SPECTRUM AUCTIONS UNDER PHYSICAL INTERFERENCE MODEL

by

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ABSTRACT

Spectrum auctions provide a platform for licensed spectrum users to share their under-utilized spectrum with unlicensed users. Existing spectrum auctions either use the protocol interference model to characterize interference relationship as binary, or do not allow the primary and secondary users to share channels simultaneously. To fill this void, we design SPA, a spectrum single-sided auction under the physical interference model, which considers the interference to be accumulative. We prove that SPA is computationally efficient, individual-rational, and truthful. Results from extensive simulation studies demonstrate that, SPA achieves higher revenue, spectrum utilization and buyer satisfaction ratio, compared with the existing auctions modified with the physical interference model.

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CHAPTER 1

INTRODUCTION

In this chapter, we will introduce the challenges of spectrum auctions and explain our motivation for choosing the physical interference model. We will then discuss the current research results and the remaining challenges. Finally, we will state our objectives and summarize our contributions.

1.1 Spectrum Auctions

Broadcasting spectrum is a critical yet scarce resource due to the substantial growth of wireless technology and applications. Indeed, Federal Communications Commission (FCC) and its counterparts across the world have released licenses of unused spectrum and collected billions of dollars in the past decade. However, a traditional FCC spectrum auction targets long-term leases. It often takes months or years to conclude, involves only a few large corporate players, and entails significant manual negotiations [1]. Moreover, spectrum is usually sold at a very high unit price recently. Due to this, small network providers may not be able to afford spectrum individually.

Since dynamic spectrum access can significantly improve spectrum utilization, it is important to motivate the primary license holders to open up their underutilized spectrum for sharing, so that the primary license holders may make profit by leasing access to spectrum resources in short-terms.

The traditional auction models players as individuals. However, due to the spatial reusability of spectrum, secondary users can be grouped into conglomerate unions such that they can share the same channel simultaneously without failing each other's transmission. The payment shall also be shared among members in one group. This will produce the most robust and effective allocation of channels.

One critical challenge is to determine the winning buyers' payments while maintaining truthfulness. A truthful auction guarantees that if a bidder bids the true valuation of the resource, its utility will not be less than that when it lies. Therefore, sharing the same channel does not indicate that each winning buyer is charged the same price. In order to prevent a dishonest bidder from improving its utility by bidding lower than its true value, it is necessary to design a mechanism meticulously in spectrum auctions. Due to spatial reusability, to determine the winners' payments is more challenging than that in traditional auction.

The followings are some desired properties that a good auction should posses:

- *Truthfulness*: An auction is truthful if, a buyer bids the true valuation of the resource, its utility will not be less than that when it lies.

- *Individual Rationality*: An auction is individually rational if all buyers have non-negative utilities by revealing their true valuations and costs.

- *Computational Efficiency*: An auction is computationally efficient if it can be conducted within polynomial time.

Another challenge is to form groups, such that users in the same group can share the same channel simultaneously without failing each other's transmission. Most of the existing spectrum auction mechanisms have simplified the step of group formation by scheduling users according to conflict graphs. In a conflict graph, whether two users can transmit at the same time depends on a fixed range of distance. The functionality of the mechanisms is the assumption that the interference relationship between any two users can be modeled based on protocol. In other words, the interference relationship is binary. But in practice for wireless networks, a conflict graph may not be precise, as the interference from other users is accumulative. Within a certain threshold of power, users may transmit simultaneously even if their locations are very close.

To solve this problem, we intend to design spectrum auctions without using the given conflict graph, but under the *physical interference model* instead. In the following section, both protocol and physical interference models are described and compared.

1.2 Interference Models

There are two widely used models to characterize interference relationship in a wireless network, namely, the physical interference model and the protocol interference model.

1.2.1 Protocol Interference Model

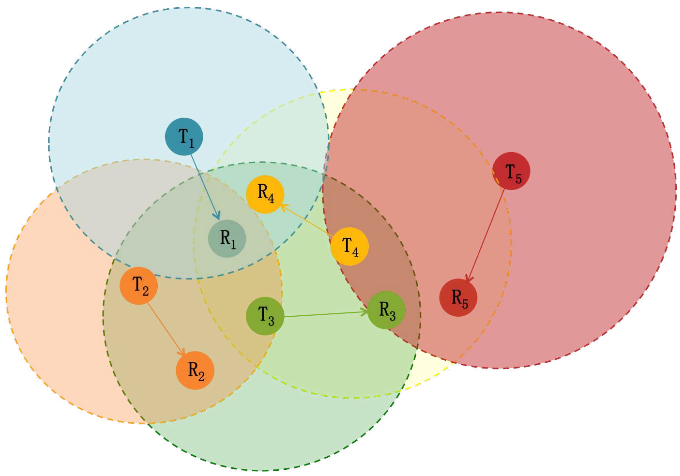
The first one is the *protocol interference model*. When two users transmit using the same channel simultaneously, they interfere with each other if the distance between them is less than a given threshold. In other words, a user can successfully receive a message from another user if and only if no node within a certain range from it is transmitting at the same channel simultaneously. Usually, a conflict graph is used to depict such interference, where each node represents a user, and an edge exists if two nodes are close enough to fail the transmissions.

For example, Figure 1.1 shows a wireless network under the protocol interference model and the corresponding conflict graph, from which the result of group formation can be $\{1, 5\}, \{2, 4\}, \{3\}$. Therefore, we need at least 3 channels to make sure all the 5 users can transmit successfully simultaneously.

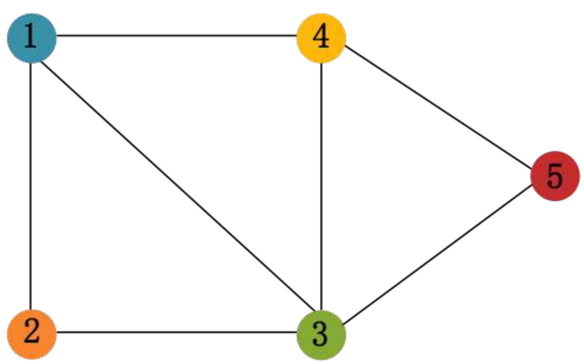
Unfortunately, this simplified interference model abstracts away the accumulative nature of interference. Even if a single transmitter far away from a receiver may not corrupt the transmission, the accumulated interference from several such nodes could still generate enough interference to prevent the receiver from successfully decoding the received message.

1.2.2 Physical Interference Model

The other model is the physical interference model [2] (a.k.a. SINR model). It computes the Signal to Interference and Noise Ratio (SINR) of each user and compares this value



(a) Wireless network



(b) Conflict graph

Figure 1.1: Protocol Interference Model

with a threshold. If the SINR value is no less than the threshold, the signal transmission is considered successful for the corresponding user, and it is considered unsuccessful otherwise.

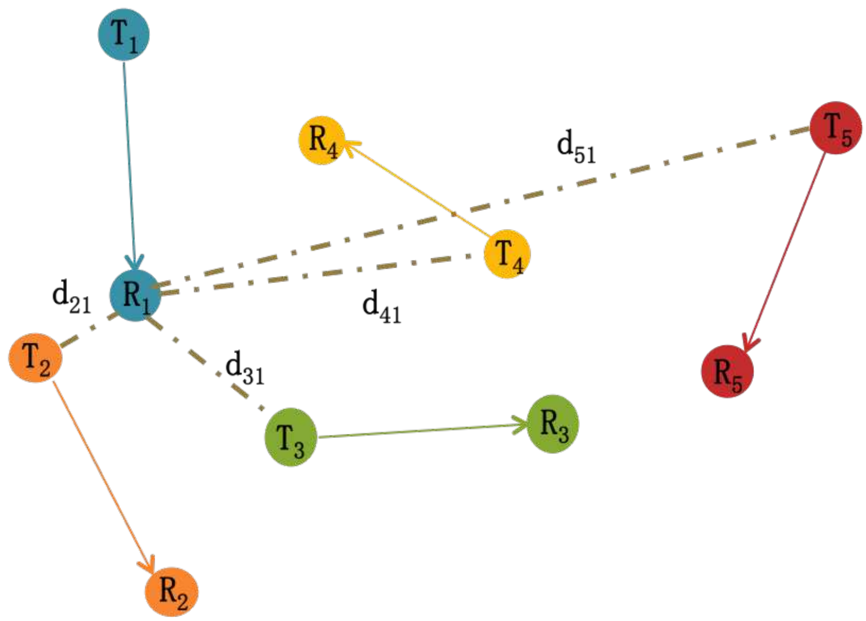


Figure 1.2: Physical Interference Model

Figure 1.2 shows an example of a wireless network. The received power P_{51} at receiver R_1 from transmitter T_5 is $P_{51} = \frac{P_5}{max\{d_{51}^{\alpha},1\}}$, where P_5 is the power level that T_5 transmits, d_{51} is the Euclidean distance from transmitter T_5 to receiver R_1, and α is the path loss exponent, whose value is usually between 2 and 4. The transmission between T_1 and R_1 is successful, if the Signal to Interference and Noise Ratio (SINR) of R_1 is no less than β_1: $SINR_1 = \frac{P_{11}}{P_{21}+P_{31}+P_{41}+P_{51}+N_0}$, where N_0 is the ambient noise power level, and β_1 is the threshold.

$$Signal\ to\ Interference\ and\ Noise\ Ratio = \frac{received\ power\ from\ its\ corresponding\ transmitter}{interference\ from\ other\ users\ +\ noise} \tag{1.1}$$

Compared with the protocol interference model, the SINR model is more precise in depicting the interference but more dynamic and complicated in terms of computation. If a group of secondary users get satisfactory SINR values, they can share the same spectrum channel at the same time without failing each other's transmission. This is called spatial reusability and it can ease the scarcity of the limited resource.

1.3 Contributions

The main contributions of this thesis are:

- In this thesis, we design the first truthful spectrum single-sided auction with spatial reusability under the physical interference model, to the best of our knowledge.
- We propose a two-staged spectrum auction, SPA, which consists of an allocation algorithm and a pricing mechanism.
- SPA allows the scenario where the primary user and secondary users can share channels simultaneously.
- We prove that SPA is truthful, individually rational, and computationally efficient.
- We successfully extend SPA to the multi-demand model, and name it SPA-x.

CHAPTER 2
PRIOR WORK

As pioneers in spectrum auction design, Zhou *et al.* [1] proposed VERITAS under the protocol model, the first truthful auction considering the spectrum reusability and computational efficiency. In [3], based on the concept of virtual valuation, Jia *et al.* designed an exponential time VCG-based auction to maximize the expected revenue. Along this line, Al-Ayyoub and Gupta [4] designed a polynomial time spectrum auction that yields approximated expected revenue. In [5], Wu and Vaidya designed SMALL to guarantee that the owners utility is non-negative in the scenario where the owner of the spectrum has a reserved price for each of the channels. Following the same design methodology, Wei *et al.* [6] designed SHIELD that improves spectrum utilization and buyer satisfaction compared with VERITAS and SMALL. Inspired by the group-buying service on the Internet, Lin *et al.* [7] designed a three-state auction, called TASG that allows a leader in each group to conduct an outer auction for aggregating the bids within the group. Along this line, Yang *et al.* [8] designed TRUBA that significantly increases revenue. In [9], Gopinathan and Li studied spectrum auctions with prior-free setting and designed a truthful auction to approximately maximize the revenue.

TRUST [10] is the first truthful double auction designed for spectrum trading. Feng *et al.* [11] extended to heterogeneous spectrum auctions and designed TAHES. In [12], a double truthful auction, called DOTA, was proposed to allow each user to bid for more than one channel. Considering the fact that secondary users may join the network in an online fashion, Wang *et al.* [13] designed TODA. In [14], Yang *et al.* proposed PROMISE for maximizing the profit without the knowledge of the users valuation distribution.

In the scenario of the physical interference model, Kakhbod *et al.* in [15] developed a truthful auction for dividing a spectrum channel into several small channels with less bandwidth, where all transmitters power levels are fixed homogeneously. In [16], a truthful single auction was studied by Bae *et al.*, where a sequential auction (an auction with multiple rounds) was used to reach a pure strategy equilibrium. Huang *et al.* also introduced a truthful auction-based spectrum sharing mechanism [17] where a group of users compete for a spectrum channel under different definitions of their utilities. In [18] Zhang *et al.* proposed TSA, a framework for truthful double auctions under the physical interference model with power control. To the best of our knowledge, there is no truthful single-sided auction for spectrum sharing under the physical interference model.

CHAPTER 3

SYSTEM MODEL AND PROBLEM STATEMENT

In this chapter, we describe the necessary concepts in cognitive radio networks and the physical interference model for the spectrum auction.

3.1 Cognitive Radio Network Model

We consider a cognitive radio network (CRN) consisting of one primary user (PU) and a set $\mathcal{S} = \{S_1, S_2, \ldots, S_n\}$ of n secondary users (SUs). The PU, e.g. a TV broadcaster, owns m licensed channels $\mathcal{C} = \{c_1, c_2, \ldots, c_m\}$, and is willing to rent the spectrum for profit. The channels are assumed to be orthogonal, which means that there is no interference among users using different channels. Let P_0 denote the transmission power of the PU's transmitter, e.g. the TV tower, denoted by T_0. SUs do not have licensed spectrum channels, but are willing to pay for channels from the PU in short term. Each $S_i \in \mathcal{S}$ is a transmitter-receiver pair (T_i, R_i). Let P_i denote the transmission power of S_i.

We allow the PU and SUs to transmit signals over the same channels simultaneously. Let $\mathcal{C}_0 \subseteq \mathcal{C}$ represent the channels that the PU is currently using. To protect the transmission of the PU from being interrupted by the transmissions of the SUs, the FCC proposed a metric, named Interference Temperature Limit (ITL) [19], which sets the maximum cumulative amount of interference that can be tolerated at certain locations. Let $\mathcal{L} = \{l_1, l_2, \ldots, l_h\}$ denote the locations where the PU measures ITL. We use γ_j to represent PU's tolerated ITL at location l_j. With this setting, the PU can lease its channels to SUs as long as the transmissions of the selected SUs do not cause more interference than γ_j, for any $l_j \in \mathcal{L}$. Assume G_k is the group of SUs assigned to the same channel c_k. The ITL constraints can be represented by

$$\mathbb{1}_{\mathcal{C}_0}(c_k) \sum_{S_i \in G_k} \frac{P_i}{d(T_i, l_j)^\alpha} \leq \gamma_j, \forall l_j \in \mathcal{L}, \tag{3.1}$$

where $\mathbb{1}_{\mathcal{C}_0}(c_k)$ is an indicator function defined as

$$\mathbb{1}_{\mathcal{C}_0}(c_k) = \begin{cases} 1, & c_k \in \mathcal{C}_0, \\ 0, & c_k \notin \mathcal{C}_0, \end{cases} \tag{3.2}$$

$d(T_i, l_j)$ is the maximum of 1 and the Euclidean distance from transmitter T_i to location l_j, and α is the path loss exponent with value between 2 and 4 usually.

We can achieve spatial reuse by assigning multiple SUs to the same channel, if they can transmit simultaneously while each obtains a satisfactory SINR value.

The Signal to Interference and Noise Ratio (SINR) [20] of S_i in G_k is:

$$\frac{\frac{P_i}{d(T_i,R_i)^\alpha}}{\mathbb{1}_{\mathcal{C}_0}(c_k)\frac{P_0}{d(T_0,R_i)^\alpha} + \sum_{S_i \neq S_j \in G_k} \frac{P_j}{d(T_j,R_i)^\alpha} + N_0} \geq \beta_i, \tag{3.3}$$

where β_i is the given threshold, N_0 is the ambient noise power level, and $\mathbb{1}_{\mathcal{C}_0}(c_k)$ is defined in Equation (3.2).

If Condition (3.3) is satisfied, the transmission is considered successful; otherwise, the transmission is considered unsuccessful. We assume that Condition (3.3) is satisfied for each S_i when it solely occupies a channel. We can preprocess this before our proposed mechanism, that if by transmitting at a power of P_i, Condition (3.3) is not satisfied, when S_i solely occupies a channel, we discard S_i out of the market since no channel could satisfy a successful transmission for S_i.

Before we formally describe our algorithm, we need the following definitions: *SU Tolerance* [21][22] and *Feasible Group.*

Definition 1 (SU Tolerance) *The tolerance τ_i indicates how much interference S_i can endure before its SINR value falls below the threshold β_i. It can be calculated by*

$$\tau_i = \frac{\frac{P_i}{d(T_i,R_i)^\alpha}}{\beta_i} - N_0. \tag{3.4}$$

Definition 2 (Feasible Group) *A group G_k of SUs is feasible with respect to S_i if, after the addition of S_i to the group, Condition* (3.3) *is satisfied $\forall S_j \in G_k \cup \{S_i\}$ and Condition* (3.1) *is satisfied for the PU.*

3.2 Auction Model

With primary and secondary users in the cognitive radio network, we aim to design a single-sided spectrum auction that is individually rational, computationally efficient, and truthful. In this setting, the PU is the seller and SUs are buyers. In the rest of the paper, we use the terminology of PU and seller, SU and buyer interchangeably. The PU contributes m *homogeneous* channels $\{c_1, c_2, ..., c_m\}$ and is using channels $c_k \in \mathcal{C}_0$. Each buyer requests at most one channel. Each S_i holds a private valuation $v_i \geq 0$ for leasing a channel, and a bid $b_i \geq 0$ as the maximum amount that it would pay for a channel.

The auction works as follows: after collecting the bids and requests from all buyers, the algorithm decides the allocation for each buyer. The algorithm also computes the payment for each winning buyer. Buyer S_i pays p_i as the corresponding charge.

The utility of S_i is defined as follows:

$$u_i = \begin{cases} v_i - p_i, & if\ S_i\ wins, \\ 0, & otherwise. \end{cases} \tag{3.5}$$

CHAPTER 4

AUCTION DESIGN OF SPA

In this chapter, we introduce the basic design of SPA in the single-demand scenario, i.e., a buyer S_i requests at most one channel.

The extension of SPA to support multi-demand auctions, where S_i requests d_i channels, is deferred to Chapter 5.

4.1 High-level Description

Our work consists of two stages: the first stage is allocation, and the second stage is pricing.

The allocation stage applies a mechanism, in which buyers are sorted based on both bids and tolerances. Then we check the feasibility of each buyer to m channels sequentially, and assign the buyer to the first feasible channel as a winner.

In the pricing stage, we decide the final payments for all winners. The pricing stage applies a mechanism that aims to find critical values.

We present the detailed algorithms in the following two subsections.

4.2 Channel Allocation

We start with an intuitive idea. When we choose a buyer from the set S to allocate a channel, the buyer with a higher bid and a higher tolerance is preferred. In other words, this buyer is more resistant to interference and is willing to pay more for the channel. This property is best characterized by the product:

$$\tilde{b}_i = b_i \cdot \tau_i. \tag{4.1}$$

Without loss of generality, we can sort all the SUs based on $\tilde{b}_i$ in a non-increasing order $\tilde{b}_1 \geq \tilde{b}_2 \geq, \ldots, \geq \tilde{b}_n$ and get a sorted list $\mathbb{S} : \mathbb{S}_1, \mathbb{S}_2, \mathbb{S}_3, \ldots, \mathbb{S}_n$.

Algorithm 1: Allocation ($\mathbb{S}$)

```
for k ← 1 to m do G_k ← ∅;
for i ← 1 to n do
    for k ← 1 to m do
        if G_k is feasible to S_i then
            G_k ← G_k ∪ {S_i}; break;
            f_ik ← 0;
        end
    end
end
𝒢 ← {G_1, G_2, ..., G_m};
return 𝒢
```

Based on the sorted list $\mathbb{S}$, Algorithm 1 allocates buyers sequentially from $\mathbb{S}_1$ to $\mathbb{S}_n$. For each buyer $\mathbb{S}_i$, the algorithm checks whether G_k is feasible to $\mathbb{S}_i$ for $k = 1$ to m. We use a binary variable f_{ik} to mark the feasibility status for $\mathbb{S}_i$. If G_k is feasible to $\mathbb{S}_i$, then $f_{ik} = 1$; otherwise 0. The algorithm assigns $\mathbb{S}_i$ to the first feasible channel. If there is no feasible channel to $\mathbb{S}_i$, the algorithm assigns $\mathbb{S}_i$ nothing.

4.3 Pricing

In this stage, we compute payments for winners.

After assigning each buyer either zero or one channel, we need to compute their payments. To maintain truthfulness, we find each buyer its critical value.

Definition 3 (Critical Value) *The smallest value such that a bidder will win when bidding higher than its critical value, and it will lose when bidding lower than that.*

Algorithm 2 illustrates payment computation of $\mathbb{S}_i$, called Pricing algorithm. The basic idea is that for each winner buyer $\mathbb{S}_i$, first take $\mathbb{S}_i$ out of the sorted list $\mathbb{S}$ and get a sorted list $\mathbb{S}^{[-i]}$ consisting of the remaining buyers.

$\mathbb{S}^{[-i]} : \mathbb{S}_1^{[-i]}, \mathbb{S}_2^{[-i]}, \mathbb{S}_3^{[-i]}, \ldots, \mathbb{S}_q^{[-i]}, \ldots, \mathbb{S}_{n-1}^{[-i]}$

Then allocate channels to remaining buyers. Each time when assigning a channel to a remaining buyer, check the feasibility of $\mathbb{S}_i$. When we find the first buyer $\mathbb{S}_q^{[-i]}$, who makes

Algorithm 2: Pricing($\mathbb{S}, \mathcal{G}$)

1 **for** $i \leftarrow 1$ **to** n **do** $p_i \leftarrow 0$;
2 $\mathcal{W} \leftarrow \bigcup_{G_k \in \mathcal{G}} G_k$;
3 **for** $\mathbb{S}_i \in \mathcal{W}$ **do**
4     $\mathbb{S}^{[-i]} \leftarrow \mathbb{S} \setminus \{\mathbb{S}_i\}$;
5   **for** $k \leftarrow 1$ **to** m **do**
6         $G_k \leftarrow \emptyset$;
7         $f_i k \leftarrow \begin{cases} 1, & \text{if } G_k \text{ is feasible to } \mathbb{S}_i, \\ 0, & \text{otherwise.} \end{cases}$
8   **end**
9   **for** $q \leftarrow 1$ **to** $n-1$ **do**
10   **for** $k \leftarrow 1$ **to** m **do**
11   **if** *G_k is feasible to $\mathbb{S}_q^{[-i]}$* **then**
12                 $G_k \leftarrow G_k \cup \{\mathbb{S}_q^{[-i]}\}$;
13   **if** *G_k is not feasible to $\mathbb{S}_i$* **then** $f_{ik} \leftarrow 0$;
14   break;
15   **end**
16   **end**
17   **if** $\sum_{k=1}^{m} f_k = 0$ **then** $p_i \leftarrow \tilde{b}_q/\tau_i$; break ;
18   **end**
19 **end**
20 **return** $\{p_1, p_2, \ldots, p_n\}$

the number of $\mathbb{S}_i$'s feasible channels become 0, its corresponding $\frac{\tilde{b}_q}{\tau_i}$ is $\mathbb{S}_i$'s critical value. Line 17 indicates the payment p_i of $\mathbb{S}_i$. If we cannot find the critical value for $\mathbb{S}_i$, then the charge is 0.

For each buyer who has been assigned one channel in Algorithm 1, we shall run Algorithm 2 to compute its payment, sequentially.

4.4 Analysis of SPA

In this section, we prove that SPA satisfies the desired properties introduced in Chapter 1.

Theorem 1 *SPA is truthful, individually-rational, and computationally efficient.*

We prove Theorem 1 by the following three lemmas.

Lemma 1 *SPA is truthful.*

Proof: It is known that an auction is truthful if the allocation algorithm of this auction is monotone while the price charged of a winner is a critical value [23].

Monotonic allocation: for each buyer S_i, if S_i wins by bidding b_i, then it also wins by bidding $b_i' > b_i$.

Suppose S_i wins by bidding b_i. With $b_i' > b_i$, we have $\tilde{b}_i' > \tilde{b}_i$. Therefore S_i's position in the sorted list $\mathbb{S}^{[-i]}$ is ranked after that in the sorted list $\mathbb{S}'^{[-i]}$ with the same τ_i. Because S_i wins by bidding b_i, there is at least one feasible channel for S_i in $\mathbb{S}^{[-i]}$. Thus S_i wins by bidding b_i'. The allocation is monotonic.

Critical Value: for each buyer S_i, p_i is its critical value, if S_i wins by bidding higher than p_i and loses by bidding lower than p_i.

We consider the following two cases separately:

- *Case 1:* $b_i > p_i$

  With $p_i = \tilde{b}_q/\tau_i$, we have $\tilde{b}_i > \tilde{b}_q$ and S_i is ranked before S_q in the sorted list. Because S_q is the first buyer who makes S_i have no feasible channel, being ranked before S_q guarantees that S_i has at last one feasible channel. Therefore, S_i wins.

- *Case 2:* $b_i < p_i$

  Similarly as the Case 1 above. S_i is ranked after S_q in the sorted list, so that S_q occupies the last feasible channel of S_i and fails S_i. Therefore, S_i loses.

Thus, p_i is the critical value of S_i.

Lemma 2 *SPA is individually rational.*

Proof: Assume that each buyer S_i bids truthfully, i.e., $b_i = v_i$. For each winning buyer S_i, Algorithm 2 returns $p_i = \tilde{b}_q/\tau_i$. According to the definition of utility, $u_i = v_i - p_i$. Because S_i is ranked before S_q, we have $\tilde{b}_i \geq \tilde{b}_q$. With $\tilde{b}_i = b_i \cdot \tau_i$ and $\tilde{b}_q = p_i \cdot \tau_i$, we get $b_i \geq p_i$. Therefore $u_i \geq 0$. For all losers, $u_i = 0$.

Thus, $u_i \geq 0$. It is individually rational.

Lemma 3 *SPA is computationally efficient.*

Proof: We now analyze the running time of the design auction model with n buyers and m channels. First, Algorithm 1 takes $O(n \log n)$ time to sort the bids. To allocate a channel to bidder S_i, Algorithm 1 needs to examine at most m channels to find the feasible lowest-indexed channel. This process takes $O(mn)$ time for n bidders. Therefore, the overall complexity of Algorithm 1 is $O(n \log n + mn)$. Second, Algorithm 2 uses the sorted bids from Algorithm 1 and hence its complexity only comes from the processes of initialization and checking feasibility for S_q, which is $O(m + mn)$ for each buyer. Therefore, the overall complexity of the Pricing Algorithm is $O(mn^2)$. In total, the overall complexity is $O(n \log n + mn^2)$.

CHAPTER 5

EXTENDING TO MULTI-DEMAND AUCTION MODEL

As shown above, we have designed a spectrum auction for the single-demand case under the physical interference model, in which each buyer requires at most one channel. In reality, it is possible that the buyers may request multiple channels. This motivates us to design spectrum auctions for the multi-demand case.

In this chapter, we show that this designed auction model can be extended to support the multi-demand auction model.

5.1 Notations and System Model

Designing spectrum auctions for the multi-demand auction is naturally more challenging than the single-demand auction. The auction we designed in Chapter 4 cannot be simply used on the multi-demand model, because the existing sorting criteria does not consider the multi-demand of buyers. In addition, for the multi-demand model, we need to consider more combination of channels. It leads to the increase of the algorithm's complexity.

An intuitive idea is to use the per-channel bid (unit bid) instead of the bid b_i for sorting. We start by introducing a set of notations.

Channel request (d_i) It represents the number of channels requested by bidder S_i.

Per-channel bid (t_i) It represents the per-channel bid submitted by bidder S_i. Therefore, $t_i = \frac{b_i}{d_i}$, and b_i is the total bid. Let $T = \{t_1, t_2, ..., t_n\}$ represent the set of bids submitted by all the bidders.

Per-channel true valuation (w_i) Each bidder S_i has a per-channel valuation w_i which describes the true price that S_i is willing to pay for each channel. In most cases, this valuation is private and is known only to the bidder itself.

In this thesis we focus on the single-minded scenario: a buyer accepts to receive either d_i channels or 0 channel. Another possible case is range-based: a buyer accepts any x_i channels if $0 \leq x_i \leq d_i$. The auction design for the range-based case will be our future work.

The auction works as follows: after collecting the bids and requests from all buyers, the algorithm decides the allocation for each buyer. The algorithm also computes the payment for each winning buyer. Buyer S_i pays p_i as the corresponding charge.

The utility of S_i is defined as follows:

$$u_i = \begin{cases} w_i \cdot d_i - p_i, & if\ S_i\ wins, \\ 0, & otherwise. \end{cases} \tag{5.1}$$

In the multi-demand model, consider a cognitive radio network (CRN) with one primary user PU and n secondary users $S = \{S_1, S_2, ..., S_n\}$. The primary user PU owns m licensed homogeneous channels $\{c_1, c_2, ..., c_m\}$. The PU is using d_0 channels, where $d_0 \leq m$. Each secondary user S_i has a demand of d_i, which indicates the number of channels that S_i requests.

5.2 Auction Design of SPA-x

In this section, we focus on the single-minded case where bidder S_i requests d_i channels and accepts to obtain either d_i channels or 0 channel.

When we choose a buyer from SUs to allocate a channel, what we are really seeking for is one that is more resistant to interference and is willing to pay higher price for the channel. This property is best characterized by the product:

$$\tilde{t}_i = t_i \tau_i. \tag{5.2}$$

Without loss of generality, we can sort all the secondary users based on the product $\tilde{t}_i$ in a non-increasing order $\tilde{t}_1 \geq \tilde{t}_2 \geq \tilde{t}_3 \geq, ..., \geq \tilde{t}_n$ and get a sorted list $\mathbb{S}$: $\mathbb{S}_1$, $\mathbb{S}_2$, $\mathbb{S}_3$, ..., $\mathbb{S}_n$.

Based on the sorted list $\mathbb{S}$, Algorithm 3 allocates buyers sequentially from $\mathbb{S}_1$ to $\mathbb{S}_n$. For each buyer $\mathbb{S}_i$, the algorithm checks whether G_k is feasible to $\mathbb{S}_i$ for $k = 1$ to m. We use a binary variable f_{ik} to mark the feasibility status for $\mathbb{S}_i$, defined as:

$$f_{ik} = \begin{cases} 1, & \text{if } G_k \text{ is feasible to } \mathbb{S}_i, \\ 0, & \text{otherwise.} \end{cases} \tag{5.3}$$

The algorithm assigns $\mathbb{S}_i$ to the first d_i feasible channels as the buyer requests. We use another binary variable a_{ik} to mark the allocation status for $\mathbb{S}_i$. If G_k is allocated to $\mathbb{S}_i$, then $a_{ik} = 1$; otherwise 0. If there are less than d_i feasible channels to $\mathbb{S}_i$, the algorithm assigns $\mathbb{S}_i$ nothing.

Algorithm 3: Strict-Allocation ($\mathbb{S}$)

for $k \leftarrow 1$ **to** m **do** $G_k \leftarrow \emptyset$;
for $i \leftarrow 1$ **to** n **do**
    for $k \leftarrow 1$ **to** m **do** Initialize f_{ik} using (5.3);
    if $\sum_{k=1}^{m} f_{ik} \geq d_i$ **then**
        for $k \leftarrow 1$ **to** m **do**
            if $f_{ik} = 1$ **and** $\sum_{k=1}^{m} a_{ik} < d_i$ **then**
                $a_{ik} \leftarrow 1$; $G_k \leftarrow G_k \cup \{\mathbb{S}_i\}$;
            end
        end
    end
end
$\mathcal{G} \leftarrow \{G_1, G_2, \ldots, G_m\}$;
return $\mathcal{G}$

With each buyer either assigned zero or d_i channels, next we need to compute their payments. To maintain truthfulness, we find each buyer its critical value.

Algorithm 4 illustrates payment computation of $\mathbb{S}_i$. The basic idea is that for each winner buyer $\mathbb{S}_i$, first take $\mathbb{S}_i$ out of the sorted list $\mathbb{S}$ and get a sorted list $\mathbb{S}^{[-i]}$ consisting of the remaining buyers. Then allocate channels to remaining buyers. Every time when assigning channels to a remaining buyer, check the feasibility of $\mathbb{S}_i$. When we find the first buyer $\mathbb{S}_q^{[-i]}$, who makes the number of $\mathbb{S}_i$'s feasible channels become less than d_i, its corresponding $\frac{\tilde{t}_q}{\tau_i}$ is $\mathbb{S}_i$'s critical value. Line 22 indicates the payment p_i of $\mathbb{S}_i$. If we cannot find the critical value for $\mathbb{S}_i$, then the charge is 0.

For each buyer who has been assigned channels in Algorithm 3, we shall run Algorithm 4 to compute its payment, sequentially.

5.3 Analysis of SPA-x

We prove that SPA-x satisfies the desired properties introduced in Chapter 1.

Algorithm 4: Strict-Pricing($\mathbb{S}, \mathcal{G}$)

for $i \leftarrow 1$ **to** n **do** $p_i \leftarrow 0$;
$\mathcal{W} \leftarrow \bigcup_{G_k \in \mathcal{G}} G_k$;
for $\mathbb{S}_i \in \mathcal{W}$ **do**
  $\mathbb{S}^{[-i]} \leftarrow \mathbb{S} \setminus \{\mathbb{S}_i\}$;
  for $k \leftarrow 1$ **to** m **do**
    $G_k \leftarrow \emptyset$;
    Initialize f_{ik} using (5.3);
  end
  for $q \leftarrow 1$ **to** $n-1$ **do**
    for $k \leftarrow 1$ **to** m **do** Initialize f_{qk} using (5.3);
    if $\sum_{k=1}^{m} f_{qk} \geq d_q$ **then**
      for $k \leftarrow 1$ **to** m **do**
        if $f_{qk} = 1$ **and** $\sum_{k=1}^{m} a_{qk} < d_q$ **then**
          $a_{qk} \leftarrow 1$; $G_k \leftarrow G_k \cup \{\mathbb{S}_q^{[-i]}\}$;
          if G_k *is infeasible to* $\mathbb{S}_i$ **then**
            $f_{ik} \leftarrow 0$;
          end
        end
      end
    end
  end
  if $\sum_{k=1}^{m} f_{ik} < d_i$ **then** $p_i \leftarrow d_i \frac{\tilde{t}_q}{\tau_i}$; break ;
end
return $\{p_1, p_2, \ldots, p_n\}$

Theorem 2 *SPA-x is truthful, individually-rational, and computationally efficient.*

We prove Theorem 2 by the following three lemmas.

Lemma 4 *SPA-x is truthful.*

Proof: It is known that an auction is truthful if the allocation algorithm of this auction is monotone while the price charged of a winner is a critical value [23].

Monotonic allocation: for each buyer S_i, if S_i wins by bidding b_i, then it also wins by bidding $b_i' > b_i$.

Suppose S_i wins by bidding b_i. With $b_i' > b_i$, we have $t_i' > t_i$ and $\tilde{t}_i' > \tilde{t}_i$. Therefore SU_i's position in the sorted list $\mathbb{S}^{[-i]}$ is ranked after its in the sorted list $\mathbb{S}'^{[-i]}$ with the same τ_i and the same d_i. Because S_i wins by bidding t_i, there are at least d_i feasible channels for SU_i in $\mathbb{S}^{[-i]}$. Thus S_i wins by bidding b_i'. The allocation is monotonic.

Critical Value: for each buyer S_i, p_i is its critical value, if S_i wins by bidding higher than p_i and loses by bidding lower than p_i.

We consider the following two cases separately:

- *Case 1:* $b_i > p_i$

  With $p_i = d_i \cdot \frac{\tilde{t}_q}{\tau_i}$, we have $\tilde{t}_i > \tilde{t}_q$ and S_i is ranked before S_q in the sorted list. Because S_q is the first buyer who makes S_i have less than d_i feasible channels, being ranked before S_q guarantees that S_i has at last d_i feasible channels. Therefore, S_i wins.

- *Case 2:* $b_i < p_i$

  Similarly as the Case 1 above. S_i is ranked after S_q in the sorted list, so that S_q occupies the d_i-th feasible channel of S_i and fails S_i. Therefore, S_i loses.

Thus, p_i is the critical value of S_i.

Lemma 5 *SPA-x is individually rational.*

Proof: Assume that each buyer S_i bids truthfully, i.e., $t_i = w_i$. For each winning buyer S_i, Algorithm 2 returns $p_i = d_i \frac{\tilde{t}_q}{\tau_i}$. According to the definition of utility, $u_i = w_i \cdot d_i - p_i$. Because S_i is ranked before S_q, we have $\tilde{t}_i \geq \tilde{t}_q$. With $\tilde{t}_i = t_i \cdot \tau_i$ and $\tilde{t}_q = \frac{p_i}{d_i} \cdot \tau_i$, we get $t_i \cdot d_i \geq p_i$. Therefore $u_i \geq 0$. For all losers, $u_i = 0$.

Thus, $u_i \geq 0$. It is individually rational.

Lemma 6 *SPA-x is computationally efficient.*

Proof: We now analyze the running time of the design auction model with n buyers and m channels. First, Algorithm 3 takes $O(n \log n)$ time to sort the bids. To allocate d_i channels to bidder S_i, Algorithm 3 needs to examine at most m channels to find the feasible channels. This process takes $O(2mn)$ time for n bidders. Therefore, the overall complexity of Algorithm 3 is $O(n \log n + 2mn)$. Second, Algorithm 4 uses the sorted bids from Algorithm 3 and hence its complexity only comes from the processes of initialization and checking feasibility for S_q, which is $O(m + 2mn)$ for each buyer. Therefore, the overall complexity of the Pricing Algorithm is $O(mn + 2mn^2)$. In total, the overall complexity is $O(n \log n + mn^2)$.

CHAPTER 6
PERFORMANCE EVALUATION

In this section, we evaluate the performance of our algorithms by comparing them with the current existing algorithms.

6.1 Environment Setup

For the homogeneous spectrum auction model, we implemented SPA and compared it with SMALL in [5]. Although originally designed under the protocol interference model, SMALL can be easily modified to adopt the physical interference model by using the SINR values group formation. Since the stage of group formation is bid-independent, we implement an effective heuristic algorithm for link scheduling in [21] to form secondary users into groups. We name the modified spectrum auction as **SMALL-SINR**.

We uniformly distributed transmitters and receivers in a 1000×1000 square region. The length of each link is randomly chosen between 100 and 300. The SINR threshold was set to 16, the environment noise $N_0 = 10^{-9}$, the path loss exponent $\alpha = 2$, and transmit power $= 0.2$. We assume that the bids from all buyers are distributed uniformly at random over $(0, 100]$. All results are averaged over 10 times for each parameter configuration.

6.2 Performance Metrics

We choose to monitor the following performance metrics.

- *Revenue*: The sum of charges to all the winning buyers.

- *Channel Utilization*: Average number of buyers allocated to each channel.

- *Buyer Satisfaction Ratio*: The percentage of buyers who get at least one channel.

6.3 Evaluation Results and Analysis of SPA

We present the simulation results of SPA in this section.

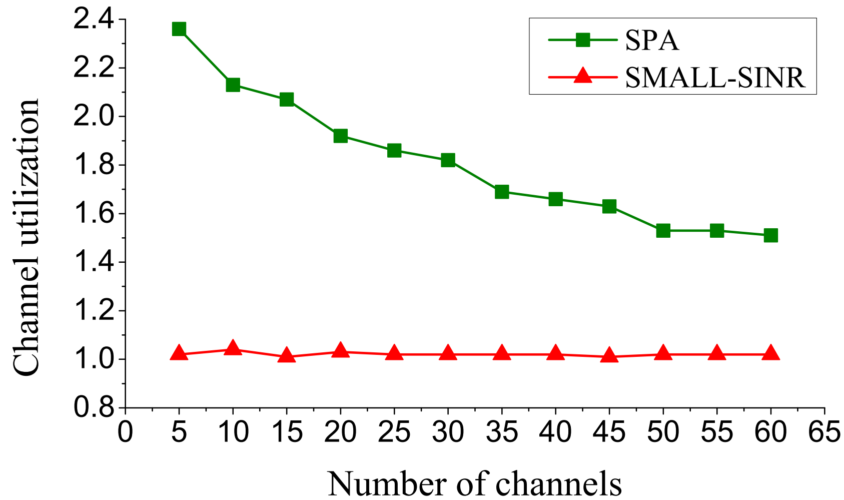
For the single-demand auctions, we modify the existing auction model SMALL in the physical interference model, by using SINR values to form groups instead of conflict graphs. After the groups are formed, SMALL defines group bid as: $\sigma_k = (|G_k| - 1) \cdot min\{b_i | S_i \in G_k\}$. Based on the group bids, all groups are sorted in a non-increasing order, and buyers in the first m groups are winners except for the buyer with the smallest bid in each group. The payment for each buyer is the smallest individual bid in each group.

Figure 6.1 illustrates revenues, channel utilizations and buyer satisfaction ratios of both SPA and SMALL-SINR. The number of buyers is 100 and the number of auctioned channels varies from 5 to 85 with an increment of 5.

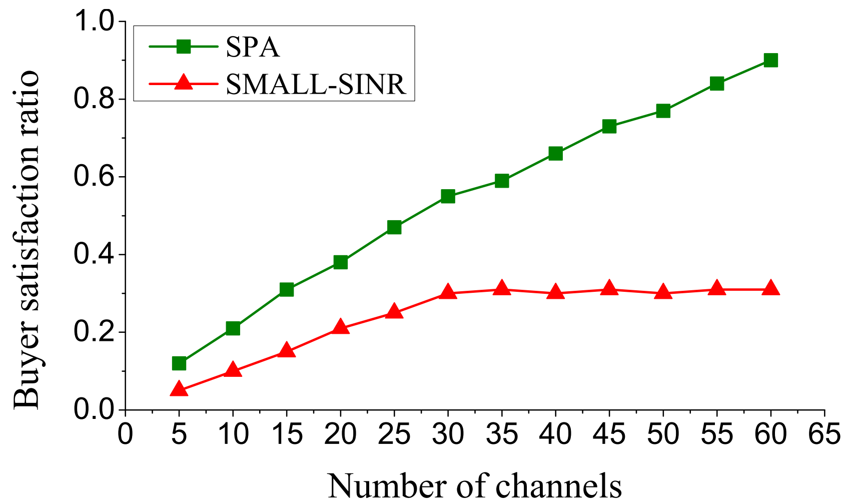
Figure 6.1(a) shows the trends of channel utilization. In SMALL-SINR, the average number of buyers in each channel stays at a steady level around 1. In SPA, initially the channel utilization is around 2.4, and falls down to a level about 1.5. Based on the interference relationship, up to 3 buyers can share the same channel and be assigned to the same group, while SMALL always sacrifices one buyer in each group to maintain truthfulness. In SPA, the competition among buyers is less intense, with more channels released.

In Figure 6.1(b), the satisfaction ratio of SMALL-SINR goes up initially, then maintains at a steady level, because groups are formed before the auction in SMALL-SINR. After the number of channels increases above a certain value, the number of winning groups remains the same, while the groups with zero bids are always losers. In SPA, the satisfaction ratio increases with the number of channels, and is above 95% since there are enough channels for almost all buyers.

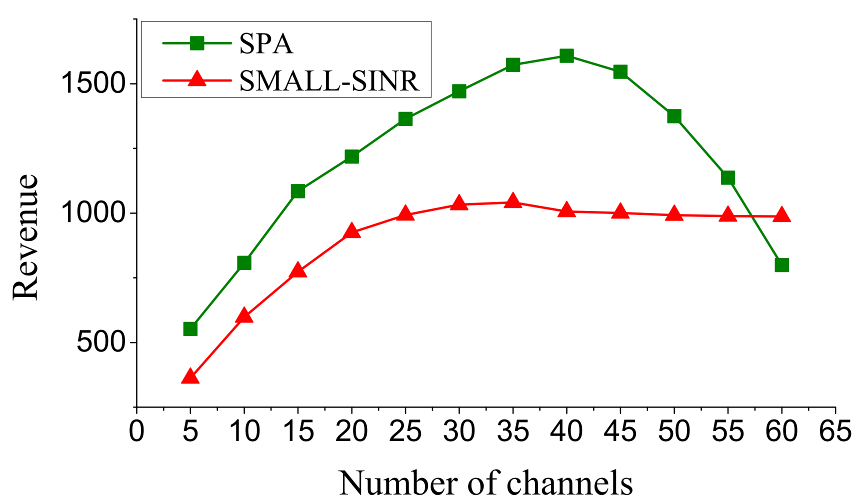
Form Figure 6.1(c) it can be observed that the revenue of SMALL-SINR slightly increases at first, then stays at a steady level. The trend shows that the revenue increases with more channels involved, but the revenue converges after the saturation of the market. On the other hand, the revenue of SPA increases rapidly at the beginning and then falls down when



(a) Channel utilization

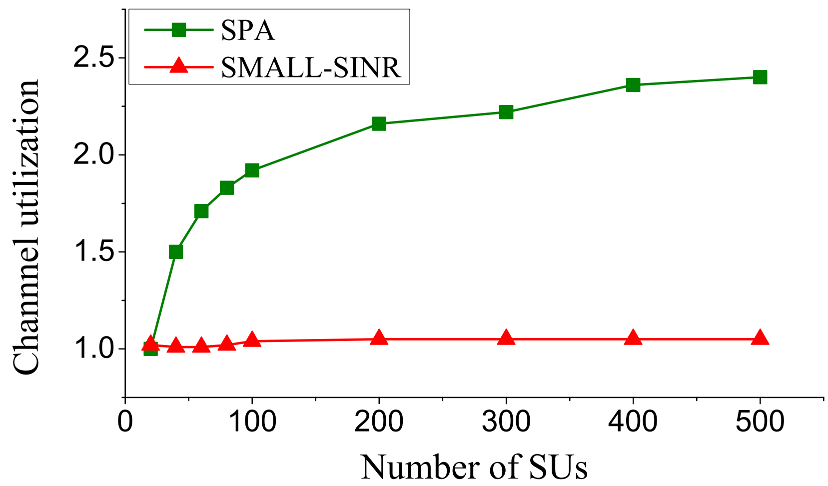


(b) Buyer satisfaction ratio

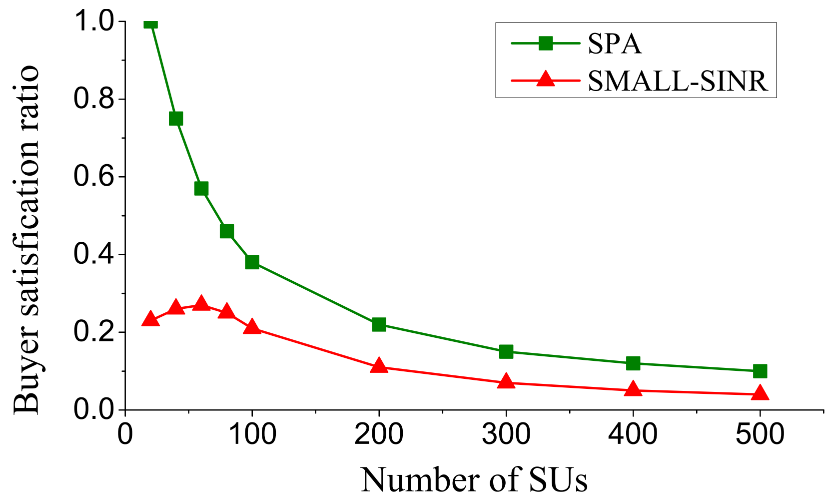


(c) Revenue

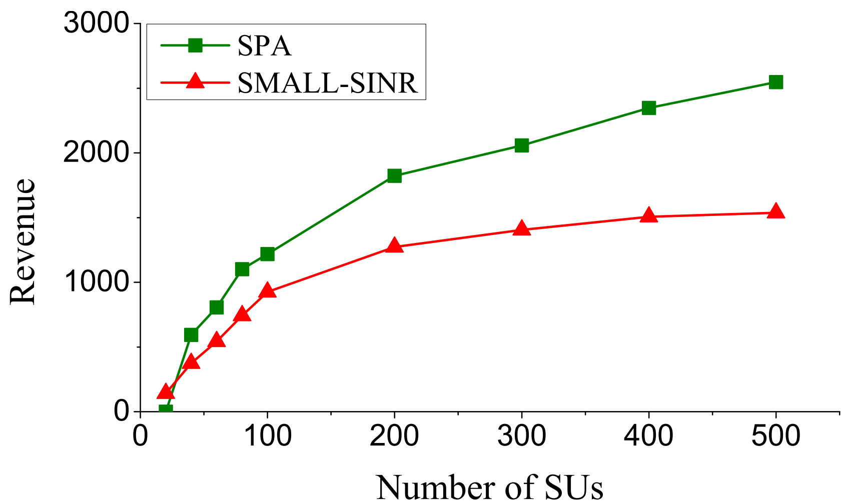
Figure 6.1: Comparing SPA and SMALL-SINR by auctioning 5-60 channels to 100 bidders.



(a) Channel utilization



(b) Buyer satisfaction ratio



(c) Revenue

Figure 6.2: Comparing SPA and SMALL-SINR by auctioning 20 channels to 100-500 buyers.

the number of auctioned channels is above 40. The fundamental reason is that, with more channels, the competition among buyers is no longer intense, which leads to a zero payment for some winners.

Figure 6.2 shows the impact of the number of SUs on revenues, channel utilizations and buyer satisfaction ratios for both SPA and SMALL-SINR. The number of auctioned channels is 20 and the number of SUs varies from 20 to 500.

Figure 6.2(a) presents the channel utilization when more buyers join the auctions. Based on the interference relationship, 2-3 buyers can share the same channel but SMALL-SINR always sacrifices one buyer in each group to maintain truthfulness. On the other hand, the group size is guaranteed to be no less than 1 due to the winner section rule. Therefore, channel utilization stays steady around 1 in SMALL-SINR. Whereas, in SPA the average number of buyers in each channel increases rapidly at first, and then remains at a level around 2.5 due to over-saturation of the market with more SUs.

From Figure 6.2(b), it can be observed that, initially the satisfaction ratio is nearly 100% in SPA. Because most buyers are winners and the market is almost saturated. When more buyers join the auction, SPA does not provide more channels to satisfy buyers. As a result, the satisfaction ratio decreases. On the contrary, SMALL-SINR can hardly achieve higher satisfaction ratio due to its sacrifice rule.

In Figure 6.2(c), the competition among buyers in SPA becomes more intense with more buyers involved. Consequently, winners' critical values are higher and the revenue increases. Similarly, when the competition between groups in SMALL-SINR grows, the seller receives more revenue. However, the sacrifice rule in SMALL-SINR inhibits significant revenue growth.

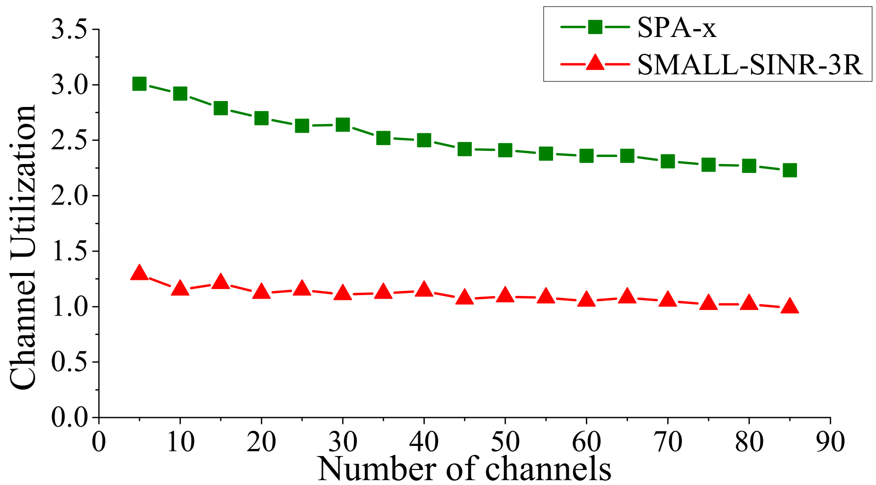
6.4 Evaluation Results and Analysis of SPA-x

For the multi-demand auctions, suppose each buyer requests at most 3 channels. we can still modify the existing auction model SMALL in the physical interference model, by using SINR values to form groups instead of conflict graphs. After the groups are formed, SMALL sacrifices the buyer with the smallest bid in each group and define group bid as: $\sigma_k = (|G_k| - 1) \cdot min\{b_i | S_i \in G_k\}$. Instead of sorting by the group bids, all groups are sorted in a non-increasing order based on the group size, and the first m groups are winner. The payment for each buyer is the smallest bid in each group. We name the modified SMALL as SMALL-SINR-3R.

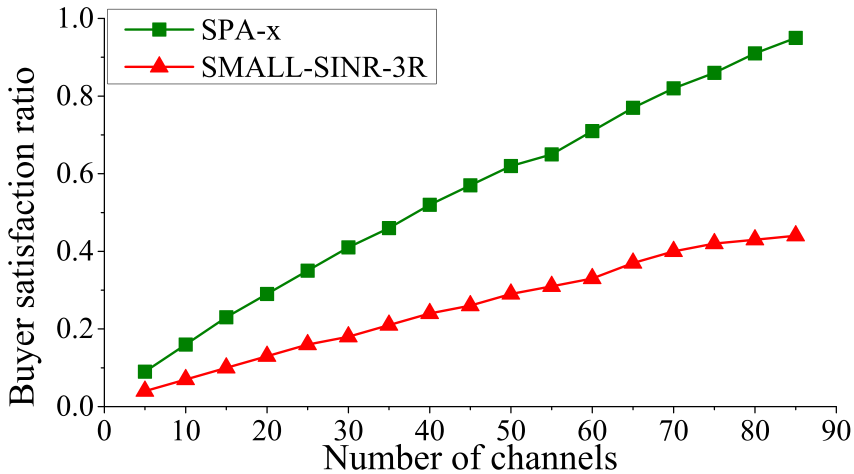
Figure 6.3 illustrates revenues, channel utilizations and buyer satisfaction ratios of both SPA-x and SMALL-SINR-3R. The number of SUs is 100 and the number of auctioned channels varies from 5 to 95.

Figure 6.4 shows the impact of the number of SUs on revenues, channel utilizations and buyer satisfaction ratios for both SPA-x and SMALL-SINR-3R. The number of auctioned channels is 50 and the number of SUs varies from 20 to 500.

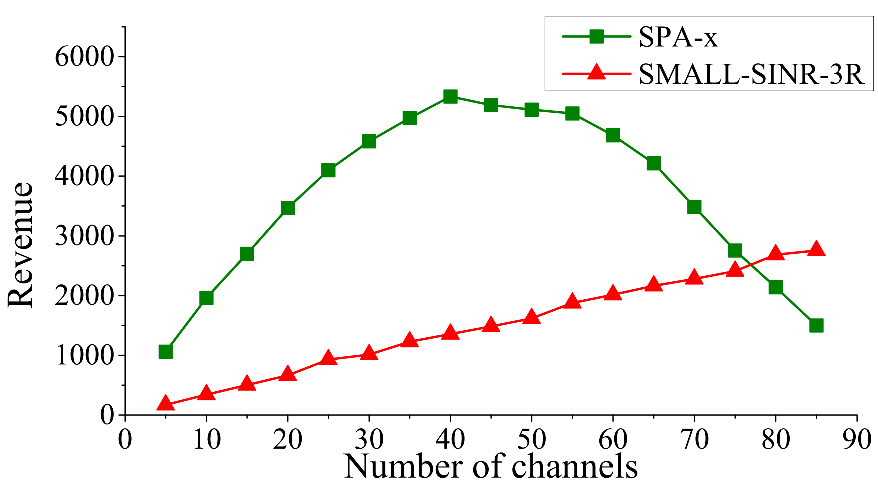
Following the same analogy in SPA, the trends in Figure 6.3 and Figure 6.4 are similar to those in Figure 6.1 and Figure 6.2. Due to the growth of channel demands in SPA-x, the speed to achieve market saturation becomes slower.



(a) Channel utilization

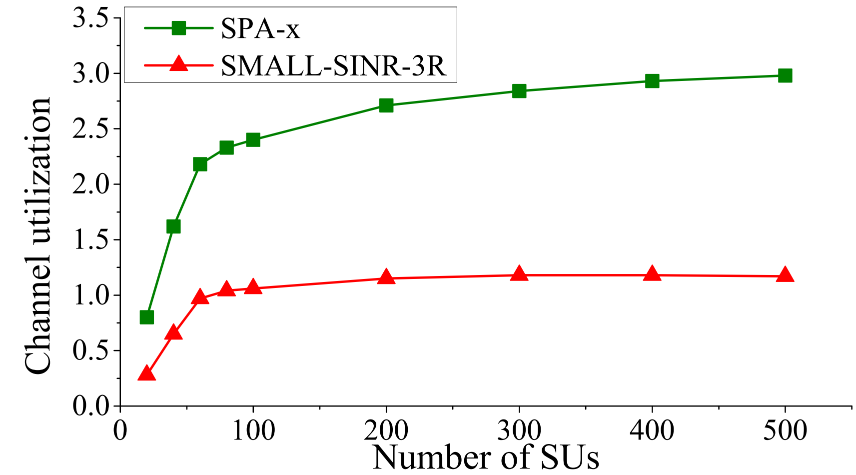


(b) Buyer satisfaction ratio

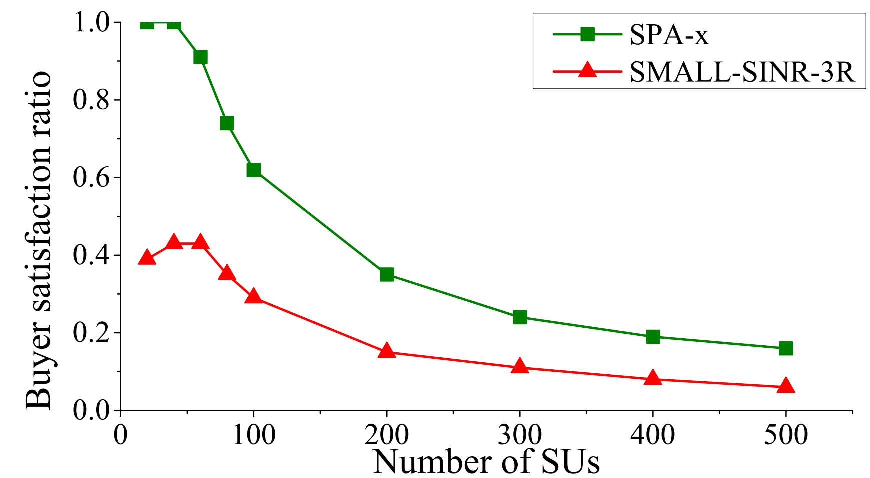


(c) Revenue

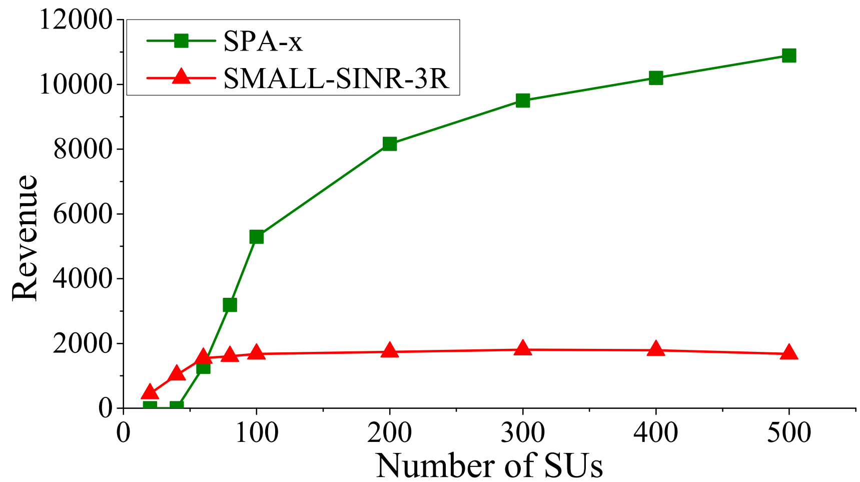
Figure 6.3: Comparing SPA-x and SMALL-SINR-3R by auctioning 5-85 channels to 100 bidders.



(a) Channel utilization



(b) Buyer satisfaction ratio



(c) Revenue

Figure 6.4: Comparing SPA-x and SMALL-SINR-3R by auctioning 50 channels to 20-500 buyers.

CHAPTER 7
CONCLUSION

In this thesis, we studied the truthful spectrum auctions under the physical interference model. We proposed SPA, which takes into consideration both bid and link tolerance in spectrum auctions. We analyzed SPA and proved that it satisfies individual rationality, computational efficiency, and truthfulness. Further performance evaluation indicates our algorithms achieve higher revenue, channel utilization and buyer satisfaction ratio, compared with the algorithms in [5] modified with the physical interference model.

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