40	L	538.7516	13160.82	4.942506	5325.567	5758.6675046
41	H	345.9754	20193.86	7.639889	5286.429	5758.5236852
42	L	534.7922	13240.99	4.975854	5322.097	5758.4033571
43	H	348.3098	20080.87	7.592912	5289.372	5758.3027453
44	L	531.5039	13308.47	5.003926	5319.213	5758.218569
45	H	350.2748	19986.91	7.553851	5291.848	5758.1481799
46	L	528.7695	13365.23	5.027533	5316.813	5758.0892905
47	H	351.9273	19908.71	7.521338	5293.929	5758.0400434
48	L	526.4937	13412.91	5.047368	5314.815	5757.998843
49	H	353.3158	19843.58	7.494255	5295.677	5757.9643869
50	L	524.5978	13452.95	5.064022	5313.150	5757.9355612
51	H	354.4815	19789.28	7.471679	5297.145	5757.9114532
52	L	523.0175	13486.55	5.077996	5311.762	5757.8912851
53	H	355.4597	19744.00	7.452849	5298.376	5757.8744172
54	L	521.6994	13514.73	5.089716	5310.604	5757.8603061
55	H	356.2801	19706.21	7.437137	5299.408	5757.8485039
56	L	520.5996	13538.35	5.099540	5309.637	5757.8386308
57	H	356.9678	19674.66	7.424020	5300.273	5757.8303728
58	L	519.6814	13558.15	5.107774	5308.830	5757.8234648
59	H	357.5441	19648.32	7.413067	5300.998	5757.8176868
60	L	518.9147	13574.74	5.114671	5308.156	5757.8128533
61	H	358.0270	19626.32	7.403918	5301.605	5757.8088104
62	L	518.2742	13588.63	5.120448	5307.593	5757.8054285
63	H	358.4314	19607.94	7.396274	5302.114	5757.8025997
64	L	517.7391	13600.26	5.125286	5307.123	5757.8002334
65	H	358.7700	19592.58	7.389886	5302.539	5757.7982542
66	L	517.2920	13610.00	5.129336	5306.730	5757.7965985
67	H	359.0535	19579.74	7.384547	5302.896	5757.7952136
68	L	516.9183	13618.15	5.132727	5306.401	5757.7940551
69	H	359.2909	19569.01	7.380084	5303.194	5757.7930861
70	L	516.6059	13624.97	5.135565	5306.126	5757.7922755
71	H	359.4895	19560.04	7.376353	5303.444	5757.7915975
72	L	516.3447	13630.69	5.137940	5305.897	5757.7910303
73	H	359.6558	19552.53	7.373234	5303.653	5757.7905559
74	L	516.1264	13635.47	5.139927	5305.704	5757.7901591
75	H	359.7949	19546.26	7.370626	5303.828	5757.7898272

76	L	515.9438	13639.46	5.141590	5305.544	5757.7895495
77	H	359.9113	19541.02	7.368445	5303.974	5757.7893172
78	L	515.7911	13642.81	5.142982	5305.410	5757.789123
79	H	360.0087	19536.63	7.366621	5304.096	5757.7889605
80	L	515.6635	13645.61	5.144146	5305.297	5757.7888245
81	H	360.0902	19532.97	7.365096	5304.199	5757.7887108
82	L	515.5567	13647.95	5.145121	5305.203	5757.7886157
83	H	360.1584	19529.90	7.363820	5304.285	5757.7885361
84	L	515.4674	13649.91	5.145936	5305.125	5757.7884696
85	H	360.2155	19527.33	7.362754	5304.356	5757.7884139
86	L	515.3927	13651.55	5.146618	5305.059	5757.7883673
87	H	360.2632	19525.19	7.361861	5304.416	5757.7883284
88	L	515.3303	13652.92	5.147188	5305.004	5757.7882958
89	H	360.3031	19523.39	7.361115	5304.467	5757.7882686
90	L	515.2781	13654.07	5.147665	5304.958	5757.7882458
91	H	360.3365	19521.89	7.360491	5304.508	5757.7882267
92	L	515.2344	13655.03	5.148064	5304.920	5757.7882107
93	H	360.3645	19520.64	7.359969	5304.544	5757.7881974
94	L	515.1978	13655.84	5.148398	5304.888	5757.7881862
95	H	360.3879	19519.59	7.359532	5304.573	5757.7881769
96	L	515.1673	13656.51	5.148678	5304.861	5757.7881691
97	H	360.4074	19518.71	7.359167	5304.598	5757.7881626
98	L	515.1417	13657.07	5.148911	5304.838	5757.7881571
99	H	360.4238	19517.97	7.358862	5304.618	5757.7881525
100	L	515.1203	13657.54	5.149107	5304.820	5757.7881487
101	H	360.4375	19517.36	7.358606	5304.635	5757.7881455
102	L	515.1024	13657.93	5.149270	5304.804	5757.7881428
103	H	360.4489	19516.85	7.358392	5304.650	5757.7881406
104	L	515.0875	13658.26	5.149407	5304.791	5757.7881387
105	H	360.4585	19516.42	7.358214	5304.662	5757.7881371
106	L	515.0749	13658.54	5.149522	5304.780	5757.7881358
107	H	360.4665	19516.06	7.358064	5304.672	5757.7881347
108	L	515.0645	13658.77	5.149617	5304.770	5757.7881338
109	H	360.4732	19515.75	7.357939	5304.680	5757.7881331
110	L	515.0557	13658.96	5.149697	5304.763	5757.7881324
111	H	360.4788	19515.50	7.357834	5304.687	5757.7881319
112	L	515.0484	13659.12	5.149764	5304.756	5757.7881314
113	H	360.4835	19515.29	7.357747	5304.693	5757.7881311
114	L	515.0423	13659.25	5.149820	5304.751	5757.7881307
115	H	360.4874	19515.12	7.357674	5304.698	5757.7881305
116	L	515.0371	13659.37	5.149867	5304.746	5757.7881303
117	H	360.4907	19514.97	7.357612	5304.702	5757.7881301
118	L	515.0329	13659.46	5.149907	5304.743	5757.7881299
119	H	360.4934	19514.85	7.357561	5304.706	5757.7881298
120	L	515.0293	13659.54	5.149939	5304.739	5757.7881297
121	H	360.4957	19514.74	7.357518	5304.709	5757.7881296
122	L	515.0263	13659.61	5.149967	5304.737	5757.7881295
123	H	360.4977	19514.66	7.357482	5304.711	5757.7881295
124	L	515.0238	13659.66	5.149990	5304.735	5757.7881294

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125 H    360.4993 19514.59 7.357452 5304.713   5757.7881294
126 L    515.0217 13659.71 5.150009 5304.733   5757.7881293
127 H    360.5006 19514.52 7.357427 5304.715   5757.7881293
128 L    515.0199 13659.75 5.150025 5304.731   5757.7881293
                                    *** OBJ FUNC VALUE ***
                                               5757.7881293
                                    *** VARIABLE VALS ***
                                    H =        7.3574279129
                                    L =        5.1500254139
                                    *** TERM WEIGHTS ***
                                    WGT  1     0.4606570773
                                    WGT  2     0.0786884721
                                    WGT  3     0.0786858454
                                    WGT  4     0.3819686052
FILE: STOPL2.TRD
SHRINK STOPE MINING PROBLEM
        19:42:41 03-18-1990
                 TOLERANCE VALUE=   9.999999999999999D-12
******* LIST OF TERMS ******
        TERM 1    70 H L
        TERM 2    2333.33 L^-1
        TERM 3    3333.33 H^-1
        TERM 4    83333.33 H^-1 L^-1
** VARIABLE START POINTS **
        H               1
        L           0.002
IT  VAR CDCOF(1) CDCOF(2) XNW(VAR) FUNVL    VLUE
  1 H       0.14 41669998 17252.32 4830.652    1171495.652
  2 L   1207663. 2338.160 0.044001 106277.1 106277.371573
  3 H   3.080082 1897222. 784.8342 4834.707   57863.521208
  4 L   54938.39 2439.509 0.210723 23153.63    23157.88564
  5 H   14.75065 398795.9 164.4257 4850.773   15923.712681
  6 L   11509.79 2840.144 0.496748 11434.94  11455.2180115
  7 H   34.77237 171091.0 70.14494 4878.213   9575.4216211
  8 L   4910.146 3521.346 0.846851 8316.327   8363.8479528
  9 H   59.27958 101737.0 41.42733 4911.590   7666.8918822
 10 L   2899.913 4344.884 1.224042 7099.236   7179.6982732
 11 H   85.68298 71413.74 28.86978 4947.299   6853.5482533
 12 L   2020.885 5219.853 1.607157 6495.760   6611.2210105
 13 H   112.5010 55184.71 22.14783 4983.306   6435.1433953
 14 L   1550.348 6095.925 1.982920 6148.433   6298.9366797
 15 H   138.8044 45358.89 18.07713 5018.371    6195.085971
 16 L   1265.399 6943.204 2.342427 5928.213   6112.6079134
 17 H   163.9699 38908.95 15.40432 5051.692   6047.8085754
 18 L   1078.302 7743.065 2.679699 5779.055   5995.4442931
 19 H   187.5789 34431.34 13.54830 5082.753   5953.4966108
 20 L   948.3812 8484.161 2.990976 5673.171   5919.2047876
 21 H   209.3683 31194.91 12.20636 5111.253    5891.376205
 22 L   854.4455 9160.368 3.274268 5595.368    5868.449579
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23	H	229.1987	28784.30	11.20654	5137.052	5849.6793287
24	L	784.4581	9769.460	3.528989	5536.689	5834.1344833
25	H	247.0292	26947.26	10.44439	5160.140	5821.3299942
26	L	731.1073	10312.09	3.755630	5491.538	5810.6884044
27	H	262.8941	25522.23	9.853009	5180.596	5801.885364
28	L	689.7106	10790.98	3.955460	5456.246	5794.551986
29	H	276.8822	24401.25	9.387685	5198.566	5788.4673142
30	L	657.1379	11210.20	4.130270	5428.315	5783.3904539
31	H	289.1189	23509.56	9.017456	5214.235	5779.1690254
32	L	631.2219	11574.66	4.282162	5405.990	5775.6431049
33	H	299.7513	22793.90	8.720245	5227.811	5772.7067708
34	L	610.4171	11889.63	4.413375	5388.000	5770.2525354
35	H	308.9362	22215.32	8.479922	5239.511	5768.2064058
36	L	593.5945	12160.46	4.526162	5373.410	5766.495465
37	H	316.8313	21744.80	8.284450	5249.547	5765.0678706
38	L	579.9115	12392.33	4.622700	5361.513	5763.8738106
39	H	323.5890	21360.31	8.124692	5258.122	5762.8769021
40	L	568.7284	12590.12	4.705032	5351.771	5762.0429363
41	H	329.3522	21044.86	7.993605	5265.424	5761.3463588
42	L	559.5524	12758.32	4.775034	5343.764	5760.7635816
43	H	334.2524	20785.21	7.885694	5271.624	5760.2766509
44	L	551.9985	12900.98	4.834399	5337.163	5759.869252
45	H	338.4079	20570.90	7.796621	5276.877	5759.5287714
46	L	545.7634	13021.72	4.884633	5331.708	5759.243898
47	H	341.9243	20393.63	7.722934	5281.317	5759.0057723
48	L	540.6053	13123.70	4.927061	5327.192	5758.8065377
49	H	344.8943	20246.72	7.661862	5285.065	5758.6399735
50	L	536.3303	13209.71	4.962841	5323.445	5758.5006147
51	H	347.3989	20124.78	7.611169	5288.224	5758.3840948
52	L	532.7818	13282.15	4.992976	5320.334	5758.2866084
53	H	349.5083	20023.44	7.569036	5290.882	5758.2050915
54	L	529.8325	13343.09	5.018327	5317.746	5758.1368918
55	H	351.2829	19939.12	7.533981	5293.118	5758.0798603
56	L	527.3786	13394.32	5.039635	5315.592	5758.0321468
57	H	352.7744	19868.91	7.504789	5294.996	5757.9922446
58	L	525.3352	13437.34	5.057530	5313.798	5757.9588624
59	H	354.0271	19810.40	7.480462	5296.573	5757.9309442
60	L	523.6323	13473.46	5.072550	5312.302	5757.9075882
61	H	355.0785	19761.62	7.460176	5297.896	5757.8880545
62	L	522.2123	13503.75	5.085149	5311.055	5757.8717131
63	H	355.9604	19720.91	7.443251	5299.006	5757.8580456
64	L	521.0276	13529.15	5.095712	5310.014	5757.846612
65	H	356.6998	19686.94	7.429125	5299.936	5757.837049
66	L	520.0387	13550.44	5.104566	5309.145	5757.8290492
67	H	357.3196	19658.58	7.417330	5300.715	5757.822358
68	L	519.2131	13568.27	5.111984	5308.419	5757.8167606
69	H	357.8389	19634.89	7.407479	5301.369	5757.8120789
70	L	518.5235	13583.22	5.118198	5307.812	5757.8081625
71	H	358.2738	19615.09	7.399249	5301.915	5757.8048866

72	L	517.9474	13595.73	5.123402	5307.306	5757.8021464
73	H	358.6381	19598.56	7.392372	5302.373	5757.7998543
74	L	517.4661	13606.20	5.127759	5306.883	5757.7979369
75	H	358.9431	19584.74	7.386625	5302.757	5757.7963332
76	L	517.0638	13614.98	5.131406	5306.529	5757.7949916
77	H	359.1984	19573.19	7.381822	5303.078	5757.7938695
78	L	516.7275	13622.32	5.134459	5306.233	5757.7929308
79	H	359.4121	19563.53	7.377806	5303.347	5757.7921456
80	L	516.4464	13628.46	5.137015	5305.986	5757.7914888
81	H	359.5910	19555.46	7.374448	5303.571	5757.7909395
82	L	516.2114	13633.60	5.139153	5305.779	5757.7904799
83	H	359.7407	19548.70	7.371641	5303.760	5757.7900955
84	L	516.0149	13637.91	5.140943	5305.606	5757.789774
85	H	359.8660	19543.06	7.369294	5303.917	5757.789505
86	L	515.8506	13641.51	5.142440	5305.462	5757.78928
87	H	359.9708	19538.34	7.367331	5304.049	5757.7890918
88	L	515.7132	13644.52	5.143693	5305.341	5757.7889344
89	H	360.0585	19534.39	7.365690	5304.159	5757.7888027
90	L	515.5983	13647.04	5.144741	5305.240	5757.7886926
91	H	360.1319	19531.09	7.364317	5304.251	5757.7886005
92	L	515.5022	13649.15	5.145618	5305.155	5757.7885234
93	H	360.1933	19528.33	7.363169	5304.328	5757.7884589
94	L	515.4218	13650.91	5.146352	5305.085	5757.788405
95	H	360.2446	19526.02	7.362209	5304.393	5757.7883599
96	L	515.3546	13652.39	5.146966	5305.026	5757.7883222
97	H	360.2876	19524.09	7.361406	5304.447	5757.7882906
98	L	515.2984	13653.63	5.147479	5304.976	5757.7882642
99	H	360.3235	19522.48	7.360734	5304.492	5757.7882421
100	L	515.2514	13654.66	5.147909	5304.935	5757.7882236
101	H	360.3536	19521.13	7.360172	5304.530	5757.7882082
102	L	515.2120	13655.52	5.148268	5304.900	5757.7881953
103	H	360.3788	19520.00	7.359702	5304.562	5757.7881844
104	L	515.1792	13656.25	5.148569	5304.871	5757.7881754
105	H	360.3998	19519.05	7.359309	5304.588	5757.7881678
106	L	515.1516	13656.85	5.148820	5304.847	5757.7881615
107	H	360.4174	19518.26	7.358981	5304.610	5757.7881562
108	L	515.1286	13657.36	5.149031	5304.827	5757.7881518
109	H	360.4321	19517.60	7.358706	5304.629	5757.7881481
110	L	515.1094	13657.78	5.149207	5304.810	5757.788145
111	H	360.4445	19517.05	7.358476	5304.644	5757.7881424
112	L	515.0933	13658.13	5.149354	5304.796	5757.7881402
113	H	360.4548	19516.58	7.358283	5304.657	5757.7881384
114	L	515.0798	13658.43	5.149477	5304.784	5757.7881369
115	H	360.4634	19516.20	7.358122	5304.668	5757.7881356
116	L	515.0685	13658.68	5.149580	5304.774	5757.7881346
117	H	360.4706	19515.87	7.357988	5304.677	5757.7881337
118	L	515.0591	13658.88	5.149666	5304.766	5757.7881329
119	H	360.4766	19515.60	7.357875	5304.685	5757.7881323
120	L	515.0512	13659.06	5.149738	5304.759	5757.7881318

121	H	360.4817	19515.37	7.357781	5304.691	5757.7881314
122	L	515.0446	13659.20	5.149799	5304.753	5757.788131
123	H	360.4859	19515.19	7.357702	5304.696	5757.7881307
124	L	515.0391	13659.32	5.149849	5304.748	5757.7881304
125	H	360.4894	19515.03	7.357636	5304.701	5757.7881302
126	L	515.0345	13659.43	5.149891	5304.744	5757.7881301
127	H	360.4924	19514.89	7.357581	5304.704	5757.7881299
128	L	515.0307	13659.51	5.149927	5304.741	5757.7881298
129	H	360.4948	19514.78	7.357535	5304.707	5757.7881297
130	L	515.0274	13659.58	5.149956	5304.738	5757.7881296
131	H	360.4969	19514.69	7.357496	5304.710	5757.7881295
132	L	515.0247	13659.64	5.149981	5304.735	5757.7881295
133	H	360.4986	19514.61	7.357464	5304.712	5757.7881294
134	L	515.0225	13659.69	5.150001	5304.733	5757.7881294
135	H	360.5001	19514.55	7.357437	5304.714	5757.7881293
136	L	515.0206	13659.73	5.150019	5304.732	5757.7881293
137	H	360.5013	19514.49	7.357415	5304.716	5757.7881293
138	L	515.0190	13659.77	5.150033	5304.730	5757.7881292

*** OBJ FUNC VALUE ***
5757.7881292
*** VARIABLE VALS ***
H = 7.3574151046
L = 5.1500336137
*** TERM WEIGHTS ***
WGT 1 0.4606570088
WGT 2 0.0786883468
WGT 3 0.0786859824
WGT 4 0.381968662

FILE: STOPL9.TRD
SHRINK STOPE MINING PROBLEM
19:41:50 03-18-1990
TOLERANCE VALUE= 9.999999999999999D-12

******* LIST OF TERMS ******
TERM 1 70 H L
TERM 2 2333.33 L^-1
TERM 3 3333.33 H^-1
TERM 4 83333.33 H^-1 L^-1

** VARIABLE START POINTS **
H 1
L 999999

IT	VAR	CDCOF(1)	CDCOF(2)	XNW(VAR)	FUNVL	VLUE
1	H	69999930	3333.413	0.006900	966102.8	966102.8954
2	L	0.483051	12078329	5000.420	4830.925	487870.296
3	H	350029.4	3349.995	0.097829	68486.40	68486.872042
4	L	6.848064	854155.4	353.1705	4837.070	38909.922749
5	H	24721.94	3569.287	0.379970	18787.20	18793.806932
6	L	26.59791	221648.7	91.28699	4856.086	13628.696735
7	H	6390.089	4246.201	0.815167	10417.98	10443.5468992
8	L	57.06172	104561.8	42.80693	4885.274	8974.4097788

9	H	2996.485	5280.055	1.327434	7955.276	8009.7850227
10	L	92.92042	65111.05	26.47108	4919.409	7430.5159853
11	H	1852.976	6481.418	1.870252	6931.064	7019.2108736
12	L	130.9176	46890.60	18.92534	4955.323	6737.6124647
13	H	1324.774	7736.595	2.416595	6402.887	6526.1789581
14	L	169.1617	36817.09	14.75277	4991.209	6370.5589279
15	H	1032.694	8981.984	2.949173	6091.188	6249.3504883
16	L	206.4421	30589.83	12.17277	5025.944	6156.2041681
17	H	852.0939	10179.21	3.456314	5890.209	6081.8943585
18	L	241.9420	26443.78	10.45456	5058.799	6023.2170815
19	H	731.8198	11304.32	3.930250	5752.470	5975.6584117
20	L	275.1175	23536.38	9.249340	5089.311	5937.4330823
21	H	647.4538	12342.98	4.366220	5653.851	5906.1217964
22	L	305.6354	21419.25	8.371442	5117.218	5880.654611
23	H	586.0009	13287.80	4.761869	5580.920	5859.6452199
24	L	333.3308	19833.45	7.713677	5142.413	5842.417933
25	H	539.9574	14136.65	5.116741	5525.645	5828.1379883
26	L	358.1719	18619.73	7.210094	5164.907	5816.3627012
27	H	504.7066	14891.19	5.431819	5482.950	5806.5704959
28	L	380.2273	17675.02	6.818021	5184.796	5798.4639297
29	H	477.2615	15555.83	5.709111	5449.478	5791.7079905
30	L	399.6377	16929.87	6.508691	5202.237	5786.0994042
31	H	455.6083	16136.72	5.951299	5422.923	5781.4185031
32	L	416.5909	16335.87	6.262045	5217.423	5777.5248102
33	H	438.3432	16641.01	6.161447	5401.657	5774.2720475
34	L	431.3013	15858.28	6.063700	5230.564	5771.5624387
35	H	424.4590	17076.31	6.342772	5384.494	5769.2974422
36	L	443.9940	15471.64	5.903093	5241.876	5767.4087002
37	H	413.2165	17450.22	6.498477	5370.556	5765.829249
38	L	454.8934	15156.84	5.772308	5251.571	5764.5111723
39	H	404.0616	17770.07	6.631637	5359.180	5763.4086653
40	L	464.2146	14899.35	5.665318	5259.847	5762.4880923
41	H	396.5723	18042.71	6.745120	5349.855	5761.7179653
42	L	472.1584	14687.93	5.577461	5266.890	5761.0746571
43	H	390.4222	18274.41	6.841549	5342.186	5760.5364394
44	L	478.9084	14513.80	5.505089	5272.868	5760.0867122
45	H	385.3562	18470.83	6.923282	5335.860	5759.710438
46	L	484.6297	14370.00	5.445321	5277.929	5759.3959551
47	H	381.1724	18636.98	6.992413	5330.631	5759.1328333
48	L	489.4689	14251.00	5.395854	5282.206	5758.9128819
49	H	377.7098	18777.28	7.050781	5326.298	5758.7288539
50	L	493.5547	14152.35	5.354841	5285.814	5758.5749984
51	H	374.8389	18895.56	7.099988	5322.704	5758.4462731
52	L	496.9992	14070.43	5.320787	5288.854	5758.3386418
53	H	372.4551	18995.17	7.141420	5319.717	5758.2485924
54	L	499.8994	14002.34	5.292477	5291.413	5758.173293
55	H	370.4734	19078.94	7.176268	5317.233	5758.1102953
56	L	502.3387	13945.67	5.268918	5293.564	5758.0576133
57	H	368.8242	19149.35	7.205552	5315.165	5758.0135389

58	L	504.3886	13898.48	5.249296	5295.370	5757.9766796
59	H	367.4507	19208.47	7.230142	5313.442	5757.9458433
60	L	506.1099	13859.15	5.232941	5296.887	5757.920054
61	H	366.3058	19258.08	7.250778	5312.005	5757.8984792
62	L	507.5545	13826.34	5.219301	5298.160	5757.8804349
63	H	365.3511	19299.70	7.268087	5310.807	5757.8653397
64	L	508.7661	13798.97	5.207920	5299.227	5757.8527144
65	H	364.5544	19334.59	7.282598	5309.807	5757.8421527
66	L	509.7818	13776.13	5.198421	5300.121	5757.8333189
67	H	363.8894	19363.83	7.294759	5308.972	5757.825929
68	L	510.6331	13757.05	5.190488	5300.871	5757.8197481
69	H	363.3342	19388.33	7.304948	5308.275	5757.8145775
70	L	511.3463	13741.12	5.183862	5301.499	5757.8102527
71	H	362.8704	19408.85	7.313482	5307.692	5757.8066349
72	L	511.9437	13727.81	5.178327	5302.024	5757.8036089
73	H	362.4829	19426.04	7.320628	5307.205	5757.8010776
74	L	512.4440	13716.68	5.173701	5302.465	5757.7989602
75	H	362.1591	19440.42	7.326612	5306.799	5757.7971891
76	L	512.8628	13707.39	5.169836	5302.833	5757.7957076
77	H	361.8885	19452.47	7.331621	5306.459	5757.7944684
78	L	513.2134	13699.61	5.166604	5303.142	5757.7934318
79	H	361.6623	19462.55	7.335813	5306.174	5757.7925647
80	L	513.5069	13693.12	5.163903	5303.400	5757.7918394
81	H	361.4732	19470.99	7.339322	5305.937	5757.7912327
82	L	513.7525	13687.69	5.161644	5303.616	5757.7907252
83	H	361.3151	19478.05	7.342259	5305.738	5757.7903007
84	L	513.9581	13683.15	5.159756	5303.797	5757.7899456
85	H	361.1829	19483.96	7.344716	5305.572	5757.7896486
86	L	514.1301	13679.35	5.158177	5303.948	5757.7894001
87	H	361.0724	19488.90	7.346772	5305.433	5757.7891923
88	L	514.2740	13676.17	5.156856	5304.075	5757.7890184
89	H	360.9799	19493.04	7.348492	5305.317	5757.788873
90	L	514.3944	13673.52	5.155752	5304.181	5757.7887514
91	H	360.9026	19496.50	7.349932	5305.220	5757.7886496
92	L	514.4952	13671.30	5.154829	5304.270	5757.7885645
93	H	360.8380	19499.40	7.351136	5305.139	5757.7884933
94	L	514.5795	13669.44	5.154056	5304.344	5757.7884338
95	H	360.7839	19501.82	7.352143	5305.071	5757.788384
96	L	514.6500	13667.89	5.153410	5304.406	5757.7883423
97	H	360.7387	19503.84	7.352986	5305.014	5757.7883074
98	L	514.7090	13666.59	5.152870	5304.458	5757.7882783
99	H	360.7009	19505.54	7.353691	5304.966	5757.7882539
100	L	514.7584	13665.50	5.152418	5304.501	5757.7882335
101	H	360.6692	19506.96	7.354281	5304.927	5757.7882164
102	L	514.7997	13664.59	5.152040	5304.537	5757.7882022
103	H	360.6428	19508.15	7.354774	5304.893	5757.7881902
104	L	514.8342	13663.83	5.151724	5304.568	5757.7881802
105	H	360.6207	19509.14	7.355187	5304.866	5757.7881719
106	L	514.8631	13663.20	5.151459	5304.593	5757.7881649

107	H	360.6021	19509.97	7.355533	5304.842	5757.788159
108	L	514.8873	13662.66	5.151238	5304.615	5757.7881542
109	H	360.5867	19510.66	7.355821	5304.823	5757.7881501
110	L	514.9075	13662.22	5.151053	5304.632	5757.7881466
111	H	360.5737	19511.24	7.356063	5304.807	5757.7881438
112	L	514.9244	13661.85	5.150899	5304.647	5757.7881414
113	H	360.5629	19511.73	7.356265	5304.793	5757.7881394
114	L	514.9385	13661.54	5.150769	5304.660	5757.7881377
115	H	360.5538	19512.14	7.356434	5304.782	5757.7881363
116	L	514.9504	13661.28	5.150661	5304.670	5757.7881351
117	H	360.5462	19512.48	7.356576	5304.772	5757.7881342
118	L	514.9603	13661.06	5.150570	5304.679	5757.7881333
119	H	360.5399	19512.76	7.356694	5304.764	5757.7881326
120	L	514.9686	13660.88	5.150495	5304.686	5757.7881321
121	H	360.5346	19513.00	7.356793	5304.757	5757.7881316
122	L	514.9755	13660.72	5.150431	5304.692	5757.7881312
123	H	360.5302	19513.20	7.356876	5304.752	5757.7881309
124	L	514.9813	13660.60	5.150378	5304.697	5757.7881306
125	H	360.5265	19513.36	7.356945	5304.747	5757.7881303
126	L	514.9861	13660.49	5.150334	5304.701	5757.7881301
127	H	360.5234	19513.50	7.357003	5304.743	5757.78813
128	L	514.9902	13660.40	5.150297	5304.705	5757.7881298
129	H	360.5208	19513.62	7.357051	5304.740	5757.7881297
130	L	514.9936	13660.33	5.150266	5304.708	5757.7881296
131	H	360.5186	19513.72	7.357092	5304.737	5757.7881296
132	L	514.9964	13660.26	5.150240	5304.711	5757.7881295
133	H	360.5168	19513.80	7.357126	5304.735	5757.7881294
134	L	514.9988	13660.21	5.150218	5304.713	5757.7881294
135	H	360.5152	19513.87	7.357154	5304.733	5757.7881293
136	L	515.0008	13660.17	5.150200	5304.714	5757.7881293
137	H	360.5140	19513.92	7.357178	5304.731	5757.7881293
138	L	515.0024	13660.13	5.150185	5304.716	5757.7881293

*** OBJ FUNC VALUE ***
5757.7881293

*** VARIABLE VALS ***
H = 7.357178294
L = 5.1501852234

*** TERM WEIGHTS ***
WGT 1 0.4606557424
WGT 2 0.0786860304
WGT 3 0.0786885151
WGT 4 0.381969712