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Compressibility of solid phase of debris flow and erosion rate

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Abstract

The change in sediment concentration of debris flow causes erosion and sedimentation of the solid phase of debris flows. Moreover, the changing affects the mobility of the flow. Therefore, knowledge of the mechanism of the changing is important to understand the mechanism of debris flow. The changing can be considered as compressibility of the flow of the solid phase. We developed a constitutive equation set of debris flow by concerning energy dissipation. A part of the energy dissipation is due to inelastic collision of particles. This process must be compressible. Therefore, we reinvestigate the process of the inelastic collisions and the effect to the compressibility. As the result, we lead internal energy to control the compressibility and so-called erosion rate equation. According to the erosion rate equation, it depends on bed gradient and energy loss gradient. A flume test is conducted to evaluate the erosion rate equation. by using a prismatic steep slope channel, which inclination is set at 12 degrees. By comparison of experimental result with the erosion rate equation, it is found that the difference between energy gradient and bed gradient to control the erosion/deposition is not so large. It means that the erosion/deposition might be very much sensitive against the unbalance of the energy gradient and bed gradient.

Keywords: Debris flow; Eroson rate, Compressibility; Flume tests

1. Introduction

Theoretical research of constitutive equations for debris flow can be categorized as (1) to define stress tensor to be satisfied mathematical requirement of tensor (Goodman & Cowin, 1972; Savage and Jeffrey, 1981; Jenkins and Savage, 1983; Iverson et al., 2001), (2) to define stress tensor based on momentum exchange, that is, interaction force, at the collisions (Bagnold, 1954; Hashimoto et al., 1983; Takahashi, 1980 & 1991) and (3) to define stress tensor by solving a simultaneous equation set of mass, momentum and kinetic energy conservation in accordance with continuum physics (Miyamoto, 1985; Egashira et al., 1997).

These theories are employed to explain the characteristics of the flow such as the resistance of the flow, sediment concentration and the transport rate of the sediment. However, in the process to derive the constitutive equations, the change in sediment concentration is not taken into account. Therefore, the simulation of the flow accompanied with changing in sediment concentration is somehow technical. We usually employ another equation to govern the change in sediment concentration. The changing causes bed aggradation and degradation. Therefore, erosion rate equation is usually introduced instead of the changing of sediment concentration. There are several researches on erosion rate (e.g., Takahashi, 1991, 2007; Egashira et al., 2001; Takahama et al., 2003). Those equations for bed entrainment are proposed conceptually.

Miyamoto (1985) discussed on the energy dissipation rate due to collision of neutral suspended particles in a simple shear flow by introducing inelasticity of particles to describe constitutive equation. In the process to derive it, it was needed to introduce the changing in sediment concentration at the instance of a collision and in a period from a collision to next collision. It means that the process must be basically compressible. Therefore, it could be applied to the changing process of sediment concentration.

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We modify the energy conservation law, so as to investigate the changing of sediment concentration and the erosion/deposition rate. A flume test to investigate the erosion rate using a prismatic steep slope channel with 12 degrees is also newly conducted. The erosion rate will be evaluated by the moving down velocity of upstream end of bed sediment which is parallelly set to the flume bed.

2. Governing Relationships of Compressibility and Erosion Rate

2.1. Governing relationships of compressibility

In the solid and liquid mixture flow, kinetic energy conservation equation is described as,

$$\frac{dK}{dt} + Ku_{i,i} = \rho_m g_i u_i + \left(u_i \sigma_{ij}\right)_{,j} - \Phi\,, \quad K = \frac{1}{2}\rho_m u_i u_i \tag{1}$$

where, K is the kinetic energy, t is the time, u_i is the velocity, i and j are the coordinate/components (=1 to 3), ρ_m is the averaged mass density of flow field, g_i is the acceleration of gravity, σ_{ij} is the stress tensor, ",i, ,j" mean the partial difference operator and Φ is the energy dissipation. Notations of i and j obey the Einstein's summation convention.

Equation (1) and mass and momentum conservation equations yield the following formula for energy dissipation.

$$\Phi = \frac{1}{3}\sigma_{ii} u_{j,j} + s_{ij} d_{ij} \tag{2}$$

where,

$$s_{ij} \equiv \frac{1}{2}\left(\sigma_{ij} + \sigma_{ji}\right) - \frac{1}{3}\sigma_{kk}\delta_{ij},\ d_{ij} \equiv \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) - \frac{1}{3}u_{k,k}\delta_{ij}$$

Equation (2) means that energy dissipation rate is divided into deviatoric and isotropic parts. where, δ_{ij} is the Kronecker's delta

Figure 1 is a schematic view of the work rate and energy dissipation rate during a collision to next collision in simple shear flow proposed by Miyamoto (1985) and Egashira et al. (1997). In the figure, W is the work by stress and $u_i{}'$ is the fluctuation component of velocity of solid particle. The space occupied by a particle is shrinking at an instance of collision and is expanding during a period from a collision to next collision. In the simple shear flow, the macroscopic flow field is incompressive. That is, both changings in occupied space by a particle, shrinking and expanding, are in balance. Moreover, in a simple shear flow, work rate by stress, dW/dt, and energy dissipation rate Φ must be in balance. That is, it must satisfy,

(Continuity in the flow field)

$$u_{i,i}{}' + n\frac{\Delta V}{V} = 0 \tag{3}$$

(Energy conservation law)

$$\frac{dW}{dt} + \Phi = 0. \tag{4}$$

in which, n is the number of the collision in a unit time, V is the space occupied by a particle, and ΔV is the volumetric change after particle to particle collision and takes negative value. The work rate and energy dissipation rate Φ in Eq. (4) is expressed by using intergranular pressure p_s and expanding/shrinking rate, $u_{i,i}$, $\Delta V/V$, as follows.

$$\frac{dW}{dt} = -p_s u_{i,i}{}',\ \Phi = -p_s n\frac{\Delta V}{V} \tag{5}$$

Equarion (4) can be enhanced to compressible flow filed. When we note the energy dissipation rate in compressible state Φ_T, Eq. (4) is re-written as,

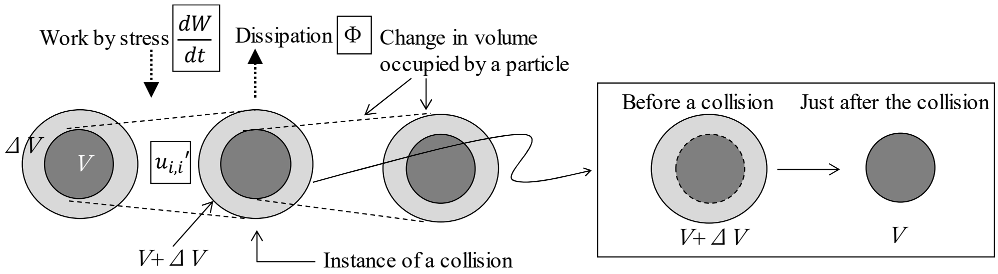

Fig. 1. Schematics of changes in space occupied by a particle and energy flow due to collisions

$$\frac{dW}{dt} + \Phi_T = 0. \tag{6}$$

Equation (5) is also rewritten as

$$\frac{dW}{dt} = -p_s u'_{i,i} = -p_s\left\{\left(u'_{i,i} + n\frac{\Delta V}{V}\right) - n\frac{\Delta V}{V}\right\} = -p_s\left(u'_{i,i} + n\frac{\Delta V}{V}\right) + p_s n\frac{\Delta V}{V}. \tag{7}$$

$$\Phi_T = -\frac{dW}{dt} = p_s\left(u'_{i,i} + n\frac{\Delta V}{V}\right) + \Phi. \tag{8}$$

The second term in brackets of right side of Eq. (8) must be equal to divergence of the macroscopic flow field, that is,

$$u'_{i,i} + n\frac{\Delta V}{V} = u_{i,i} \tag{9}$$

When we introduce Φ_i as follows, we can define something like an internal energy, E_i , too,

$$\Phi_i\left(=\frac{dE_i}{dt}\right) \equiv p_s u_{i,i}, \tag{10}$$

and Φ_T is expressed by sum of internal energy changing rate and energy dissipation rate, as follows.

$$\Phi_T = \Phi_i + \Phi. \tag{11}$$

Herein, Φ_i depend on the divergence of flow field, so that E_i is reversible energy because the value of E_i increases as positive divergence, and E_i decreases as negative divergence.

The continuity equation of flow field is expressed by using sediment concentration in the sediment-water mixture layer as follows,

$$\frac{dc}{dt} + c u_{i,i} = 0 \tag{12}$$

where, c is the volumetric concentration. Eq. (12) can be re-rewritten as

$$u_{i,i} = -\frac{1}{c}\frac{dc}{dt} = -\frac{d}{dt}(\log c). \tag{13}$$

Substituting of Eq. (13) into Eq. (10) yields

$$\Phi_i = \frac{dE_i}{dt} = -p_s\frac{d}{dt}(\log c). \tag{14}$$

Using the condition that $c = c* \rightarrow E_i = 0$ in Fig. 2, integration of Eq. (14) along the plane, p_s=constant, yields

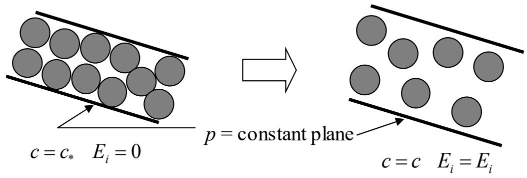

Fig. 2. Schematics of flow field

$$E_i = -p_s \log\left(\frac{c}{c*}\right). \tag{15}$$

E_i depends on concentration profile. Sediment concentration profile depends on bed gradient. In following discussion, we assume that the sediment moving layer does not reach to the flow surface. It means that the gradient is relatively gentle, it must be less than 10 to 12 degrees. We also assume that the sediment concentration in sediment moving layer is as shown in Fig. 3, That is, the concentration at the bed is c_*, and 0 at the surface of sediment moving layer, and the profile is linear. It yields the average sediment concentration over the sediment moving layer, c_s, should be equal to $c_*/2$. And, it must be noted that $E_i = 0$ at the bed.

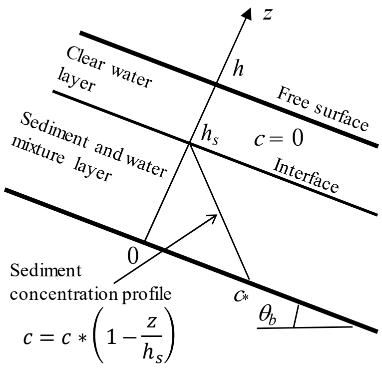

Fig. 3. Schematics figure of debris flow with two-layer, sediment moving layer and clear water layer

Pressure profile with a linear profile of sediment concentration in the sediment moving layer is expressed as

$$p_s = c_s s\rho g h_s \left(1 - \frac{z}{h_s}\right)^2 \cos\theta_b, \quad s = \frac{\sigma}{\rho} - 1. \tag{16}$$

where, σ is the mass density of sediment, that is solid phase, ρ is the mass density of clear water, that is liquid phase, z is the vertical axis normal to the bed, h_s is the thickness of sediment and water mixture layer and θ_b is the bed slope.

Substituting of Eq. (16) to Eq. (15) and integrating it over the depth, E_i of the flow, E_{iT}, is obtained as follows.

$$E_{iT} = \frac{1}{9} c_s\, s\rho g {h_s}^2 \cos\theta_b \tag{17}$$

2.2. Equations for erosion rate

From Eq. (17), erosion rate can be derived. The time differential of the E_{iT}, Eq. (17), is

$$\frac{dE_{iT}}{dt} = \frac{2}{9} c_s s\rho g h_s \cos\theta_b \frac{dh_s}{dt} = \frac{2}{9} c_s s\rho g h_s \cos\theta_b \left(\frac{\partial h_s}{\partial t} + u_{si} h_{s,i}\right). \tag{18}$$

Herein, the bed elevation changing, that is erosion/deposition rate, is expressed by using the continuity relation for solid phase, as follows.

$$\frac{\partial h_s}{\partial t} + (h_s u_{si})_{,i} = -\frac{c_*}{c_s}\frac{\partial z_b}{\partial t} \tag{19}$$

where, z_b is the bed elevation and u_{si} is the velocity of sediment that is solid phase.

Substituting of Eq. (19) into Eq. (18) under the assumption of slow and gentle changing of the flow, $u_{si}h_{s,i} \cong 0$, the relationship between the changing of E_{iT} and the bed elevation changing is expressed as

$$\frac{dE_{iT}}{dt} = \Phi_i \cong -\frac{2}{9}c_s s\rho g h_s \cos\theta_b \frac{c_*}{c_s}\frac{\partial z_b}{\partial t} = -\frac{4}{9}c_s s\rho g h_s \cos\theta_b \frac{\partial z_b}{\partial t} \tag{20}$$

The work rate, that is shown in Eq. (6), in the energy conservation law in compressible flow can be expressed by using bed gradient as

$$\frac{dW}{dt} \cong \int_0^{h_s}(1 + c_s s)\rho g \sin\theta_b\, u_s\, dz. \tag{21}$$

From Eq. (6), therefore, Φ_T can be evaluated as

$$\Phi_T \cong (1 + c_s s)\rho g h_s \sin\theta_b\, \bar{u}_s. \tag{22}$$

where, $\bar{u}_s$ is the representative velocity in the sediment moving layer. Equation (11) corresponds to real energy dissipation, Φ, and that in in Eq. (5) can be expressed by using energy slope, $\sin\theta_e$,

$$\Phi \cong (1 + c_s s)\rho g h_s \sin\theta_e\, \bar{u}_s. \tag{23}$$

From Eq. (11), Φ_i is derived as follows.

$$\Phi_i = \Phi_T - \Phi = (1 + c_s s)\rho g h_s \bar{u}_s(\sin\theta_b - \sin\theta_e) \tag{24}$$

Then, substituting of Eq. (24) into Eq. (20) yields the following deposition rate equation.

$$\frac{\partial z_b}{\partial t} = -\frac{9}{4}\frac{(1+c_s s)}{c_s s}\bar{u}_s \frac{(\sin\theta_b - \sin\theta_e)}{\cos\theta_b}. \tag{25}$$

Introducing an approximation, $\theta \cong \theta_b \cong \theta_e$, yields

$$\frac{\partial z_b}{\partial t} = -\frac{9}{4}\frac{(1+c_s s)}{c_s s}\bar{u}_s(\tan\theta_b - \tan\theta_e). \tag{26}$$

3. Flume Tests and Those Results

To evaluate the characteristics of erosion rate equation, a flume experiment was conducted. The flume dimensions are around 8 m in length, 10 cm in width and 40cm in depth. Bed slope of the flume is 12 degrees. Sediment is set on the bed with longitudinally constant thickness, 10 cm, and is saturated. Only clear water is supplied steadily at 1.0 *l*/s from upstream end on the saturated sediment. A permeable weir is set at the downstream reach. The value of equilibrium flux sediment concentration correspond to bed gradient 12 degree is 0.220 (e.g., Egashira et al., 1997). The physical property of the sediment is as follows: d_{60} is 1.47 mm, d_{max}=4.75 mm, σ/ρ= 2.63, ϕ_s= 36.9 degrees and c_*=0.547, where d_{60} is the 60% diameter, d_{max} is the maximum diameter of sediment and ϕ_s is the interparticle friction angle.

Figure 4 shows temporal changes of flux sediment concentration at downstream end. Herein, time "0" is the time that clear water was sullied at the upstream end. In the figure, equilibrium flux sediment concentration (0.220) is also shown. This figure shows that equilibrium state is established for around 80 sec. since the flow is reaching to downstream end. Fig. 5 shows temporal changes of longitudinal bed shapes and the time evolution of upper end of movable bed. In Fig. 5 (a), coordinate x is set along to the flume bed. From Fig. 5 (b), the velocity of erosion of upper end along to the flume bed, u_b, is obtained and is almost constant, $u_b = 0.052$ m/s. If the velocity of erosion along to the flume bed is the same over the bed, bed profile is not change. If the profile is maintained, transforming coordinate from x to X, $X = x - u_b \Delta t$, $\Delta t = t_0 - t$, all of bed profiles must be plotted in one line. Fig. 6 is the result of the transformation. The reference time, t_0, in Fig. 6 is 47 s, and it is found that all of bed profiles fall on the same line.

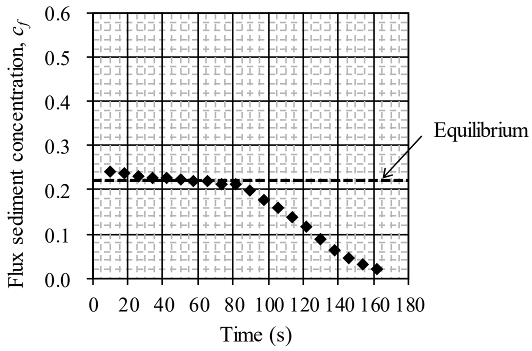

Fig. 4. Temporal changes of flux sediment concentration measured at downstream end

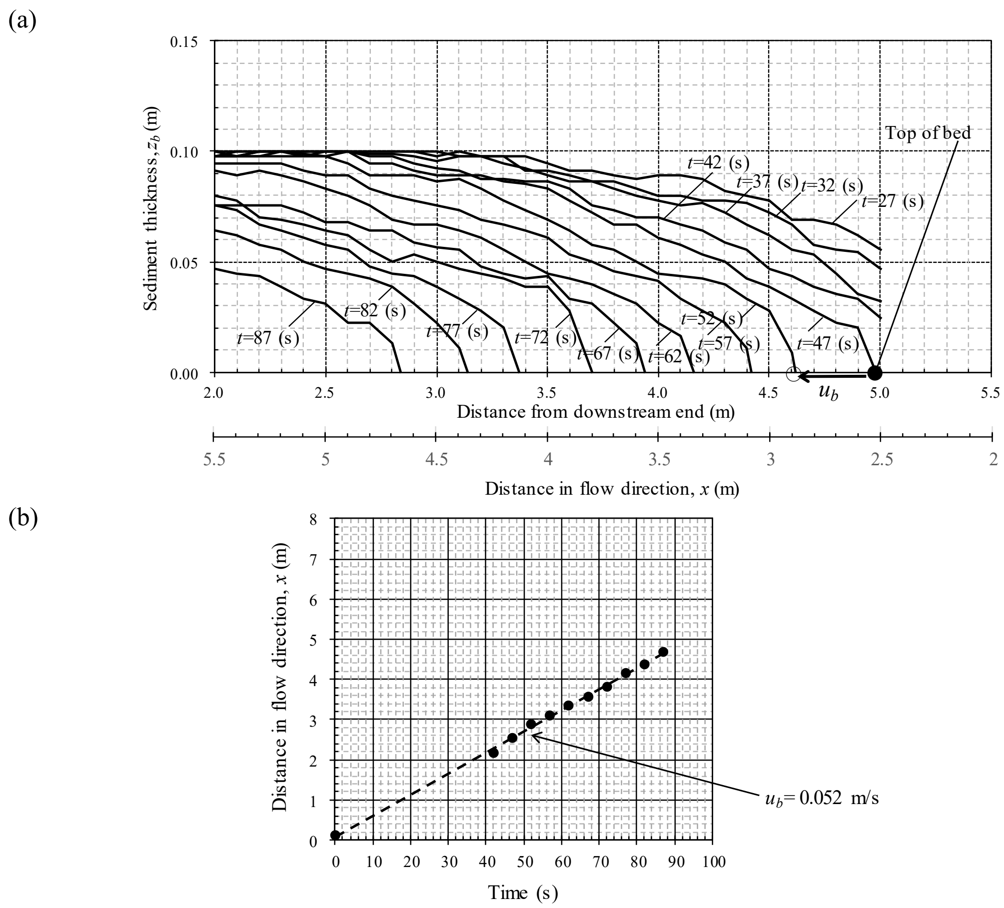

Fig.5. (a) Temporal changes of longitudinal bed shapes and (b) Time evolution of upper end point of movable bed

Present tests were carried out for bed erosion over the constant sediment thickness on the rigid bed. In case of erosion of sediment on the rigid bed as shown in Fig. 7, the erosion rate is schematically expressed as

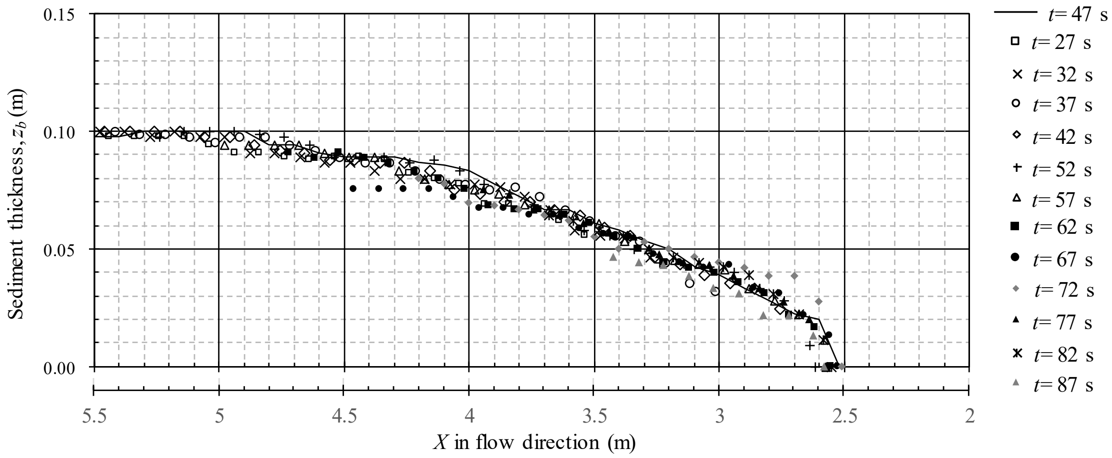

Fig. 6. Longitudinal bed shapes obtained by transformed distance in flow direction

$$\frac{\partial z_b}{\partial t} = -u_b \tan(\theta_\infty - \theta_b). \tag{27}$$

where, u_b is the erosion rate along the direction of rigid bed slope with θ_∞.

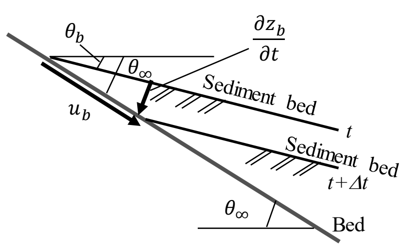

Fig. 7. Schematics of bed erosion

Relation between θ_e and θ_b as shown in Fig. 7 is discussed using flume data for u_b, $\bar{u}_s$,and proposed erosion rate, Eq. (26). Substituting of Eq. (27) into Eq. (26) yields

$$\frac{u_b}{\bar{u}_s} = \frac{9}{4}\frac{(1+c_s s)}{c_s s}\frac{\tan\theta_b - \tan\theta_e}{\tan\left(\theta_\infty - \theta_b\right)}. \tag{28}$$

Introducing following approximations, $\tan\left(\theta_\infty - \theta_b\right) \cong \tan\theta_\infty - \tan\theta_b$, $\tan\theta_b - \tan\theta_e \cong \tan(\theta_b - \theta_e)$, Eq. (28) is also expressed

$$\frac{u_b}{\bar{u}_s} = \frac{9}{4}\frac{(1+c_s s)}{c_s s}\frac{\tan\theta_b - \tan\theta_e}{\tan\theta_\infty - \tan\theta_b} \quad \text{or} \quad \frac{u_b}{\bar{u}_s} = \frac{9}{4}\frac{(1+c_s s)}{c_s s}\frac{\tan(\theta_b - \theta_e)}{\tan\left(\theta_\infty - \theta_b\right)} \tag{29}$$

If the bed erosion can be progressed with maintaining of the bed shape, the bed erosion rate along the direction of θ_∞, $u_b(x)$, can take constant value. Supposing that $\bar{u}_s$ takes constant value over the bed, the following takes constant values.

$$\frac{\tan\theta_b - \tan\theta_e}{\tan(\theta_\infty - \theta_b)} \cong \frac{\tan\theta_b - \tan\theta_e}{\tan\theta_\infty - \tan\theta_b} \cong \frac{\tan(\theta_b - \theta_e)}{\tan(\theta_\infty - \theta_b)} \tag{30}$$

When the constant value of $u_b/\bar{u}_s$ is obtained by flume tests, Eq. (30) can be determined experimentally. To evaluate the relation in Eq. (30), u_s must be known in addition to u_b. The flow surface velocity, u_w, is measure instead of u_s. And u_b/u_w takes almost constant value 1/27 to 1/28 (≅1/30). Free surface velocity u_w is larger than u_s, then u_b/u_w is smaller than u_b/u_s. Supposing $u_s \cong u_w/2$, $u_b/u_s \cong 1/15$ is satisfied, and then the value of Eq. (30) takes constant value about 0.01. This means that the difference of bed gradient and energy gradient is extremely smaller than the difference of two angles θ_∞ and θ_b, and is less than $0.01 \tan\theta_\infty$. Consequently, θ_e is almost same as θ_b, and the state is close to equilibrium. It means that small difference of energy gradient with bed gradient, such as disturbance, leads to relatively erosion/sedimentation. Therefore, we may find easily anti-dune formation under equilibrium state.

4. Conclusions

The constitutive equation of the erosion rate is newly derived based on the energy conservation law considering the compressibility of solid phase, and the erosion rate is evaluated by flume tests.

(1) Erosion rate, Eq. (26), is derived based on energy conservation law focused on compressibility of flow field. We investigate the effect of the inelasticity to the flow and define some “internal energy” to control the compressibility. The time differential of the depth integrated internal energy, and difference between the time differential and work rate yield erosion rate by using continuity of solid phase in the sediment moving layer. The ratio of erosion rate to the flow velocity of sediment moving layer is proportional to difference between bed slope and equilibrium bed slope.

(2) Flume tests are carried out using a prismatic steep slope channel of 12 degrees in slope. Erosion rate is evaluated by erosion of bed sediment around upstream reach of parallelly deposited sediment to the flume bed in steady flow. A speed of the upper endpoint of movable bed, u_b, due to erosion process is almost constant, and longitudinal bed shape during erosion process keeps the shape. Then, by using moving coordinate with u_b in flow direction, the bed profile has similarity in geometry.

(3) In comparison of experimental data with the erosion rate equation, energy gradient is not so much different with bed gradient. It means that small disturbance in energy gradient may cause something like anti-dune formation.

As future issues, we need to investigate the longitudinal bed profile, sedimentation process as well as more cases of erosion.

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