A GEOMETRIC PROGRAMMING FORMULATION OF THE BEST CONSTANT BIT WEIGHT AND ROTARY SPEED FOR ROTARY ROCK BITS

By

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INTRODUCTION

Drilling costs can be influenced by many variables. The relationship of those variables to the costs are strictly nonlinear and therefore difficult to firmly establish. With techniques from the geometric programming optimization methods, bounds can be placed on the percentage cost contribution of each of these variables. With this knowledge, an engineer can better determine which variables to change, if any, in order to improve the drilling costs.

E. M. Galle and H. B. Woods of Hughes Tool Company have established equations to find the optimal constant weight and rotary speed assuming proper attention is given to the other variables such as bit selection, drilling fluids and hydraulics[2,3]. The problem has been formulated from their equations into geometric programming format to aid in obtaining cost percentage boundaries on the constant bit weight and rotary speed variables.

Because the problem contains a necessary integration, consideration is given to three cases based upon bit wear:

1) flat crested bit tooth wear [$p = 1.0$]

2) tungsten carbide studded bits which have little tooth wear during the bit life [$p = 0.0$], and

3) self-sharpening or chipping type bit tooth wear [$p = 0.5$].

PROBLEM STATEMENT

Galle and Woods have formulated the cost per foot (expressed in rig hours per foot) of a bit run [C] and have multiplied it by the formation drillability parameter [C_f] to arrive at a dimensionless objective function [K]. Since C_f is a constant for the formation being drilled, minimizing K will result in a minimum cost per foot for the bit run.

The objective function is composed of three variable ,

1) D_f (final bit tooth dullness, measured between 0.0 and 1.0),

2) $\overline{W}$ (equivalent 7-7/8" bit weight used in drilling allowed to run between 0 and 79 thousand pounds), and

3) N (rotary speed, RPM).

The problem in its simplest form will have an objective function to be minimized subject to the following five constraints.

1) D_f must be less than 1.

2) The bearing life expended is a function of D_f, $\overline{W}$ and N and is measured between 0 and 1. Thus, the final fraction of bearing life expended must be less than 1.

3) As D_f increases from 0 to 1, K initially decreases and then increases as the drilling slows down because of a dull bit. The bit should be pulled from the hole and replaced before K begins to rise. This occurs where the slope of the curve on a graph of D_f versus K is non positive (i.e., $\partial K/\partial D_f \leq 0$).

4) $\overline{W}$ must be less than or equal to 79 thousand pounds.

5) D_f, $\overline{W}$, and N must be nonnegative values for an optimal solution.

To ease the burden of computation, first the general case will be introduced and rearranged as much as possible before developing the three cases, In the general case, the constants and common polynomials will be replaced with "C" and "R" subscripted terms, respectively, to reduce the size of the problem.

In each of the three cases, the problem will be reduced to this format[1].

$$\text{Minimize} \quad K = \sum_{t=1}^{To} A_t \prod_{n=1}^{N} X_n^{b_{tn}}$$

subject to the constraints

$$0 \leq \sum_{t=1}^{Tm} C_{tm} \prod_{n=1}^{N} X_{nm}^{d_{tnm}} \leq 1 \; ; \; m = 1 \cdots M$$

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[1]For further explanation of the format of a geometric programming problem, consult Duffin and others, 1967.

$$X_i \geq 0 \; ; \; i = 1 \cdots N$$

where

X is a variable

N is the number of variables

M is the number of constraints

To is the number of terms in objective equation

Tm is the number of terms in constraint "m"

"A" and "C" are coefficients

"b" and "d" are exponents

An example using this format would be

Minimize $K = 3X_1^2X_2 + 2X_1X_2^3$

Subject to

$$2X_1X_2^{-2} + X_1X_2 \leq 1$$

$$5X_1^{-2}X_2^4 \leq 1$$

$$X_1, \; X_2 \geq 0.$$

To force each of the three cases into this format, the integrations must be performed and logarithmic, exponential, and some other functions must be approximated by a variable raised to a power.

GENERAL CASE

The general form of the problem can be expressed as

Minimize

$$K = \left[A_n + \frac{(1359.1-1646.1\ln\overline{W})}{(N+4.348\times10^{-5}N^3)} \int_0^{D_f}(0.928125D^2+6.0D+1)\,dD\right] \Bigg/ \left[\frac{(1359.1-1646.1\ln\overline{W})(\overline{W})^k(e^{-100/N^2})}{(N+4.348\times10^{-5}N^3)} \int_0^{D_f}(0.928125D^2+6.0D+1)^{1-p}\,dD\right]$$

Subject to

1) $D_f \leq 1$

2) $$\left[(N)(1359.1-1646.1\ln\overline{W}) \Big/ (S_n)(C_{11}e^{-C_{12}\overline{W}}+C_{13})(N+4.348\times10^{-5}N^3)\right] \int_0^{D_f}(0.928125D^2+6.0D+1)\,dD \leq 1$$ [2]

3) $\partial K/\partial D_f \leq 0$

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[2] In the paper by Galle and Woods (1960a), a tabulated function of $\overline{W}$, which they called L, appeared in the denominator of constraint 2. I have approximated this tabulated function by $(C_{11}e^{-C_{12}\overline{W}}+C_{13})$.

4) $\overline{W} \leq 79$

5) D_f, N, $\overline{W} \geq 0$.

As a simplification, for each numerical constant introduce a subscripted "C".

Minimize

$$K = \left[C_1 + \frac{(C_2 - C_3 \ln \overline{W})}{(N + C_4 N^3)} \int_0^{D_f} (C_5 D^2 + C_6 D + 1)\, dD\right] \Bigg/$$

$$\left[\frac{(C_2 - C_3 \ln \overline{W})(\overline{W})^{C_7}\left(e^{-100/N^2} N^{C_8} + C_9 N\left(1 - e^{-100/N^2}\right)\right)}{(N + C_4 N^3)} \int_0^{D_f} (C_5 D^2 + C_6 D + 1)^{1-p}\, dD\right]$$

Subject to

1) $D_f \leq 1$

2) $\dfrac{(N)(C_2 - C_3 \ln \overline{W})(C_{10})}{(C_{11} e^{-C_{12}\overline{W}} + C_{13})(N + C_4 N^3)} \displaystyle\int_0^{D_f} (C_5 D^2 + C_6 D + 1)\, dD \leq 1$

3) $\partial K / \partial D_f \leq 0$

4) $(C_{14})(\overline{W}) \leq 1$

5) D_f, N, $\overline{W} \geq 0$

Perform the integration appearing in the numerator of the objective function and in constraint 2.

$$\int_0^{D_f}(C_5D^2+C_6D+1)\,dD=\frac{C_5}{3}D_f^{\,3}+\frac{C_6}{2}D_f^{\,2}+D_f = C_{15}D_f^{\,3}+C_{16}D_f^{\,2}+D_f$$

Selectively, common polynomials in the problem can be replaced by a subscripted "R". Later, the values of the "R" terms will be brought into the minimization.

Minimize

$$K = \frac{C_1+(R_1)(R_2)^{-1}(C_{15}D_f^{\,3}+C_{16}D_f^{\,2}+D_f)}{(R_1)(\overline{W})^{C_7}(R_2)^{-1}(R_3)\int_0^{D_f}(C_5D^2+C_6D+1)^{1-p}dD}$$

Subject to

1) $D_f \leq 1$

2) $\dfrac{(N)(R_1)(C_{10})(R_2)^{-1}(R_5)}{(C_{11}e^{-C_{12}\overline{W}}+C_{13})} \leq 1$

3) $\partial K/\partial D_f \leq 0$

4) $(C_{14})(\overline{W}) \leq 1$

5) $D_f,\ N,\ \overline{W} \geq 0$

Constraint 2 can be reduced to two constraints in order to remove the exponential term. This is accomplished by using the relationship

$$e^u = \lim_{\varepsilon\to\infty}(1 + u/\varepsilon)^{\varepsilon}.$$

This justifies the replacement of $e^{C_{12}\overline{W}}$ with $(1-C_{12}\overline{W}/\varepsilon_1)^{\varepsilon_1}$ where ε_1 is sufficiently large.

Introducing a new variable $t_1 = 1 - C_{12}\overline{W}/\mathcal{E}_1$, $e^{-C_{12}\overline{W}}$ can be replaced with $(t_1)^{\mathcal{E}_1}$. This necessitates imposing another constraint $1 - C_{12}\overline{W}/\mathcal{E}_1 \leq t_1$ which is equivalent to $(t_1)^{-1} - (\mathcal{E}_1)^{-1}(C_{12})(t_1)^{-1}(\overline{W}) \leq 1$.

Constraint 2, with the introduction of t_1, is

$$\frac{(N)(R_1)(C_{10})(R_2)^{-1}(R_5)}{(C_{11})(t_1)^{\mathcal{E}_1} + (C_{13})} \leq 1.$$

Rearranging, this becomes

$$(N)(R_1)(C_{10})(R_2)^{-1}(R_5) \leq (C_{11})(t_1)^{\mathcal{E}_1}+(C_{13}),\ \text{then}$$

$$(C_{10})(N)(R_1)(R_2)^{-1}(R_5)-(C_{13}) \leq (C_{11})(t_1)^{\mathcal{E}_1},\ \text{then}$$

$$(C_{10})(C_{11})^{-1}(N)(R_1)(R_2)^{-1}(R_5)(t_1)^{-\mathcal{E}_1}-(C_{11})^{-1}(C_{13})(t_1)^{-\mathcal{E}_1} \leq 1,$$

and finally, after defining new constants,

$$(C_{17})(N)(R_1)(R_2)^{-1}(R_5)(t_1)^{-\mathcal{E}_1}-(C_{18})(t_1)^{-\mathcal{E}_1} \leq 1.$$

Also present will be the additional constraint

$$(t_1)^{-1}-(C_{19})(t_1)^{-1}(\overline{W}) \leq 1.$$

The general statement of the problem is then reduced to

Minimize

$$K = \frac{(C_1)+(R_1)(R_2)^{-1}(C_{15}D_f^{\,3}+C_{16}D_f^{\,2}+D_f)}{(R_1)(\overline{W})^{C_7}(R_2)^{-1}(R_3)\int_0^{D_f} (C_5D^2+C_6D+1)^{1-p}dD}$$

Subject to

1) $D_f \leq 1$

2) $(C_{17})(N)(R_1)(R_2)^{-1}(R_5)(t_1)^{-\varepsilon_1}-(C_{18})(t_1)^{-\varepsilon_1} \leq 1$

3) $(t_1)^{-1}-(C_{19})(t_1)^{-1}(\overline{W}) \leq 1$

4) $\partial K/\partial D_f \leq 0$

5) $(C_{14})(\overline{W}) \leq 1$

6) $D_f,\ N,\ \overline{W},\ t_1 \geq 0$

This is the simplest statement of the general case problem. For each of the three cases (p=0, .5, and 1.0), the term in the objective function must be integrated, constraint 4 must be reduced, and bounds must be placed on the "R" polynomials.

CASE I

p = 1.0 (FLAT CRESTED BIT TOOTH WEAR)

Performing the integration in the denominator of the objective function,

$$\int_{0}^{D_f} (C_5D^2+C_6D+1)^{1-p}dD = \int_{0}^{D_f} dD = D_f \,.$$

The objective function will be

$$K = \frac{(C_1)+(R_1)(R_2)^{-1}(C_{15}D_f^{\,3}+C_{16}D_f^{\,2}+D_f)}{(R_1)(\overline{W})^{C_7}(R_2)^{-1}(R_3)(D_f)} \text{ , or,}$$

rearranging,

$$K = (C_1)(D_f)^{-1}(\overline{W})^{-C_7}(R_1)^{-1}(R_2)(R_3)^{-1}$$
$$+ (D_f)^{-1}(\overline{W})^{-C_7}(R_3)^{-1}(R_5)$$

Constraint 4 from the general case would be

$$\partial K/\partial D_f = \Big[(R_1)(\overline{W})^{C_7}(R_2)^{-1}(R_3)(D_f)(R_1)(R_2)^{-1}(R_4)$$
$$- \big[(C_1)+(R_1)(R_2)^{-1}(R_5)\big]\,(R_1)(\overline{W})^{C_7}(R_2)^{-1}(R_3)\Big]$$
$$\Big/\Big[(R_1)^2(\overline{W})^{2C_7}(R_2)^{-2}(R_3)^2(D_f)^2\Big] \leq 0.$$

The denominator is positive and may be removed without changing the sense of the inequality. From the numerator, factor $(R_1)(\overline{W})^{C_7}(R_2)^{-1}(R_3)$.

$$\left[(R_1)(\overline{W})^{C_7}(R_2)^{-1}(R_3)\right]\left\{(D_f)(R_1)(R_2)^{-1}(R_4) - \left[(C_1) + (R_1)(R_2)^{-1}(R_5)\right]\right\} \leq 0$$

(R_1), $(\overline{W})^{C_7}$, $(R_2)^{-1}$, and (R_3) are positive and may be removed without changing the sense of the inequality.

$$(D_f)(R_1)(R_2)^{-1}(R_4) - (C_1) - (R_1)(R_2)^{-1}(R_5) \leq 0$$

Expand R_4 since it does not appear in the objective function.

$$(D_f)(R_1)(R_2)^{-1}(C_5D_f^{\,2}+C_6D_f+1) - (C_1)$$
$$-(R_1)(R_2)^{-1}(R_5) \leq 0$$

Perform the multiplication in the first term.

$$(C_5)(D_f)^3(R_1)(R_2)^{-1} + (C_6)(D_f)^2(R_1)(R_2)^{-1}$$
$$+ (D_f)(R_1)(R_2)^{-1} - (C_1) - (R_1)(R_2)^{-1}(R_5) \leq 0$$

Add C_1 to both sides of the inequality and then divide both sides of the inequality by C_1.

$$(C_1)^{-1}(C_5)(D_f)^3(R_1)(R_2)^{-1} + (C_1)^{-1}(C_6)(D_f)^2(R_1)(R_2)^{-1}$$
$$+ (C_1)^{-1}(D_f)(R_1)(R_2)^{-1} - (C_1)^{-1}(R_1)(R_2)^{-1}(R_5) \leq 1$$

By defining new constants, reduce constraint 4 to

$$(C_{21})(D_f)^3(R_1)(R_2)^{-1} + (C_{22})(D_f)^2(R_1)(R_2)^{-1} + (C_{20})(D_f)(R_1)(R_2)^{-1} - (C_{20})(R_1)(R_2)^{-1}(R_5) \leq 1.$$

The "R" polynomial terms must be bounded by the introduction of a constraint for each.

R_1 must be maximized to help minimize the objective function. Therefore, R_1 must have an upper bound to help ensure a bounded objective function.

$$C_2 - C_3 \ln\overline{W} \geq R_1$$

This can be rewritten as

$$R_1 + C_3 \ln\overline{W} \leq C_2.$$

If $\mathcal{E}_2$ is sufficiently small, $\ln\overline{W}$ can be approximated by $(\mathcal{E}_2)^{-1}(\overline{W})^{\mathcal{E}_2} - (\mathcal{E}_2)^{-1}$.

Substituting,

$$(R_1) + (C_3)[(\mathcal{E}_2)^{-1}(\overline{W})^{\mathcal{E}_2} - (\mathcal{E}_2)^{-1}] \leq (C_2).$$

Expanding,

$$(R_1) + (C_3)(\mathcal{E}_2)^{-1}(\overline{W})^{\mathcal{E}_2} - (C_3)(\mathcal{E}_2)^{-1} \leq (C_2)$$

Add the constant $(C_3)(\mathcal{E}_2)^{-1}$ to both sides.

$$(R_1) + (C_3)(\mathcal{E}_2)^{-1}(\overline{W})^{\mathcal{E}_2} \leq (C_2) + (C_3)(\mathcal{E}_2)^{-1}$$

Divide both sides by the quantity $(C_2) + (C_3)(\mathcal{E}_2)^{-1}$ and define new constants.

$$(C_{23})(R_1) + (C_{24})(\overline{W})^{\mathcal{E}_2} \leq 1$$

R_2 must be minimized to help minimize the objective function and must have a lower bound to obtain a feasible solution to the problem.

$$N + C_4N^3 \leq R_2$$

This can be rewritten as

$$(N)(R_2)^{-1} + (C_4)(N)^3(R_2)^{-1} \leq 1.$$

R_3 must be maximized to minimize the objective function and must have an upper bound.

$$(e)^{-100/N^2}(N)^{C_8} + (C_9)(N)(1-e^{-100/N^2}) \geq R_3$$

Expanding,

$$(e)^{-100/N^2}(N)^{C_8} + (C_9)(N) - (C_9)(N)(e)^{-100/N^2} \geq R_3.$$

Using the same procedure as on the bearing life constraint in the general case, $(e)^{-100/N^2}$ can be replaced by $(1 - 100/N^2 \big/ \mathcal{E}_3)^{\mathcal{E}_3}$, where $\mathcal{E}_3$ is taken to be sufficiently large. Define a new variable $t_2 = 1 - 100/N^2 \big/ \mathcal{E}_3$ and introduce the new constraint $(t_2)^{-1} + (\mathcal{E}_3)^{-1}(-100)(N)^{-2}(t_2)^{-1} \leq 1.$

The R_3 constraint is then

$$(t_2)^{\mathcal{E}_3}(N)^{C_8} + (C_9)(N) - (C_9)(N)(t_2)^{\mathcal{E}_3} \geq R_3,$$

rearranging,

$$(R_3) - (t_2)^{\varepsilon_3}(N)^{C_8} + (C_9)(N)(t_2)^{\varepsilon_3} \leq (C_9)(N),$$

and by dividing both sides by $(C_9)(N)$,

$$(C_9)^{-1}(N)^{-1}(R_3) - (C_9)^{-1}(N)^{C_8-1}(t_2)^{\varepsilon_3} + (t_2)^{\varepsilon_3} \leq 1.$$

Defining new constants, the two constraints become

$$(C_{25})(N)^{-1}(R_3) - (C_{25})(N)^{C_{26}}(t_2)^{\varepsilon_3} + (t_2)^{\varepsilon_3} \leq 1,$$

and

$$(t_2)^{-1} - (C_{27})(N)^{-2}(t_2)^{-1} \leq 1.$$

R_5 must be minimized to minimize the objective function and must have a lower bound.

$$(C_{15})(D_f)^3 + (C_{16})(D_f)^2 + (D_f) \leq R_5$$

Divide both sides by R_5.

$$(C_{15})(D_f)^3(R_5)^{-1} + (C_{16})(D_f)^2(R_5)^{-1} + (D_f)(R_5)^{-1} \leq 1$$

With these additional constraints and variables, Case I (for p = 1.0) will have 24 terms and 9 variables, resulting in a minimization problem with a degree of difficulty of 14.

The full problem for p = 1.0 is

Minimize

$$K = (C_1)(D_f)^{-1}(\overline{W})^{-C_7}(R_1)^{-1}(R_2)(R_3)^{-1} + (D_f)(\overline{W})^{-C_7}(R_3)^{-1}(R_5)$$

Subject to

1) $D_f \leq 1$

2) $(C_{17})(N)(R_1)(R_2)^{-1}(R_5)(t_1)^{\varepsilon_1} - (C_{18})(t_1)^{-\varepsilon_1} \leq 1$

3) $(t_1)^{-1} - (C_{19})(t_1)^{-1}(\overline{W}) \leq 1$

4) $(C_{21})(D_f)^3(R_1)(R_2)^{-1} + (C_{22})(D_f)^2(R_1)(R_2)^{-1}$

$+ (C_{20})(D_f)(R_1)(R_2)^{-1}$

$- (C_{20})(R_1)(R_2)^{-1}(R_5) \leq 1$

5) $(C_{14})(\overline{W}) \leq 1$

6) $(C_{23})(R_1) + (C_{24})(\overline{W})^{\varepsilon_2} \leq 1$

7) $(N)(R_2)^{-1} + (C_4)(N)^3(R_2)^{-1} \leq 1$

8) $(C_{25})(N)^{-1}(R_3) - (C_{25})(N)^{C_{26}}(t_2)^{\varepsilon_3} + (t_2)^{\varepsilon_3} \leq 1$

9) $(t_2)^{-1} - (C_{27})(N)^{-2}(t_2)^{-1} \leq 1$

10) $(C_{15})(D_f)^3(R_5)^{-1} + (C_{16})(D_f)^2(R_5)^{-1}$

$+ (D_f)(R_5)^{-1} \leq 1$

11) $D_f,\ N,\ \overline{W},\ R_1,\ R_2,\ R_3,\ R_5,\ t_1,\ t_2 \geq 0.$

The dual function for this problem is

$$K = (C_1/d_1)^{d_1}\ (1/d_2)^{d_2}\ (C_{17}/d_4)^{d_4}(C_{18}/d_5)^{-d_5}$$

$$(1/d_6)^{d_6}\ (C_{19}/d_7)^{-d_7}\ (C_{21}/d_8)^{d_8}\ (C_{22}/d_9)^{d_9}$$

$$(C_{20}/d_{10})^{d_{10}}\ (C_{20}/d_{11})^{-d_{11}}\ (C_{14})^{d_{12}}(C_{23}/d_{13})^{d_{13}}$$

$$(c_{24}/d_{14})^{d_{14}} (1/d_{15})^{d_{15}} (c_4/ d_{16})^{d_{16}} (c_{25}/d_{17})^{d_{17}}$$

$$(c_{25}/d_{18})^{-d_{18}} (1/d_{19})^{d_{19}} (1/d_{20})^{d_{20}} (c_{27}/d_{21})^{-d_{21}}$$

$$(c_{15}/d_{22})^{d_{22}} (c_{16}/d_{23})^{d_{23}} (1/d_{24})^{d_{24}} (d_4-d_5)^{d_4-d_5}$$

$$(d_6+d_7)^{d_6+d_7} (d_8+d_9+d_{10}-d_{11})^{d_8+d_9+d_{10}-d_{11}}$$

$$(d_{13}+d_{14})^{d_{13}+d_{14}} (d_{15}+d_{16})^{d_{15}+d_{16}}$$

$$(d_{17}-d_{18}+d_{19})^{d_{17}-d_{18}+d_{19}} (d_{20}-d_{21})^{d_{20}-d_{21}}$$

$$(d_{22}+d_{23}+d_{24})^{d_{22}+d_{23}+d_{24}} .$$

The normalization condition is

$$d_1 + d_2 = 1,$$

and the orthogonality condition is

$$-c_7d_1-c_7d_2-d_7+d_{12}+\varepsilon_2d_{14} = 0$$

$$-d_1+d_2+d_3+3d_8+2d_9+d_{10}+3d_{22}+2d_{23}+d_{24} = 0$$

$$d_4+d_{15}+3d_{16}-d_{17}-c_{26}d_{18}+2d_{21} = 0$$

$$-d_1+d_4+d_8+d_9+d_{10}-d_{11}+d_{13} = 0$$

$$-d_1-d_4-d_8-d_9-d_{10}+d_{11}-d_{15}-d_{16} = 0$$

$$-d_1-d_2+d_{17} = 0$$

$$d_2+d_4-d_{11}-d_{22}-d_{23}-d_{24} = 0$$

$$-\varepsilon_1d_4+\varepsilon_1d_5-d_6+d_7 = 0$$

$$\varepsilon_3d_{18}+\varepsilon_3d_{19}-d_{20}-d_{21} = 0.$$

CASE II

p = 0.0 (TUNGSTEN CARBIDE STUDDED BITS)

Working with the general case, perform the integration in the denominator of the objective function.

$$\int_0^{D_f} (C_5D^2+C_6D+1)^{1-p}dD = \int_0^{D_f} (C_5D^2+C_6D+1)dD$$

$$= C_5/3\ D_f^{\ 3} + C_6/2\ D_f^{\ 2} + D_f$$

$$= C_{15}D_f^{\ 3}+C_{16}D_f^{\ 2}+D_f$$

The objective function will be

$$K = \frac{(C_1)+(R_1)(R_2)^{-1}(C_{15}D_f^{\ 3}+C_{16}D_f^{\ 2}+D_f)}{(R_1)(\overline{W})^{C_7}(R_2)^{-1}(R_3)(C_{15}D_f^{\ 3}+C_{16}D_f^{\ 2}+D_f)},$$

or, rearranging,

$$K = (C_1)(\overline{W})^{-C_7}(R_1)^{-1}(R_2)(R_3)^{-1}(R_5)^{-1}+(\overline{W})^{-C_7}(R_3)^{-1}.$$

Constraint 4 from the general case would be

$$\partial K/\partial D_f = -(C_1)(\overline{W})^{-C_7}(R_1)^{-1}(R_2)(R_3)^{-1}(R_4)(R_5)^{-2} \le 0.$$

(C_1), $(\overline{W})^{-C_7}$, $(R_1)^{-1}$, (R_2), $(R_3)^{-1}$, (R_4), and $(R_5)^{-2}$ are all positive and the constraint can be removed because it is not binding.

As in Case I, R_1 must be maximized, R_2 minimized, and R_3 maximized to help ensure minimizing the objective function. Since R_5 appears in the objective function with a negative subscript, it must be maximized. Thus R_5 has an upper bound if the objective function is to be bounded.

$$(C_{15})(D_f)^3+(C_{16})(D_f)^2+(D_f) \geq R_5$$

Rearranging the terms,

$$(R_5)-(C_{15})(D_f)^3-(C_{16})(D_f)^2 \leq D_f .$$

Dividing both sides by D_f,

$$(R_5)(D_f)^{-1}-(C_{15})(D_f)^2-(C_{16})(D_f) \leq 1.$$

Case II (for $p = 0.0$) will have 20 terms and 9 variables, resulting in a minimization problem with a degree of difficulty of 10.

The full problem for $p = 0.0$ is

Minimize

$$K = (C_1)(\overline{W})^{-C_7}(R_1)^{-1}(R_2)(R_3)^{-1}(R_5)^{-1}+(\overline{W})^{-C_7}(R_3)^{-1}$$

Subject to

1) $D_f \leq 1$

2) $(C_{17})(N)(R_1)(R_2)^{-1}(R_5)(t_1)^{-\varepsilon_1}-(C_{18})(t_1)^{-\varepsilon_1} \leq 1$

3) $(t_1)^{-1}-(C_{19})(t_1)^{-1}(\overline{W}) \leq 1$

4) $(C_{14})(\overline{W}) \leq 1$

5) $(C_{23})(R_1)+(C_{24})(\overline{W})^{\varepsilon_2} \leq 1$

6) $(N)(R_2)^{-1}+(C_{24})(N)^3(R_2)^{-1} \leq 1$

7) $(C_{25})(N)^{-1}(R_3)-(C_{25})(N)^{C_{26}}(t_2)^{\varepsilon_3}+(t_2)^{\varepsilon_3} \leq 1$

8) $(t_2)^{-1}-(C_{27})(N)^{-2}(t_2)^{-1} \leq 1$

9) $(R_5)(D_f)^{-1}-(C_{15})(D_f)^2-(C_{16})(D_f) \leq 1$

10) $D_f,\ N,\ \overline{W},\ R_1,\ R_2,\ R_3,\ R_5,\ t_1,\ t_2 \geq 0$

The dual function for this problem is

$$K = (C_1/d_1)^{d_1}\ (1/d_2)^{d_2}\ (C_{17}/d_4)^{d_4}\ (C_{18}/d_5)^{-d_5}$$
$$(1/d_6)^{d_6}\ (C_{19}/d_7)^{-d_7}\ (C_{14})^{d_8}\ (C_{23}/d_9)^{d_9}$$
$$(C_{24}/d_{10})^{d_{10}}\ (1/d_{11})^{d_{11}}\ (C_4/d_{12})^{d_{12}}\ (C_{25}/d_{13})^{d_{13}}$$
$$(C_{25}/d_{14})^{-d_{14}}\ (1/d_{15})^{d_{15}}\ (1/d_{16})^{d_{16}}\ (C_{27}/d_{17})^{-d_{17}}$$
$$(1/d_{18})^{d_{18}}\ (C_{15}/d_{19})^{-d_{19}}\ (C_{16}/d_{20})^{-d_{20}}$$
$$(d_4-d_5)^{d_4-d_5}\ (d_6+d_7)^{d_6+d_7}\ (d_9+d_{10})^{d_9+d_{10}}$$
$$(d_{11}+d_{12})^{d_{11}+d_{12}}\ (d_{13}-d_{14}+d_{15})^{d_{13}-d_{14}+d_{15}}$$
$$(d_{16}-d_{17})^{d_{16}-d_{17}}\ (d_{18}-d_{19}-d_{20})^{d_{18}-d_{19}-d_{20}}.$$

The normalization condition is

$$d_1+d_2 = 1$$

and the orthogonality condition is

$$-C_7d_1-C_7d_2-d_7+d_8+\varepsilon_2d_{10} = 0$$

$$d_3-d_{18}-2d_{19}-d_{20} = 0$$

$$d_4+d_{11}+3d_{12}-d_{13}-C_{26}d_{14}+2d_{17} = 0$$

$$-d_1+d_4+d_9 = 0$$

$$d_1-d_4-d_{11}-d_{12} = 0$$

$$-d_1-d_2+d_{13} = 0$$

$$-d_1+d_4+d_{18} = 0$$

$$-\varepsilon_1d_4+\varepsilon_1d_5-d_6+d_7 = 0$$

$$-\varepsilon_3d_{14}+\varepsilon_3d_{15}-d_{16}+d_{17} = 0.$$

CASE III

p = 0.5 (SELF-SHARPENING OR CHIPPING TYPE BIT TOOTH WEAR)

Working with the general case, perform the integration in the denominator of the objective function.

$$\int_0^{D_f} (C_5D^2+C_6D+1)^{1-p}dD = \int_0^{D_f} (C_5D^2+C_6D+1)^{\frac{1}{2}}dD$$

$$= \frac{[(2)(C_5)(D_f)+(C_6)](R_4)^{\frac{1}{2}}}{(4)(C_5)} + \left[\frac{(4)(C_5)-(C_6)^2}{(8)(C_5)(C_5)^{\frac{1}{2}}}\right.$$

$$\left.\ln\left((R_4)^{\frac{1}{2}}+(C_5)^{\frac{1}{2}}(D_f)+\frac{(C_6)}{(2)(C_5)^{\frac{1}{2}}}\right)\right] - \frac{(C_6)}{(4)(C_5)}$$

$$+ \left[\frac{(4)(C_5)-(C_6)^2}{(8)(C_5)(C_5)^{\frac{1}{2}}} \ln\left(1 + \frac{(C_6)}{(2)(C_5)^{\frac{1}{2}}}\right)\right]$$

$$= (\tfrac{1}{2})(D_f)(R_4)^{\frac{1}{2}}+(C_{28})(R_4)^{\frac{1}{2}}+(C_{29})\ln[(R_4)^{\frac{1}{2}}+(C_{30})(D_f)+(C_{31})]$$
$$+(C_{32})$$

$$= (R_6)$$

The objective function will be

$$K = \frac{(C_1)+(R_1)(R_2)^{-1}(R_5)}{(\overline{W})^{C_7}(R_1)(R_2)^{-1}(R_3)^{-1}(R_6)}$$

$$K = (C_1)(\overline{W})^{-C_7}(R_1)^{-1}(R_2)(R_3)^{-1}(R_6)^{-1}$$

$$+ (\overline{W})^{-C_7}(R_3)^{-1}(R_5)(R_6)^{-1}$$

Constraint 4 from the general case would be

$$\partial K/\partial D_f = -\Big[(C_1)(\overline{W})^{-C_7}(R_1)^{-1}(R_2)(R_3)^{-1} + (\overline{W})^{-C_7}(R_3)^{-1}(R_5)\Big](R_6)^{-2}$$

$$\Big[(\tfrac{1}{2})(R_4)^{\frac{1}{2}} + (\tfrac{1}{4})(R_4)^{-\frac{1}{2}}\big[(2)(C_5)(D_f)^2 + (C_6)(D_f)\big]$$

$$+ (\tfrac{1}{2})(C_{28})(R_4)^{-\frac{1}{2}}\big[(2)(C_5)(D_f) + (C_{26})\big] +$$

$$\Big\{\big[(\tfrac{1}{2})(C_{29})(R_4)^{-\frac{1}{2}}\big[(2)(C_5)(D_f) + (C_6)\big] + (C_{29})(C_{30})\big] /$$

$$\big[(R_4)^{\frac{1}{2}} + (C_{30})(D_f) + (C_{31})\big]\Big\}\Big] \leq 0.$$

Place the terms over a common denominator.

$$\partial K/\partial D_f = -\Big[(C_1)(\overline{W})^{-C_7}(R_1)^{-1}(R_2)(R_3)^{-1}$$

$$+ (\overline{W})^{-C_7}(R_3)(R_5)\Big]\ (R_6)^{-2}\ \Big[(\tfrac{1}{2})(R_4)$$

$$+ (\tfrac{1}{2})(C_{30})(D_f)(R_4)^{\frac{1}{2}} + (\tfrac{1}{2})(C_{31})(R_4)^{\frac{1}{2}}$$

$$+ (\tfrac{1}{2})(C_5)(D_f)^2 + (\tfrac{1}{2})(C_5)(C_{30})(D_f)^3(R_4)^{-\frac{1}{2}}$$

$$+ (\tfrac{1}{2})(C_5)(C_{31})(D_f)^2(R_4)^{-\frac{1}{2}}$$

$$+ (\tfrac{1}{4})(C_6)(D_f) + (\tfrac{1}{4})(C_6)(C_{30})(D_f)^2(R_4)^{-\frac{1}{2}}$$

$$+ (\tfrac{1}{4})(C_6)(C_{31})(D_f)(R_4)^{-\frac{1}{2}}$$

$$+ (C_5)(C_{28})(D_f) + (C_5)(C_{28})(C_{30})(D_f)^2(R_4)^{-\frac{1}{2}}$$

$$+ (C_5)(C_{28})(C_{31})(D_f)(R_4)^{-\frac{1}{2}} + (\tfrac{1}{2})(C_6)(C_{28})$$

$$+ (\tfrac{1}{2})(C_6)(C_{28})(C_{30})(D_f)(R_4)^{-\frac{1}{2}} + (\tfrac{1}{2})(C_6)(C_{28})(C_{31})(R_4)$$

$$+ (C_5)(C_{29})(D_f)(R_4)^{-\frac{1}{2}} + (\tfrac{1}{2})(C_6)(C_{29})(R_4)^{-\frac{1}{2}}$$

$$+ (C_{29})(C_{30})\Big]\Big[(R_4)^{\frac{1}{2}}+(C_{30})(D_f)+(C_{31})\Big]^{-1} \leq 0$$

The quantities $[(C_1)(\overline{W})^{-C_7}(R_1)^{-1}(R_2)(R_3)^{-1} + (\overline{W})^{-C_7}(R_3)(R_5)]$, $(R_6)^{-2}$, and $[(R_4)^{\frac{1}{2}} + (C_{30})(D_f) + (C_{31})]^{-1}$ are positive and may be removed without changing the direction of the inequality. Regroup the remaining terms

$$-\Big[(\tfrac{1}{2})(R_4) + (\tfrac{1}{2})(C_{31})(R_4)^{\frac{1}{2}} + (\tfrac{1}{2})(C_{30})(D_f)(R_4)^{\frac{1}{2}}$$

$$+ (\tfrac{1}{2})(C_6)(C_{28})(C_{31})(R_4)^{-\frac{1}{2}} + (\tfrac{1}{2})(C_6)(C_{29})(R_4)^{-\frac{1}{2}}$$

$$+ (\tfrac{1}{4})(C_6)(C_{31})(D_f)(R_4)^{-\frac{1}{2}} + (C_5)(C_{28})(C_{31})(D_f)(R_4)^{-\frac{1}{2}}$$

$$+ (\tfrac{1}{2})(C_6)(C_{28})(C_{30})(D_f)(R_4)^{-\frac{1}{2}} + (C_5)(C_{29})(D_f)(R_4)^{-\frac{1}{2}}$$

$$+ (\tfrac{1}{2})(C_5)(C_{31})(D_f)^2(R_4)^{-\frac{1}{2}} + (\tfrac{1}{4})(C_6)(C_{30})(D_f)^2(R_4)^{-\frac{1}{2}}$$

$$+ (C_5)(C_{28})(C_{30})(D_f)^2(R_4)^{-\frac{1}{2}}$$

$$+ (\tfrac{1}{2})(C_5)(C_{30})(D_f)^3(R_4)^{-\frac{1}{2}} + (\tfrac{1}{4})(C_6)(D_f)$$

$$+ (C_5)(C_{28})(D_f) + (\tfrac{1}{2})(C_5)(D_f)^2\Big] \leq (\tfrac{1}{2})(C_6)(C_{28})$$

$$+ (C_{29})(C_{30})$$

Divide both sides by the right hand side and define new constants.

$$- (C_{33})(R_4) - (C_{34})(R_4)^{\frac{1}{2}} - (C_{35})(D_f)(R_4)^{\frac{1}{2}}$$

$$- (C_{36})(R_4)^{-\frac{1}{2}} - (C_{37})(D_f)(R_4)^{-\frac{1}{2}} - (C_{38})(D_f)^2(R_4)^{-\frac{1}{2}}$$

$$- (C_{39})(D_f)^3(R_4)^{-\frac{1}{2}} - (C_{40})(D_f) - (C_{41})(D_f)^2 \leq 1$$

As in Case I, R_1 must be maximized, R_2 minimized, R_3 maximized, and R_5 minimized if the objective function is to be minimized. R_4 will be maximized and must have an upper bound.

$$C_5 \, D_f^2 + C_6 \, D_f + 1 \geq R_4.$$

Rearranged, this becomes

$$R_4 - C_5 \, D_f^2 - C_6 \, D_f \leq 1 .$$

R_6 must also be maximized and have an upper bound.

$$(\tfrac{1}{2})(D_f)(R_4)^{\frac{1}{2}} + (C_{28})(R_4)^{\frac{1}{2}} + (C_{29}) \ln (R_7)$$
$$+ (C_{32}) \geq R_6$$

If $\mathcal{E}_3$ is sufficiently small, $\ln (R_7)$ can be approximated by $(\mathcal{E}_3)^{-1}(R_7)^{\mathcal{E}_3} - (\mathcal{E}_3)^{-1}$. Substituting,

$$(\tfrac{1}{2})(D_f)(R_4)^{\frac{1}{2}} + (C_{28})(R_4)^{\frac{1}{2}} + (C_{29}) \left[(\mathcal{E}_3)^{-1}(R_7)^{\mathcal{E}_3} \right.$$
$$\left. - (\mathcal{E}_3)^{-1} \right] + (C_{32}) \geq R_6 .$$

Expanding,

$$(\tfrac{1}{2})(D_f)(R_4)^{\frac{1}{2}} + (C_{28})(R_4)^{\frac{1}{2}} + (C_{29})(\mathcal{E}_3)^{-1}(R_7)^{\mathcal{E}_3}$$
$$-(C_{29})(\mathcal{E}_3) + (C_{32}) \geq R_6 .$$

Rearranging the terms,

$$(\tfrac{1}{2})(D_f)(R_4)^{\frac{1}{2}} + (C_{28})(R_4)^{\frac{1}{2}} - (R_6) - (C_{29})(\mathcal{E}_3)$$
$$+ (C_{32}) \geq -(C_{29})(\mathcal{E}_3)^{-1}(R_7)^{\mathcal{E}_3}.$$

Dividing both sides of the inequality by the terms on the right side of the inequality and defining new constants,

$$-(C_{42})(D_f)(R_4)^{\frac{1}{2}}(R_7)^{-\mathcal{E}_3} - (C_{43})(R_4)^{\frac{1}{2}}(R_7)^{-\mathcal{E}_3}$$
$$+ (C_{44})(R_6)(R_7)^{-\mathcal{E}_3} + (C_{45})(R_7)^{-\mathcal{E}_3} \leq 1.$$

R_7 must be maximized in order to maximize R_6.

$$(R_4)^{\frac{1}{2}} + (C_{30})(D_f) + (C_{31}) \geq R_7.$$

Rearranging the terms,

$$(R_7) - (R_4)^{\frac{1}{2}} - (C_{30})(D_f) \leq C_{31}.$$

Dividing both sides by C_{31},

$$(C_{46})(R_7) - (C_{46})(R_4)^{\frac{1}{2}} - (C_{47})(D_f) \leq 1.$$

Case III (for p = 0.5) will have 39 terms and 12 variables, resulting in a minimization problem with a degree of difficulty of 26.

The full problem for p = 0.5 is

Minimize

$$K = (C_1)(\overline{W})^{-C_7}(R_1)^{-1}(R_2)(R_3)^{-1}(R_6)^{-1} + (\overline{W})^{-C_7}(R_3)^{-1}(R_5)(R_6)^{-1}$$

Subject to

1) $D_f \leq 1$

2) $(C_{17})(N)(R_1)(R_2)^{-1}(R_5)(t_1)^{-\varepsilon_1} - (C_{18})(t_1)^{-\varepsilon_1} \leq 1$

3) $(t_1)^{-1} - (C_{19})(t_1)^{-1}(\overline{W}) \leq 1$

4) $-(C_{33})(R_4) - (C_{34})(R_4)^{\frac{1}{2}} - (C_{35})(D_f)(R_4)^{\frac{1}{2}}$
$- (C_{36})(R_4)^{-\frac{1}{2}} - (C_{37})(D_f)(R_4)^{-\frac{1}{2}} - (C_{38})(D_f)^2(R_4)^{-\frac{1}{2}}$
$- (C_{39})(D_f)^3(R_4)^{-\frac{1}{2}} - (C_{40})(D_f)$
$- (C_{41})(D_f)^2 \leq 1$

5) $(C_{14})(\overline{W}) \leq 1$

6) $(C_{23})(R_1) + (C_{24})(\overline{W})^{\varepsilon_2} \leq 1$

7) $(N)(R_2)^{-1} + (C_4)(N)^3(R_2)^{-1} \leq 1$

8) $(C_{25})(N)^{-1}(R_3) - (C_{25})(N)^{C_{26}}(t_2)^{\varepsilon_3} + (t_2)^{\varepsilon_3} \leq 1$

9) $(t_2)^{-1} - (C_{27})(N)^{-2}(t_2)^{-1} \leq 1$

10) $(R_4) - (C_5)(D_f)^2 - (C_6)(D_f) \leq 1$

11) $(C_{15})(D_f)^3(R_5)^{-1} + (C_{16})(D_f)^2(R_5)^{-1}$

$+ (D_f)(R_5)^{-1} \leq 1$

12) $- (C_{42})(D_f)(R_4)^{\frac{1}{2}}(R_7)^{-\mathcal{E}_3} - (C_{43})(R_4)^{\frac{1}{2}}(R_7)^{-\mathcal{E}_3}$

$+ (C_{44})(R_6)(R_7)^{-\mathcal{E}_3} + (C_{45})(R_7)^{-\mathcal{E}_3} \leq 1$

13) $(C_{46})(R_7) - (C_{46})(R_4)^{\frac{1}{2}} - (C_{47})(D_f) \leq 1$

14) $\overline{W}$, N, D_f, R_1, R_2, R_3, R_4, R_5, R_6, R_7, t_1, t_2,

$t_3 \geq 0.$

The dual function for this problem is

$$K = (C_1/d_1)^{d_1} (1/d_2)^{d_2} (C_{17}/d_4)^{d_4} (C_{18}/d_5)^{-d_5} (1/d_6)^{d_6}$$

$$(C_{19}/d_7)^{-d_7} (C_{32}/d_8)^{-d_8} (C_{34}/d_9)^{-d_9} (C_{35}/d_{10})^{-d_{10}}$$

$$(C_{36}/d_{11})^{-d_{11}} (C_{37}/d_{12})^{-d_{12}} (C_{38}/d_{13})^{-d_{13}}$$

$$(C_{39}/d_{14})^{-d_{14}} (C_{40}/d_{15})^{-d_{15}} (C_{41}/d_{16})^{-d_{16}}$$

$$(C_{14})^{d_{17}} (C_{23}/d_{18})^{d_{18}} (C_{24}/d_{19})^{d_{19}} (1/d_{20})^{d_{20}}$$

$$(C_4/d_{21})^{d_{21}} (C_{25}/d_{22})^{d_{22}} (C_{25}/d_{23})^{-d_{23}}$$

$$(1/d_{24})^{d_{24}} (1/d_{25})^{d_{25}} (C_{27}/d_{26})^{-d_{26}} (1/d_{27})^{d_{27}}$$

$$(C_5/d_{28})^{-d_{28}} (C_6/d_{29})^{-d_{29}} (C_{15}/d_{30})^{d_{30}}$$

$$(C_{16}/d_{31})^{d_{31}} (1/d_{32})^{d_{32}} (C_{42}/d_{33})^{-d_{33}}$$

$$(C_{43}/d_{34})^{-d_{34}} (C_{44}/d_{35})^{d_{35}} (C_{45}/d_{36})^{d_{36}}$$

$$(C_{46}/d_{37})^{d_{37}} (C_{46}/d_{38})^{-d_{38}} (C_{47}/d_{39})^{-d_{39}}$$

$$(d_4-d_5)^{d_4-d_5} (d_6+d_7)^{d_6+d_7}$$

$$(-d_8-d_9-d_{10}-d_{11}-d_{12}-d_{13}-d_{14}-d_{15}-d_{16})^{\emptyset\ 3}$$

$$(d_{18}+d_{19})^{d_{18}+d_{19}} (d_{20}+d_{21})^{d_{20}+d_{21}}$$

$$(d_{22}-d_{23}+d_{24})^{d_{22}-d_{23}+d_{24}} (d_{25}-d_{26})^{d_{25}-d_{26}}$$

$$(d_{27}-d_{28}-d_{29})^{d_{27}-d_{28}-d_{29}} (d_{30}+d_{31}+d_{32})^{d_{30}+d_{31}+d_{32}}$$

$$(-d_{33}-d_{34}+d_{35}+d_{36})^{d_{33}-d_{34}+d_{35}+d_{36}}$$

$$(d_{37}-d_{38}-d_{39})^{d_{37}-d_{38}-d_{39}}$$

The normalization condition is

$$d_1 + d_2 = 1,$$

and the orthogonality condition is

$$-C_7d_1-C_7d_2-d_7+d_{17}+ 2d_{19} = 0$$

$$d_3-d_{10}-d_{12}-2d_{13}-3d_{14}-d_{15}-2d_{16}-2d_{28}-d_{29}+3d_{30}+2d_{31}$$
$$+d_{32}-d_{33}-d_{39} = 0$$

$$d_4+d_{20}+3d_{21}-d_{22}-C_{25}d_{23}+2d_{26} = 0$$

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[3] $\emptyset$ Represents $(-d_8-d_9-d_{10}-d_{11}-d_{12}-d_{13}-d_{14}-d_{15}-d_{16})$

$$-d_1+d_4+d_{18} = 0$$

$$d_1-d_4-d_{20}-d_{21} = 0$$

$$-d_1-d_2+d_{22} = 0$$

$$-d_8-\tfrac{1}{2}d_9-\tfrac{1}{2}d_{10}+\tfrac{1}{2}d_{11}+\tfrac{1}{2}d_{12}+\tfrac{1}{2}d_{13}+\tfrac{1}{2}d_{14}+d_{27}-\tfrac{1}{2}d_{33}-\tfrac{1}{2}d_{34}$$

$$-\tfrac{1}{2}d_{38} = 0$$

$$d_2+d_4-d_{30}-d_{31}-d_{32} = 0$$

$$-d_1-d_2+d_{35} = 0$$

$$\mathcal{E}_3d_{33}+\mathcal{E}_3d_{34}-\mathcal{E}_3d_{35}-\mathcal{E}_3d_{36}+d_{37} = 0$$

$$-\mathcal{E}_1d_4+\mathcal{E}_1d_5-d_6+d_7 = 0$$

$$-\mathcal{E}_3d_{23}+\mathcal{E}_3d_{24}-d_{25}+d_{26} = 0$$

CONCLUSION

The problem has been set up and analyzed by breaking it into three cases based on the bit tooth wear. It has been formulated in the geometric programming format shown in the problem statement on page 4. The normality and orthogonality conditions were also written to check for errors which would produce infeasible results. With this formulation as input, a geometric programming computer code could now be used to obtain the optimal values of D_f, N, and $\overline{W}$ for comparison with Galle and Woods and with field results. Using the d's as cost coefficients, we could better determine the contributions of the three variables to the total cost.

SUBSTITUTION POLYNOMIALS

$$R_1 = (C_2)-(C_3)\ln(\overline{W})$$

$$R_2 = (N)+(C_4)(N)^3$$

$$R_3 = (e)^{-100/N^2}(N)^{C8}+(C_9)(N)(1-e^{-100/N^2})$$

$$R_4 = (C_5)(D_f)^2+(C_6)(D_f)+1$$

$$R_5 = (C_{15})(D_f)^3+(C_{16})(D_f)^2+(D_f)$$

$$R_6 = (\tfrac{1}{2})(D_f)(R_4)^{\frac{1}{2}}+(C_{28})(R_4)^{\frac{1}{2}}+(C_{29})\ln\left[(R_4)^{\frac{1}{2}}+(C_{30})(D_f)+(C_{31})\right]+(C_{32})$$

$$R_7 = (R_4)^{\frac{1}{2}}+(C_{30})(D_f)+(C_{31})$$

SUBSTITUTION CONSTANTS

$C_1 = A_n$

$C_2 = 1359.1$

$C_3 = 1646.1$

$C_4 = 4.348 \times 10^{-5}$

$C_5 = 0.928125$

$C_6 = 6.0$

$$C_7 = k = \begin{cases} 0.6, & \text{for very soft formations} \\ 1.0, & \text{for most other formations} \end{cases}$$

$$C_8 = \begin{cases} 0.428, & \text{for hard formations} \\ 0.75, & \text{for soft formations} \end{cases}$$

$$C_9 = \begin{cases} 0.2, & \text{for hard formations} \\ 0.5, & \text{for soft formations} \end{cases}$$

$C_{10} = S_n^{-1}$

$C_{11} = 18225.7637$

$$c_{12} = 0.075825$$

$$c_{13} = 548.3858$$

$$c_{14} = (79)^{-1}$$

$$c_{15} = c_5/3$$

$$c_{16} = c_6/2$$

$$c_{17} = (c_{10})(c_{11})^{-1}$$

$$c_{18} = (c_{11})^{-1}(c_{13})$$

$$c_{19} = (\varepsilon_1)^{-1}(c_{12})$$

$$c_{20} = (c_1)^{-1}$$

$$c_{21} = (c_1)^{-1}(c_5)$$

$$c_{22} = (c_1)^{-1}(c_6)$$

$$c_{23} = [c_2+(\varepsilon_2)^{-1}(c_3)]^{-1}$$

$$c_{24} = [(c_3)(\varepsilon_2)^{-1}] \,/\, [c_2+(\varepsilon_2)^{-1}(c_3)]$$

$$c_{25} = c_9^{-1}$$

$$c_{26} = c_8-1$$

$$c_{27} = 100\,\varepsilon_3^{-1}$$

$$c_{28} = c_6/4c_5$$

$$C_{29} = \frac{(4)(C_5)-(C_6)^2}{(8)(C_5)^{3/2}}$$

$$C_{30} = (C_5)^{\frac{1}{2}}$$

$$C_{31} = \frac{(C_6)}{(2)(C_5)^{\frac{1}{2}}}$$

$$C_{32} = -C_{28}+C_{29}\ln(1+C_{31})$$

$$CC = (\tfrac{1}{2})(C_6)(C_{28})+(C_{29})(C_{30})$$

$$C_{33} = \frac{\frac{1}{2}}{CC}$$

$$C_{34} = \frac{(\frac{1}{2})(C_{31})}{CC}$$

$$C_{35} = \frac{(\frac{1}{2})(C_{30})}{CC}$$

$$C_{36} = \frac{(\frac{1}{2})(C_6)(C_{28})(C_{31})+(\frac{1}{2})(C_6)(C_{29})}{CC}$$

$$C_{37} = \frac{(\frac{1}{4})(C_6)(C_{31})+(C_5)(C_{28})(C_{31})+(\frac{1}{2})(C_6)(C_{28})(C_{30})+(C_5)(C_{29})}{CC}$$

$$C_{38} = \frac{(\frac{1}{2})(C_5)(C_{31})+(\frac{1}{4})(C_6)(C_{30})+(C_5)(C_{28})(C_{30})}{CC}$$

$$C_{39} = \frac{(\frac{1}{2})(C_5)(C_{30})}{CC}$$

$$C_{40} = \frac{(\frac{1}{4})(C_6)+(C_5)(C_{28})}{CC}$$

$$C_{41} = \frac{(\frac{1}{2})(C_5)}{CC}$$

$$CB = (C_{29})(\mathcal{E}_3)^{-1}$$

$$C_{42} = \tfrac{1}{2}/CB$$

$$C_{43} = \frac{C_{28}}{CB}$$

$$C_{44} = 1/CB$$

$$C_{45} = \frac{-(C_{29})(\mathcal{E}_3)+(C_{32})}{CB}$$

$$C_{46} = 1/C_{31}$$

$$C_{47} = \frac{C_{30}}{C_{31}}$$

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