Coupled Porous-Medium and Free-Flow across Wavy Surface

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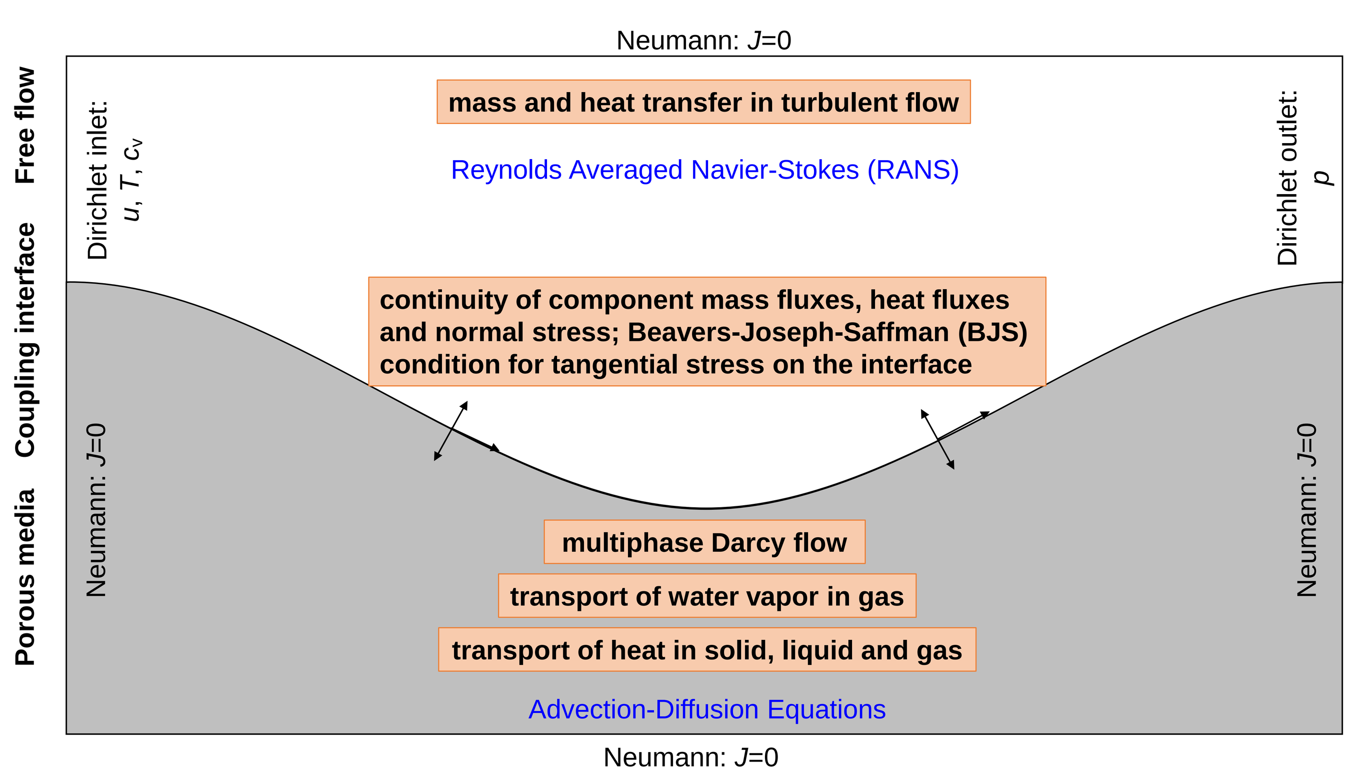
1. Introduction

- Evaporation from soil is a complex process which couples the mass and heat transfer between the porous media and the atmosphere.
- Main mechanisms controlling evaporation: surface roughness, turbulence, boundary layers, heterogeneity…
- Tilled soil surface is more common in nature. The free fluid flow near this geometry is easy to transform to turbulent behavior.

2. Objective

- Develop mathematical model for subsurface-turbulent atmosphere interaction with simplified wavy surface based on the coupling of Navier-Stokes free flow and Darcy flow in porous media under non-equilibrium phase change conditions;
- Study the effect of flow pattern and the parameters of both porous media and free-flow region.

3. Model Concept



Free-flow region:

Two phases: liquid, gas
Two components: dry air, water vapor

Porous media:

Single phase: gas (incompressible flow)
Two components: dry air, water vapor

Scale of the 2D vertical domain:

Width: 0.6m
Height: 0.3m
Wave length: λ=0.6m
Amplitude: γ=0.05m

4. Mathematical Model

Free flow equations

- Mass balance

$$\nabla \cdot \boldsymbol{u}_g^{ff} = 0$$

- Momentum balance – RANS

$$\rho_g \frac{\partial \boldsymbol{u}_g^{ff}}{\partial t} + \rho_g \left(\boldsymbol{u}_g^{ff} \cdot \nabla\right)\boldsymbol{u}_g^{ff} = \nabla \cdot \left[-p_g^{ff} + \left(\mu_g + \mu_{g,T}\right)\left(\nabla \boldsymbol{u}_g^{ff} + \left(\nabla \boldsymbol{u}_g^{ff}\right)^T\right) - \frac{2}{3}\rho_g k \boldsymbol{I}\right] + \rho_g \boldsymbol{g}$$

$$\rho_g \frac{\partial k}{\partial t} + \rho_g \left(\boldsymbol{u}_g^{ff} \cdot \nabla\right) k = \nabla \cdot \left[\left(\mu_g + \frac{\mu_{g,T}}{\sigma_k}\right)\nabla k\right] + P_k - \rho_g \varepsilon$$

$$\rho_g \frac{\partial \varepsilon}{\partial t} + \rho_g \left(\boldsymbol{u}_g^{ff} \cdot \nabla\right) \varepsilon = \nabla \cdot \left[\left(\mu_g + \frac{\mu_{g,T}}{\sigma_\varepsilon}\right)\nabla \varepsilon\right] + C_{\varepsilon 1}\frac{\varepsilon}{k}P_k - C_{\varepsilon 2}\rho_g \frac{\varepsilon^2}{k}$$

$$\mu_{g,T} = \rho_g C_\mu \frac{k^2}{\varepsilon} \qquad P_k = \mu_{g,T}\left[\nabla \boldsymbol{u}_g^{ff} : \left(\nabla \boldsymbol{u}_g^{ff} + \left(\nabla \boldsymbol{u}_g^{ff}\right)^T\right)\right]$$

- Component mass balance

$$\frac{\partial\left(\rho_g w_v\right)}{\partial t} + \nabla \cdot \left(\rho_g w_v \boldsymbol{u}_g^{ff}\right) - \nabla \cdot \left[D_v^{ff} \nabla\left(\rho_g w_v\right)\right] = 0$$

- Energy balance

$$\left(\rho_g c_{p,g}\right)\frac{\partial T}{\partial t} + \left(\rho_g c_{p,g}\right)\nabla \cdot \left(\boldsymbol{u}_g^{ff} T\right) - \nabla \cdot \left(\lambda_g \nabla T\right) = 0$$

Porous media equations

- Mass balance

$$\phi \frac{dS_w}{dP_c}\frac{\partial\left(\rho_w P_c\right)}{\partial t} + \nabla \cdot \left(\rho_w \boldsymbol{u}_w^{pm}\right) = -f_{vw}$$

$$\phi \frac{dS_g}{dP_c}\frac{\partial\left(\rho_g P_c\right)}{\partial t} + \nabla \cdot \left(\rho_g \boldsymbol{u}_g^{pm}\right) = f_{vw}$$

$$\boldsymbol{u}_w^{pm} = -\frac{K_{int} k_{rw}}{\mu_w}\left(\nabla p_w^{pm} + \rho_w \boldsymbol{g}\right)$$

$$\boldsymbol{u}_g^{pm} = -\frac{K_{int} k_{rg}}{\mu_g}\left(\nabla p_g^{pm} + \rho_g \boldsymbol{g}\right)$$

- Component mass balance

$$\phi \frac{\partial\left(\rho_g w_v S_g\right)}{\partial t} + \nabla \cdot \left(\rho_g w_v \boldsymbol{u}_g^{pm}\right) - \nabla \cdot \left[D_v^{pm} \nabla\left(\rho_g w_v\right)\right] = f_{vw}$$

- Energy balance

$$\left(\rho c_p\right)_{eff}\frac{\partial T}{\partial t} + \nabla \cdot \left[\left(\rho_g c_{p,g}\right)\boldsymbol{u}_g^{pm} T + \left(\rho_w c_{p,w}\right)\boldsymbol{u}_w^{pm} T\right] - \nabla \cdot \left(\lambda_{eff} \nabla T\right) = -L f_{vw} - Q_s$$

$$f_{vw} = \frac{b\left(\theta_w - \theta_r\right)RT}{M_w}\left(\rho_g c_v - \rho_{vs} H_r\right)$$

Coupling conditions on the interface

- Continuity of mass flux

$$\left(\rho_g \boldsymbol{u}_g^{ff}\right) \cdot \boldsymbol{n}^{ff} = -\left(\rho_g \boldsymbol{u}_g^{ff}\right) \cdot \boldsymbol{n}^{pm}$$

- Continuity of normal stress

$$\left(p_g^{ff} \boldsymbol{I} - \mu_g \nabla \boldsymbol{u}_g^{ff}\right) \cdot \boldsymbol{n}^{ff} = p_g^{pm}$$

- Jump of tangential stress - BJS

$$\left(\boldsymbol{u}_g^{ff} + \frac{\sqrt{K_{int}}}{\alpha_{BJ}\mu_g}\left(\boldsymbol{\tau} + \boldsymbol{\tau}_T\right)\boldsymbol{n}^{ff}\right) \cdot \boldsymbol{t}_i^{ff} = 0, \quad i \in \{1, 2, \ldots, d-1\}$$

- Continuity of energy (temperature and heat flux)

$$T^{ff} = T^{pm}$$

$$\left\{\left[\left(\rho_g c_{p,g}\right)\boldsymbol{u}_g^{ff} T - \lambda_g \nabla T\right] \cdot \boldsymbol{n}\right\}^{ff} = -\left\{\left[\left(\rho_g c_{p,g}\right)\boldsymbol{u}_g^{pm} T + \left(\rho_w c_{p,w}\right)\boldsymbol{u}_w^{pm} T - \lambda_{eff} \nabla T\right] \cdot \boldsymbol{n}\right\}^{pm}$$

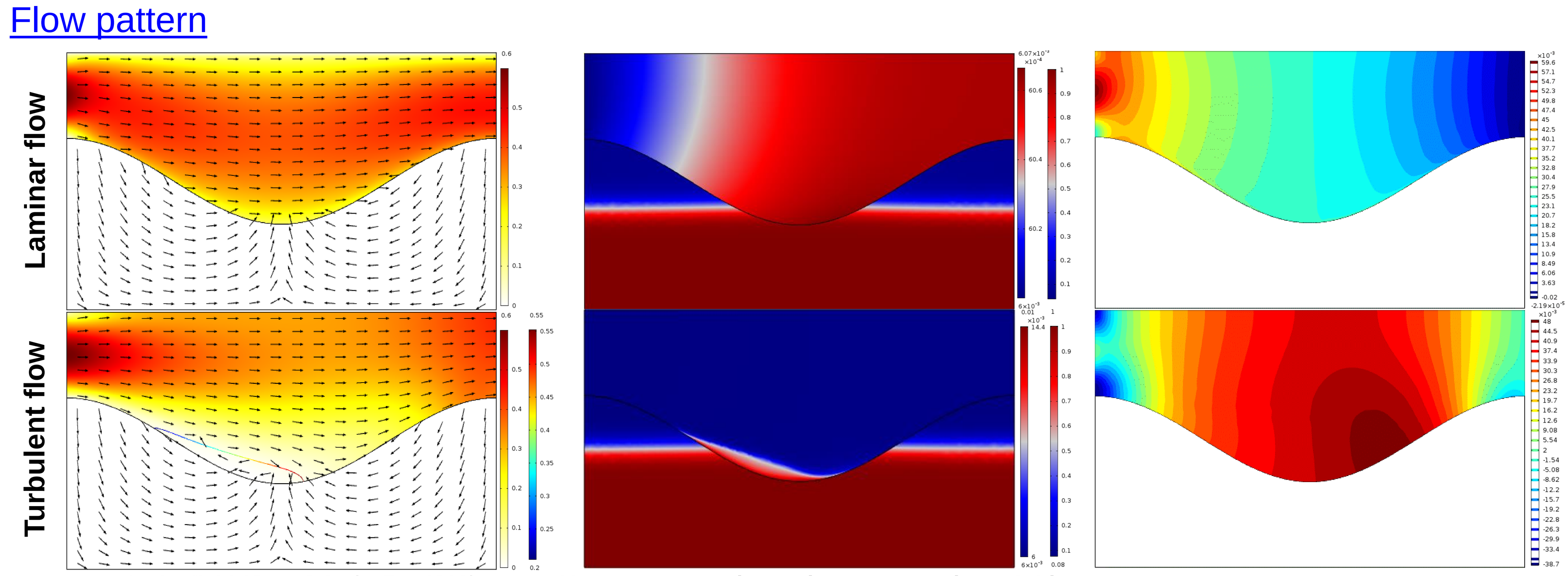
- Continuity of transport (vapor concentration and flux)

$$\left[w_v\right]^{ff} = \left[w_v\right]^{pm}$$

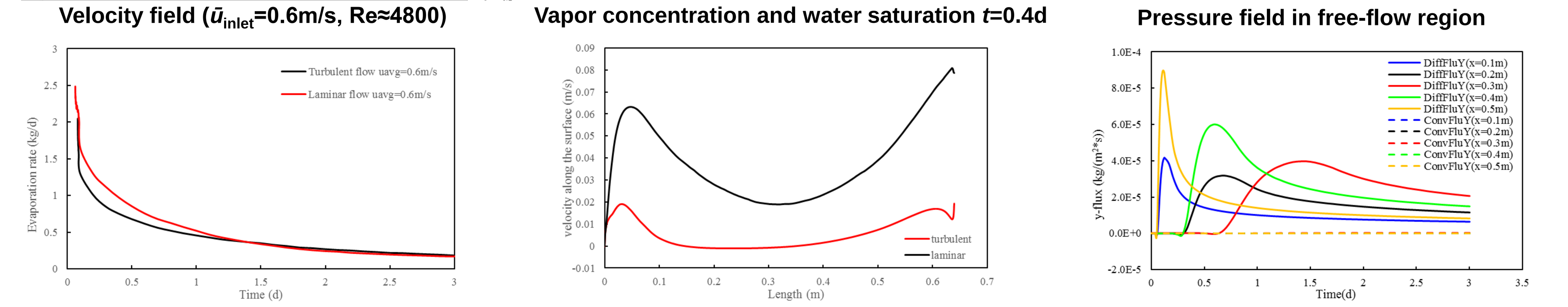
$$\left\{\left[\rho_g w_v \boldsymbol{u}_g^{ff} - D_v^{ff} \nabla\left(\rho_g w_v\right)\right] \cdot \boldsymbol{n}\right\}^{ff} = -\left\{\left[\rho_g w_v \boldsymbol{u}_g^{pm} - D_v^{pm} \nabla\left(\rho_g w_v\right) \cdot \boldsymbol{n}\right]\right\}^{pm}$$

5. Results and Analysis

Flow pattern



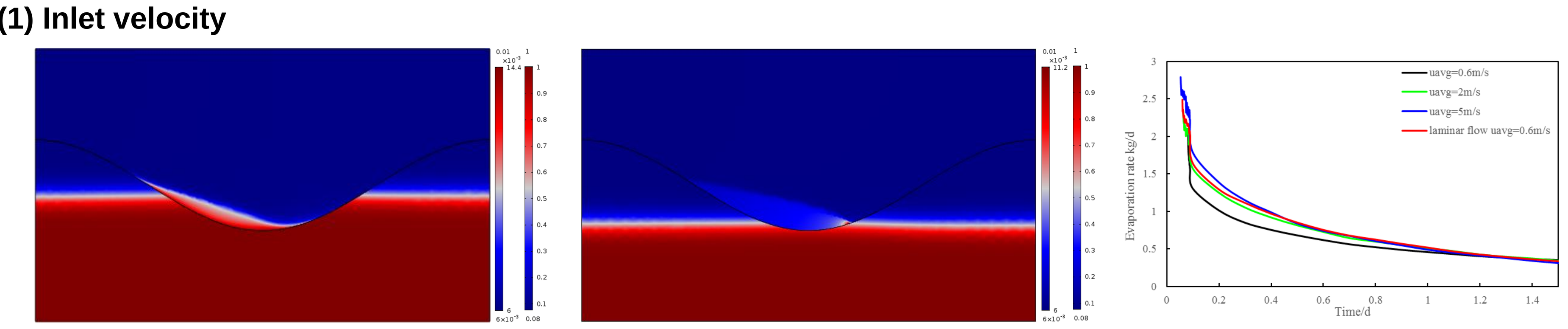
Velocity field ($\bar{u}_{inlet}$=0.6m/s, Re≈4800)   Vapor concentration and water saturation t=0.4d   Pressure field in free-flow region



- The comparison of the velocity field shows that there is an obvious recirculation area enveloped by partial interface and recirculation line, which is non-dimensionalized by x/λ, in turbulent case. The gradient of velocity, concentration and temperature is smaller in this area.
- Vapor concentration becomes larger from the inlet to the outlet in the laminar case and some vapor resides at the valley finally; while in the turbulent flow, vapor concentration always keeps a higher value at the recirculation area.
- The pressure decreases from left to right in laminar flow; But in turbulent case, pressure is larger at the upstream part and decreases to the two sides, which impedes the vapor motion and results in high vapor concentration in the recirculation zone.
- The free-flow velocity along the interface is larger in laminar case, which leads to a thinner boundary layer, thus the concentration gradient within the boundary layer is larger and the evaporation rate is larger at the beginning. Before the time when the evaporation rate of laminar flow equals that of turbulent flow, the interface of two regions has contact with the drying front, which guarantees enough water sources. After the drying front detaches the surface in laminar flow, the water still connects with the surface in turbulent flow. So the laminar evaporation rate becomes smaller than the turbulent later. But the difference is slight due to the surface velocity.
- Diffusive flux is always larger than the convective flux at different positions. Diffusion is the dominant mechanism of vapor transport. The diffusive flux of upstream surface is larger than that of downstream part due to the exist of recirculation area.

Influence of parameters of free-flow

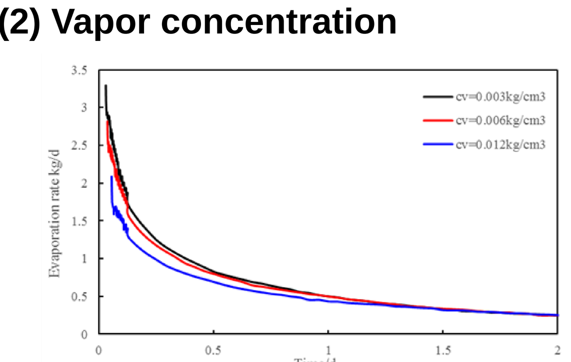
(1) Inlet velocity



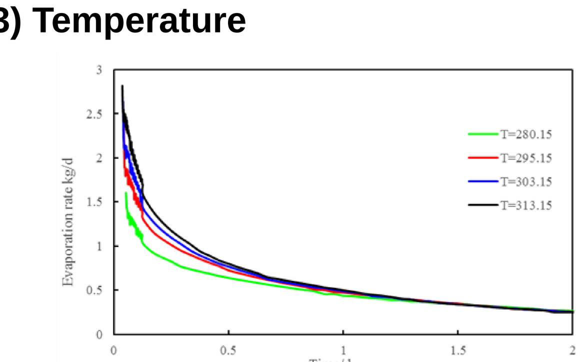
Vapor concentration and water saturation (u_{avg}=0.6m/s, t=0.4d)   Vapor concentration and water saturation (u_{avg}=5m/s, t=0.4d)

- Increasing airflow velocity leads to thinner boundary layer, thus larger concentration gradient within the boundary layer and higher evaporation rate.
- As the increase of inflow velocity, convection plays a more important role, which enhances the interaction of the atmosphere and the soil. The vapor concentration difference between the recirculation zone and the other area becomes narrow due to the turbulence.

(2) Vapor concentration



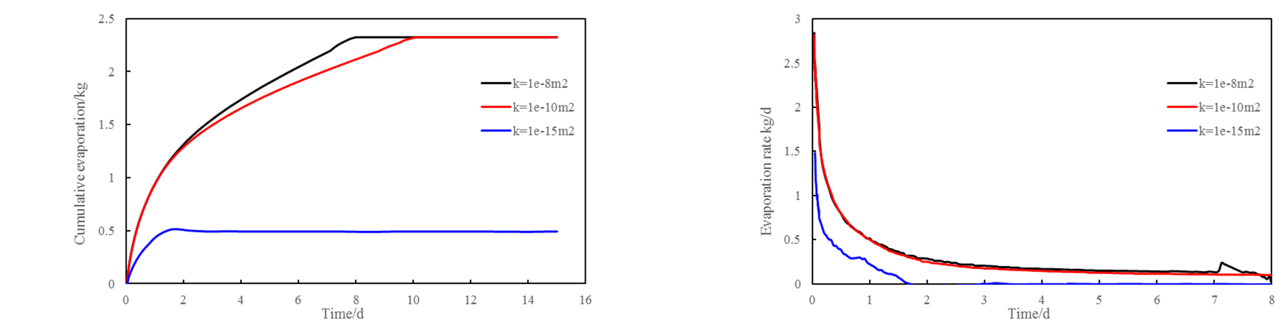
(3) Temperature



- Lower inflow vapor concentration leads to higher concentration gradient within the boundary layer, thus evaporation rate will be higher.
- Higher temperature corresponds to higher evaporation rate. Reasonably, when the temperature of air is higher than that of the porous media, the evaporation rate will increase; while if the temperature of air is lower, the evaporation rate will decrease.

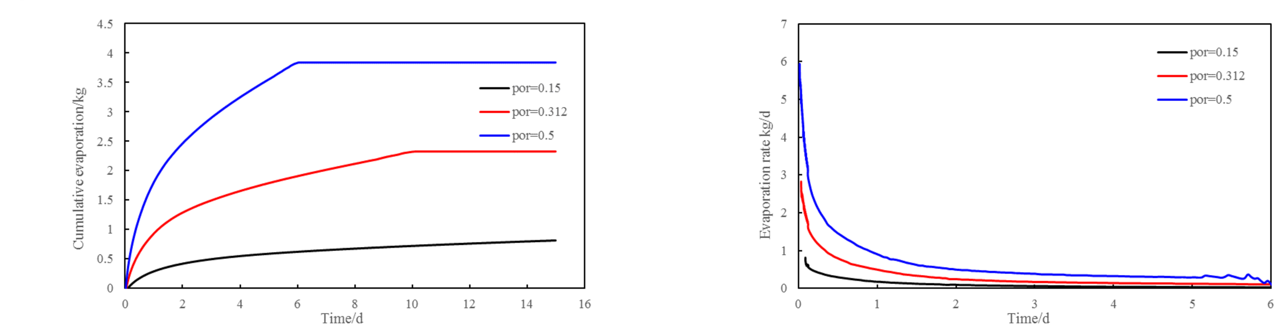
Influence of parameters of porous media

(1) Permeability



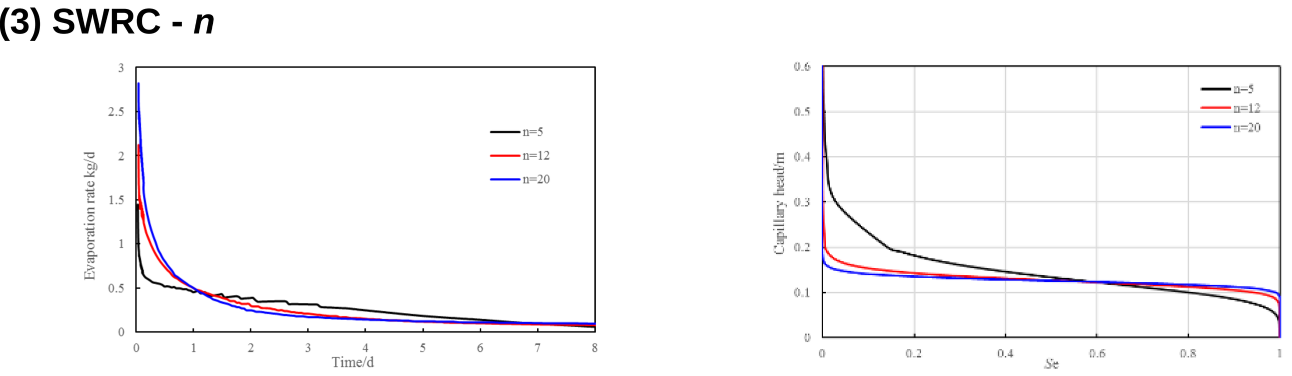
- Permeability denotes the ability of the porous media to allow fluid flow through. When permeability is very low, the porous media cannot provide enough water to the soil surface to evaporate, the drying front will move slowly.

(2) Porosity



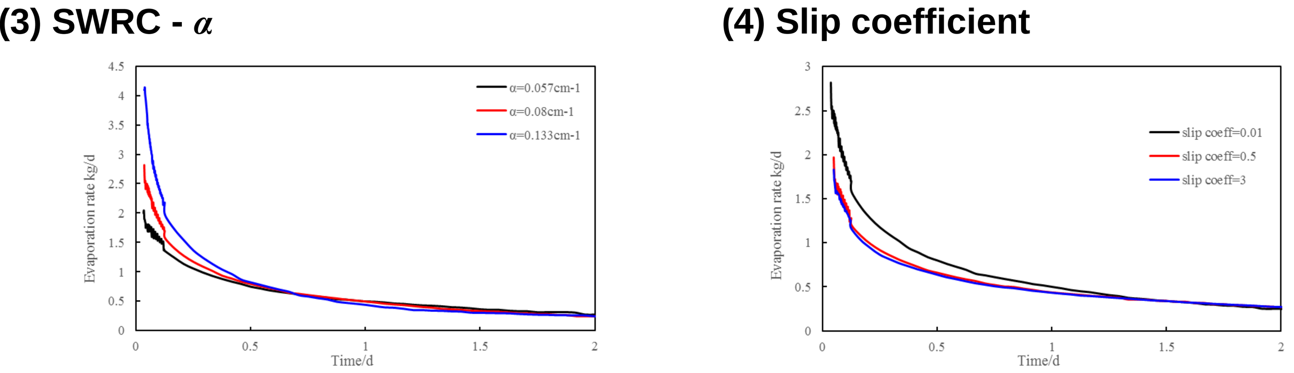
- Porous media with higher porosity provide more spaces to storage and transmit water, thus more evaporation and evaporation rate.

(3) SWRC - *n*



- *n* decides both the slope of SWRC and air-entry value. At the beginning, the porous media is high saturated, larger *n* indicates stronger power forces air to enter the porous media and push water to the soil surface; when the porous media is dryer, it shows the opposite.

(3) SWRC - α   **(4) Slip coefficient**



- α decides the air-entry pressure: larger α indicates it is much easier for air come into the porous media and push water to the soil surface.
- A larger slip velocity on the interface indicates a narrow difference of velocity near the interface, thus the concentration gradient within the boundary layer is smaller and evaporation rate will be lower.

6. Conclusion

- Mathematical models (coupled single-phase two-components RANS and two-phase two-components Darcy model) are developed for turbulent airflow across permeable wavy soil surface. The influences of flow pattern, parameters of free-flow and parameters of porous media on the evaporation are considered in this study.
- The structure of the soil surface has a great influence on the flow and interaction behavior between the atmosphere and the subsurface. It is easy to form turbulent flow at the downstream zone and the valley. So airflow turbulence has to be taken into consideration when the surface is not flat even if the free flow velocity is not very high.
- There is a recirculation area formed at the downstream part. The vapor accumulates in this area partially due to the higher pressure at the upstream side which impedes the vapor motion. Compared with the upstream zone, the boundary layer in this area is thicker. Accordingly, the velocity, vapor concentration and temperature gradients are also smaller. Thus the diffusive flux is lower in this area.
- Diffusion is the dominant mechanism of vapor transport. When the airflow velocity increases, the influence of convection will increase accordingly. Higher velocity leads to thinner boundary layer, thus the gradients of velocity, vapor concentration and temperature will become larger. It will also enhance the interaction between the atmosphere and the porous media in the meantime.
- When the drying front detaches the soil surface, the evaporation rate will decline since there is no direct water supply.
- The parameters of porous media, including permeability, porosity and SWRC, influence the evaporation though the amount of water storage or water supply to the soil surface.
- The parameters of free-flow, including inlet velocity, vapor concentration, influence the evaporation though the boundary layer and the gradients formed in the boundary layer.
- Next study will focus on the effects of diffusion/convection and boundary layer.

7. Reference

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