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OPTIMIZATION OF CYCLIC STEAM INJECTION USING GEOMETRIC PROGRAMMING

by

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ABSTRACT

This thesis compares geometric programming as a viable alternative for determining cycle times for cyclic steam injection. This thesis discusses cyclic steam injection, results that it has had in the past. Next there is a review of geometric programming techniques with the reader. Using the geometric programming techniques cyclic steam injection problems were solved and compared with the heuristic method. The results of the geometric programming method cut cycle time from 5 to 20 percent, and increased cycle time by 20 percent where the total production from the start of the cycle to the maximum product rate was small (36 barrels/day) It appears that geometric programming is an alternative for the calculation of cycle time.

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INTRODUCTION

The purpose of this thesis is to help solve a problem in the petroleum industry in the area of secondary recovery. This is, what should be the length of time needed to inject steam into wells to increase the amount of production and at the same time achieve the maximum net profit possible to the oil producer. Secondarily, in this time of petroleum shortages at home, and oil embargoes from abroad, it is desirable to be able to produce as much as possible from our own resources.

This thesis deals with the use of geometric programming as an optimization technique to solve the problem. The thesis will cover in general the nature of cyclic steam injection and that of geometric programming. It will also encompass the combination of these two areas to achieve a solution. The paper will deal primarily with the optimiazation technique rather than the petroleum engineering problems related to cyclic steam injection.

The information used to develop this thesis has come from literature in the areas of cyclic steam injection and geometric programming. Having looked at other techniques and methods to solve this problem, this method would appear to be a better way for the petroleum industry to meet its goals for the future.

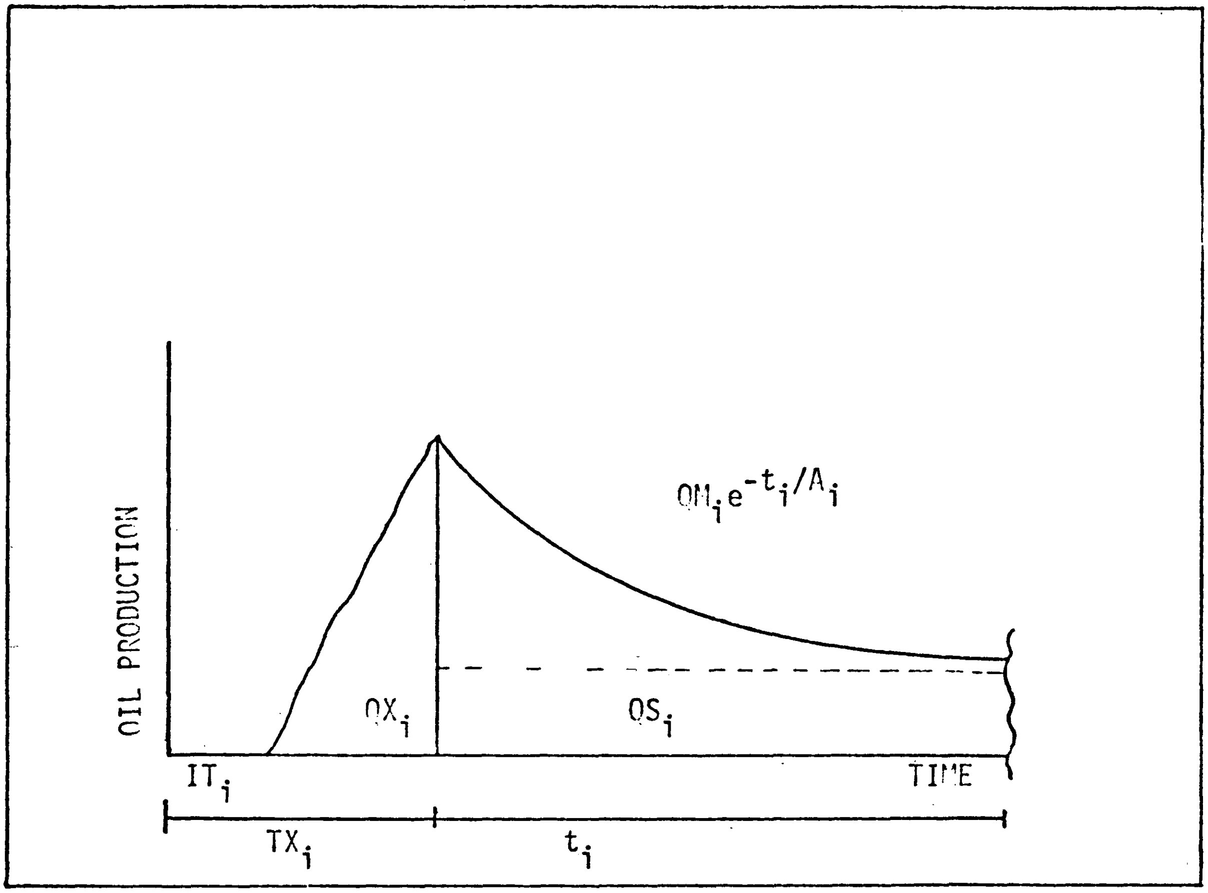
CYCLIC STEAM INJECTION

Cyclic steam injection is a secondary recovery method that has become popular in the past fifteen years. It is a method in which steam is pumped into wells to increase their production or to increase the total life of the reservoir. Several companies in different parts of the world have used and are using cyclic steam injection to increase their recovery rates.

> Tidewater used cyclic steam injection on their Kern River property. Tidewater injects steam into wells for five days and then returns the wells to production. Tidewater usually returns these wells to steam injection every 124 days. A second operator on the Kern River field (Cresmont Oil) produced 15,381 barrels during the first cycle period, which was the equivalent of 94.2% of the cumulative production over a seven year period. Their production record can be seen in Table 1. (1)

As seen from their production record, cyclic steam injection is impressive on the additional recovery. This does not imply that such ventures will be profitable in all cases, only that production should increase.

> A third example is the Wilmington Field. The payout period for this field is averaging 3 1/2 months, and some wells are paying out in four weeks. The payout period is the time needed to recover investment. The injected heat lasts for about six months. (1)



Ill. 1. Characteristic production cycle (2)

Here it can be seen that the company is getting the increased production rate for about 2 1/2 months at no cost to them.

After the steam has been injected into the well the rate of production will increase to a point and then decline until it reaches its steady-state level. When the steady state is reached, steam is again injected into the well. This process is repeated several times depending upon the condition of the reservoir. Since production cannot take place during the period of steam injection, it is important to keep the injection time as low as possible to maximize profits before taxes. It is therefore desirable to have a method that can be used to determine the times for steam injection and for production after injection. The reason for wanting to know how long the cyclic will last is so that there will be steamers on hand to start the process over again. Also, the number of steamers can vary from well to well to achieve the rate of production for an optimum solution. The cashed line on the right-hand side represents the steady-state flow.

Some general considerations that are made when cyclic steam injection is used are such things as

> The oil in place should be greater than 1,200 barrels per acre foot. The production interval should be in excess of fifty feet. Most of the wells that have been using steam injection have been producing about twelve to fifteen years. It has been found that there is no advantage to using steam injection for wells that are producing from depths greater than 3,000 feet.

The reason for this is that the heat loss of the pipe length is too great and wells at this depth tend to have enough heat and therefore do not benefit from this type of stimulation. It has been found that reservoirs that are producing at 10 percent of their primary recovery rate can be increased to 15 percent. The reason for the increase in the rate of production is that we are increasing the pressure of the well causing oil to flow more readily. (1)

In conclusion, this is a viable method of secondary recovery in which we would like to know steaming times and production times for maximum petroleum production. By maximizing production we maximize profits. A technique to calculate these times and related factors is geometric programming which is discussed in Chapter 2.

TABLE 1 (1)

Actual Production Results

Well Number

	5-K (Fig. 5)	7-K (Fig. 6)	8-K (Fig. 7)	9-K (Fig. 8)	12-C (Fig. 9)	15-C (Fig.10)	18-K (Fig.11)	Total	Average
Zone Thickness, net ft	230	225	195	502	18	20	195	1,087	155
Cumulative Pri. Pro.									
Bbl oil	27,186	28,392	31,915	29,707	16,325	20,370	32,420	186,315	26,616
% Water Cut	61.7	45.8	66.4	64.4	80.3	64.0	88.0	--	76.0
First Cycle Injection Data									
Steam injected, bbl	16,070	13,666	14,109	15,009	4,553	4,298	8,269	75,977	10,854
Injection time, days	17	19	23	21	20	17	10	127	18
Soak time, days	11	7	12	3	7	7	1	50	7
Injection rate, B/D	945	719	613	715	228	253	827	--	598
Wellhead pressure, psi	500	400	320	320	650	350	305	--	406
First Cycle Production Data									
Oil Produced, bbl	10,661	7,295	7,156	6,106	15,381	7,770	6,561	60,930	8,704
Time, days	145	140	121	218	162	137	134	1,057	151
BOPD	74	52	59	28	95	57	49	414	59
% cum primary	39.4	25.7	22.4	20.6	94.2	38.1	20.2	--	32.7
Bbl oil/bbl steam injected	0.66	0.53	0.51	0.41	3.38	1.81	0.79	--	0.80
% water cut	42.8	47.8	40.6	68.5	18.0	8.7	34.1	--	44.4
Bbl water/bbl steam injected	0.50	0.49	0.35	.89	0.74	0.17	4.14	--	0.94

Table 1 Continued

	5-K (Fig. 5) 230	7-K (Fig. 6) 225	8-K (Fig. 7) 195	9-K (Fig. 8) 502	12-C (Fig. 9) 18	15-C (Fig.10) 20	18-K (Fig.11) 195	Total 1,087	Average 155
Second Cycle Production Data									
Steam injection, bbl	19,868	17,594	18,156	00	5,694	5,308	17,811	84,431	14,072
Injection time, days	27	18	18	--	23	28	36	150	25
Soak time, days	6	6	12	--	8	7	12	51	9
Injection rate,B/D	736	977	1,009	--	248	190	495	--	609
Wellhead pressure,psi	350	350	350	--	350	550	400	--	392
*Second Cycle Production Data									
Oil produced,bbl	4,040	--	1,042	--	--	6,478	7,656	19,216	490
Time,days (not total 2nd cycle)	51	--	20	--	--	93	125	289	72
1st cycle production, equivalent time	5,117	--	826	--	--	6,686	6,561	19,190	480

*Second cycle not complete - first and seconc cycle data compared on an equivalent time basis.

GEOMETRIC PROGRAMMING

Geometric programming is a mathematical programming technique used to optimize nonlinear problems. This is done with the use of linear equations using the dual. This will obviously make calculations easier. In G.P. there are several rules and concepts which we must observe when attempting to solve a problem. First, look at the degrees of difficulty, which will be defined later. If it is zero or a positive number, it is a well-formulated problem. The problem can be solved by hand for zero D.D. problems. For problems with positive D.D., depending upon the degrees of difficulty we can solve by hand, get bounds to the problem or solve with the aid of a computer. If the problem has a negative D.D., then it is a poorly formulated problem.

In geometric programming, the dual is easier to solve than the primal, and at optimality both have the same solution. The G.P. primal problem should be in the form of minimizing the objective function subject to constraints which are less than or equal to one or negative one.

The following is a step-by-step problem set up and solution procedure. [3]

STEP 0. Formulate the Geometric Programming primal problem.

Minimize: $$\sum_{t=1}^{T_0} S_{ot}\, C_{ot} \prod_{n=1}^{N} X_n^{a_{otn}}$$

Subject to: $$g_m(X) = \sum_{t=1}^{T_m} S_{mt}\, C_{mt} \prod_{n-1}^{N} X_n^{a_{mtn}} \leq R_m$$

There are N primal variables (X_n) and M primal constraints (g_m) The constraint functions $g_m(X)$ must be dimensionless (with no constant terms)

Example:

Minimize: $5X_1{}^2X_2 - 3X_2{}^3X_3$

Subject to: $+ 1/8X_1{}^{-4} + 1/8^{-4}X_3{}^2 \leq 1$

In terms of the general notation

$N=3$ $T_0=2$ $c_{01}=5$ $s_{01}=1$

$c_{02}=3$ $s_{02}=-1$

$M=1$ $T_1=2$ $R_1=-1$ $c_{11}=8$ $s_{11}=1$

$c_{12}=1$ $s_{12}=1$

$a_{011}=2$ $a_{012}=1$ $a_{013}=0$

$a_{021}=0$ $a_{022}=3$ $a_{023}=1$

$a_{111}=4$ $2_{112}=0$ $a_{113}=0$

$a_{121}=0$ $a_{122}=0$ $a_{123}=2$

STEP 1. Write the Geometric Programming dual objective function.

MAXIMIZE

$$R_o \left[\prod_{m=0}^{M} \prod_{t=1}^{T_m} (C_{mt}\ W_{mo}/W_{mt})^{S_{mt}\ W_{mt}} \right]^{R_0}$$

As the primal objective function is positive or negative at the optimum, R_0 is equal to +1 or -1. The dual variables are the w_{0t}, w_{mt}, and w_{m0} above. These may also be called weights.

EXAMPLE

MAXIMIZE:

$$(5/w_1)^{w_1}\ (3/w_2)^{-w_2}\ (8/w_3)^{+w_3}\ (1/w_4)^{w_4}\ (w_3+w_4)^{w_3+w_4}$$

where w_i = the weight of the ith term.

STEP 2. Write the dual normality equation for the terms of the primal objective function.

$$\sum_{t=1}^{T_0} S_{0t}\ W_{0t} = R_0$$

EXAMPLE

$$w_1 = w_2 \qquad = 1$$

STEP 3. Write the N dual orthogonality equations, one for each primal variable.

$$\sum_{m=0}^{M} \sum_{t=1}^{T_m} S_{mt}\ a_{mtn}\ w_{mt} \qquad = 0$$

EXAMPLE

$$2w_1 - 4w_3 - 4w_4 = 0$$

$$w_1 - 3w_2 = 0$$

$$-w_2 + 2w_3 = 0$$

STEP 4. Write the M linear inequality constraints, one for each primal constraint.

$$W_{m0} = R_m \sum_{t=1}^{T_m} S_{mt} W_{mt} \geq 0$$

EXAMPLE

$$W_{10} = -(-3/4+1/4) = 1/2 \leq 0$$

for the other system this term has already been incorporated into the objective function.

STEP 5. Find the degree-of-difficulty of the primal problem by the following formula:

DEGREE OF DIFFICULTY =

independent primal terms - # primal variables - 1

EXAMPLE

D.D. = 0 = 4 - 3 - 1

STEP 6. If the degree of difficulty is zero, solve the dual equality constraints for the dual variables. If the degree of difficulty is negative, the problem is badly formulated. If the degree of difficulty is positive, attempt to reduce it with condensations or approximation. Bound the dual variables and estimate the primal minimum. If an exact solution is necessary, resort to a computer program.

STEP 7. Find the minimum value g_0 of the dual objective function; at optimality the primal and dual objective function are equal.

EXAMPLE

$$g_0(x) = [5/(3/2)]^{3/2}[3/(1/2)]^{-1/2}[8/(1/8)]^{1/8}*$$

$$*[1/(5/8)]^{5/8}\ [1/8+5/8]^{(1/8+5/8)}$$

STEP 8. Write the T_0 primal-dual optimality relations for the terms of the objective function.

$$W_{0t} = (C_{0t} \sum_{n=1}^{N} Y_n^{a_{otn}})/(R_0\ g_0) \quad t = 1, \ldots, T_0$$

EXAMPLE

$$w_1 = (5x_1^2\ x_2)/g_0 = 3/2$$

$$w_2 = (3x_2^3\ x_3)/g_0 = 1/2$$

STEP 9. Write the $\sum_{m=1}^{m} T_m$ primal-dual optimality relations

for the terms of the primal constraints.

$$C_{mt} \prod_{n=1}^{N} X_n^{a_{mtn}} = W_{mt}/W_{m0} \quad t = 1, \ldots, T_m$$

$$m = 1, \ldots, M$$

EXAMPLE

$$8x_1^4 = w_3/(w_3 + w_4) = (1/8)/(3/4)$$

$$x_3^2 = w_4/(w_3 + w_4) = (5/8)/(3/4)$$

STEP 10. Choose any N of the primary optimality relations. These are a system of N linear equations in the logarithms of the primal variables. Solution of this system completes the problem.

Using the above techniques, we will solve the cyclic steam injection problem in the next chapter.

In the Appendix will be found the Quick and Dirty rules for geometric programming. These are easier to follow than the 10-step method and were used to work the problem.

CYCLIC STEAM INJECTION USING GEOMETRIC PROGRAMMING

In applying geometric programming to the solution of a steam injection problem there are some assumptions which must be noted.

Assumption 1: If there is nothing done to the production rate it will produce at the steady state. Before starting the injection the well is producing at the steady state.

Assumption 2: There is no production during steam injection, and there is a soak period after steam injection before production begins.

Assumption 3: The production rate will increase until it reaches a peak and then decline to the steady state.

Looking at the model of Curry, Chang, and Curry, Jr., the total profit per cycle is P(t) where $T = T_x + t$.

$$P(t) = VBO.A.QM\,(1-e^{-t/A}) + QS.t.VBO + VBO.QX - CBS.BSTR + CSC.$$

A and QM are curve fit parameters for the production decline rate and are known for the model. The following are constants and are known for the model:

VBO, QX, QS, CBS; BSTR, CSC.

where $PPD(t) = \frac{P(T)}{T}$

Rewriting $PPD(t) = \frac{P(t)}{t}$ where $t = T-TX$

$$P(t) = VBO.A.QX\ (1-e^{-t/A}) + QS.t.VBO + VBO.QX - CBS.BSTR - CSC.$$

Cleaning up the equation where

$K_1 = VBO.A.QM$ $K_2 = QS.VBO$

$K_3 = VBO.QX$ $K_4 = CBS.BSTR + CSC$

$$P(t) = K_1\ (1-e^{-t/A}) + K_2 t + K_3 - K_4$$

let $K_3 - K_4 = K_5$

then $PPD(t) = \frac{P(t)}{t} = \frac{K_1(1-e^{-t/A})}{t} + K_2 + K_{5/t}$

$$= \frac{K_1 + K_5}{t} - \frac{K_1 e^{-t/a}}{t} + K_2$$

let $K_1 + K_5 = K_6$

In order to find the optimum cycle period for each well, the production decline response curve for each well must be known. There are in general three basic classifications of production decline response, i.e.

1. constant percentage decline,
2. hyperbolic decline
3. harmonic decline (2)

The first classification was selected for the following reasons:

1. It gave a better fit for the sample data and has been recommended by Dunn.
2. It has a simpler mathematical expression than the others; moreover, it is easier to find the optimum period of time for each well. (2)

The profit per day is PPD(t)

$$= \frac{K_6}{t} - \frac{K_1 e^{-t/a}}{t} + K_2$$

Rewriting:

$$\frac{P(t)}{t} = PPD^1(t) = PDD(t) - K_2 = \frac{K_6}{t} - \frac{K_1}{t}e^{-t/a}$$

$$PPD^1(t) = \frac{1}{t}\,(K_6 - K_1 e^{-t/a})$$

let t= lnx

$$= \frac{K_6 - K_1 e^{-\ln x/a}}{\ln X}$$

$$a^u = e^{\ln a}$$

$$= \frac{K_6 - K_1 X^{-1/a}}{\ln x}$$

However we can minimize the reciprocal to maximize then:

$$\text{MIN:} \quad \frac{\ln x}{K_6 - K_1 X^{-1/p}} = \mu V^{-1}$$

$$\text{where } \mu \geq \Sigma^{-1} X^{\Sigma} - \Sigma^{-1}$$

$$v \leq K_6 - K_1 X^{-1/A}$$

Rewriting in the form of ≤ 1

$$\frac{K_1}{K_6} X^{-1/a} + \frac{V}{K_6} \leq 1$$

and $\Sigma^{-1} X^{\Sigma} \mu^{-1} - \Sigma^{-1} \mu^{-1} \leq 1$

Writing the dual geometric programming problem.

$$\text{Max } g_0 = \left(\frac{1}{W_1}\right)^{W1} \left(\frac{1}{W_2}\right)^{W2} \left(\frac{1}{W_3}\right)^{W3} \left(\frac{1}{W_4}\right)^{W4} *$$

$$* \left(\frac{1}{W_5}\right)^{-W_5} (W_2 + W_3)^{W_2+W_3} (W_4+W_5)^{W_4 + W_5}$$

The dual normality equation is:

$$W_1 = 1$$

The dual orthogonality equations are:

$$(U)\ W_1 - W_5 + W_5 = 0$$

$$(V)\ -W_1 + W_3 = 0$$

$$(X)\ -\frac{1}{A} W_2 + \Sigma W_4 = 0$$

Rearranging:

$$W_1 = 1 \qquad W_3 = 1$$

$$W_2 = -A - AW_5$$

$$W_4 = 1 + W_4 \text{ or } W_5 = W_4 - 1$$

$$W_4 = -\frac{1}{A\Sigma} W_2 \text{ or } W_5 = \frac{-W_2}{\Sigma A} - 1$$

Writing the primal-dual optimality relations:

$$W_2 = \left(\frac{K_1}{K_6} X^{-1/A}\right)(W_2 + W_3)$$

$$W_4 = (\Sigma^{-1} X^{\Sigma} \mu^{-1})(W_4 + W_5)$$

$$W_5 = (\Sigma^{-1} \mu^{-1})(W_4 + W_5)$$

Since X is the variable that relates the cycle time, solution will be for X.

(1) $W_2 = \left(\frac{K_1}{K_6} S^{-1/A}\right)(W_2 + W_3)$

(2) $W_3 = 1$

(3) $W_4 - W_5 = 1$

(4) $W_4 = \frac{-1}{AC} W_2$

(5) $W_4 = (\Sigma^{-1} X^2 \mu^{-1})(W_4 + W_5)$

(6) $W_5 = (\Sigma^{-1} \mu^{-1})(W_4 + W_5)$

$$W_5 = (\Sigma^{-1} \mu^{-1})(W_4 + W_5)$$

$$\mu^{-1} = \frac{W_5}{(W_4 + W_5)\Sigma^{-1}}$$

$$W_4 = \frac{\Sigma^{-1} X^{\Sigma} W_5}{(W_4 + W_5)\Sigma^{-1}}(W_4 + W_5)$$

$$W_4 = X^{\Sigma} W_5$$

$$W_5 = W_4 - 1$$

$$W_4 = X^{\Sigma} W_4 - 1$$

$$W_4 = \frac{-X^{\Sigma}}{1 - X^{\Sigma}}$$

$$W_4 = \frac{-1 W_2}{A\Sigma}$$

$$W_2 = \frac{-X^{\Sigma}}{1 - X^{\Sigma}} \; \frac{A\Sigma}{(-1)} = \frac{X^{\Sigma} A\Sigma}{1 - X^{\Sigma}}$$

$$W_3 = 1$$

$$W_2 = \left(\frac{K_1}{K_6} X^{-1/A}\right)(W_2 + W_3)$$

$$\frac{\Sigma A X^{\Sigma}}{1-X^{\Sigma}} = \left(\frac{K_1}{K_6} X^{-1/A}\right)\left(\frac{\Sigma A X^{\Sigma}}{(1 - X^{\Sigma})} + 1\right)$$

$$= \frac{K_1 X^{-1/A} \Sigma A X^{\Sigma}}{K_6\ (1-X^{\Sigma})} + \frac{K_1}{K_6} X^{-1/A}$$

$$\Sigma A X^{\Sigma} = \frac{K_1}{K_6} \Sigma X^{-1/A} A X^{\Sigma} + \frac{K_1}{K_6} X^{-1/A}(1 - X^{\Sigma})$$

$$1 = \frac{K_1}{K_6} X^{-1/A} + \frac{K_1 X^{-1/A}(1 - X\Sigma)}{K_6\ \Sigma A X^{\Sigma}}$$

$$1 = \frac{K_1}{K_6} X^{-1/A} + \frac{K_1 X^{-1/A}}{K_6\ A X^{\Sigma}} - \frac{K_1 X^{-1/A} X^{\Sigma}}{K_6\ \Sigma A X^{\Sigma}}$$

$$\frac{K_6}{K_1} = X^{-1/A} + \frac{X^{-1/A}}{\Sigma A X^{\Sigma}} - \frac{X^{-1/A} X^{\Sigma}}{\Sigma A X^{\Sigma}}$$

$$\frac{K_6}{K_1} = X^{-1} + \frac{X^{-1/A}\ (1 - X^{\Sigma})}{\Sigma A X^{\Sigma}}$$

$$\frac{K_6}{K_1} = X^{-1/A} + \frac{1}{A} \Sigma^{-1} X^{-1/A\ -\Sigma} - \frac{1}{A} X^{-1/A} \Sigma^{-1}$$

$$= X^{-1/A} + \frac{1}{A} X^{-1/A} (\Sigma^{-1} X^{-\Sigma} - \Sigma^{-1})$$

if $\ln x^{-1} = \lim_{\Sigma \to 0} (\Sigma^{-1} X^{-\Sigma} - \Sigma^{-1})$

then

$$\frac{K_6}{K_1} = X^{-1/A} + \frac{1}{A} X^{-1/A} \ln X^{-1}$$

$$\frac{K_6}{K_1} = X^{-1/A} \left(1 + \frac{1}{A} \ln X^{-1}\right)$$

$$\frac{K_6 A}{K_1} = A\, X^{-1/A} - X^{-1/A} \ln x$$

$$- \frac{K_6\, A}{K_1\, X^{-1/A}} + A = \ln x$$

$$t = \ln x$$

$$A - \frac{K_6}{K_1} A e^{-t/A} = t$$

$$K_1 = VBO.A.QM$$

$$K_1 = (1.16)(32.34 \text{ X } 7)\left(\frac{60}{7}\right)$$

$$K_1 = 2250.864$$

$$K_6 = K_1 + VBO.QX - CBX.BSTR - CSC$$

$$= K_1 + (1.16)(187) - (.17)(3969) - 350$$

$$K_6 = 1442.954$$

$$A = 226.38$$

TABLE 2

Nonlinear Regression Results

Well	QM	A	Corr. Coef.	Stand. Dev.
1	60.	32.34	0.997	16.4
2	73.	46.84	0.999	22.9
3	66.	35.78	0.998	36.5
4	44.	33.68	0.989	52.5

TABLE 3

Data Used in Sample Schedule

Lease Number	Number Wells	Number Steamers	Length Schedule	Value Oil	Cost Steam	Setup Steam	Steam Util.	Gen Cap Per Day
1	4	1	365	1 160	0.170	350.000	1.00	1050.

Well	Cycle Time	Steamer Time	Delay Max Prod	Last Steam	Prod To Max	Max Increase	Decline Factor	Reg. Steam	Stable Rate
1	67	4	16	155	187.	60.	32.34	3969.	0.
1	67	4	16	0	187.	60.	32.34	3969.	0.
2	72	4	15	168	103.	73.	46.84	4382.	0.
2	72	4	15	0	103.	73.	46.84	4382.	0.
3	51	2	11	157	155.	66.	35.78	2330.	0.
3	51	2	11	0	155.	66.	35.78	2330.	0.
4	88	4	12	144	36.	44.	33.68	4485.	0.
4	88	4	12	0	36.	44.	33.68	4485.	0.

The equation is now

$$226.38 - \frac{(1442.954)(226.38)}{2250.864} e^{(.00442)(t)} = t$$

$$226.38 - 145.124[e^{(.00442)(t)}] = t$$

solve for t by iteration.

TABLE 4

Results Using Geometric Programming

Well	QM	A	t	T_G	T_H	Well	% Change
1	60	32.34	47.42	63.42	67	1	5.34
2	73	46.84	44.64	59.64	72	2	17.16
3	66	35.78	28.16	39.16	51	3	23.21
4	44	33.68	99.90	111.90	88	4	27.15

CONCLUSIONS

Geometric programming gives an optimal solution for cycle time for cyclic steam injection. Looking at the previous chapter, geometric programming cuts the cycle time from 5 to 20 percent. In one case geometric programming increases the cycle time by 20 percent. The increase occurs where the total production from the start at the cycle until time TX which is QX is small (36 bbls/day) The difference in the solutions between the heurestic method and the geometric programming method are not dependent upon just one parameter but upon changes in the total production from the start of the cycle until time TX (QX) and the curve fit parameters, QX and A.

The geometric programming solution will be within 1 percent of the heuristic solution when the values of QX (total production from the start of the cycle until time TS), QM and A (curve fit parameters) are (150, +1, -15), (60, +.07, -.49), and (30, +.29, -.71), respectively. When these numbers were used to calculate the cycle time for well 2, the cycle time was within a half day of the heuristic after the first iteration. When using the original values of the parameters QX, QM, and A, the geometric programming solution took six iterations to converge and differed from the heuristic model by 17 percent. It would appear that the nature of the geometric programming model is such that when the above values are used for the parameter QX, QM, and A, the model behaves like the heuristic model. If QX (the total production from the start of the cycle until time TX) is decreased, keeping all other things equal, it will force the cycle time to increase. Keeping all other things equal and decreasing the value of either curve fit parameters, QM or A, the cycle time will increase in both cases.

The optimum cycle time for the maximization of daily profit from wells using cyclic steam injection can be found with the use of geometric programming. Economic and engineering data will now be available to calculate when and how many steamers should be brought on line.

The time that steamers are in use is known and since, in this formulation, steaming time is given, any increase or decrease will only increase or decrease cycle time by the same amount. The loss of production from the well can be calculated during the steaming and soak period. The manpower requirements for the operation of steamers at a given time will be known, from past history and experience.

The geometric programming solution provides information such that, a conversational computer program can be written to allow the operators to change the values of any of the constants. These are the most important constants that one may wish to change:

(a) the value of a barrel of oil,

(b) cost of steam,

(c) total production from the start of the cycle to time TX (QX),

(d) QM, curve fit parameter,

(e) A, curve fit parameter.

The program must provide for any combination of changes of the above constants.

Another recommendation for future work in the area of cyclic steam injection using geometric programming is that the problem should be formulated using only the injection time as a given. The reason is that the soak period should be a function of the injection time, because the maximum heat created during the soak period is dependent on the amount of steam injected. Secondly, the total production from the start of the cycle to time TX which is QX, should be a function of the maximum heat executed during the soak period. This method would give a better solution for cycle time. A variation of this would be if the cycle time is dictated, then it would be possible by working backward to find the appropriate injection and soak times as well as the decline time t.

Finally, an opportunity cost could be added to account for the loss of production during the injector and soak periods.

On the basis of this thesis, geometric programming appears to be a viable alternative method for the calculation of the optimum cycle time for cyclic steam injection.

APPENDICES

NOMENCLATURE

IT_i - steam injection time (fixed for given i) (days)

TX_i - time from the start of injection until the maximum production rate has been reached (fixed for given i) (days)

QX_i - total production from the start of the cycle until time TX_i (fixed for given i) (bbl)

QS_i - well stable production state (fixed for given i) (bbl/ day)

t_i - production time since maximum rate has been reached (day)

QM_i - curve fit parameter for production decline rate (bbl/ week)

A_i - curve fit parameter for production decline rate (weeks)

VBO - value of a barrel of oil ($/bbl)

CBS - cost of a barrel of steam ($/bbl)

CSC - constant cost of setting up a steaming operation ($)

$BSTR_i$ - barrels of steam required (fixed for given i) (bbl)

T_g - total cyclic time using geometric programming (days)

T_h - total cyclic time using heuristic method (days)

QUICK AND DIRTY FOR SIGNOMAL GEOMETRIC PROGRAMMING

1. The Primal Geometric Programming Problem may be written in the form:

$$\text{Minimize: } g_0(X) = \sum_{t=1}^{T_0} \sigma_{0t} C_{0t} \prod_{n=1}^{N} X_k^{\partial_{0tm}}$$

$$\text{Subject To: } g_m(X) = \sum_{t=1}^{T_m} \sigma_{mt} C_{mt} \prod_{n=1}^{N} X_k^{\sigma_{mtm}} \leq \sigma_m;\ m=1, \quad ,m$$

Where $\sigma_{mt} = \_\ 1;\ m = 0, \quad ,M;\ t = 1, \quad ,T_m$

$C_{mt} > 0\ ;\ m = 0, \ldots ,M;\ t = 1, \ldots ,T_m$

$x_n \geq 0;\ n = 1, \quad ,N \quad X = (x_1, x_2, \quad ,x_n)$

a_{mtn} = arbitrary real number; $m = 0,\ldots,M;\ t = 1,. \quad ,Tm;$

$n = 1,\ldots,N$

2. The Dual Geometric Programming Problem is then:

$$\text{Maximize: } \sigma_0 \prod_{t=1}^{T_0} \left(\frac{C_{0t}}{0t}\right)^{\sigma_{0t}\delta_{0t}} \prod_{m=1}^{m} \left(\frac{\delta_{mt\ m0}}{\delta_{mt}}\right)^{\sigma_{mt}\delta_{mt} \quad \sigma_0}$$

Subject to:

$$\sum_{t=1}^{T_0} \sigma_{ot}\delta_{ot} = \sigma_0 \qquad \text{(NORMALITY)}$$

$$\sum_{m=0}^{M} \sum_{t=1}^{T_m} \sigma_{mt}\, a_{mtn} \delta_{mt} = 0; \; n = 1, \quad , N \text{ (ORTHOGONALITY)}$$

$$\delta_{mt} \geq 0; \; m = 0, \quad , M; \; t=1, \quad , T_m \text{ (POSITIVITY)}$$

$$\delta_{mo} = \sigma_m \sum_{t=1}^{t=T_m} \sigma_{mt} \delta_{mt} \geq 0; \; m=1, \quad , M \text{ (POSITIVITY)}$$

QUICK AND DIRTY GEOMETRIC PROGRAMMING RULES FOR POSYNOMIALS

1. Optimum value of the objective function is always of the form:

$$g_0(x)^* = (\text{coeff. of first term}/w_1)^{w_1} *$$
$$(\text{coeff. of second term}/w_2)^{w_2} *$$
$$\ldots * \ (\text{coeff. of last term}/w_{last})^{w_{last}} *$$
$$(\Sigma w\text{'s in 1st constraint})^{\Sigma w\text{'s in 1st constr.}}$$
$$\ldots * \ (\Sigma w\text{'s in last constraint})^{\Sigma w\text{'s in last constraint}}$$

2. Equations generated for a geometric program are:

Σw's in objective function = 1, (normality constraint),

and

For each primal variable x_j, given in variables and m terms:

$$\sum_{i=1}^{i=m} (\text{exponent on } x_j \text{ in } i^{th} \text{ term}) * (w_i) = 0, \quad \text{for } j=1,2,3, \quad ,n.$$

(orthogonality constraint)

3. Primal variables may be found by (objective function rule):

$$g_0(x)^* = (\text{1st term in obj. fun.}/w_1) =$$
$$(\text{2nd term in obj. fun.}/w_2) =$$
$$\ldots = (\text{last term in obj. fun.}/w_{\text{last term in obj fun.}})$$

4. Primal variables may be found by (constraint rule):

Let w_i be the weight of the i^{th} term in a constraint, then:

$$w_i = (i^{th} \text{ term in constraint}) *$$
$$(\text{sum of all } w\text{'s in that constraint})$$

DERIVATIVE OF PROFIT FUNCTION

$$P(t) = \frac{K_1}{t} - \frac{k_1 e^{-t/A}}{t} + K_2 + \frac{K_3}{t} - \frac{K_4}{t}$$

$$\frac{dP(-t)}{dT} = \frac{K_1}{t^2} + \frac{K_1 E^{-t/A}}{t^2} + \frac{K_1 E^{-t/A}}{At} - \frac{K_3 - K_4}{t^2} = 0$$

$$\frac{(-K_1 - K_3 + K_4)}{t^2} + \frac{K_1 e^{=t/A}}{t^2} (1 + \frac{t}{A}) = 0$$

$$K_1 e^{-t/A} = \frac{K_1 - K_3 + K_4}{(1 + \frac{t}{A})}$$

$$e^{-t/A} = \frac{K_1 - K_3 + K_4}{K_1 (1 + \frac{t}{A})}$$

$$\frac{K_6}{K_1} = e^{-t/A} \; (1 - t/A)$$

$$e^{-t/A} = \frac{K_6}{K_1} \left[\frac{A}{(A-t)} \right]$$

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