MODELING COUPLED THERMAL-HYDROLOGIC-MECHANICAL PROCESSES IN FRACTURED GEOTHERMAL RESERVOIRS USING EMBEDDED DISCRETE FRACTURE METHOD

by

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ABSTRACT

The Enhanced Geothermal System (EGS) energy resources have been regarded as a clean substitute for fossil energy. Exploration and research have been widely conducted in geothermal fields where the temperature gradient is promising for energy recovery. The EGS development relies on artificially created hydraulic fractures through which the injected low-temperature fluid can be flowing while being heated by the surrounding rocks. The injection/production induces the pore pressure and temperature change in the field, which in turn causes rock deformation under the geo-stress. The mechanical deformation affects the fluid flow and heat transfer by changing the hydraulic parameters such as porosity and permeability. Therefore, a coupled Thermal-Hydrologic-Mechanical (THM) model is essential to simulate the fracture-dominated EGS development for better planning and management.

In this dissertation, the Embedded Discrete Fracture Model (EDFM) is introduced to explicitly and mathematically model the fluid flow and heat transfer processes dominated by hydraulic fractures. A fully coupled model is firstly developed in which mass and energy balance, as well as momentum balance equations, are all solved by the Integrated Finite Difference (IFD) method. In the fully coupled scheme, stress tensor components (except for the principal stress in the z-direction) and the mean stress are treated as primary variables so that the normal stresses acting on the fracture faces can be assessed, from which the fracture aperture can then be determined. The assumption made in this scheme is that discrete fractures are subject to the same stress state as their containing mesh grid, which is not completely rigorous from the perspective of fracture mechanics. Therefore, secondly, a sequentially coupled model that adopts the eXtended Finite Element Method (XFEM) as its mechanical solver is developed. XFEM is widely used in fracture propagation simulations and is capable of obtaining the fracture aperture directly under dynamic pressure, temperature and stress states. Lastly, to accommodate the natural fractures induced by artificial stimulation, Multiple INteracting Continua (MINC) model is
integrated into the sequentially coupled approach. Several intermediate-scale synthetic fractured geothermal reservoir models are established to investigate how various important parameters or fracture models impact the production rate and temperature in both fully and sequentially coupled models, such as permeability, thermal conductivity, injection rate/temperature, and natural fracture spacing. The fully coupled model is also applied to a field experimental EGS project and results are compared with existing literature and measured data.

To the best of our knowledge, despite plenty of research on the fully and sequentially coupled THM model using EDFM and XFEM, there is a lack of model development, investigations, and applications in the field of three-dimensional (3D) fully or sequentially coupled model for EDFM, 3D EDFM with XFEM and 3D EDFM-MINC with XFEM. This research will fill in this gap between 2D and 3D, propose a methodology for modeling the coupled THM process in fractured geothermal reservoirs and provide insights into the impact of THM parameters on the geothermal energy recovery.
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<tr>
<td>3D</td>
<td>Three-Dimensional</td>
</tr>
<tr>
<td>DFN</td>
<td>Discrete Fracture Network</td>
</tr>
<tr>
<td>DOE</td>
<td>Department of Energy</td>
</tr>
<tr>
<td>DP</td>
<td>Double Porosity</td>
</tr>
<tr>
<td>DTS</td>
<td>Distributed Temperature Sensing</td>
</tr>
<tr>
<td>EDFM</td>
<td>Embedded Discrete Fracture Model (or Method)</td>
</tr>
<tr>
<td>EGS</td>
<td>Enhanced Geothermal System</td>
</tr>
<tr>
<td>EOR</td>
<td>Enhanced Oil Recovery</td>
</tr>
<tr>
<td>FD</td>
<td>Finite Difference</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite Element Method</td>
</tr>
<tr>
<td>FORGE</td>
<td>Frontier Observatory for Research in Geothermal Energy</td>
</tr>
<tr>
<td>FVM</td>
<td>Finite Volume Method</td>
</tr>
<tr>
<td>HPC</td>
<td>High Performance Computing</td>
</tr>
<tr>
<td>IFD</td>
<td>Integrated Finite Difference</td>
</tr>
<tr>
<td>LGR</td>
<td>Local Grid Refinement</td>
</tr>
<tr>
<td>MINC</td>
<td>Multiple INteracting Continua</td>
</tr>
<tr>
<td>SRV</td>
<td>Stimulated Reservoir Volume</td>
</tr>
<tr>
<td>THM</td>
<td>Thermal-Hydrologic-Mechanical</td>
</tr>
<tr>
<td>THMC</td>
<td>Thermal-Hydrologic-Mechanical-Chemical</td>
</tr>
<tr>
<td>XFEM</td>
<td>eXtended Finite Element Method</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Biot’s coefficient</td>
</tr>
<tr>
<td>$\alpha_k, K_k, \beta_{T,k}, D_k$</td>
<td>Biot’s coefficient, drained bulk modulus, skeleton thermal expansion coefficient and drained elasticity tensor of the $k^{th}$ continuum</td>
</tr>
<tr>
<td>$\alpha_{up}p_{up}$</td>
<td>Upscaled pressure term in the force vector</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Fluid phase index</td>
</tr>
<tr>
<td>$\beta_f$</td>
<td>Fluid thermal expansion coefficient</td>
</tr>
<tr>
<td>$\beta_\phi$</td>
<td>Pore volumetric thermal expansion coefficient</td>
</tr>
<tr>
<td>$3\beta_{up}K_{dr}(T_{up} - T_{ref})$</td>
<td>Upscaled temperature term in the force vector</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Strain tensor</td>
</tr>
<tr>
<td>$\varepsilon_t$</td>
<td>Total strain tensor</td>
</tr>
<tr>
<td>$\varepsilon_v$</td>
<td>Volumetric strain</td>
</tr>
<tr>
<td>$\varepsilon_{xx,yy,zz}$</td>
<td>Strains in $x,y$ and $z$ direction</td>
</tr>
<tr>
<td>$\lambda_T$</td>
<td>Thermal conductivity of the rock</td>
</tr>
<tr>
<td>$\lambda,G$</td>
<td>Lamé parameters</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>$\nu_u$</td>
<td>Undrained Poisson’s ratio</td>
</tr>
<tr>
<td>$\nabla p_\beta$</td>
<td>Phase $\beta$ pressure gradient</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Porosity</td>
</tr>
<tr>
<td>$\phi_f, \phi_{f0}$</td>
<td>Current and initial fracture porosity</td>
</tr>
<tr>
<td>$\rho_R$</td>
<td>Rock solid density</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Stress tensor</td>
</tr>
<tr>
<td>$\sigma_{m,\text{int}}$</td>
<td>Mean stress at the interface</td>
</tr>
<tr>
<td>$\sigma'$</td>
<td>Effective stress</td>
</tr>
<tr>
<td>$\sigma_{zz'}$</td>
<td>Total stress on the normal direction of fracture face</td>
</tr>
</tbody>
</table>
\( \tau \) : Boundary traction

\( \xi, \eta, \zeta \) : Local coordinates of a 3D finite element

\( \Gamma \) : Surface of an integration element

\( \Gamma_T \) : Constant temperature boundary

\( \Gamma_p \) : Constant pressure boundary

\( \Gamma_u \) : Prescribed displacement boundary

\( \Omega \) : Volume of an integration element

\( \Psi_{\sigma}^{+,-} \) : Stress flux from ‘+’ grid to the contact interface, and stress flux from the interface to ‘−’ grid

\( \bar{a}_j \) : Additional Heaviside degrees of freedom

\( b \) : Body force

\( \bar{b}_{\alpha K} \) : Additional Asymptotic degrees of freedom

\( b, b_i, b_{\text{max}} \) : Current, initial and maximum mechanical aperture under normal effective stress

\( b_k \) : Coupling coefficient between fluid flow and mechanics for the \( k^{th} \) continuum of the MINC model

\( \tilde{b}_k \) : Coupling coefficient between heat transfer and mechanics for the \( k^{th} \) continuum of the MINC model

\( d_0 \) : Initial fracture aperture

\( d_{ff} \) : Grid distance between intersected fracture elements

\( d_{fm} \) : Grid distance between a discrete fracture and its containing box

\( d_{j,j+1} \) : Grid distance between neighboring continuum \( j \) and \( j + 1 \)

\( g \) : Gravity acceleration

\( h_\beta \) : Specific enthalpy of phase \( \beta \)

\( h (p, T)_{\text{int,}+,−} \) : Thermal and pressure terms evaluated at the interface and each side of the interface.
$k_f, k_{f0}$  
Current and initial fracture permeability

$k_{r\beta}$  
Relative permeability of phase $\beta$, $k_{r\beta} = 1$

$n_\Gamma$  
Normal vector of the domain boundary on which the traction is applied

$n_{\Gamma_d}$  
Discontinuity normal vector

$n_{\Gamma_d}^+$  
Normal vector pointing towards the domain defined by the normal vector $n_{\Gamma_d}$

$p$  
Pressure

$\bar{p}$  
Specified values of constant pressure boundary

$q^k$  
Sink and source term of the $k^{th}$ component in mass/balance equations

$[u]$  
Fracture aperture change, jump in the displacement field across the discontinuity

$\bar{u}$  
Prescribed displacement on the boundary $\Gamma_u$

$u_\beta$  
Specific internal energy of phase $\beta$

$u$  
Displacement Vector

$u^{+,-}$  
Displacement at the $+, -$ faces of the discontinuity

$u_{x,y,z}$  
Displacements in $x$, $y$ and $z$ direction

$v_\beta$  
Darcy’s velocity of phase $\beta$

$A_{j,j+1}$  
Interface area between neighboring continuum $j$ and $j+1$

$A_{nm}$  
Interface area between the $n^{th}$ grid block of interest and its neighbor, $m^{th}$ grid block

$B$  
Skempton coefficient

$B(r, \theta)$  
Asymptotic function

$B_{std, Hev, Tip}$  
Partial derivative operation on shape functions $N_{std, Hev, Tip}$

$C_R$  
Rock specific heat
$C_f$  
Fluid compressibility

$D$  
stress-strain relation matrix (elasticity tensor)

$D_{up}$  
Upscaled drained elasticity tensor for the MINC model element

$E_s$  
Proppant Young’s modulus

$FI$  
Fracture transmissivity index

$F^k$  
Flux term of the $k^{th}$ component in mass/balance equations

$F \cdot n_F$  
Specified mass/heat flux with the unit normal vector $n_F$ to the boundary $\Gamma_F$

$F^k_{nm}$  
Flux of the $k^{th}$ between $n^{th}$ grid block and $m^{th}$ grid block

$F_{n,p,int}$  
Force vector integrated from internal fluid pressure

$F_{n,t}$  
Force vector integrated from external traction

$H(x)$  
Heaviside function

$I$  
Unit matrix

$J$  
Jacobian matrix, transformation from local to global coordinates

$J_{tri}$  
Jacobian matrix for 2D triangles

$K$  
Bulk modulus

$K_{HF}$  
Stiffness of hydraulic fractures

$K_f$  
Fluid bulk modulus

$K_{nm}$  
Stiffness matrix of the $nm^{th}$ section

$K_s$  
Bulk modulus of rock solid grain

$K_u$  
Undrained bulk modulus

$L, L_0$  
Natural fracture spacing (current and initial)

$M$  
Biot’s modulus

$M^k$  
Accumulation term of the $k^{th}$ component in mass/energy balance equations
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{1,2,3,4,5,6,7,8}$</td>
<td>Continuous shape functions for standard or Galerkin FEM in a 3D hexahedron local coordinate system</td>
</tr>
<tr>
<td>$N_G$</td>
<td>Number of Gauss quadrature points</td>
</tr>
<tr>
<td>$NK$</td>
<td>Component number in mass/energy balance equations</td>
</tr>
<tr>
<td>$[N]$</td>
<td>Jump of shape functions</td>
</tr>
<tr>
<td>$N_{tri}$</td>
<td>Shape functions for 2D triangles</td>
</tr>
<tr>
<td>$PROX(x)$</td>
<td>Proximity function of MINC model</td>
</tr>
<tr>
<td>$S_\beta$</td>
<td>Phase $\beta$ saturation</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature</td>
</tr>
<tr>
<td>$\bar{T}$</td>
<td>Specified values of constant temperature boundary</td>
</tr>
<tr>
<td>$T_{f,i}$</td>
<td>Fracture temperature, current and initial</td>
</tr>
<tr>
<td>$W_{i,j,k}$</td>
<td>Weight of a specific Gauss integration point with the subscripts $i, j$ or $k$ indicating the dimensional axis</td>
</tr>
<tr>
<td>$X_{\beta}^k$</td>
<td>Mass fraction of component $k$ in phase $\beta$, $\beta = water$ and $X_{\beta}^k = 1$</td>
</tr>
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</table>
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CHAPTER 1
INTRODUCTION

The background, motivation and general literature review are included in this first chapter. Geothermal energy is considered clean and renewable, attracting a lot of research and development in academia. Among three types of geothermal resources, EGS has a promising potential to produce consistent and sustainable heat for electricity generation. The first section below introduces the basics of geothermal energy and why coupled THM simulation is a must in EGS development. The second section briefly summarizes various types of coupling schemes, followed by the approaches of modeling fractures in the coupled model. The last section points out that the proposed methodology is motivated by a commonly used fracture mechanics approach and what novelty is achieved specifically in this study.

1.1 Geothermal Reservoirs and Enhanced Geothermal Systems

Geothermal energy, originating from the decay of natural radioactive isotopes during the formation of the earth, is clean renewable energy that can be used in the form of direct heating or electricity generation (Blackwell et al. 2006; Ghassemi 2012; Pandey et al. 2018). The energy heats the rock and fluids within the rock, forming the conventional type of geothermal resources as summarized in Table 1.1: those hydrothermal types where an abundant amount of producible water can be extracted, generally with a temperature around 30-90 °C for direct use and 90-150 °C for both direct use and electricity production (Pandey et al. 2018). However, there are several practical limits to the corresponding geothermal exploration and development, such as the large depth of effective temperature, the insufficient amount of high-temperature water, and the conductive hydrological properties of the reservoir rocks, which might reduce the economy of geothermal energy utilization. The temperature needs to be higher than 150 °C such that the electricity generation is economical.
Lu (2018) approximated theoretically that the energy reserves in the upper 10 km of the earth’s crust are $1.3 \times 10^{27}$ J which can supply the global energy utilization for 2.17 million years based on the annual global energy consumption of $6.0 \times 10^{20}$ J/year.

However, the geothermal energy consumption in the US accounts for only 2% of the total renewable energy, with 68% utilized as electric power generation, as shown in Figure 1.1.
There is still an enormous potential for the geothermal energy utilization according to these data. 70% of the current geothermal reservoirs have a temperature lower than the economical temperature, located in a special geological location or depth such as active volcanic areas (Pandey et al. 2018).

Figure 1.2: Locations of identified hydrothermal sites, the co-located near-hydrothermal field EGS resource, and the relative favorability of the deep EGS resource (NREL 2018).

Therefore, a better heat resource is sought for accessing the abundant heat energy contained within the earth’s crust. EGS can be developed by artificially fracturing and inducing the preexisting fractures in the geothermal formations that are not as permeable as conventional resources, offering much more flexibility for exploration. Figure 1.2 shows favorable potential EGS fields across the United States. Working fluid is circulated by injection and flows through the fractures or fracture networks for heating until produced as hot water or steam, as illustrated by Figure 1.3. EGS is also named Hot Dry Rock (HDR) due to its deficiency in water content and rock permeability (Ghassemi 2012). The technology of reservoir stimulation is employed to create fractures in potential reservoirs but also increases the cost of EGS development in terms of drilling and stimulation. Ghassemi (2012) pointed out that the costs can be reduced and risks mitigated by reservoir
development technologies such as coupled THM model based reservoir simulation. The fluid flow and heat transfer are dominated by the fractures or fracture networks in EGS, and are also subject to the impact of mechanical deformation. It is emphasized in Pandey et al. (2018) that understanding the complex interaction and coupling between chemical, thermal and mechanical processes could help understand the reservoir behavior and optimize the heat recovery performances. Reservoir scale modeling is an important tool for forecasting and optimization.

Figure 1.3: EGS field: cold fluid injection into fracture networks and hot fluid produced (DOE 2016a).

1.2 Coupled Fluid Flow and Geomechanics Model and Coupled THM Model

Reservoir scale modeling plays an important role in the EGS development according to our introduction in the last section. More specifically, it is essential to couple the thermal, hydrologic, and mechanical processes in the reservoir model to accurately capture their interactions (Ghassemi and Zhou 2011; Hofmann et al. 2016; Pandey et al. 2017; Salimzadeh et al. 2018; Wang et al. 2022, 2019). Generally, fluid flow and heat transfer are driven by the pore pressure and temperature gradients respectively but these two processes
are tightly coupled. Artificial injection and production perturb the pore pressure and temperature rapidly during the EGS development. Under a confined stress state, the pore pressure field alteration causes the rock matrix deformation governed by linear elasticity (if we treat it as a linear elastic model), which in turn changes the stress/strain dependent hydrologic properties such as porosity and permeability. Thermally induced strain adds a factor to the deformation, as shown in Figure 1.4. On top of the coupling relations above, the fractures stimulated in the reservoir increase the complexity of the model by dominating the fluid flow/heat transfer. Fracture aperture is the relative distance between the rock face discontinuity and significantly depends on the rock matrix deformation thus influences the production rate and temperature.

Figure 1.4: Tightly coupled three processes: thermal, hydrologic, mechanical, under the fracture dominated fluid flow and heat transfer.

Coupling fluid flow and geomechanics (without considering heat transfer) can be achieved by several schemes: (1) fully coupled; (2) iteratively coupled; (3) explicitly coupled; (4) loosely coupled (Kim et al. 2011). Among these schemes, fully and iteratively coupled approaches are commonly implemented for accurate modeling results but they have respective advantages and disadvantages. A fully coupled model is unconditionally stable and converges relatively smoothly within a time step but requires a tremendous effort of software development on integrating the numerical discretizations of mass/energy balance with momentum balance equations (Hu et al. 2013; Huang et al. 2015; Wang and Wu 2022; Winterfeld and Wu 2016; Yu et al. 2019). An alternative flexible approach would
be iteratively executing two separate programs sequentially until convergence, each of which is responsible for one module: fluid flow/heat transfer or mechanics (Beck et al. 2020; Dana et al. 2018; Jha and Juanes 2007; Kim and Moridis 2013; Minkoff et al. 2003; Rutqvist et al. 2002). A general structure of a single iteration and data transfer procedure has been illustrated in Figure 1.5. Within one time step, such iterations would be performed until convergence. The stability of an iteratively coupled model depends on the organization of coupling modules but the effort of code development can be saved by deploying two well-developed programs. ‘Sequentially coupling’ and ‘iteratively coupling’ will be used interchangeably in this thesis, indicating the same solution process (sequential actually covers the definition of iterative and refers more toward the execution sequence of the two modules).

Figure 1.5: General structures of iteratively coupled simulators: data has been transferred between two modules for essential coupling parameters.

In this dissertation, a fully coupled model, TOUGH2-CSM (Winterfeld and Wu 2016), has firstly been extended for the capability of modeling hydraulically fractured EGS using an EDFM approach. A key assumption in this model is: the stress acting on the discrete fracture embedded is determined from the stress state of the containing primary grid (Li, S.-B. et al. 2016; Moinfar et al. 2013). Afterward, a sequentially coupled model has been developed based on TOUGH2-EGS and an XFEM code, for fluid flow/heat transfer and...
geomechanics respectively. The loose assumption made in the fully coupled model is mitigated by applying the rigorous discontinuous fracture mechanics. The sequentially coupling is implemented in a two-way iterative manner (Jha and Juanes 2014; Kim et al. 2012; Minkoff et al. 2003). Both models are validated against analytical solutions of classic coupled fluid flow and geomechanics.

1.3 Embedded Discrete Fracture Model (EDFM) and Multiple INteracting Continua (MINC)

Quite a number of models were proposed for fluid flow between fracture and matrix in naturally fractured reservoir: the classical double porosity model (Kazemi 1969; Warren and Root 1963), the Multiple-INteracting-Continua (MINC) model (Pruess 1983) and the multiple continuum model (Wu 1999; Wu et al. 2004). These models are adopted to describe a reservoir with densely and well-distributed fractures so that multiple continua (fracture and matrix) can be treated as a uniformly-distributed ‘sugar cubes’ or nested cubes, as shown in Figure 1.6. The outermost layer mostly represents the natural fractures while all the other inner layers are partitioned to approximate the uniform pressure drop towards the matrix media.

Later on, the occurrence of hydraulic fracturing technology motivated engineers and scientists to put forward a new methodology to model these long and discrete hydraulic fractures. The Discrete fracture model (DFM) (Garipov et al. 2016; Karimi-Fard et al. 2004; Kim and Deo 2000) and EDFM (Lee et al. 2001; Li and Lee 2008; Moinfar et al. 2014; Wang et al. 2019; Xu et al. 2017) were brought in to accurately capture the irregular shape and distribution of hydraulic fractures. EDFM embeds the fractures with arbitrary shapes and strikes into the reservoir grids without sacrificing the accuracy (as shown in Figure 1.7), which prevails over DFM where mesh grids are specially designed and refined in order to conform to the shape of fractures.

EDFM and MINC techniques can be integrated to handle the tight gas and oil reservoirs where both discrete hydraulic and natural fractures conduct fluid flow (Ding et al. 2018; Jiang and Younis 2016; Yan et al. 2018). Natural fractures might be
reactivated in the EGS stimulation process and need to be considered in a geothermal reservoir simulation (Baria et al. 1999; Guo et al. 2019; Huang et al. 2017; Li et al. 2019; Shi et al. 2019). It would be impossible to use EDFM-XFEM for all hydraulic and natural fractures due to the computational costs. A combined EDFM and MINC methodology for the hydraulically and naturally fractured EGS is developed for the sequentially coupled THM model and the effects of complex fracture networks on production behaviors are investigated in this dissertation.

Figure 1.6: Illustration of MINC model conceptualization: left, sugar cubes MINC model with a potential embedded discrete fracture (red line); right, naturally fractured reservoir (preexisting fractures, black lines) with a potential embedded discrete fracture (red line).

Figure 1.7: Pink polygons model the embedded discrete fractures which intersect with primary grids. Multiple fractures could also intersect with each other within a grid.
1.4 Extended Finite Element Method (XFEM) for Sequentially Coupled Model

In the fully coupled model we developed, the momentum balance equation was discretized by the Integral Finite Difference (IFD) method which is not used commonly in mechanical analysis. Most mechanics computations are performed based on the Finite Element Method (FEM) which offers a continuous displacement field. XFEM was proposed initially for fracture propagation simulation by Moës et al. (1999) and Sukumar et al. (2000) where the strong or weak discontinuity is approximated by enriching nodes of the elements that contain such discontinuity. XFEM enables us to compute the discontinuous displacement field without conforming the mesh to the surface of the discontinuity. It is obvious that EDFM is well compatible with XFEM, neither of which needs the mesh conformation. Therefore, a lot of studies coupled EDFM and XFEM to conduct hydraulic-mechanical or hydraulic fracturing modeling in artificially fractured unconventional reservoirs and geothermal reservoirs (Li et al. 2021; Ren et al. 2016; Wang et al. 2020; Yan et al. 2018). Through the enrichment of the nodes in XFEM, as shown in Figure 1.8, the relative displacement between two faces of the discontinuity can be obtained, which is exactly the fracture aperture needed for modeling fluid flow/heat transfer in fractured EGS. When the thermal effect is taken into consideration, the thermal strain on the two faces of the discontinuity would expand the fracture with cold fluid injected and increase the fracture permeability.

In the sequentially coupled model of this dissertation, XFEM is employed as the mechanical module of the coupled model. To the best of our knowledge, there are no studies that focus on a 3D coupled EDFM-XFEM model yet. The integrated EDFM-XFEM methodology provides a new approach to modeling coupled THM processes for EGS development and investigating the impacts of various parameters on the simulation results.
1.5 Structure of this Dissertation

The dissertation will be divided into seven chapters:

1. The major referenced literature will be reviewed in Chapter 2, although some background literature has been introduced in this chapter.

2. Chapter 3 will elaborate on the mathematical models including governing equations of mass/energy balance, momentum balance, and constitutive relations such as stress/strain dependent fracture/matrix permeability.

3. Numerical discretization, IFD, and XFEM will be explained in Chapter 4, as well as the algorithms of the two coupling schemes. The EDFM will also be briefly revisited, with a special focus on its relation with XFEM. The extension from EDFM-XFEM to EDFM-MINC-XFEM will be illustrated in this chapter as well.

4. Multiple computational cases for thermal, hydrologic, and mechanical modules will be run for validating the developed models in Chapter 5.

5. The fully coupled model will be applied on a synthetic intermediate-scale EGS and a series of sensitivity analyses will be performed to study the key impacts on the production behavior, in Chapter 6. This model will also be applied to the EGS-Collab field experiments to match existing simulation results.
6. In Chapter 7, the sequentially coupled model will be firstly used to simulate a synthetic intermediate-scale EGS with a dominating single fracture. Then the single-fracture model will be integrated with a MINC model to achieve Stimulated Reservoir Volume (SRV). Numerical studies will be conducted for both models to investigate key impact factors.

7. The last chapter, Chapter 8, will summarize all the work, obtain the conclusions, and propose recommendations for future work.
CHAPTER 2
LITERATURE REVIEW

The general background of the dissertation has been reviewed in Chapter 1 with several listings of literature. The literature to be reviewed in this chapter, by contrast, put more focus on those that are primarily referenced in terms of their methodology and approaches. The fracture mechanics, coupling schemes for discretely fractured continuum and multi-porosity continuum, and EGS model sensitivity studies will be recapitulated in their work.

2.1 Fracture Mechanics and Constitutive Relations

In this section, approaches of fracture handling in mechanical simulators are introduced: from the classical uniformly distributed natural fractures to the large-scale long discrete fractures. Their responses to the geomechanics are modeled by essential constitutive relations which are proposed by studies reviewed here.

2.1.1 Natural Fractures Modeled by Double or Multiple Porosity

Berryman (2002) proposed a poroelastic model for double-porosity continua and derived the mechanical constants for the two types of solid constituents. It was mentioned that multi-porosity geomechanics can be studied in a similar manner, which provides a potential extension path for the naturally fractured reservoir that is modeled by MINC. An important theory in this model is that the total stress exerted on the fracture continuum is equal to the matrix continuum under the uniform expansion and contraction scenario:

\[ \delta \sigma_v = \delta \sigma^1_v = \cdots = \delta \sigma^k_v = \cdots = \delta \sigma^{n_m}_v \]  

(2.1)

in which \( n_m \) is the number of multiple continua.

Following this fundamental work, Kim et al. (2012) determined the constitutive relations of poroelasticity/thermoelasticity and coupling coefficients between fluid flow and geomechanics, by upscaling the multi-porosity continuum properties. Fracture continuum
has a Lagrange porosity which directly depends on the mechanical deformation:

\[ \Phi_f = f(\varepsilon_v, p, T) \]  \hspace{1cm} (2.2)

where \( \Phi_f \) is the Lagrange porosity of the fracture continuum, \( \varepsilon_v \) is the volumetric strain, \( p \), \( T \) are pressure and temperature respectively. Lagrange porosity is defined as the ratio of pore volume in the deformed configuration to the bulk volume in the reference (initial) configuration (Dana et al. 2018; Garipov et al. 2018; Li, S.-B. et al. 2016; Yan et al. 2018). Yan et al. (2018) applied the MINC model with the discrete fracture model into the fixed-stress split scheme, to simulate the shale gas reservoir production where the SRV contains both hydraulic fractures and induced/natural fractures.

Rutqvist et al. (2002) developed a sequentially coupled program combining TOUGH2 and FLAC3D for deformable fractured reservoirs. In their work, the dynamic hydraulic properties of fracture and matrix were listed and explained. Those properties were directly or indirectly correlated with the normal effective stress acting on the fracture:

\[ w_f = f(\sigma'_n) \]  \hspace{1cm} (2.3)

in which fracture width or aperture \( w_f \) is dependent on the effective normal stress \( \sigma'_n \). The fractured model was also considered as a multiple porosity continuum. In the EGS simulation conducted by Wang et al. (2016), fracture aperture was also affected by thermal strain induced by the cold water injection. They used a multiple continuum model for fractured EGS as well and observed the enhanced fracture permeability due to the low-temperature injection.

The multiple porosity model or MINC model is one major target of this dissertation to improve the hydraulically fractured geothermal reservoirs by incorporating induced or natural fractures in SRV. Effects of natural fractures on EGS productions will be investigated with respect to fracture spacing, MINC partition number, and multi-continuum properties.
2.1.2 Discrete Fracture Model

When fracture geometries become more complicated, or long explicit hydraulic fractures are stimulated or induced in the reservoir, multiple continua are not accurate to reflect fracture-matrix interaction anymore. A discrete fracture model is indispensable in this scenario. Moreover, fractures are quite sensitive to mechanical deformation thus a coupled model considering fracture dynamic response is also necessary for reservoir modeling.

2.1.2.1 FEM as a Fracture Mechanics Solver: Conforming Fracture

Garipov et al. (2016) mixed FVM and FEM to achieve a coupled model for discrete fractures defined by unstructured grids. Specifically, pressures are unknowns at the center of each element including fracture elements while displacements at nodes of each element and fracture element Gauss points (computed as fracture apertures). Fractures and their associated elements take the fluid pressure and normal stress as boundary tractions. Similar approaches can be found in Jiang and Yang (2018) where they modeled shale gas production under various stress-dependent parameters such as matrix and fracture porosity and permeability.

2.1.2.2 Non-conforming Fracture

It was proposed in Moinfar et al. (2013) that embedded discrete fractures have dynamic aperture following a nonlinear relation dependent on effective normal stresses which are approximated by reservoir boundary tractions and pore pressure change. The transformation from boundary normal tractions to fracture face normal directions can be accomplished by three-dimensional stress transformation (Jaeger et al. 2007). Additionally, Li, S.-B. et al. (2016) utilized a pseudo-continuum approach in the hydraulically fractured reservoir to treat the fractures in the crushed grid as a set, as shown in Figure 2.1. The normal stresses on each fracture are calculated by the global-local transformation in a similar way to Moinfar et al. (2013)’s work. Fracture normal closure is then obtained through the constitutive relations from Barton et al. (1985). These two works inspired the fully coupled of this dissertation:
1. the discrete hydraulic fractures are embedded into the matrix grid where stresses are solved as primary variables during Newton’s iterations;

2. primary grids with fractures are treated as crushed grids where the fracture pseudo-continuum shares the same stress state as its containing grid but the normal stresses need to be calculated based on the local coordinate transformation, as shown in Figure 2.1 (a);

3. the constitutive relation of fracture aperture-normal stress is used to calculate porosity and permeability;

The second statement above is a loose assumption since the discrete fracture in this work might not be treated as a continuum when a primary grid has only one or two fracture grids. But this assumption also gives a reasonable dynamic response of the fracture aperture: enlarged by the temperature reduction and compacted by the pressure decay.

Figure 2.1: Fracture normal stresses (a) embedded fractures share the same stress state as its containing grid; (b) fracture set pseudo-continuum model in Li, S.-B. et al. (2016).

2.1.2.3 XFEM as a Fracture Mechanics Solver

The increasing application of EDFM in unconventional reservoirs has driven the combining of EDFM and XFEM since they are naturally compatible without the need to remesh or conform to the fracture shapes. XFEM was more utilized to model fracture propagation but it is also capable of providing the fracture aperture for coupled fluid flow and geomechanics modeling. Ren et al. (2016) and Ren et al. (2018) fully coupled EDFM
with XFEM for multiphase fluid flow in fractured porous media. Yan et al. (2018) incorporated XFEM with EDFM and MINC in a sequentially coupled model for a stimulated shale gas reservoir. The comparison between standard local refinement and their approach shows that EDFM-XFEM is efficient to obtain accurate results of pressure distribution and fracture apertures. An EGS model was considered in Li et al. (2021) based on XFEM and EDFM. Fracture propagation was also simulated by XFEM.

However, in the above models, only two-dimensional cases were studied. A three-dimensional EDFM-XFEM model is developed in an iteratively coupled manner in this dissertation. Any arbitrarily shaped fracture can be embedded into the reservoir. The limitation is that the current model is only capable of handling a single hydraulic fracture. This is mainly because there is no published research discussing how to compute the fracture tip and fracture tip/joint intersection enrichment in 3D space yet.

### 2.2 Coupled Model of Fluid Flow/Heat Transfer and Geomechanics

In this section, the coupling schemes of fluid/heat flow and geomechanics proposed by the researchers are reviewed. Two popular schemes are fully coupled and sequentially coupled models.

#### 2.2.1 Fully Coupled Model

Compared to the iteratively coupled model, although a fully coupled model suffers from tremendous code development effort, it provides unconditionally stability and efficient convergence. The quasi-static equilibrium equation is usually discretized by FEM in a fully coupled model while the mass/energy balance equations are discretized by FVM or IFD (Garipov et al. 2016; Li, S. et al. 2016; Li, S.-B. et al. 2016). The displacements are solved on the element vertices and the pressure/saturation/temperature at the center of the element.

Hu et al. (2013) and Winterfeld and Wu (2016) developed a fully coupled TOUGH2-EGS, TOUGH2-CSM where momentum balance equation has been discretized by IFD, the advantage of which is easier to maintain the major structure of mass/energy
balance solving. Wang et al. (2021) applied this methodology to the Carbon Dioxide Enhanced-Oil-Recovery (EOR). The momentum balance equations were transformed so that the mean stress could be treated as a primary variable in addition to pressure, saturation, and temperature. For linear elasticity and especially when the deformation is small in geo-engineering problems, this solution procedure is quite efficient. This fully coupled model has been validated against existing numerical simulations and analytical solutions (Winterfeld and Wu 2012, 2016). The fully coupled model in this dissertation is based on TOUGH2-CSM and extended to incorporate all stress tensor components (except principal stress in the z-direction) as primary variables. The details of governing equations and numerical implementations are found in Chapter 3 and Chapter 4.

2.2.2 Iteratively Coupled Model

The iteratively coupled model can be classified into different schemes with respect to the order in which fluid flow/heat transfer and geomechanics are solved in the workflow. This order and its associated algorithm have an impact on numerical stability of the sequential solutions.

2.2.2.1 Drained and Undrained Split Scheme

As mentioned earlier, the coupling approaches can be classified into four types, among which the fully coupled model and some of the iteratively coupled methods are unconditionally stable. Jha and Juanes (2007) adopted the constitutive equation that relates fluid mass content variation with volumetric strain and pressure (Biot 1941; Coussy 2004):

\[ \zeta = \frac{\delta m}{\rho_{fl,0}} = \alpha \varepsilon_v + M^{-1} \delta p \]  

(2.4)

in which \( m \) is the mass per unit bulk volume, \( \alpha \) is the Biot coefficient, \( \varepsilon_v \) is volumetric strain and \( M \) is the Biot modulus. This equation can be utilized to express the stress-strain relation in both undrained and drained forms, leading to the undrained and drained split sequential approach respectively. They employed the mixed FEM where the pressure and saturation are interpolated by piecewise constant functions within an element.
and displacement by the FEM shape functions. The interpolation approach for pressure and saturation is equivalent to the IFD and Finite Volume Method (FVM).

2.2.2.2 Fixed-Stress Split Scheme

Kim et al. (2011) analyzed the numerical stability of four strategies in the family of iteratively coupled methods, including drained split, undrained split, and the other two approaches: fixed-strain split and fixed-stress split. They proved that the undrained split and the fixed-stress split are unconditionally stable and the fixed-stress split converges faster than the undrained split. The fixed-stress split is accomplished by solving the flow problem first with total stress fixed, followed by the mechanical problem. Another form of Equation 2.4 can be derived to achieve the fixed-stress split scheme.

In the following sequential model work (Kim 2018; Kim et al. 2012; Kim and Moridis 2013; Kim et al. 2015, 2012), they extended the fixed-stress split to be capable of modeling coupled thermal-hydrologic-mechanical processes with multi-porosity continua. To incorporate the thermal-hydrologic response induced by mechanical deformation, two constitutive equations of fluid mass change and elastic system entropy change have been integrated into the governing equations. It is also emphasized in Kim et al. (2012) and Kim (2018) that the conventional fixed-stress split scheme they adopted is only a two-way coupling, ignoring the direct heat source from the geomechanical deformation. This effect is ignorable when heat capacity is large where most Earth science problems belong. TOUGH family code for fluid flow/heat transfer is coupled with mechanical analysis code in Kim et al. (2012), Kim et al. (2012), Kim and Moridis (2013) and Kim et al. (2015).

Compared to the porosity correction and fixed stress rate in the coupling algorithm in Kim et al. (2012) and Kim et al. (2015), Dana et al. (2018), Mikelić and Wheeler (2013), Liu, L.-J. et al. (2020), Liu, Y.-Z. et al. (2020) and Garipov et al. (2018) employed a variant of the fixed-stress split where total volumetric stress is fixed as constant in the flow iterations. Their method is implemented in this work and the details will be elaborated in Chapter 4.
In this dissertation, TOUGH2-EGS is coupled with XFEM to establish the coupled THM model. Jha and Juanes (2014) developed their model based on fixed-stress split by coupling Stanford’s General Purpose Research Simulator (GPRS) and PyLith as flow and mechanics simulators respectively. They pointed out that the conventional reservoir simulator needs to be modified on the accumulation term for poroelasticity. This modification was achieved by using Lagrange porosity.

2.3 Iteratively Coupled Schemes for Discrete Fractured Reservoir Model

The sequentially coupled schemes introduced above are only applicable to conventional poromechanics problems such as finite strain geomechanics and thermoporomechanics but remain unclear for the discrete fractured systems. Yoon, S. et al. (2021) and Kim (2021) proposed a fixed-stress split scheme for fracture propagation modeling where fluid flow in fractures is directly comparable to that in reservoir matrix. The fracture aperture was updated according to the pressure variation using fracture stiffness as a stabilization term. It was demonstrated that the fracture stiffness would strongly affect the convergence performance of the coupled simulation. Liu et al. (2021) also utilized the fracture normal stiffness as a stabilization term to update the fracture porosity in the flow step of the fixed-stress split algorithm:

\[
\begin{align*}
    k_{nn} &= \frac{\partial \sigma_n}{\partial \zeta_n} = \frac{(\sigma_n + k_{ni}\zeta_m)^2}{k_n\zeta_m^2} \quad (2.5) \\
    \sigma_n &= \frac{k_{ni}\zeta_n}{1 - \zeta_n/\zeta_m} \quad (2.6)
\end{align*}
\]

where \(\sigma_n\) is the normal stress on fracture, \(\zeta_n\) is the fracture normal closure, \(\zeta_m\) is the maximum fracture closure and \(k_{ni}\) is the initial normal stiffness.

In this dissertation, the above ideas are referenced to stabilize the sequential coupling process. It can be observed that the fracture stiffness term is necessary to achieve better convergence performance but does not affect the simulation results. The details will be discussed in Chapter 7.
2.4 EGS Coupled THM Modeling and Sensitivity Studies

EGS injection/production has been modeled by plenty of literature using various models: coupled hydro-thermal model, coupled THM model, coupled Thermal-Hydrologic-Mechanical-Chemical (THMC) model, with Discrete Fracture Network (DFN), EDFM, and hybrid model of hydraulic fractures and natural fractures. Most of the studies investigated the key parameters that affect the EGS development processes using sensitivity analysis.

Pandey et al. (2018) summarized numerous works and listed their modeling results. It showed that the coupled THM model usually provides a lower production after a long-term water injection due to the enhanced fracture permeability, however, this effect is also diminished by the injection rate. Sun et al. (2017) discussed how injection temperature, thermal conductivity, reservoir elastic modulus, and fracture permeability influence the production temperature. Li et al. (2021) integrated EDFM and XFEM in a 2D model and studied the effect of matrix permeability, heterogeneity, fracture aperture on the production temperature in an EGS reservoir with multiple intersected fractures. Pandey et al. (2017) compared the THM model with the TH model to study the mechanical impact on the production temperature and investigated the effect of rock matrix thermal expansion coefficient, and fracture stiffness on an EGS reservoir with a single fracture.

Hybrid fracture models were established in Yan et al. (2018), Ding et al. (2018), Jiang and Younis (2016) and Moinfar et al. (2014) to study the hydraulic and natural fracture impact on tight gas/oil reservoir production. Induced or reactivated natural fractures are also essential factors in EGS development. The increase of MINC number usually improves the modeling accuracy of a tight gas/oil reservoir, especially compared to the Double Porosity (DP) model. The production rate and accumulative gas production are key indices to compare various models. It is necessary to establish and compare such models in EGS as well.
Following the ideas of the above research, parameters such as injection temperature, matrix thermal expansion coefficient, injection rate, matrix permeability, and presence of natural fractures will be discussed in Chapter 6 and Chapter 7.
CHAPTER 3
MATHEMATICAL MODELS AND GOVERNING EQUATIONS

In this Chapter, the mathematical model and governing equations of the two coupled models will be introduced and derived. The mathematical model of fluid flow/heat transfer is the same for the fully and iteratively coupled models but this is not the case for mechanics. Hence there will be two separate subsections that explain these two mechanical models respectively. Constitutive relations control the deformation of the matrix, hydraulic fractures, and natural fractures, which dominates the coupling between fluid flow/heat transfer and mechanics. They will be explained in detail following the mathematical model. Density, viscosity, enthalpy, and specific internal energy should also be included in the constitutive relations but the reservoir fluid in this dissertation is assumed to be single-phase liquid water whose physical properties can be directly computed using existing correlations.

Prior to the details of governing equations, the basic assumptions for fluid flow/heat transfer and mechanics need to be stated and emphasized:

1. Geothermal reservoir matrix, hydraulic and natural fractures are porous media saturated by high temperature and high pressure single-phase fluid, water. The fluid flow obeys Darcy’s Law;
2. Heat transfer processes involve heat advection by fluid flow and heat conduction by temperature gradient;
3. The rock matrix and multi-porosity continuum are assumed to be linear elastic based on the small strain assumption.

3.1 Mass/Energy/Momentum Balance Equations

Three types of governing equations will be introduced in this section. The coupled THM model obviously involves three components: fluid flow governed by the mass balance equations, heat transfer by the energy balance equation, and quasi-static equilibrium by the momentum balance equation. The fully coupled model in this dissertation is built on
the parallel framework of TOUGH2-CSM (Winterfeld and Wu 2016), a coupled model of fluid/heat flow and geomechanics for THM processes. The newly developed model will be called TOUGH2-THM hereinafter. TOUGH2-CSM is established on TOUGH2-MP (Zhang et al. 2008), a massively parallel version of TOUGH2, which is used for conducting non-isothermal multi-phase multi-component fluid flow simulations. TOUGH2-CSM contains several modules of different types of reservoir fluids. The water/air module for the geothermal reservoir simulation is used in this work. The sequentially coupled model employs TOUGH2-EGS (Fakcharoenphol et al. 2013) with the same reservoir fluid module, as the Thermal-Hydrologic (TH) simulator. Since these two types of code belong to the same family, the governing equations originate from the same form. The mechanical governing equations will be explained in the following subsections: the momentum balance equation is derived from the quasi-static equilibrium equation for the fully coupled model while the iteratively coupled model directly employs the quasi-static equilibrium equation for further discretization.

3.1.1 Fluid Flow/Heat Transfer (Mass/Energy Balance)

Mass and energy are to be balanced when the system is solved and in general form, they can be expressed as this equation:

$$\frac{d}{dt} M^k = - \nabla \cdot F^k + q^k \tag{3.1}$$

In Equation 3.1, $M^k$, $F^k$ and $q^k$ are accumulation term, flow term and sink/source term respectively. For the fluid flow mass balance equation, the accumulation consists of all terms for mass calculation, i.e., the component mass in a certain phase:

$$M^k = \phi \sum_\beta S_\beta \rho_\beta X^k_\beta \tag{3.2}$$

in which $\rho_\beta$, $\phi$, $S_\beta$, are density of phase $\beta$, porosity and phase $\beta$ saturation; $X^k_\beta$ represents the mass fraction of component $k$ in phase $\beta$. In this dissertation, the model has only one phase one component with $\beta = water$ and $X^k_\beta = 1$. For the energy balance equation, the
accumulation term is computed by:

\[ M^{N^K+1} = (1 - \phi) \rho_R C_R T + \phi \sum_\beta S_\beta \rho_\beta u_\beta \quad (3.3) \]

where \( N^K + 1 \) indicates the last equation in addition to \( N^K \) component \( (N^K = 1) \) mass equations. \( \rho_R \) and \( C_R \) are the rock density and rock specific heat respectively. Temperature is denoted as \( T \). Phase \( \beta \) specific internal energy is expressed by \( u_\beta \). The above term calculates energy by summing the internal energy of both rock and fluid. Advective mass flux of component \( k \) is summed over all phases (there is only one phase in the system of this study):

\[ F^k_{\text{adv}} = \sum_\beta X^k_\beta F_\beta \quad (3.4) \]

Multiphase Darcy’s law dominates the mass flux of phase \( \beta \):

\[ F_\beta = \rho_\beta v_\beta = -k r_\beta \rho_\beta \frac{\mu_\beta}{\mu} (\nabla p_\beta - \rho_\beta \bar{g}) \quad (3.5) \]

in which Darcy’s velocity \( v_\beta \) multiplied by the phase density gives the mass fluxes. Essentially, Darcy’s velocity is calculated by pressure gradient \( \nabla p_\beta \) as well as capillary pressure and the gravity term. Multiphase flux is described using the relative permeability of phase \( \beta \), \( k r_\beta \) and its associated viscosity, \( \mu_\beta \). The single-phase fluid flow assumption gives: \( k r_\beta = 1 \) and capillary pressure is zero. Both conduction and convection are considered in the heat flux:

\[ F^{N^K+1} = -\lambda_T \nabla T + \sum_\beta h_\beta F_\beta \quad (3.6) \]

in which \( \lambda_T \) is the thermal conductivity of the rock and the phase specific enthalpy is marked by \( h_\beta \).

The boundary conditions for fluid flow/heat transfer are Dirichlet (prescribed pressure/temperature) boundary conditions:

\[ p \text{ or } T = \bar{p} \text{ on } \bar{T} \text{ or } \Gamma_p \text{ on } \Gamma_T \quad (3.7) \]

or Neumann (prescribed flux) boundary condition

\[ F \cdot n_F = \bar{F} \text{ on } \Gamma_F \quad (3.8) \]
where \( p \) and \( T \) denote pressure and temperature, with \( \bar{p} \) and \( \bar{T} \) are specified values on the boundaries \( \Gamma_p \) and \( \Gamma_T \), \( F \) is the specified mass/heat flux with the unit normal vector \( n_F \) orthogonal to the boundary \( \Gamma_F \).

### 3.1.2 Quasi-static Equilibrium (Momentum Balance)

Momentum balance is the governing equation behind the mechanics solver. Both fully and sequentially coupled models are using the same mechanics governing equation but end up with different derivations for numerical discretization. This section will focus on the fundamental quasi-static equilibrium which can be derived further for the fully coupled model and directly utilized in the sequentially coupled model.

#### 3.1.2.1 Fully Coupled Model

In the fully coupled model, we used the engineering sign convention (compression is positive and tension is negative) which is consistent with the original work of TOUGH2-CSM. In the following section for the iterative coupling, the mechanics sign convention (compression is negative and tension is positive) will be reused.

The mechanical equation starts from the linear elasticity, Hooke’s law, considering thermal and pore pressure effects:

\[
\sigma - h(p, T) I = 2G \varepsilon + \lambda (tr \varepsilon) I \tag{3.9}
\]

in which \( \sigma \) is the stress tensor and \( \varepsilon \) is the strain tensor which can be calculated by displacement vector, \( \lambda \) and \( G \) in Equation 3.9 are the Lamé parameters, \( I \) is the unit matrix and \( tr \varepsilon \) is the trace of the strain tensor. The thermal and pore pressure term is expressed as below:

\[
h(p, T) = \alpha p + 3\beta_T K (T - T_{ref}) \tag{3.10}
\]

in which \( \alpha \) is Biot’s coefficient, \( \beta_T \) is the skeleton thermal expansion coefficient and \( K \) is the bulk modulus.
The quasi-static equilibrium states that stress tensor and body force, $b$ should satisfy:

$$\nabla \cdot \sigma + b = 0$$  \hspace{1cm} (3.11)

It should be noted that $b$ is the summation of both rock and fluid:

$$b = \phi (\rho_g S_g + \rho_w S_w) + (1 - \phi) \rho_r g$$  \hspace{1cm} (3.12)

where $\rho_g$, $S_g$, $\rho_w$, $S_w$ are density and saturation of gas and water respectively; $\rho_r$ is the rock density, $\phi$ is the porosity of rock and $g$ is the gravity acceleration vector.

Combining Equation 3.9 and 3.11 leads to the thermo-poro-elastic Navier equation:

$$\nabla [h (p, T)] + (\lambda + G) \nabla (\nabla \cdot u) + G \nabla^2 u + b = 0$$  \hspace{1cm} (3.13)

Taking the divergence of Equation 3.13 yields:

$$\nabla^2 [h (p, T)] + (\lambda + 2G) \nabla^2 (\nabla \cdot u) + \nabla \cdot b = 0$$  \hspace{1cm} (3.14)

Since the divergence of displacement vector, $\nabla \cdot u$ is the volumetric strain, $\varepsilon_v$:

$$\nabla \cdot u = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$$  \hspace{1cm} (3.15)

in which $\varepsilon_{xx}$, $\varepsilon_{yy}$ and $\varepsilon_{zz}$ are the strains in $x$, $y$ and $z$ direction, $u_x$, $u_y$ and $u_z$ are displacements in the $x$, $y$ and $z$ direction. Taking the trace of Equation 3.9 gives:

$$K \varepsilon_v = \sigma_m - h (p, T)$$  \hspace{1cm} (3.16)

where $\sigma_m$ is the mean stress. Combining Equation 3.14, 3.15 and 3.16 provides (Winterfeld and Wu 2016):

$$\frac{3 (1 - \nu)}{1 + \nu} \nabla^2 \sigma_m + \nabla \cdot b - \frac{2 (1 - 2\nu)}{1 + \nu} \nabla^2 [h (p, T)] = 0$$  \hspace{1cm} (3.17)

in which $\nu$ is Poisson’s ratio. The form of this mean stress equation is much similar to Equation 3.1 but without an accumulation term. This is the reason why TOUGH2-CSM selected mean stress as an additional primary variable and employed the same spatial discretization approach for solving the mean stress as for the pressure and temperature.
Winterfeld and Wu (2016) mentioned the formulations for the stress tensor components based on the derivatives of components of Equation 3.13:

\[
\begin{align*}
\frac{\partial^2}{\partial x^2} [h(p, T)] + \frac{3}{2(1+\nu)} \frac{\partial^2}{\partial x^2} (\sigma_m - h(p, T)) + \\
\frac{1}{2} \nabla^2 \left( \sigma_{xx} - h(p, T) - \frac{3\nu}{1+\nu} (\sigma_m - h(p, T)) \right) + \frac{\partial}{\partial x} b_x = 0 \\
\frac{\partial^2}{\partial y^2} [h(p, T)] + \frac{3}{2(1+\nu)} \frac{\partial^2}{\partial y^2} (\sigma_m - h(p, T)) + \\
\frac{1}{2} \nabla^2 \left( \sigma_{yy} - h(p, T) - \frac{3\nu}{1+\nu} (\sigma_m - h(p, T)) \right) + \frac{\partial}{\partial y} b_y = 0 \\
\frac{\partial^2}{\partial z^2} [h(p, T)] + \frac{3}{2(1+\nu)} \frac{\partial^2}{\partial z^2} (\sigma_m - h(p, T)) + \\
\frac{1}{2} \nabla^2 \left( \sigma_{zz} - h(p, T) - \frac{3\nu}{1+\nu} (\sigma_m - h(p, T)) \right) + \frac{\partial}{\partial z} b_z = 0 \\
\frac{\partial^2}{\partial x\partial y} [h(p, T)] + \frac{3}{2(1+\nu)} \frac{\partial^2}{\partial x\partial y} (\sigma_m - h(p, T)) + \\
\frac{1}{2} \nabla^2 \sigma_{xy} + \frac{1}{2} (\frac{\partial}{\partial x} b_x + \frac{\partial}{\partial y} b_y) = 0 \\
\frac{\partial^2}{\partial z\partial y} [h(p, T)] + \frac{3}{2(1+\nu)} \frac{\partial^2}{\partial z\partial y} (\sigma_m - h(p, T)) + \\
\frac{1}{2} \nabla^2 \sigma_{zy} + \frac{1}{2} (\frac{\partial}{\partial z} b_z + \frac{\partial}{\partial y} b_y) = 0 \\
\frac{\partial^2}{\partial x\partial z} [h(p, T)] + \frac{3}{2(1+\nu)} \frac{\partial^2}{\partial x\partial z} (\sigma_m - h(p, T)) + \\
\frac{1}{2} \nabla^2 \sigma_{xz} + \frac{1}{2} (\frac{\partial}{\partial x} b_x + \frac{\partial}{\partial z} b_z) = 0
\end{align*}
\]

Equations 3.18-3.23 are the differential equations for stress components: \(\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{yz}, \sigma_{xz}\), known as Beltrami-Michell equations. As can be seen, these six components derived from Equation 3.13 can also be regarded as governing equations and then the stress components can be solved during Newton’s iterations. TOUGH2-THM is designed to use pressure, saturation/mass fraction, temperature, mean stress, x-normal stress, y-normal stress, and three shear stresses as primary variables.

Boundary tractions in the fully coupled model are set to be constant as the stresses of initial conditions:

\[
\sigma_{\alpha\beta} = \sigma_{\alpha\beta,0}
\] (3.24)
in which \( \alpha, \beta \in \{m, x, y, z\} \) and represent mean, \( xx, yy, zz, xy, zy, xz \) stresses. There are no prescribed displacement boundary conditions used in this model.

The main objective of making these modifications is to compute the normal stress acting on embedded discrete fractures. When embedded discrete fractures intersect with the reservoir grids, the normal stress on the fracture surface is of more concern on how the fluid flow in fractures would be affected. In this study, it is assumed that a discrete fracture embedded in a reservoir grid has the same stress state with this grid block, enabling the normal stress to be obtained by stress transformation in three-dimensional space using all stress tensor components, same as the approach used by Moinfar et al. (2013).

Embedded discrete fracture permeability is strongly dependent on normal stress and thermal strain, which could impact the stability and convergence of numerical solutions if the permeability is not updated on Newton’s iteration level. In order to avoid this unfavorable condition, the primary variable list is extended so that fracture porosity and permeability can be updated implicitly by normal stress.

3.1.2.2 Sequentially Coupled Model

In the sequentially coupled model, the mechanical equations are solved by XFEM where the quasi-static Equation 3.11 is treated as a strong form from which a weak form can be derived for discretization. The sign convention we used for stress was: negative for compression and positive for tension. Note that in the fluid flow module (TOUGH2-EGS), the sign of stress was reversed to compute the volumetric strain.

Boundary conditions for mechanics include two parts: (1) external boundaries with prescribed displacement or applied traction; (2) internal boundary fluid pressure. The traction boundary condition can be expressed as:

\[
\tau n_\Gamma \text{ on } \Gamma_t
\]  

(3.25)

where \( \tau \) is the boundary traction and \( n_\Gamma \) is the normal vector of the domain boundary on which the traction is applied. The prescribed displacement boundary is:

\[
u = \bar{\nu} \text{ on } \Gamma_u
\]  

(3.26)
where $\vec{u}$ is the prescribed displacement on the boundary $\Gamma_u$, as shown in Figure 3.1 (a).

Fluid pressure is imposed on the discontinuity, i.e., fracture faces:

$$\sigma \cdot n_{\Gamma_d}^+ = -\sigma \cdot n_{\Gamma_d}^- = -(p + p_s)n_{\Gamma_d}^+$$  \hspace{1cm} (3.27)

where the $n_{\Gamma_d}^+$ is the normal vector pointing towards the domain defined by the normal vector $n_{\Gamma_d}$ and $n_{\Gamma_d}^-$ points to the opposite direction, as shown in Figure 3.1 (b). In the program, the normal vector of the fracture plane is firstly calculated using its vertex coordinates, which is utilized as the vector pointing to the positive domain. $p$ is the fluid pressure within fracture. $p_s$ is the stress generated by a virtual proppant within the fracture. This stress will prevent the relative displacement from changing drastically and buffer the displacement of the discontinuity face (Yan et al. 2018):

$$p_s = -E_s \frac{[\vec{u}]}{d_0} \cdot n_{\Gamma_d}$$  \hspace{1cm} (3.28)

where $E_s$ is the proppant Young’s modulus and $d_0$ is the initial fracture aperture.

Compared to the definition in Yan et al. (2018), the above constitutive relation is modified by removing the condition when relative displacement is positive, i.e., the expansion of the fracture. This modification indicates that when a fracture tends to expand there will be a tension imposed on the surface avoiding the unrealistic large positive displacement, which might potentially propagate the existing fracture. $[\vec{u}]$ is the relative displacement between two faces of the discontinuity and can be expressed as:

$$[\vec{u}] = \vec{u}^+ - \vec{u}^-$$  \hspace{1cm} (3.29)

### 3.2 Constitutive Relations

Constitutive relations, including the relations between stress, strain and displacement, porosity/permeability and stress/strain, fracture responses to stress/strain are introduced, explained and derived in this section.
3.2.1 Stress, Strain and Displacement in the Sequentially Coupled Model

Part of the constitutive relations for the fully coupled model has been explained in the above section for the purpose of derivation. In this section, some of them will be revisited using the mechanics sign convention (tensile stress is positive) for the sequentially coupled model. Equations can be transformed into the fully coupled model by adding a negative sign in front of stresses and strains. Effective stress is the stress that deforms the rock matrix, removing the pore pressure support from the total stress:

\[ \sigma' = \sigma + \alpha p I \]  

(3.30)

where \( \sigma' \) is the effective stress and \( \alpha = 1 - \frac{K}{K_s} \) is the Biot’s coefficient. \( K \) and \( K_s \) are bulk modulus of rock and rock solid grain respectively. Effective stress deforms the matrix based on the thermo-elastic relation (Kim and Hosseini 2015; Li et al. 2021; Salimzadeh et al. 2018):

\[ \sigma' = D \varepsilon_t \]  

(3.31)

where \( D \) is the stress-strain relation matrix and \( \varepsilon_t \) represents the total strain in Equation 3.31. Incorporate the thermal strain caused by the temperature change:

\[ \varepsilon_t = \varepsilon - \beta_T (T - T_{ref}) \]  

(3.32)

where \( \beta_T \) is the skeleton thermal expansion coefficient of the rock matrix. Strain tensor can be computed from the displacement in the context of infinitesimal deformation, as shown...
3.2.2 Matrix Porosity and Permeability (in Fully and Sequentially Coupled Models)

Coussy (2004) described the difference between Eulerian and Lagrangian porosity: the porous space is better captured by the Lagrangian porosity which refers the current porous volume to the initial volume. In TOUGH2-CSM, the grid volume in the mass/energy accumulation term of Equation 3.1 is assembled by multiplying the discretized grid volume with the volumetric strain. Then geomechanical porosity formulation will be multiplied to complete the accumulation term. Lagrangian porosity replaces this computation by referring the initial volume, which can be simply implemented in both TOUGH2-THM and TOUGH2-EGS. Lagrangian porosity is widely used in coupled TH or THM models (Dana et al. 2018; Garipov et al. 2018; Kim et al. 2015; Li, S.-B. et al. 2016; Liu, L.-J. et al. 2020; Liu, Y.-Z. et al. 2020; Yan et al. 2018). It can be derived from the thermo-poro-elastic state equations to reach an incremental form expressed by:

\[ \delta \phi = \frac{1}{N} \delta p - 3 \beta_\phi \delta T + \alpha \delta \varepsilon_v \]  

(3.34)

by considering the volumetric strain in Equation 3.15, where \( \beta_\phi \) is the pore volumetric thermal expansion coefficient, \( N \) is the matrix Biot’s modulus and \( \alpha \) is the Biot coefficient. In Equation 3.34,

\[ \frac{1}{N} = \frac{\alpha - \phi}{K_s}, \quad \alpha = 1 - \frac{K}{K_s}, \quad \beta_\phi = \beta_T(\alpha - \phi) \]  

(3.35)

Combining Equation 3.34 and 3.35 gives Lagrangian porosity:

\[ \delta \phi = \frac{(1 - \alpha)(\alpha - \phi)}{K} \delta p - 3 \beta_\phi \delta T + \alpha \delta \varepsilon_v \]  

(3.36)

Coussy (2004) also derived the constitutive equations for the porous material in addition to the matrix skeleton and showed that the mass change per unit porous volume is:

\[ \frac{\delta m_f}{\rho_f} = \left( \frac{1}{N} + \frac{\phi}{K_f} \right) \delta p - 3(\beta_\phi + \phi \beta_f) \delta T + \alpha \delta \varepsilon_v \]  

(3.37)
in which $K_f$ is fluid bulk modulus and $\beta_f$ is fluid thermal expansion coefficient. Since the mass change can also be expressed by plugging Equation 3.34 into the accumulation mass term:

$$\delta m_f = \delta (\rho_f \phi) = \rho_f \delta \phi + \phi \delta \rho_f$$

$$= \rho_f \left( \frac{1}{N} \delta p - 3\beta_f \delta T + \alpha \delta \varepsilon_v \right) + \phi \left( \frac{\partial \rho_f}{\partial p} \delta p + \frac{\partial \rho_f}{\partial T} \delta T \right)$$

$$(3.38)$$

It is obvious that Equation 3.37 and Equation 3.38 are the same, and the term $\phi \rho_f \left( \frac{\delta p}{K_f} - 3\beta_f \delta T \right)$ is automatically computed in the fully implicit discretization through fluid property interpolation. This equivalence also indicates that the application of Lagrangian porosity implements thermo-poro-elasticity equations automatically, in both fully coupled and sequentially coupled models in this dissertation.

Permeability is usually a function of porosity given initial porosity and permeability (Liu et al. 2011) based on the Kozeny-Carman equation:

$$k_m = k_{m0} \left( \frac{\phi_m}{\phi_{m0}} \right)^3 \left( \frac{1 - \phi_{m0}}{1 - \phi_m} \right)^2$$

(3.39)

in which the subscript 0 represents initial states and $m$ indicates matrix.

Matrix Continuum in Multi-porosity Model

The multi-porosity thermo-poro-elasticity assumes volumetric stresses in Equation 3.45 are all equal for all continua, as shown in Equation 2.1 (Berryman 2002; Kim et al. 2012). The total stress in Equation 3.30 - Equation 3.32 can be rewritten as (Kim et al. 2012; Yan et al. 2018):

$$\sigma = D_{up} \epsilon + \sum_k K_{dr} \tilde{b}_k (T_k - T_{ref,k}) I + \sum_k K_{dr} b_k p_k I$$

(3.40)

where $D_{up}$ is the upscaled drained elasticity tensor for the MINC model/element, $b_k$ is the coupling coefficient between fluid flow and mechanics for the $k^{th}$ continuum of the MINC model while $\tilde{b}_k$ is between heat transfer and mechanics, and $K_{dr}$ is the volume-weighted harmonic average drained bulk modulus of the subelements.
Kim et al. (2012) derived the coupling coefficient, $\tilde{b}_k$ and $b_k$ in Equation 3.40 for each continuum and the weighted average bulk modulus:

$$b_k = -\alpha_k \frac{\eta_k}{K_k}$$  \hspace{1cm} (3.41)$$

$$\tilde{b}_k = -3K_{dr} \beta_{T,k} \frac{\eta_k}{K_k}$$  \hspace{1cm} (3.42)$$

$$\frac{1}{K_{dr}} = \sum_{k=1}^{n_m} \frac{\eta_k}{K_k}$$  \hspace{1cm} (3.43)$$

in which $\alpha_k$, $K_k$, $\beta_{T,k}$ are Biot coefficient, drained bulk modulus, and skeleton thermal expansion coefficient of the $k^{th}$ continuum respectively. $\eta_k$ is volume fraction of the $k^{th}$ continuum. The upscaled drained elasticity modulus tensor is also obtained as:

$$D_{up} = K_{dr} \sum_{k=1}^{n_m} \frac{\eta_k}{K_k} D_k$$  \hspace{1cm} (3.44)$$

in which $D_k$ is the drained modulus tensor of the $k^{th}$ continuum in the MINC model.

Equations 3.40 - 3.44 replace Equations 3.30 - 3.32 in the SRV where MINC model is applied. Accordingly, the porosity will also be calculated in a different manner from Equation 3.36. Lagrangian porosity of each continuum has been derived and applied in Kim et al. (2012), Kim et al. (2015) and Yoon, H. C. et al. (2021):

$$\delta \phi_k = \left( \frac{\alpha_k^2}{K_k} + \frac{\alpha_k - \phi_k}{K_{s,k}} \right) \delta p_k - 3\beta_{T,k} \alpha_k \delta T_k - \frac{b_k}{\eta_k} \delta \sigma_v^k$$  \hspace{1cm} (3.45)$$

Equation 3.45 is utilized both within flow iterations and after the mechanical solver updates total stresses:

1. The flow iteration assumes fixed stress where $\delta \sigma_v^k$ equals to zero, which simplifies the equation to be dependent on the pressure and temperature only;

2. After mechanical solver is invoked, stress of each continuum will be updated:

$$\delta \sigma_v^k = \delta \sigma_v^{k,l+1} - \delta \sigma_v^{k,l} \text{ where } l \text{ represents the } l^{th} \text{ coupling iteration level; }$$

Permeability of the MINC continuum can also be computed by the Kozeny-Carman Equation 3.39 except for the natural fracture which is the outermost subelement of a MINC element. The permeability of natural fracture will be discussed in the next section.
3.2.3 Fracture Porosity and Permeability

In comparison to the last subsectin, the fracture porosity and permeability in response to the stress/strain are derived in this section, for both sequentially and fully coupled models.

3.2.3.1 Sequentially Coupled Model

Hydraulic Fractures

In the sequentially coupled model, XFEM provides the fracture aperture results so that the fracture porosity and permeability can be directly related to the aperture \([u]\):

\[
\phi_f = \phi_{f0} \frac{d_0 + [u] \cdot n_{r_d}}{d_0} \quad (3.46)
\]

\[
k_f = k_{f0} \left( \frac{d_0 + [u] \cdot n_{r_d}}{d_0} \right)^2 \quad (3.47)
\]

where \(n_{r_d}\) is the normal vector of the fracture face.

Natural Fractures

Natural fracture porosity obeys Equation 3.45 but the permeability is a special case since it is strongly dependent on the fracture spacing. Yan et al. (2018) provides an approach to compute natural fracture permeability which has been extended to the 3D case:

\[
1 + \varepsilon_f = \frac{V_f}{V_{f0}} = \frac{L^3 - (L - d_f)^3}{L_0^3 - (L_0 - d_{f0})^3} = \frac{d_f (3L^2 - 3Ld_f + d_f^2)}{d_{f0} (3L_0^2 - 3L_0d_{f0} + d_{f0}^2)} \quad (3.48)
\]

in which \(L\) and \(L_0\) are current and initial fracture spacing respectively, \(d_f\) and \(d_{f0}\) are the current and initial natural fracture apertures. Since the relation \(\varepsilon_f = \varepsilon_v K_{dr}/K_f\) (Yan et al. 2018) and \(k_f \propto d_f^3/L\) (Pruess 1983; Witherspoon et al. 1980) are given,

\[
k_f = k_{f0} \left( \frac{d_f}{d_{f0}} \right)^3 \frac{L_0}{L} = k_{f0} \left( 1 + \frac{\varepsilon_v K_{dr}}{K_f} \right)^3 (1 + \varepsilon_v)^{-\frac{3}{2}} \quad (3.49)
\]

in which \(k_{f0}\) is the initial permeability of the natural fracture. Equation 3.49 is used to update natural fracture permeability based on the volumetric strain obtained from the mechanics solver.
### 3.2.3.2 Fully Coupled Model

*Fracture Aperture Response to Normal Stresses*

In the fully coupled model, the fracture aperture is a stress-dependent value (Min et al. 2004; Rutqvist et al. 2002):

$$b = b_i + \Delta b = b_i + b_{\text{max}} \left( e^{-d\sigma'_n} - e^{-d\sigma'_i} \right)$$  \hspace{1cm} (3.50)

in which $b$ is the current aperture under normal effective stress $\sigma'_n$, $b_i$ is the initial aperture under the initial normal effective stress $\sigma'_i$, $b_{\text{max}}$ is the maximum mechanical aperture, and $d$ is a coefficient measured by the laboratory. Figure 3.2 shows the fracture aperture response to the normal stresses acting on the fracture face, with parameters provided in Table 3.1. Fracture aperture increases rapidly when the tensile stress is exerted. Barton et al. (1985) correlated fracture closure with normal stress:

$$\Delta b = \frac{\Delta \sigma'_n}{k_n - \frac{\Delta \sigma'_n}{\Delta b_{\text{max}}}}$$  \hspace{1cm} (3.51)

in which $\Delta b$ is the change of fracture aperture, $\Delta b_{\text{max}}$ is the maximum closure and $k_n$ is the fracture stiffness.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Aperture</td>
<td>1.5×10^{-5}</td>
<td>m</td>
</tr>
<tr>
<td>Mechanical Aperture</td>
<td>4×10^{-5}</td>
<td>m</td>
</tr>
<tr>
<td>Stress Coefficient, $d$</td>
<td>$1\times10^{-8}/2\times10^{-8}/3\times10^{-8}$</td>
<td>1/Pa</td>
</tr>
</tbody>
</table>

Given the aperture of a fracture, fracture permeability can be calculated by:

$$k_f = \frac{b^2}{b_i^2} k_i$$  \hspace{1cm} (3.52)

and porosity:

$$\phi_f = \frac{b}{b_i} \phi_i$$  \hspace{1cm} (3.53)

in which the subscript $i$ indicates the initial condition and $f$ denotes the current state.
Figure 3.2: Fracture aperture change under compressive(positive)/tensile(negative) stresses with various coefficients.

**Fracture Aperture Response to Thermal Strain**

Compared to the sequentially coupled model where fracture aperture can be directly computed from displacement solution, the fully coupled model needs to consider the thermal stress due to the cold water injection. A key assumption of the fully coupled model is that during the solution process, embedded discrete fracture elements share the same stress state as their containing reservoir grid block. Hence, the normal stress on a fracture surface becomes a secondary parameter and can be calculated by the stress transformation in the three-dimensional space according to Jaeger et al. (2007):

\[
\sigma_{z'z'} = l_{31}^2 \sigma_{xx} + l_{32}^2 \sigma_{yy} + l_{33}^2 \sigma_{zz} + 2l_{31}l_{32} \sigma_{xy} + 2l_{31}l_{33} \sigma_{xz} + 2l_{32}l_{33} \sigma_{yz} \tag{3.54}
\]

where subscript \(z'z'\) is the normal direction of fracture face after stress transformation, and \(l_{31}, l_{32}\) and \(l_{33}\) are related to the longitudinal \((\lambda_y)\) and zenith angles \((\theta_y)\) of the coordinate.
transformation:
\[ l_{31} = \sin \theta_g \cos \lambda_g, \ l_{32} = \sin \theta_g \sin \lambda_g, \ l_{33} = \cos \theta_g \]  (3.55)

Longitudinal and zenith angles can be specified by the plane normal vector when
generating EDFM mesh, and then transferred to the coupled model for simulation. As can
be seen in Equation 3.54, the normal stress on fractures is dependent on the normal shear
stresses of the matrix grid block. This explains, again, the reason why the primary variable
list needs to be extended.

Normal stress on a fracture face is a total stress without accounting for the pore
pressure and thermal stress yet. Wang et al. (2016) modeled fracture aperture change by
computing the matrix displacement. McDermott and Kolditz (2006) pointed out that
fracture face is being ‘pulled’ by rock matrix when the temperature is reduced, and the
thermal stress can be expressed by:

\[ \sigma_{thermal} = 3 \beta_{Tf} K_f (T_f - T_i) \]  (3.56)

in which \( \beta_{Tf} \) and \( K_f \) are thermal expansion coefficient and bulk modulus of matrix grid
and \( T_f \) and \( T_{fi} \) are current and initial temperatures of fracture element. The fracture
thermal expansion coefficient and bulk modulus are set to be equal to the matrix in order
to model the ‘pulling’ by the matrix. The thermal expansion coefficient and bulk modulus
won’t affect the hydraulic property calculation since the porosity and permeability are both
directly dependent on the normal stresses. Temperature reduction generates a negative
(tensile in engineering sign convention) stress on the contact surface between matrix and
surface, as shown in Figure 3.3. Normal effective stress on fracture surface now becomes:

\[ \sigma'_n = \sigma_{z'z'} - \alpha_f p_f + \sigma_{thermal} \]  (3.57)
Figure 3.3: Normal stress and thermal stress on red fracture plane: yellow arrow points towards the direction of stress; blue arrows represent total stress of matrix grid block. Fracture red and green share the same stress state with the box but the stress transformation will provide different normal stresses for them.
CHAPTER 4
NUMERICAL DISCRETIZATION AND COUPLING ALGORITHM

In this chapter, numerical discretization for fluid flow, heat transfer, and mechanics will be discussed. The fluid flow and heat transfer governing equations, i.e. mass and energy balance equations, are discretized using the IFD method in space, a similar manner to the Finite Volume Method (FVM), both of which ensure the mass/energy conservation. The momentum balance equation can be discretized by either IFD or FEM. In the model where IFD is employed for fluid flow/heat transfer spatial discretization, the same approach used towards mechanics will save a lot of programming efforts and will be considered accurate when the strain is assumed to be small and finite. FEM, on the other hand, providing continuous displacement space, is widely accepted by academia and industry to solve mechanical deformation. XFEM, specially proposed for the discontinuous mechanics based on FEM, has also become a powerful method in fractured reservoir modeling, such as hydraulic fracture propagation. The sequentially coupled approach enables us to combine two programs for fluid flow/heat transfer and mechanics respectively without a tremendous programming effort.

The IFD discretization for the fluid flow/heat transfer and mechanics in the fully coupled model will be illustrated first and then the XFEM discretization for the mechanics in the sequentially coupled model will be discussed. The EDFM will be briefly reviewed in terms of its implementation and compatibility with XFEM afterward. The single hydraulic fracture model is extended to incorporate natural or induced fractures by introducing the MINC model into the sequentially coupled model. The meshing steps, fluid flow/heat transfer, and mechanics implementation related to MINC will be demonstrated following EDFM. In the end, the coupling algorithm for the sequentially coupled approach, and the program structure for the fully coupled model will be explained following the fracture (EDFM and MINC) model.
4.1 Numerical Discretization

The approaches for numerical discretization are demonstrated in this section. The fluid flow/heat transfer equations are discretized by IFD in both fully and sequentially coupled model. The mechanics, however, are discretized in different methods: IFD for the fully coupled model and FEM for the sequentially coupled model.

4.1.1 Fully Coupled Model

TOUGH2-THM discretizes the space and time using IFD and fully-implicit backward Finite Difference (FD) method respectively. IFD approach ensures the local mass and energy conservation. The governing equation of geomechanics can be discretized by IFD as well. The general form of governing equation, derived from Equation 3.1 by applying IFD, can be expressed as:

\[
\frac{d}{dt} \int_{\Omega} M^k d\Omega = \int_{\Gamma} F^k \cdot n d\Gamma + \int_{\Omega} q^k d\Omega \quad (4.1)
\]

where \(\Omega\) is the volume of an integration element, \(\Gamma\) is its surface area, \(n\) is the normal vector pointing outwards on the surface. A further step of discretizing Equation 4.1 achieves:

\[
\frac{d}{dt} \left( M^k_n V_{n,0} \left( 1 - \varepsilon_{v,n} \right) \right) = \sum_m A_{nm} F^k_{nm} + q^k_n V_{n,0} \left( 1 - \varepsilon_{v,n} \right) \quad (4.2)
\]

in which \(V_{n,0}\) is the grid block area at zero strain, \(A_{nm}\) is the contact area at zero strain between the \(n^{th}\) grid block of interest and its neighbor, \(m^{th}\) grid block, \(F^k_{nm}\) is the flux between the \(n^{th}\) grid block and the \(m^{th}\) grid block, \(\varepsilon_{v,n}\) is the volumetric strain calculated from x, y and z normal strains. However, note that the mathematical derivation shown in Equations 3.34-3.38 proves that the Lagrangian porosity can replace the volumetric strain in the accumulation term. Hence, Equation 4.2 can be written as the conventional form (\(\phi\) is in \(M^k_n\)) if the Lagrangian porosity formula is used:

\[
\frac{d}{dt} \left( M^k_n V_{n,0} \right) = \sum_m A_{nm} F^k_{nm} + q^k_n V_{n,0} \quad (4.3)
\]

The approach of IFD can be demonstrated by Figure 4.1.
Figure 4.1: Discretized grid and its relationship with neighbors: demonstrating how flux flow in/out to the grid and flux between grid n and grid m.

IFD approximates the advective mass flux for component k using Darcy’s law, as shown in the following formulation:

\[ A_{nm} F_{nm}^k = \sum_{\beta} -k_{nm} \left( \frac{k_{r\beta} \rho_\beta X_\beta^k}{\mu_\beta} \right)_{nm} \left( \frac{p_n + p_{c\beta,n} - p_m - p_{c\beta,m}}{D_n + D_m} - \rho_{\beta,nm} g_{nm} \right) A_{nm} \]  

(4.4)

which represents the multiphase Darcy’s law. In this dissertation, the single-phase is presumed so \( k_{r\beta} = 1, \ X_\beta^k = 1 \) and \( p_{c\beta} = 0 \). Similarly, heat flux can be expressed by:

\[ A_{nm} F_{nm}^{NK+1} = \lambda_{nm} \frac{T_n - T_m}{D_n + D_m} A_{nm} + \sum_{\beta} -k_{nm} \left( \frac{k_{r\beta} \rho_\beta X_\beta^k}{\mu_\beta} \right)_{nm} \left( \frac{p_n + p_{c\beta,n} - p_m - p_{c\beta,m}}{D_n + D_m} - \rho_{\beta,nm} g_{nm} \right) h_\beta A_{nm} \]  

(4.5)

in which, again, \( h_\beta \) is the specific enthalpy of phase \( \beta \).

Discretization of stress equations is slightly different from flow and heat equations. Aiming to take heterogeneity into consideration, stress equations are discretized in a manner called the flux and interface approach. This approach will be explained here by taking mean stress, x-direction normal stress and xz shear stress as examples. As was mentioned above, Equation 3.17 governs the mean stress and can be written in the discretized form similar to Equation 4.1 without accumulation term:

\[ \int_{\Gamma} P^{NK+2} \cdot n d\Gamma = 0 \]  

(4.6)
in which
\[ F^{NK+2} = \frac{3(1-\nu)}{1+\nu} \nabla \sigma_m + b - \frac{2(1-2\nu)}{1+\nu} \nabla [h(p,T)] \quad (4.7) \]

After the numerical integration, Equation 4.6 becomes:
\[ \sum_m A_{nm} \Psi_{\sigma, nm} = 0 \quad (4.8) \]

The mean stress flux is defined as:
\[ \Psi_\sigma = \frac{3(1-\nu)}{1+\nu} \nabla \sigma_m + b - \frac{2(1-2\nu)}{1+\nu} \nabla [h(p,T)] \quad (4.9) \]

As a result, there is a stress flux from \(n^{th}\) grid to the contact interface, and a stress flux from the interface to \(m^{th}\) grid, which are defined as \(\Psi_{\sigma^+}\) and \(\Psi_{\sigma^-}\) respectively along the normal vector direction. \(\Psi_{\sigma^+}, \Psi_{\sigma^-}\) can be computed using \(m^{th}\) and \(n^{th}\) grid properties as follows:
\[ \Psi_{\sigma,+} = \frac{3(1-\nu_+)}{1+\nu_+} \frac{\sigma_{m,+} - \sigma_{m,int}}{s_+} + b_+ - \frac{2(1-2\nu_+)}{1+\nu_+} \frac{h(p,T)_+ - h(p,T)_{+,int}}{s_+} \quad (4.10) \]
\[ \Psi_{\sigma,-} = \frac{3(1-\nu_-)}{1+\nu_-} \frac{\sigma_{m,int} - \sigma_{m,-}}{s_-} + b_- - \frac{2(1-2\nu_-)}{1+\nu_-} \frac{h(p,T)_{-,int} - h(p,T)_-}{s_-} \quad (4.11) \]

where + and − represent \(m^{th}\) and \(n^{th}\) grids respectively and \(s_+\) and \(s_-\) are used to replace \(D_m\) and \(D_n\) for an alternative description of the two-grid connection. \(h(p,T)_{+,int}\) and \(h(p,T)_{-,int}\) are two thermal and pressure terms evaluated at two sides of the interface (using their respective properties).

Equation 4.10 and 4.11 are combined to solve for \(\sigma_{m,int}\), under the condition of \(\Psi_{\sigma,+} = \Psi_{\sigma,-} = \Psi_{\sigma}\), resulting in:
\[ \Psi_{\sigma} = \frac{\sigma_{m,+} - \sigma_{m,-} + \frac{s_+ (1+\nu_+)}{3(1-\nu_+)} b_+ + \frac{s_- (1+\nu_-)}{3(1-\nu_-)} b_-}{\frac{s_+ (1+\nu_+)}{3(1-\nu_+)} + \frac{s_- (1+\nu_-)}{3(1-\nu_-)}} \quad (4.12) \]
\[ \sigma_{m,int} = \sigma_{m,+} - \frac{s_+ (1+\nu_+)}{3(1-\nu_+)} \left( \Psi_{\sigma} - b_+ + \frac{2(1-2\nu_+)}{1+\nu_+} \frac{h(p,T)_+ - h(p,T)_{+,int}}{s_+} \right) \quad (4.13) \]
where $\sigma_{m,\text{int}}$ is the mean stress at the interface. Normal stresses and shear stresses can be discretized as well by the same approach as Equation 3.18. Examples below are shown for x-direction normal stress only and y-direction normal stress shares the same formulations:

\[
\Psi_{xx} = \sigma_{xx,+} - \sigma_{xx,-} + 2s_+ b_{x,+} + 2s_- b_{x,-} - \frac{3\nu_+}{1+\nu_+} (\sigma_{m,+} - \sigma_{m,\text{int}}) - \frac{3\nu_-}{1+\nu_-} (\sigma_{m,\text{int}} - \sigma_{m,-}) +
\]

\[
\frac{3s_+}{1+\nu_+} \frac{\sigma_{m,+} - \sigma_{m,\text{int}}}{x_+} + \frac{3s_-}{1+\nu_-} \frac{\sigma_{m,\text{int}} - \sigma_{m,-}}{x_-} +
\]

\[
\frac{2\nu_+ - 1}{1 - \nu_+} (h(p, T)_+ - h(p, T)_{+,\text{int}}) + s_+ \frac{2\nu_+ - 1}{1 - \nu_+} \frac{h(p, T)_+ - h(p, T)_{+,\text{int}}}{x_+} +
\]

\[
\frac{2\nu_- - 1}{1 - \nu_-} (h(p, T)_{-,\text{int}} - h(p, T)_-) + s_- \frac{2\nu_- - 1}{1 - \nu_-} \frac{h(p, T)_{-,\text{int}} - h(p, T)_-}{x_-} +
\]

\[
\frac{2}{2(s_+ + s_-)}
\]

(4.14)

where $x_+$ and $x_-$ terms exist only when x-direction connections are under calculation, otherwise the related terms are regarded as zero. Equation 4.14 illustrates that the interface mean stress facilitates the computation of x-direction normal stress flux and so do the interface pressure and temperature. In TOUGH2-CSM and TOUGH2-THM, interface quantities are updated at each Newton iteration level by Equation 4.12 and 4.13 for stress flux assemblage.

\[\text{Figure 4.2: Schematic plot of two neighbors in x-direction for shear stress flux illustration:} \ A, B, C, D \text{are faces of grid II, B is the interface between II and EE.}\]
On the other hand, shear stress $\sigma_{xy}$, $\sigma_{yz}$ and $\sigma_{xz}$ have a slightly different form as shown in Equations 3.18 - 3.23. Based on Equations 3.21 - 3.23 and the shear stress flux between two grids: one flux from one grid center to the common interface is equal to the other flux from the interface to the other grid center, the interface shear stress ($\sigma_{xz}$ as an example) can be eliminated and the flux is expressed as:

$$\Psi_{xz} \cdot n = \frac{\sigma_{xz,+} - \sigma_{xz,-}}{s_+ + s_-} + \frac{1}{s_+ + s_-} \left[ s_+ \frac{\partial h(p,T)_+}{\partial z} + \frac{3s_+}{2(1+\nu_+)} \frac{\partial (\sigma_{m,+} - h(p,T)_+)}{\partial z} + s_+ b_{z,+} + \right. \\
\left. s_- \frac{\partial h(p,T)_-}{\partial z} + \frac{3s_-}{2(1+\nu_-)} \frac{\partial (\sigma_{m,-} - h(p,T)_-)}{\partial z} + s_- b_{z,-} \right] \mathbf{i} \cdot n +$$

(4.15)

in which $\mathbf{i}$ and $\mathbf{k}$ are the unit vectors in the x and z direction. The partial derivative term, such as $\frac{\partial h(p,T)_+}{\partial z}$, needs to be calculated on one of the grid:

$$\frac{\partial F}{\partial z} = \frac{F_C - F_D}{d_{CD}}$$

(4.16)

in which the subscript $C$ and $D$ denote the faces as shown in Figure 4.2, $F$ is the term to be evaluated and $d_{CD}$ is the distance between face $C$ and $D$ (same as the grid dimension in z-direction). When the flux of shear stress $\sigma_{xz}$ is calculated between two grids, $II$ and $EE$, which are neighbors in the x direction as shown in Figure 4.2. Grid $EE$ is the ‘+’ grid so

$$\frac{\partial h(p,T)_+}{\partial z} = \frac{h(p,T)_{EE,C} - h(p,T)_{EE,D}}{\Delta z_{EE}}$$

(4.17)

where the interface properties are needed, such as $C$ and $D$, and $d_{CD} = \Delta z_{EE}$. Therefore the discretized Equation 4.15 is derived as Equation 4.18, in which is the flux of shear stress $xz$ for grid $EE$ in the x-direction. In Equation 4.18, $s_+ = d_{EI}$ and $s_- = d_{IE}$. For grid $EE$, there are up to 6 faces on which the summation of a shear stress flux is zero. The assemblage of shear stress flux is computed using a formula like Equation 4.18. Other shear stresses $\sigma_{yz}$ and $\sigma_{xy}$ are discretized in the same forms of flux as above.
\[
\Psi_{xz,EE} \cdot i = \frac{\sigma_{xz,EE} - \sigma_{xz,II}}{d_{EI} + d_{IE}} + \frac{1}{d_{EI} + d_{IE}} \\
\left[ d_{EI} \left( \frac{h(p, T)_{EE,C} - h(p, T)_{EE,D}}{\Delta z_{EE}} + \frac{3}{2(1 + \nu_{EE})} \left( \sigma_{m,EE,C} - h(p, T)_{EE,C} \right) - \left( \sigma_{m,EE,D} - h(p, T)_{EE,D} \right) + b_{z,EE} \right) + \\
d_{IE} \left( \frac{h(p, T)_{II,C} - h(p, T)_{II,D}}{\Delta z_{II}} + \frac{3}{2(1 + \nu_{II})} \left( \sigma_{m,II,C} - h(p, T)_{II,C} \right) - \left( \sigma_{m,II,D} - h(p, T)_{II,D} \right) + b_{z,II} \right) \right] \cdot i
\]

4.1.2 Sequentially Coupled Model

A mixed FEM is utilized to discretize the sequentially coupled model, that is, the pressure, saturation, and temperature as primary variables of fluid flow/heat transfer equations are interpolated at the center of the mesh grid whereas the displacements are interpolated on the mesh grid nodes. The shape function of pressure, saturation, and temperature is 1 and constant within the grid but 0 outside, making this interpolation equivalent to a mass-conservative FVM or IFD method. The shape function of displacement is continuous across the grid. Once the shape function is the same for test functions and trial solutions, the discretization is deemed as the Galerkin FEM.

Since TOUGH2-EGS is used as the flow simulator in the sequentially coupled model, the numerical discretization for fluid flow/heat transfer is the same as what has been introduced in the previous subsection. In this subsection, more focus will be put on the mechanics simulator in which 3D XFEM is employed.

4.1.2.1 Shape and Enrichment Functions

The continuous shape functions \( N(x) \) for standard or Galerkin FEM in a 3D hexahedron local coordinate system are used for the eight nodes of a 3D hexahedron element to interpolate the test functions and trial solutions in FEM, hence the displacement \( u \) at any position \( x \) within 3D space is expressed by:

\[
u(x) = \sum_{i=1}^{8} N_i(x) u_i
\]
XFEM enriches the Equation 4.19 by two types of discontinuous functions: (1) Heaviside jump functions and (2) crack tip asymptotic functions (Khoei 2014; Sukumar et al. 2000).

Heaviside functions are defined by determining which side of the discontinuity the point of interest is located on. The signed distance function is expressed as:

$$
\varphi(x) = \min \|x - x^*\| \text{sign} ((x - x^*) \cdot n_{\Gamma_d})
$$

(4.20)

where \(x^*\) is the point on the discontinuity with the shortest distance to the point of interest, \(n_{\Gamma_d}\) is the unit normal vector of the discontinuity in Figure 3.1. Given Equation 4.20, Heaviside function can be defined as:

$$
H(x) = \begin{cases} 
-1, & \text{if } \varphi(x) < 0 \\
1, & \text{if } \varphi(x) > 0
\end{cases}
$$

(4.21)

As shown in Figure 3.1, once the normal vector, \(n_{\Gamma_d}\) has been determined from the vertices of the fracture plane, the Heaviside function can be calculated using this vector.

On the other hand, crack tip asymptotic functions are defined in the local coordinate of the crack tip. In 3D space, the fracture/discontinuity can be treated as a plane, as shown in Figure 4.3. The plane normal vector points to the positive space of the domain and the global coordinate system XYZ can be rotated around axis x and y to achieve that the XY plane aligns with the fracture plane, i.e., the Z-axis aligns with the normal vector, \(n_{\Gamma_d}\).

With the rotational angle, \(\beta\) and \(\gamma\) around Y- and X-axis computed, the rotation matrix \(R\) from global coordinate system XYZ to local coordinate system X’Y’Z’ can be assembled by:

$$
R = \begin{bmatrix}
cos\alpha & -sin\alpha & 0 \\
sin\alpha & cos\alpha & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
cos\beta & 0 & sin\beta \\
0 & 1 & 0 \\
-sin\beta & 0 & cos\beta
\end{bmatrix} \begin{bmatrix}
1 & 0 & 0 \\
0 & cos\gamma & -sin\gamma \\
0 & sin\gamma & cos\gamma
\end{bmatrix}
$$

(4.22)

where \(\alpha\) can be set to zero. If the point of interest is marked as P in Figure 4.3, the projected point of P can be located on the plane, P’, from which the closest point, O’, on the fracture plane boundary can be found (origin of X’Y’Z’). Lastly, the vector O’P’ can facilitate the computation of \(\alpha\) which is the rotation around the Z-axis. Note that the
objective of Z-axis rotation is that O’X’ is approximately tangent to the discontinuity boundary.

Figure 4.3: Schematic plot of a discontinuous plane in 3D space: x’y’z’ is the local coordinate system of the fracture tip and r and θ transform the local coordinate system into a polar coordinate system.

The rotation matrix is helpful when the derivative of the asymptotic function is needed, which will be elaborated later. The crack tip asymptotic function within the polar coordinate system is written as:

\[ \mathcal{B}(r, \theta) = \{ B_1, B_2, B_3, B_4 \} = \left\{ \sqrt{r} \sin \frac{\theta}{2}, \sqrt{r} \cos \frac{\theta}{2}, \sqrt{r} \sin \frac{\theta}{2} \sin \theta, \sqrt{r} \cos \frac{\theta}{2} \sin \theta \right\} \]  

in which r and θ are parameters as shown in Figure 4.3.

4.1.2.2 Nodal Enrichment

Heaviside and crack tip asymptotic functions are utilized to enrich the standard FEM nodes of which elements contain the fractures. The elements that are completely cut through by fractures need to be enriched by Heaviside functions and those containing the
fracture tips enriched by asymptotic functions, as shown in Figure 1.8 and Figure 4.4. The enriched displacement field is:

\[
\mathbf{u}(x) = \sum_{i=1}^{N} N_i(x) \mathbf{u}_i + \sum_{j=1}^{M} N_j(x) (H(x) - H(x_j)) \bar{a}_j + \sum_{k=1}^{K} N_k(x) \sum_{\alpha=1}^{4} (B_{\alpha}(x) - B_{\alpha}(x_k)) \bar{b}_{\alpha k}
\]  

(4.24)

where \(N\) is the number of the standard FEM nodes (8 in the 3D hexahedron element), \(M\) is the number of Heaviside enriched nodes and \(K\) is the number of crack tip enriched nodes. The subtraction of \(H(x_j)\) and \(B_{\alpha}(x_k)\) is to maintain the displacement value unchanged at element nodes. \(\bar{a}_j\) and \(\bar{b}_{\alpha k}\) are additional degrees of freedom which describe the displacement across the discontinuity.

![Figure 4.4: 3D node enrichment within a domain containing a fracture plane, circle: Heaviside enrichment; star: asymptotic enrichment. Left: 3D view, right: top view](image)

**4.1.2.3 Weak Form of the Governing Equations**

The weak form of the governing Equation 3.11 is derived by multiplying a test function and integrating it with respect to the domain or boundary surface, or by the minimum potential energy approach. The arbitrariness of the test function keeps the trial solution exact but simplifies the solution process by reducing the order of differential equation, and it has been proved that the weak form is equivalent to the strong form, Equation 3.11. The
weak form can be written as:

\[ \int_\Omega \delta \varepsilon : \sigma \mathrm{d}\Omega + \int_{\Gamma_d} \delta \mathbf{u} \mathbf{n} \cdot \mathbf{n} \mathrm{d}\Gamma = \int_{\Gamma_t} \delta \mathbf{u} \cdot \mathbf{\tau} \mathrm{d}\Gamma + \int_\Omega \delta \mathbf{u} \cdot \mathbf{b} \mathrm{d}\Omega \]  \hspace{1cm} (4.25)

where \( \delta \varepsilon \) comes from the test function multiplied and \( \llbracket \mathbf{u} \rrbracket \) is the jump in the displacement field across the discontinuity face:

\[ \llbracket \mathbf{u} \rrbracket = \mathbf{u}(x^+) - \mathbf{u}(x^-) \]  \hspace{1cm} (4.26)

If the constitutive relations, Equation 3.30 - 3.32, are plugged into the Equation 4.25, the following equations will be obtained (Li et al. 2021; Ren et al. 2018; Yan et al. 2018):

\[ \int_\Omega \delta \varepsilon : \mathbf{D} \varepsilon \mathrm{d}\Omega = \int_\Omega \delta \varepsilon : \alpha \mathbf{p} \mathbf{I} \mathrm{d}\Omega + \int_\Omega \delta \varepsilon : \beta \mathbf{T} (\mathbf{T} - \mathbf{T}_{\text{ref}}) \mathbf{I} \mathrm{d}\Omega + \int_{\Gamma_d} \llbracket \delta \mathbf{u} \rrbracket \alpha (p + p_s) \mathbf{n} \mathbf{r}_d \mathrm{d}\Gamma + \int_{\Gamma_t} \delta \mathbf{u} \cdot \mathbf{\tau} \mathrm{d}\Gamma + \int_\Omega \delta \mathbf{u} \cdot \mathbf{b} \mathrm{d}\Omega \]  \hspace{1cm} (4.27)

In Equation 3.33, the relation between strain and displacement has been derived, and further integration of the displacement approximated by shape functions gives:

\[ \varepsilon(x) = \mathbf{B}^{\text{std}}(x) \bar{\mathbf{u}} + \mathbf{B}^{\text{Hev}}(x) \bar{\mathbf{a}} + \mathbf{B}^{\text{Tip}}(x) \bar{\mathbf{b}} \]  \hspace{1cm} (4.28)

if Equation 4.24 is expressed in an alternative and simplified form:

\[ \mathbf{u}(x) = \mathbf{N}^{\text{std}}(x) \bar{\mathbf{u}} + \mathbf{N}^{\text{Hev}}(x) \bar{\mathbf{a}} + \mathbf{N}^{\text{Tip}}(x) \bar{\mathbf{b}} \]  \hspace{1cm} (4.29)

where \( \mathbf{B}^{\text{Type}} \) can be defined as the result of an operation applied on \( \mathbf{N}^{\text{Type}} \):

\[ \mathbf{B}^{\text{Type}} = \nabla_S \mathbf{N}^{\text{Type}} = \begin{bmatrix} \frac{\partial N_{\text{Type}}}{\partial x} & 0 & 0 \\ 0 & \frac{\partial N_{\text{Type}}}{\partial y} & 0 \\ 0 & 0 & \frac{\partial N_{\text{Type}}}{\partial z} \end{bmatrix} \]  \hspace{1cm} (4.30)

where \( \text{Type} \) is \( \text{std} \), \( \text{Hev} \) or \( \text{Tip} \) in Equation 4.29.
If Type is Hev,  

\[
B_i^{Hev} = \begin{bmatrix}
\frac{\partial N_i}{\partial x} (H(x) - H(x_i)) & 0 & 0 \\
0 & \frac{\partial N_i}{\partial y} (H(x) - H(x_i)) & 0 \\
0 & 0 & \frac{\partial N_i}{\partial z} (H(x) - H(x_i)) \\
\frac{\partial N_i}{\partial y} (H(x) - H(x_i)) & \frac{\partial N_i}{\partial x} (H(x) - H(x_i)) & 0 \\
0 & \frac{\partial N_i}{\partial z} (H(x) - H(x_i)) & \frac{\partial N_i}{\partial y} (H(x) - H(x_i)) \\
\frac{\partial N_i}{\partial z} (H(x) - H(x_i)) & 0 & \frac{\partial N_i}{\partial x} (H(x) - H(x_i)) \\
\end{bmatrix}
\]  

(4.31)

in which \(i\) is the enriched node index and \(N_i\) is the shape function corresponding to node \(i\).

If Type is Tip,  

\[
B_{\alpha i}^{Tip} = \begin{bmatrix}
\frac{\partial (N_i (B_\alpha(x) - B_\alpha(x_i)))}{\partial x} & 0 & 0 \\
0 & \frac{\partial (N_i (B_\alpha(x) - B_\alpha(x_i)))}{\partial y} & 0 \\
0 & 0 & \frac{\partial (N_i (B_\alpha(x) - B_\alpha(x_i)))}{\partial z} \\
\frac{\partial (N_i (B_\alpha(x) - B_\alpha(x_i)))}{\partial y} & \frac{\partial (N_i (B_\alpha(x) - B_\alpha(x_i)))}{\partial x} & 0 \\
0 & \frac{\partial (N_i (B_\alpha(x) - B_\alpha(x_i)))}{\partial z} & \frac{\partial (N_i (B_\alpha(x) - B_\alpha(x_i)))}{\partial y} \\
\frac{\partial (N_i (B_\alpha(x) - B_\alpha(x_i)))}{\partial z} & 0 & \frac{\partial (N_i (B_\alpha(x) - B_\alpha(x_i)))}{\partial x} \\
\end{bmatrix}
\]  

(4.32)

where each term in the matrix block can be transformed by the chain rule:

\[
\frac{\partial (N_i (B_\alpha(x) - B_\alpha(x_i)))}{\partial x} = \frac{\partial N_i}{\partial x} (B_\alpha(x) - B_\alpha(x_i)) + N_i \frac{\partial B_\alpha(x)}{\partial x}
\]  

(4.33)

In Equation 4.33, the derivative of the asymptotic function with respect to \(x\), \(y\) or \(z\) can be computed analytically if the local coordinate can be transformed properly. The details of the analytical derivative can be found in Khoei (2014). The coordinate transformation needs attributes in the rotation matrix computed by Equation 4.22. The attributes in the rotation matrix provide the direct derivative of the rotated coordinates and they can be placed into the corresponding position for calculating the transformed derivatives. For example, the derivative of the asymptotic function with respect to the global \(x\) axis is:

\[
\frac{\partial B_\alpha(x)}{\partial x} = \frac{\partial B_\alpha(x)}{\partial x'} \frac{\partial x'}{\partial x} + \frac{\partial B_\alpha(x)}{\partial y'} \frac{\partial y'}{\partial x} + \frac{\partial B_\alpha(x)}{\partial z'} \frac{\partial z'}{\partial x}
\]

(4.34)
in which \( x', y', \) and \( z' \) are rotated axes in Figure 4.3. According to the rotation matrix below:

\[
\begin{bmatrix}
x' \\
y' \\
z'
\end{bmatrix} =
\begin{bmatrix}
R_{11} & R_{12} & R_{13} \\
R_{21} & R_{22} & R_{23} \\
R_{31} & R_{32} & R_{33}
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
\tag{4.35}
\]

the two coordinate derivatives should be (zero with respect to \( x' \)):

\[
\frac{\partial y'}{\partial x} = R_{21}, \quad \frac{\partial z'}{\partial x} = R_{31}
\tag{4.36}
\]

Equation 4.34 - 4.36 can be used to compute the derivatives of the tip functions analytically in Equation 4.33.

A linear equation system can be obtained to solve for displacement and enriched displacement at element nodes after Equation 4.28 and 4.29 substitute the strain and displacement term in Equation 4.27:

\[
\begin{bmatrix}
K_{rr} & K_{rs} & K_{rt} \\
K_{sr} & K_{ss} & K_{st} \\
K_{tr} & K_{ts} & K_{tt}
\end{bmatrix}
\begin{bmatrix}
\bar{u} \\
\bar{a} \\
\bar{b}
\end{bmatrix} =
\begin{bmatrix}
F_r \\
F_s \\
F_t
\end{bmatrix}
\tag{4.37}
\]

in which the stiffness matrix on the left-hand-side can be assembled by:

\[
K_{nm} = \int_{\Omega} (B^n)^T DB^m d\Omega
\tag{4.38}
\]

and \( r, s \) and \( t \) correspond to the types of \textit{std, Hev, Tip} and the according degrees of freedom are \( \bar{u}, \bar{a} \) and \( \bar{b} \) in Equation 4.37. \( n \) and \( m \) take the values of \( r, s \) or \( t \). The right-hand-side of Equation 4.37 is considered as the force vector:

\[
F_n = \int_{\Omega} (B^n)^T (\alpha p I + \beta (T - T_{\text{ref}}) I) d\Omega + \int_{\Omega} (N^n)^T b d\Omega + \int_{\Gamma_t} (N^n)^T \tau d\Gamma + \int_{\Gamma_d} [N^n]^T p n_{\Gamma_d} d\Gamma
\tag{4.39}
\]

combining all terms of pore pressure, body force, boundary traction, and internal pressure. Among all these terms, the internal pressure term involves a jump of shape function \([N^n]\) that can be written as:

\[
[N] = N^+ - N^-
\tag{4.40}
\]
The standard shape function will be canceled out since $N$ has the same value across the discontinuity as long as the local coordinate is unchanged. Heaviside shape function can be computed by:

$$[N]_{Hev} = N^+ - N^- = 2N$$  \hspace{1cm} (4.41)

since the Heaviside function is 1 on one face and $-1$ on the other hence the subtraction is $2N$. Crack tip asymptotic function is special due to the fact that only the first function is effective since the sine function is continuous across the discontinuity:

$$[N]_{Tip} = N^+ - N^- = 2N \sqrt{r}$$  \hspace{1cm} (4.42)

And additionally, Equation 4.41 and 4.42 can be used to calculate the relative displacement extension Equation 3.29:

$$w = n_{\Gamma_d} \cdot [u] = n_{\Gamma_d} \cdot (u^+ - u^-) = 2n_{\Gamma_d} \cdot \sum_{j=1}^{M} N_j(x) + 2n_{\Gamma_d} \cdot \sqrt{r} \sum_{k=1}^{K} N_k(x)b_{1k}$$  \hspace{1cm} (4.43)

The above equation can be utilized to compute the fracture aperture after obtaining the displacement solution from the linear system in Equation 4.37. Note that only $b_{1k}$ is relevant in Equation 4.43. In order to incorporate the virtual proppant term into the stiffness matrix, Equation 4.41 and 4.42 are also essential:

$$K_{nm} = \int_{\Omega} (B^n)^T DB^m d\Omega + \int_{\Gamma_d} [N^n]^T E_{prop} [N^m] d\Gamma$$  \hspace{1cm} (4.44)

in which $E_{prop} = (E_s/d_{HF0}) n_{\Gamma_d} n_{\Gamma_d}^T$ is the proppant Young’s modulus to model a spring that connects the discontinuous faces and prevents the displacement of either face from penetrating the other.

### 4.1.2.4 Numerical Integration for the Stiffness Matrix and Force Vectors

There are multiple integrations with respect to either a surface or a volume in the discretized Equations 4.39 and 4.44. In FEM, integrations are usually handled by numerical integration accurately. The volumetric integration in the stiffness matrix can be computed by:

$$K_{nm} = \sum_{i=1}^{N_G} \sum_{j=1}^{N_G} \sum_{k=1}^{N_G} W_i W_j W_k B^{nT} (\xi_i, \eta_j, \zeta_k) DB^m (\xi_i, \eta_j, \zeta_k) |J (\xi_i, \eta_j, \zeta_k)|$$  \hspace{1cm} (4.45)
in which \( N_G \) is the number of Gauss quadrature points and \( W \) is the weight of a specific point with the subscripts \( i, j \) or \( k \) indicating the dimensional axis. The matrix \( B^n \) needs to be evaluated at a specific Gauss point. The above equation is general for any type of hexahedron element hence the Jacobian matrix, \( J \), results from the transformation from the local coordinate system to the global one. However, the integration domain in XFEM is slightly different from the standard FEM in that some of the elements are subdivided by the cutting discontinuity. Hence, Equation 4.45 cannot be directly employed for integration.

Khoei (2014) summarized two approaches to tackle the problem: (1) subdivide the positive and negative domains by triangulation and add up all integrations of the sub-triangles; (2) subdivide the whole element into sub quadrilateral (or hexahedron) elements and add up all contributions from these sub-elements. The first scheme has been adopted widely for 2D XFEM code (Bordas et al. 2007; Pais et al. 2010) but might not be a good choice for the 3D case due to the complexity of subdividing the 3D hexahedron element.

In this study, the second option was implemented by subdividing the brick element into 125 or 1000 subelements (5 or 10 divisions in each axis) as shown in Figure 4.5 and Equation 4.46. The more subdivisions made potentially give more accurate integrations and hence less numerical error in the solution.

\[
K_{nm} = \sum_{l=1}^{N_{+\text{sub}}} \left( \sum_{k=1}^{N_G} W_k B^n T(\xi, \eta, \zeta) DB^m(\xi, \eta, \zeta) \right)_l + \sum_{l=1}^{N_{-\text{sub}}} \left( \sum_{k=1}^{N_G} W_k B^n T(\xi, \eta, \zeta) DB^m(\xi, \eta, \zeta) \right)_l
\]  

(4.46)

The above equation provides the integration approach using the sub-element subdivision, which is quite similar to the standard FEM. The only difference is to transform the local Gauss point in a sub-element into the local coordinate system of the whole element then compute \( B(\xi, \eta, \zeta) \) in the global coordinate system. \( W_k \) in the above equation is the
multiplication of three weights in Equation 4.45 and the determinant of the Jacobian matrix.

In terms of the pore pressure and temperature term, pressure and temperature are both constant within the element as stated above. The equation to compute the pore pressure term is shown as:

\[ F_{n,p} = \int_{\Omega} (B^n)^T \alpha p I d\Omega \]

\[ = \sum_{i=1}^{N_G} \sum_{j=1}^{N_G} \sum_{k=1}^{N_G} W_i W_j W_k B^{nT} (\xi_i, \eta_j, \zeta_k) \alpha p (\xi_i, \eta_j, \zeta_k) |J (\xi_i, \eta_j, \zeta_k)| \]

which is similar to the integration in Equation 4.46.

Figure 4.5: (a) subdivide the element containing discontinuity into sub-elements, red crosses are Gauss points for integration in each sub-element; (b) cut fracture segment and subdivision for internal traction integration, red crosses are Gauss points in each triangle. The number of Gauss points is only three in each triangle for the ease of plotting. Seven is used in the program for accuracy.

Two types of surface or areal integration need to be resolved: (1) integration on the external surface for external traction; (2) integration on the internal surface for internal traction. The first type of integration on a surface perpendicular to the Z-axis, for example, is calculated by:

\[ F_{n,t} = \int_{-1}^{1} \int_{-1}^{1} N^{nT} (\xi, \eta, 1) |J_{sub}| d\xi d\eta = \sum_{i=1}^{N_G} \sum_{j=1}^{N_G} W_i W_j N^{nT} (\xi_i, \eta_j, 1) |J_{sub}| \]

Note that \( \zeta \) is set to be 1 (Z face) in the shape function and the Jacobian matrix is a submatrix accounting for the specific face of the element. The second type of integration
needs to evaluate the jump shape function, as shown in Equation 4.41 and 4.42. The Gauss
points for such integration are located on the discontinuous plane, as shown in Figure 4.5 (b). The cut fracture plane segment is divided into triangles in each of which Gauss points are collected for their coordinates and corresponding weights. The coordinates can be obtained locally on the triangle and transformed into the hexahedron global coordinates and the weights are computed with the data of triangular areas. The equation for internal pressure traction is shown as below:

\[ F_{n,\text{int}} = \sum_{i=1}^{N_{tri}} \sum_{j=1}^{N_G} W_{ij} [N^n]^T (\xi, \eta, \zeta) \rho \mathbf{n}_{\Gamma_d} |J_{tri}| \]  

where the subscript \( \text{int} \) represents internal pressure, \( \text{tri} \) marks the triangles that are subdivided into. \( J_{tri} \) is the Jacobian transformation from triangles back to the global coordinate system.

### 4.2 Embedded Discrete Fracture Model

The concept of embedded discrete fracture is originally proposed by Lee et al. (2001) for modeling fluid flow between rock matrix and long fractures. Moinfar et al. (2014) extended the model and applied it to three-dimensional compositional simulation. Wang et al. (2019) developed an EDFM preprocessor in which discrete fractures of arbitrary shape and strike can be embedded into mesh grids. EDFM enables us to generate complex fracture geometry and model the fluid flow in fractured reservoirs. It is intuitive to treat the embedded discrete fracture as a well in its connected grid block and Equation 4.3 becomes:

\[ \frac{d}{dt} \left( M_k^V V_{n,0} \right) = \sum_m A_{nm} F_{nm}^k + q_{nm}^k V_{n,0} + q_{mf} \]  

in which \( q_{mf} \) is the flow between discrete fracture and the connected matrix grid. This flow term can be computed by:

\[ q_{mf} = FI \rho_{k} \frac{k_{\nu k}}{\mu_{\beta}} (p_m - p_f + P_{cm} - P_{cf}) \]  

Compared with Darcy’s flow, FI is the fracture index and should be expressed as \( \frac{A}{D_m + D_f} \). The fracture index is used to describe the connection area and distance between fracture and matrix, similar to those between two normal grid blocks.
EDFM is employing geometrical calculations to obtain such properties so that a set of new mesh beyond reservoir grids can be built. Wang et al. (2019) regarded discrete fracture element as a grid block that has a relatively small volume, and connections between fracture elements and primary grid blocks were established for fluid/heat flow simulation. The improved model in this dissertation is based on the program developed by Wang et al. (2019). The principles of EDFM will be explained in this section, followed by the improvement of EDFM.

4.2.1 Geometrical Computation

EDFM approach is basically computing the intersection between an arbitrary plane and a box in three-dimensional space, which becomes an analytical geometric problem. As shown in Figure 3.3, two planes in three-dimensional space intersect with primary grid blocks and with each other at the same time. The fracture plane has been cut into discrete pieces within this grid block, generating two fracture elements: red and green polygons. It is intuitive that the contact area between the fracture element and the box is just the area of polygon and the contact area between two intersected fractures is fracture width multiplied by the length of line segment AB. Distance between a fracture element and a box can be calculated using (Li and Lee 2006):

\[ d_{fm} = \frac{\int \left( \frac{x_v}{V} \right) dV}{V} \]  

(4.52)

in which \( x_v \) is the distance from an infinitesimal volume of the matrix grid box to the fracture plane. Such distance can be computed using the Hessian form of the plane and the coordinate of a point:

\[ \mathbf{n}_{\Gamma_d} \cdot \mathbf{x} = -p \]  

(4.53)

\[ |d| = \mathbf{n}_{\Gamma_d} \cdot \mathbf{x}_0 + p \]  

(4.54)

in which, \( \mathbf{n}_{\Gamma_d} \) is the normal vector of the fracture plane, \( \mathbf{x}_0 \) is the coordinate of the point and Equation 4.53 is the Hessian form of a plane.
The connection distance between two intersected fractures can be calculated by (Xu et al. 2017):

\[ d_{ff} = \frac{\int_{S_1} x_n dS_1 + \int_{S_2} x_n dS_2}{S_1 + S_2} \] (4.55)

where \( S_1 \) is the area of one part cut by the intersecting line segment and \( S_2 \) is the other, such as in Figure 3.3, the red polygon is cut into two pieces by line segment AB. The distance from the red fracture element to the interface AB is calculated by Equation 4.55, similarly to the green fracture element.

The challenging task of establishing EDFM is to find the coordinates of the cut polygons, with which contact area and distance calculation above would not be an obstacle. The procedure proposed by Wang et al. (2019) for box fracture intersection includes:

1. Check if one or more of the polygon vertices is inside the box, save the vertex inside the box;
2. Check the intersection between box edge and fracture polygon and if the intersection point is within the polygon, save vertex if yes;
3. Check if polygon edge intersects with box face and if the intersection point is within box face, save vertex if yes;
4. Check if any pair of the saved vertices is the same vertex and eliminate the duplicated vertex;
5. Sort vertices counter-clockwise and calculate discrete fracture area and contact distance.

The above procedure is performed between each fracture plane and grid box to locate all fracture elements. Wang et al. (2019) integrated all fracture polygons into a single one within each primary grid box. However, since it is desired to consider the geomechanical effect on each discrete fracture polygon, EDFM was modified to treat fracture polygons individually. Hence, beyond the steps stated above, additional geometrical calculations are necessary:

6. Check if polygons are along the box surfaces, save these polygons for further use;
7. Then check polygon intersection on the connected face between two boxes, calculate contact area and distance if intersected;
8. Check polygon intersection on the connected edge between two boxes, calculate contact area and distance if intersected;
9. Eliminate duplicates of polygons along the box surfaces and check if intersected with polygons inside the box, calculate contact area and distance if yes.

The modification of fracture connection considers conditions illustrated in Figure 4.6, making the EDFM more general for arbitrary fracture embedment. Although fractures intersected with each other could reach equilibrium fast due to high conductivity, this may not be true when geomechanics is taken into account or when fractures have extremely different conductivities.

Figure 4.6: Fracture-fracture connection conditions considered by the modification of EDFM: (a) Partially intersection inside a box; (b) No intersection inside a box; (c) Connection on box faces; (d) Connection on edge of box faces; (e) Connection on box edges; (f) Connection of different fracture planes on box edges; (g) Polygons along box faces; (h) Connection between polygons along box faces and polygons inside boxes.

### 4.2.2 Relation between EDFM and XFEM

The implementation of 3D EDFM facilitates that of XFEM in that the geometrical computation performed in EDFM can be directly utilized by XFEM. XFEM relies on the Level Set Method (Moës et al. 1999; Sukumar et al. 2000) to detect the relative position
between element nodes and fracture discontinuity, which is extremely useful for fracture propagation modeling that requires the fracture tip localization.

In this study, fracture propagation is not considered so the relative position between fracture grids and primary grids is a cheap one-time computation. The Level Set functions such as \( \varphi \) and \( \vartheta \) in Figure 3.1 are still computed, which is useful in obtaining the asymptotic functions. During the EDFM procedures, an element may be classified into three categories: (1) element without fracture cutting through; (2) element cut through by fracture; (3) element containing the fracture tip. Instead of the traditional fracture tip detection (Bordas et al. 2007; Pais et al. 2010) using Level Set values, counting the number of polygon edges on the box element surface is performed: if the number of edges on box surfaces is less than the total number of the polygon edges, the polygon is a fracture tip, as shown in Figure 4.7.

![Figure 4.7: Two types of enriched elements: (a) Heaviside enrichment when element is completely cut through by fracture; (b) Crack tip enrichment when polygon edges are not all on box element surfaces.](image)

The nodes belonging to the fracture tip elements will be enriched by crack tip asymptotic functions and the nodes belonging to the cut-through elements and not enriched by crack tip functions will be enriched by Heaviside functions. Once the enrichment type has been determined for a node, it will not be changed in the simulation when propagation is not considered. In Section 4.1.2.1, the importance of EDFM geometrical computation in local coordinate system transformation has also been stated.
EDFM is naturally compatible with XFEM in such a coupled THM model.

4.3 Multiple INteracting Continua (MINC) Model

MINC model has been utilized as a powerful tool to model fractured reservoirs with uniformly distributed natural/induced fractures. The improvement from DP to MINC provides a more accurate model to describe the mass/energy transfer between matrix and natural fractures. The MINC technique has been adopted widely in tight/shale gas/oil modeling and even integrated with hydraulic fracture models, such as DFM or EDFM, to achieve a hybrid fracture model. EGS projects involve hydraulic fracturing which highly likely induces or reactivates natural fractures within the sites. Hence MINC model is incorporated into the EDFM to simulate water circulation processes in SRV. The meshing preprocessing has been implemented in EDFM in this dissertation so the implementation details will be elaborated in this section. Constitutive relations for multiple continua within MINC were introduced in Section 3.2.2. MINC is only introduced into the iteratively coupled model in this dissertation.

4.3.1 Meshing Computation for MINC

Theory and implementation of MINC mesh were proposed by Pruess (1983). The steps of generating such mesh are revisited briefly in this section since it is also realized in EDFM to integrate two mesh systems. The volume fraction of each continuum should be prepared and used to determine the connection area and distance between neighbor continuums. Fracture spacing is needed to define the proximity function which describes the matrix volume fraction within a distance $x$ from the fracture:

$$PROX(x) = \frac{V(x)}{V_m} = \frac{V(x)}{(1 - \phi_1)V_0}$$

in which $\phi_1$ is the volume fraction of fractures and $V_0$ is the domain (or grid) volume. If the spacing of three perpendicular sets of plane, parallel, equidistant, infinite fractures are $a$, $b$ and $c$, then the matrix volume with a distance $x$ from the fractures is
\[ V(x) = abc - (a - 2x)(b - 2x)(c - 2x) \]

Pruess (1983) defined the proximity function as:

\[ \text{PROX}(x) = uvw - (uv + uw + vw) + (u + v + w) \] when \( u = \frac{2x}{a}, v = \frac{2x}{b}, w = \frac{2x}{c} \) (4.57)

Given a volume fraction, the proximity function can be calculated by Equation 4.56 and based on its relation with \( x \), Equation 4.57 is inverted to obtain the distance \( x_j \) from the \( j^{th} \) continuum to the fracture. The inverse of the proximity function is accomplished by the bi-section method.

The interface area between two neighboring continuums \( j \) and \( j + 1 \) can be expressed as:

\[ A_{j,j+1} = (1 - \phi_1)V_n \frac{d\text{PROX}(x)}{dx} \bigg|_{x_j} \] (4.58)

in which the derivative is evaluated at \( x_j \) and the total volume of the grid, \( V_n \) is needed to be multiplied. The distance between them is:

\[ d_{j,j+1} = \frac{x_{j+1} - x_j}{2} + \frac{x_j - x_{j-1}}{2} = \frac{1}{2} (x_{j+1} - x_{j-1}) \] (4.59)

with \( x_j \) and \( x_{j+1} \) calculated by inverting the proximity function as described in Equation 4.57. Two special cases are the first and the last continuum:

\[ d_{1,2} = \frac{x_2}{2} \text{ and } d_{n_{m-1},n_m} = \frac{x_{n_m-1} - x_{n_m-2}}{2} + D_{n_m} \] (4.60)

in which \( n_m \) is the partition number of the MINC grid, \( D_{n_m} \) is the distance from the innermost interface area, calculate analytically by approximating the pressure/temperature gradient:

\[ D_{n_m} = \frac{3uvw}{10(uv + vw + uw)} \] (4.61)

with:

\[ u = a - 2x_{n_m-1}, \ v = b - 2x_{n_m-1}, \ w = c - 2x_{n_m-1} \] (4.62)
4.3.2 Mesh Preprocessing for EDFM and MINC

EDFM combined with MINC are employed to model SRV as shown in Figure 4.8. The nested grid area is SRV where hydraulic fractures coexist with natural fractures. There are researches (Ding et al. 2018) discussing that discrete fracture cannot directly connect the matrix in the MINC grid since that would violate the theoretical foundation of the MINC method: the pressure gradient from the outermost layer to the innermost layer should be retained. Therefore, in this study, only natural fractures are connected to hydraulic fractures as a highly conductive path while the matrix within the MINC grid serves as a storage volume for mass and energy, as shown in the right of Figure 4.8. There is no global connection between the matrix in two neighboring MINC grids, for example, grid m1m3 is not connected to m2m3. The same assumption has also been used in Yan et al. (2018). Outside of the SRV, the matrix without fractures is connected to the natural fracture in SRV.

Figure 4.8: Reservoir model with SRV: hydraulic fractures, MINC grids, and matrix grids. The connected line indicates the connection between two grids. Left: red hydraulic fracture (HF) induces natural fractures (F) which are modeled by MINC (nested grids). The outermost layer of MINC grids is a natural fracture (F) and then multiple continua (m1m1,m1m2,m1m3); Right: connection relation between hydraulic fractures, natural fractures, and matrix. Hydraulic fractures connect only to natural fractures which connect to its nested multiple continua. The matrix outside of SRV connects to natural fractures only.
The preprocessing steps to generate the EDFM mesh follow the procedure in Section 4.2.1: the MINC partition doesn’t involve hydraulic fractures except the outermost natural fracture layer so MINC meshing can be performed after EDFM meshing. The procedure is summarized as below:

1. Iterate the original element list to find the to-be-partitioned box elements and assign a new volume of natural fracture;
2. Create multiple continuum matrix elements with corresponding volumes and assign global element index;
3. Iterate the original connection list to re-assign the global element index of two connected elements for non-SRV matrix and natural fractures, such as m1 and F1 in Figure 4.8;
4. Compute the distance and interface area using Equations 4.58-4.61;
5. Extend the connection list by iterating all to-be-partitioned grids and assign connection distances and area values acquired from the calculations in section 4.3.1.

4.3.3 MINC Extension in Mechanics Solver

The fluid flow/heat transfer solver, TOUGH2-EGS, treats all grids as independent grids hence partitioned MINC elements don’t need to be specially handled. The only extended implementation is updating the porosity using Equation 3.45. The pressure and temperature are solved independently and transferred to the mechanics solver, XFEM.

The capability of the mechanics solver is to be extended for MINC grids. There are two extensions compared to the original mechanics solver, (1) the elasticity tensor for assembling the stiffness matrix; (2) the pressure, temperature, and body force term on the right-hand side:

1. the elasticity tensor needs to be upscaled using Equation 3.40;
2. the pressure, temperature, and body force term need to be weight-averaged using the upscaled drained bulk modulus and the continuum bulk modulus:
\[ \alpha_{up}p_{up} = K_{dr} \sum_{k=1}^{n_m} \frac{\alpha_k \eta_k}{K_k} p_k \]

\[ 3\beta_{up}K_{dr}(T_{up} - T_{ref}) = 3K_{dr}^2 \sum_{k=1}^{n_m} \frac{\beta_k \eta_k}{K_k}(T_k - T_{ref}) \]  \hspace{1cm} (4.63)

4.4 Program Structure and Coupling Algorithm

The program structure and coupling algorithm are demonstrated in this section on a high level, focusing on the workflow and organization of the main computing and programming routines.

4.4.1 Fully Coupled Model

The fully coupled model, TOUGH2-THM is developed based on a parallel framework of TOUGH2-MP. The program relies on Message Passing Interface (MPI) for parallel implementation, run with multiple processes on multiple processors simultaneously. When performing parallel computation, TOUGH2-MP code partitions the reservoir domain into sub-domains using the partitioning algorithm from the METIS software package (Karypis and Kumar 1998). The subdomains are handled by each processor to compute secondary thermophysical properties, assemble mass, energy and stress residual equations and build local Jacobian linear systems. The local systems are solved in parallel by multiple processors through the AZTEC linear solver package. Information is exchanged during each iteration for assembling the Jacobian matrix and also for solving the linear equations (Wu et al. 2002).

In this work, due to the extension of the primary variable list, parallel computation would help to mitigate the increased burden of solving large linear systems of equations. In this subsection, the parallel structure and overall program structure will be briefly introduced, but the parallel performance will not be a focus of this study.
4.4.1.1 Grid Partitioning

In the TOUGH family code, all grid blocks are given specific names (or indices) that are used for setting up connection data. Assembling mass, energy and mechanical equations for a grid block needs the primary and secondary parameters of all its connected neighbors after assigning partitioned subdomains to different processors. Therefore, each of the processors is required to have access to the grid blocks that are directly connected to the border of its assigned subdomain.

TOUGH2-MP, after partition, divides the subdomain into three sets of grids: internal, border, and external. This approach can be illustrated by Figure 4.9. The grids in the red box have been assigned to the current processor and global indices (before partitioning) are shown in Figure 4.9 (a). This processor is connected to other four processors that contain neighbors of the current processor. The partitioning process assigns local indices to these grids in the red box from 1 to 16. The grids in the green box are called internal sets which don’t need any data from other processors; the grids between the green box and the red box are called border sets which connect to the external sets.

Local indices are first assigned to each of the three sets, in ascending global index order. Then local connections are established using the local indices. Input parameters are distributed to different processors from the master processor according to global and local indices, and global and local connections. External sets are prepared for data exchange.

Figure 4.9: Grid indices before and after domain partition: (a) global indices before partition; (b) local indices after partition; (c) local indices after divided into three sets.
between connected subdomains. After the local connection is set up and data is distributed, each processor should be able to conduct the main computation. The overall structure of parallel processing is shown in Figure 4.10.

**Figure 4.10: The structure of parallel processing for TOUGH2-THM.**

### 4.4.1.2 Fully Coupling and Program Structure

Each processor, in the main computation module, will initialize the stress, temperature, and pore pressure first, by either equilibrium state computation or data input file. Then external and border data sets are exchanged before starting time looping. Secondary variables are updated first based on primary variables from the last iteration or time step. During Newton’s iteration, mass, energy, and stress equations are assembled, and the Jacobian matrix is built for the solution. Afterward, the AZTEC solver takes charge of solving the local linear system for the increment towards the next iteration. Convergence will be checked when residual equations are computed and whether the system is converged determines the following step of entering the next iteration or time step. The whole structure is illustrated in Figure 4.11.

It should be noted that exchanging data sets are time-consuming, reducing the efficiency of parallel performance. TOUGH2-MP exchanges only primary variables and only once for each iteration, using non-blocking communication. In this dissertation,
efficiency is sacrificed to a certain degree because the interface quantities are necessary for stress equation assembly. As is shown in the shear stress Equation 4.18, for the two connected grids of interest, the cross derivative should be approximated by performing FD using stress and pressure on the block surface that may not be relevant to the current connection. Thus, it is possible that interface quantities of external grids are needed, which cannot be computed in the current processor. Consequently, this is a required data exchange procedure if no modification of the local grid setup is desired.

Figure 4.11: Program structure of the fully coupled model computation (time looping and iteration).

4.4.2 Sequentially Coupled Model

The scheme to couple fluid flow/heat transfer with geomechanics adopted in the sequentially coupled model is the fixed-stress split proposed by Kim et al. (2011). This coupling method belongs to the sequential-implicit family, provides unconditional stability while solving such a coupled system, and is also believed to give the same results as a fully coupled scheme. The model in this study aims at cases where strains and hydraulic properties are sensitive to both temperature and pressure, so unconditional stability is essential.
The fixed-stress split has been analyzed (Mikelić and Wheeler 2013) and implemented (Beck et al. 2020; Dana et al. 2018; Garipov et al. 2018; Jha and Juanes 2014; Liu, L.-J. et al. 2020; Liu, Y.-Z. et al. 2020; Yan et al. 2018) in plenty of research. Kim et al. (2012, 2015, 2011) stated that the fixed-stress split freezes the volumetric stress rate during the flow step, which can be achieved by correcting the porosity using stress or strain in the last two converged time steps. Beck et al. (2020); Dana et al. (2018); Garipov et al. (2018); Mikelić and Wheeler (2013); Yan et al. (2018), however, freeze the total stress instead of the stress rate in the flow step. The analysis from Mikelić and Wheeler (2013) and Dana et al. (2018) shows that this scheme also ensures unconditional stability. This fixed-stress split is what has been adopted in this study.

### 4.4.2.1 Matrix Porosity Update in Flow Iterations

Dana et al. (2018) and Liu, L.-J. et al. (2020) used an equation which is different from Lagrangian porosity Equation 3.36 to update matrix porosity under the fixed stress condition:

\[
\phi_{k+1,m} = \phi_{k,m} + \left( \frac{\alpha(1 + \varepsilon_{v,m-1}) - \phi_{m-1}}{K} \right) (p_{k+1,m} - p_{k,m})
\]  

(4.64)

in which the subscript \( k \) denotes the flow iteration number and \( m \) indicates the coupling iteration number. Garipov et al. (2018) proposed that if stresses are fixed in the flow solving:

\[
\sigma_{mean,m+1} - \sigma_{mean,m} = 0
\]

\[
\sigma_{mean,m+1} - \sigma_{mean,m} = K(\varepsilon_{v,m+1} - \varepsilon_{v,m}) - \alpha(p_{m+1} - p_m) - 3K\beta_T(T_{m+1} - T_m) = 0
\]

(4.65)

in which \( m \) is the coupling iteration number. Therefore, the strain change between iterations is written as:

\[
\varepsilon_{v,m+1} - \varepsilon_{v,m} = \frac{\alpha}{K} (p_{m+1} - p_m) - 3\beta_T(T_{m+1} - T_m)
\]

(4.66)

In this study, the Lagrangian porosity calculated in each flow iteration is expressed by:

\[
\phi_{m+1} = \phi_m + \left( \frac{\alpha - \phi_m}{K} \right) (p_{m+1} - p_m) + \alpha(\varepsilon_{v,m+1} - \varepsilon_{v,m}) - 3\beta_T(\alpha - \phi_m)(T_{m+1} - T_m)
\]

(4.67)
Plug Equation 4.66 into Equation 4.67 and the porosity in \( m + 1 \) coupling iteration becomes:

\[
\phi_{m+1} = \phi_m + \frac{(\alpha - \phi_m)(1 - \alpha)}{K} (p_{m+1} - p_n) - 3\beta_T (\alpha - \phi_m) (T_{m+1} - T_n) \\
+ \frac{\alpha^2}{K} (p_{m+1} - p_m) - 3\alpha \beta_T (T_{m+1} - T_m)
\] (4.68)

where \( n \) is the last converged time step index. Equation 4.68 is used in this study to update the matrix porosity within the \( m + 1 \)th coupling iteration. Note that within the \( m + 1 \)th coupling iteration, there will be \( k \) flow iterations until convergence, which is not labeled in the above equations. The variable with \( m \) as its subscript is fixed and obtained from the previous coupling iteration. The reason why there are two pressure and temperature terms evaluated at \( n \)th time step on the right hand side of Equation 4.67 but porosity and strain at \( m \)th coupling iteration is that: each coupling iteration is conducted in order to adjust the porosity and strain distribution so the updated porosity and strain are reflected by using the \( m \)th coupling iteration as the initial value for the next \( m + 1 \)th coupling iteration. In TOUGH2-EGS, the volumetric strain is computed based on the mean stress on each iteration level during a flow step when the mean stress is fixed. Equation 4.68 is equivalent to directly calculating volumetric strain based on the fixed stress value and plugging the volumetric strain into Equation 4.67.

The comparison between Dana et al. (2018) and Garipov et al. (2018) in updating matrix porosity is conducted qualitatively and the simulation results are the same. Dana et al. (2018)’s method has a slightly better convergence performance, i.e., the coupling iteration number is less for most of the time steps.

### 4.4.2.2 Fracture Porosity Update in Flow Iterations

Liu, L.-J. et al. (2020) and Yoon, S. et al. (2021) updated the fracture porosity using fracture stiffness. The equation in Yoon, S. et al. (2021) has a variable stiffness at different locations of the fracture. The constant fracture porosity was initially tested in this study to model the single-fracture EGS case, the results gave a bad convergence performance where most of the time steps cannot converge within a predefined maximum coupling iteration number, and the time step size became extremely small.
In this dissertation, to achieve a better convergence performance, the method in Yoon, S. et al. (2021) is adopted:

\[
\phi_{HF,m+1} = \left(\frac{d_{max} - d_{HF,m}}{d_{max}}\right)^2 \left(\frac{p_{m+1} - p_n}{K_{HF}}\right) + \phi_{HF,m} \tag{4.69}
\]

where the subscript \( HF \) denotes hydraulic fracture, \( d_{max} \) is the maximum fracture aperture change and \( d_{HF,m} \) is the fracture aperture closure obtained from the previous coupling iteration, \( m \) (from mechanics solver), \( K_{HF} \) is the initial fracture stiffness provided as an input parameter. Equation 4.69 has been proposed based on the form in Yoon, S. et al. (2021) where the larger fracture aperture results in larger fracture stiffness. The application of this porosity updating equation improves the convergence substantially, which will be discussed in Chapter 7.

Table 4.1 summarizes the equations adopted to update matrix/fracture porosity and the corresponding convergence performance for both literature and this work (NA indicates no fracture has been modeled in the study).

<table>
<thead>
<tr>
<th>Work</th>
<th>Equations for Matrix/Fracture</th>
<th>Convergence Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dana et al. (2018)</td>
<td>4.64 and 4.68/NA</td>
<td>Good</td>
</tr>
<tr>
<td>Garipov et al. (2018)</td>
<td>4.68/NA</td>
<td>Good</td>
</tr>
<tr>
<td>Yoon, S. et al. (2021)</td>
<td>4.68 with correction/4.69-like form</td>
<td>Good</td>
</tr>
<tr>
<td>This study</td>
<td>4.68/NA</td>
<td>Fair</td>
</tr>
<tr>
<td>This study</td>
<td>4.64 and 4.68/NA</td>
<td>Good</td>
</tr>
<tr>
<td>This study</td>
<td>4.68/Constant</td>
<td>Bad</td>
</tr>
<tr>
<td>This study</td>
<td>4.68/4.69</td>
<td>Good</td>
</tr>
</tbody>
</table>

4.4.2.3 Program Structure and Workflow

Mass/energy balance equations are solved iteratively within TOUGH2-EGS by Newton’s method. The mechanical Equations 4.37 - 4.39 can be directly solved without Newton’s iteration. The XFEM formulation is implemented by MATLAB. Given input data of pressure, saturation, fluid density and temperature, the XFEM program will solve
for the displacement and in turn compute the stress by:

$$\sigma'(x) = DB(x)u$$  \hfill (4.70)

where $B = [B^{\text{std}}, B^{\text{Hev}}, B^{\text{Tip}}]$ and $u$ are the solved discrete values on the element nodes including enriched degrees of freedom. Stresses at 6 by 6 by 6 Gauss points are collected within a single element and the weighted average is calculated at the element-level.

Fracture aperture can be calculated with the enriched degrees of freedom by Equation 4.44. Similarly, fracture apertures at Gauss points on the fracture segments are collected and averaged by weights. Then stresses and fracture apertures are transferred to TOUGH2-EGS where the volumetric strain of matrix elements and fracture elements are obtained and utilized for fluid flow/heat transfer.

The fixed-stress split coupling approach has been shown in Figure 4.12. TOUGH2-EGS calls XFEM for the initialization of stress, strain, and fracture aperture. XFEM writes the stress and associated data into a file that can be read by TOUGH2-EGS. After the simulation enters time looping, within each time step, the flow problem is solved first assuming that total stress has been fixed. Within the flow problem, porosity of matrix and fracture are updated by Equation 4.68 and 4.69 respectively. Volumetric strain is updated as a secondary variable since pressure and temperature are updated in the iterations using Equations 4.66 and 4.68. Once the flow model becomes converged, pressure and temperature convergence criteria are checked. If pressure and temperature variations are below the criteria, only one XFEM execution is needed before the next convergence check, and the flow problem proceeds. If the criterion is not satisfied, XFEM is called and update the total stress which is used to compute the porosity based on Equation 4.67 and 3.46 for the next coupling iteration afterward. The outer coupling iterations are performed until pressure and temperature converge again. TOUGH2-EGS writes pressure, saturation, fluid density and temperature into a file for XFEM to read in before solving mechanics.

Essentially, the coupling iteration in a time step keeps refining the porosity and permeability based on the converged state variables (pressure, temperature and stress),
starting from the state of the last time step. Extreme cautions are needed to implement the steps of updating primary variables, performing coupling iterations, and computing secondary variables for Jacobian matrices.

Figure 4.12: Flow chart of sequentially coupled model.
CHAPTER 5
MODEL VALIDATION

In this chapter, validations are conducted for both fully and sequentially coupled THM models.

- Both models rely on EDFM for fluid flow and heat transfer computation so the coupled TH model using EDFM is to be validated, especially the implementation of EDFM has been improved to model general fracture shapes and locations;
- The fully coupled model, TOUGH2-THM is developed to incorporate all stress tensor components as primary variables based on the TOUGH2-CSM framework which has been well validated and documented. Hence it is to be validated against TOUGH2-CSM, as well as analytical poroelasticity solutions.
- The mechanics module of the sequentially coupled model is developed based on an in-house XFEM solver. The capabilities extended include:
  1. stiffness matrix assembly using a fast coordinate transformation approach;
  2. internal pressure traction integration on the 3D discontinuity;
  3. general traction and constant displacement boundaries as well as body force terms for FEM.
These three capabilities are to be validated against the original XFEM solver, open-source FEM code, and analytical poroelasticity solutions.
- The sequential coupling scheme connecting TOUGH2-EGS and the in-house XFEM simulator in the sequentially coupled model needs to be validated against analytical poroelasticity solutions such as the Terzaghi’s problem and Mandel’s problem.

5.1 Fluid Flow and Heat Transfer Modeling by EDFM

Wang et al. (2019) have validated EDFM using an analytical solution and Local Grid Refinement (LGR) for hydrologic problems. Since EDFM in this work has been modified to explicitly include all discrete fracture elements and will be used on a coupled THM model, validation should be performed for the newly developed model. The simulation
result of LGR was compared with EDFM in this case. The reservoir model is shown in Figure 5.1 as a two-dimensional case containing three fractures with intersections. LGR model had exactly the same geometrical parameters (fracture dimensions and locations), except that the near-fracture area was refined to capture the transient behavior of mass/energy transfer between fracture and matrix.

In total, EDFM has 2480 grids including matrix and fracture elements and LGR has 5325 grids. The dimension of the model was 106 m by 106 m by 20 m. Injection and production of water were both at fracture elements and conducted for 500 days. Input parameters of this case were shown in Table 5.1. The simulation was run by TOUGH2-THM without geomechanics coupling. EDFM took 55 seconds to finish the computation and LGR took 3600 seconds due to the small volumes of fracture elements. Simulation results are shown in Figure 5.2. Note that the flow rate in (a) of Figure 5.2 is the result of injecting cold water with 0.1 kg/s into a less permeable reservoir for both
EDFM and LGR approaches, such that the transient rate can be prolonged for better observation.

Table 5.1: Input parameters for EDFM validation cases

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix Permeability</td>
<td>2×10⁻¹⁴</td>
<td>m²</td>
</tr>
<tr>
<td>Fracture Permeability</td>
<td>2×10⁻¹¹</td>
<td>m²</td>
</tr>
<tr>
<td>Initial Reservoir Pressure</td>
<td>4.53×10⁷</td>
<td>Pa</td>
</tr>
<tr>
<td>Initial Reservoir Temperature</td>
<td>300</td>
<td>°C</td>
</tr>
<tr>
<td>Production (constant pressure)</td>
<td>1×10⁷</td>
<td>Pa</td>
</tr>
<tr>
<td>Injection (constant rate)</td>
<td>1</td>
<td>kg/s</td>
</tr>
<tr>
<td>Injection Specific Enthalpy</td>
<td>3×10⁵</td>
<td>J/kg</td>
</tr>
<tr>
<td>Rock/Fracture Porosity</td>
<td>0.05</td>
<td>Unitless</td>
</tr>
<tr>
<td>Rock/Fracture Heat Conductivity</td>
<td>5</td>
<td>W/(m·°C)</td>
</tr>
<tr>
<td>Rock/Fracture Specific Heat</td>
<td>1000</td>
<td>J/(kg·°C)</td>
</tr>
<tr>
<td>Production Well Index</td>
<td>4×10⁻¹⁰</td>
<td>m³</td>
</tr>
</tbody>
</table>

Figure 5.2: Comparison of (a) production rate and (b) production temperature for LGR and EDFM cases (0.1 kg/s of injection for (a)).

It is shown in the comparison in Figure 5.3 that simulation results of EDFM match well with those of LGR on pressure/temperature distribution, temperature production, and transient flow rate. The matched results validate the accuracy of the fluid/heat flow calculation of the modified model. Moreover, EDFM exhibits a better performance with much fewer grids and faster computation than LGR.
5.2 Mechanics Module Validation

The validation for mechanics modules in the fully coupled and sequentially coupled modes is conducted in this section, by comparing with the existing software and analytical solutions.

5.2.1 Fully Coupled Model

TOUGH2-THM computes all of the stress tensor components fully implicitly and fully coupled with fluid/heat flow. Compared with TOUGH2-CSM, the improvement mainly lies in solving normal stresses on arbitrary-strike embedded discrete fractures. TOUGH2-CSM updates the stress tensor for each grid block after a time step convergence and has been validated by analytical solutions and existing modeling work. Thus, in order to validate
this newly developed fully coupled model, it is desired to compare TOUGH2-THM with TOUGH2-CSM as a benchmark.

Table 5.2: Input parameters for geomechanical validation cases

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix Permeability</td>
<td>$2 \times 10^{-15}$</td>
<td>$\text{m}^2$</td>
</tr>
<tr>
<td>Fracture Permeability</td>
<td>$2 \times 10^{-11}$</td>
<td>$\text{m}^2$</td>
</tr>
<tr>
<td>Initial Reservoir Pressure</td>
<td>$4.53 \times 10^7$</td>
<td>$\text{Pa}$</td>
</tr>
<tr>
<td>Initial Reservoir Temperature</td>
<td>300</td>
<td>$^\circ \text{C}$</td>
</tr>
<tr>
<td>Production (constant pressure)</td>
<td>$1 \times 10^7$</td>
<td>$\text{Pa}$</td>
</tr>
<tr>
<td>Injection (constant rate)</td>
<td>0.1</td>
<td>$\text{kg/s}$</td>
</tr>
<tr>
<td>Injection Specific Enthalpy</td>
<td>$3 \times 10^5$</td>
<td>$\text{J/kg}$</td>
</tr>
<tr>
<td>Rock/Fracture Porosity</td>
<td>0.05</td>
<td>Unitless</td>
</tr>
<tr>
<td>Rock/Fracture Heat Conductivity</td>
<td>5</td>
<td>$\text{W/(m} \cdot ^\circ \text{C})$</td>
</tr>
<tr>
<td>Rock/Fracture Specific Heat</td>
<td>1000</td>
<td>$\text{J/(kg} \cdot ^\circ \text{C})$</td>
</tr>
<tr>
<td>Production Well Index</td>
<td>$4 \times 10^{-14}$</td>
<td>$\text{m}^3$</td>
</tr>
<tr>
<td>Rock Expansion Coefficient</td>
<td>$4 \times 10^{-6}$</td>
<td>$1/^\circ \text{C}$</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>0.25</td>
<td>Unitless</td>
</tr>
<tr>
<td>Young’s Modulus</td>
<td>30</td>
<td>$\text{GPa}$</td>
</tr>
<tr>
<td>Biot’s Coefficient</td>
<td>1</td>
<td>Unitless</td>
</tr>
<tr>
<td>Permeability Correlation Coefficient</td>
<td>2</td>
<td>Unitless</td>
</tr>
</tbody>
</table>

Figure 5.4: Geothermal model for coupled model validation (comparing TOUGH2-THM and TOUGH2-CSM).

A coarse grid problem was built for comparing coupled modeling results of two programs: a homogeneous model of 10 by 10 by 4 grids with dimensions of 100 m $\times$ 80 m $\times$ 24 m. Grid blocks were all of dimensions 10 m $\times$ 8 m $\times$ 6m, as shown in Figure 5.4. Injection of cold water into a hot geothermal reservoir induced stress field alteration,
especially at injection and production points. Stress states, as well as pressure and
temperature, were compared between two models at the production grid. Input parameters
are shown in Table 5.2. The reference stress state at the top of the reservoir was:
\[ \sigma_{xx} = 1.27 \cdot 10^8 \text{Pa}, \quad \sigma_{yy} = 0.96 \cdot 10^8 \text{Pa}, \quad \sigma_{zz} = 1.21 \cdot 10^8 \text{Pa} \]
and all boundary conditions were constant stress for mechanical computation. The observation was set up at the production
grid and the results of the comparison are shown in Figure 5.5. The matched results
validate the newly developed coupled fluid/heat flow and geomechanics model.

![Fig 5.5: Comparison of pore pressure and stress state of production grid for validation case (10*10*4 coarse grid model).](image.png)

### 5.2.2 Sequentially Coupled Model

In this section, the extended capabilities of the mechanics module in the sequentially
coupled model are validated. The implementations to be validated include: (1) general
boundary traction and body force in XFEM; (2) newly developed XFEM stiffness matrix
assembly; (3) internal traction boundary for XFEM.

Table 5.3: Input parameters for FEM boundary traction and body force validation

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir dimension in X direction</td>
<td>25*4</td>
<td>m</td>
</tr>
<tr>
<td>Reservoir dimension in Y direction</td>
<td>20*4</td>
<td>m</td>
</tr>
<tr>
<td>Reservoir dimension in Z direction</td>
<td>10*4</td>
<td>m</td>
</tr>
<tr>
<td>Rock Young’s modulus</td>
<td>90</td>
<td>GPa</td>
</tr>
<tr>
<td>Rock Poisson’s ratio</td>
<td>0.3</td>
<td>unitless</td>
</tr>
<tr>
<td>Rock solid density</td>
<td>2650</td>
<td>kg/m³</td>
</tr>
<tr>
<td>Stress on top</td>
<td>-4×10⁶</td>
<td>Pa</td>
</tr>
<tr>
<td>Stress on YZ face top</td>
<td>-1×10⁶</td>
<td>Pa</td>
</tr>
<tr>
<td>Stress gradient on YZ face</td>
<td>2.5×10⁴</td>
<td>Pa/m</td>
</tr>
<tr>
<td>Stress on XZ face top</td>
<td>-1.5×10⁶</td>
<td>Pa</td>
</tr>
<tr>
<td>Stress gradient on YZ face</td>
<td>3.75×10⁴</td>
<td>Pa/m</td>
</tr>
</tbody>
</table>

Figure 5.6: Boundary conditions and tractions for boundary condition validations: the dark blue columns are zero displacement boundaries in the corresponding direction; red arrows are applied tractions.

5.2.2.1 Body Force and Boundary Traction

The implementations of body force and external traction are to be validated first. These two terms in the right-hand-side force vector are essential in the reservoir-scale modeling in which boundary tractions are usually applied to reflect the geo-stress conditions. Body force is also a factor that deforms the rock matrix. The benchmark software for validation is SfePy, an open-source Python-based FEM analysis tool that has been widely used in academia and industry. The input data is summarized in Table 5.3.
Figure 5.7: Result comparison between XFEM implementation and SfePy: The column on the left is the result from this work, the column on the right is the result from SfePy; The first row is displacement in the X-direction, the second in the Y-direction and the third in the Z-direction.

Note that no fluid or heat is considered in this case, only elastic deformation caused by the body force and boundary tractions. The XZ face at y=0, the YZ face at x=0 are fixed zero displacement boundaries in corresponding directions. The top of the reservoir is imposed by constant stress and on XZ, YZ face where tractions are imposed, stresses are
increasing as depth with a constant gradient, as shown in Figure 5.6.

The plotting of the results from our model and SfePy is shown in Figure 5.7. The plotting from our implementation is visualized in MATLAB and SfePy in PyVista. The same color scale is used for the two results although the displayed colors seem a bit different. The good match between the two results validated the implementation of external tractions and body force.

5.2.2.2 Assembly of the Stiffness Matrix

Compared to the original XFEM program, the assembly of the stiffness matrix is improved by adopting the integration approach of subdivision into small elements and also the way of computing the derivative matrix $B$, especially $B^{tip}$. These implementations accelerate the computational processes but need to be validated using the original code (Wang et al. 2020). The boundary tractions are kept the same as the above case except that the direction of traction on YZ face is reversed and becomes tension instead of compression. The stress gradient also remains unchanged. See Table 5.3 for the input data. Discontinuity has been introduced into this validation, as shown in Figure 5.8.

![Figure 5.8: Fracture discontinuity and reservoir model for validating the new implementation of the stiffness matrix.](image-url)
The results with the above boundary traction are shown in Figure 5.9. The plotting dots in the first row of this figure are located at the Gauss points where the numerical integration is evaluated. Those locations are recycled for plotting purposes. The same color scale is used in both fracture width plottings and a good match is observed. The second row shows the displacement in the X-direction. The displacements in different directions have also been plotted and compared. A good match between the new model and the original program indicates that the implementation for stiffness matrix assembly can be validated.

Figure 5.9: Validation of the new implementation on the stiffness matrix assembly by comparing fracture aperture distribution and displacement in X-direction: the first row is fracture width plotting, the second is displacement in X-direction; the left column is our model and the right column is from the original program.
5.2.2.3 Internal Pressure Traction

The original code has the capability of computing the displacement with internal pore pressure as traction. A new efficient approach is implemented to replace the original one to numerically integrate the jump shape function. This implementation needs to be validated. In this case, the input data is as shown in Table 5.3 where all boundary tractions are compressive. The additional internal traction imposed on fracture is $4 \times 10^6$ Pa, modeling a uniform pore pressure in the fracture.

![Validation for the new implementation of internal pressure traction](image)

The results are shown in Figure 5.10. The first row shows the fracture aperture closure (relative discontinuity displacement) at all Gauss integration points and it can be observed that the fracture apertures of the two results don’t match perfectly near the fracture tip.
but the overall discrepancy is acceptable. The displacement of the matrix in the 
X-direction, as illustrated in the second row, matches well between the new model and the 
original program. The consistent matrix nodal displacement will be able to provide 
accurate matrix stress results as well.

5.3 Coupled Model Validation using Analytical Solutions

Besides the validation compared against existing models, it is also necessary to validate 
the coupled model using analytical solutions. Commonly adopted solutions for validating 
coupled model are Terzaghi’s problem (Terzaghi et al. 1996) and Mandel’s problem 
(Abousleiman et al. 1996). Terzaghi’s problem is used to test the numerical code for 
fluid-to-solid coupling while Mandel’s problem is used as a benchmark for the coupled 
poroelasticity (Jha and Juanes 2014). In our fully coupled model, the poroelasticity effect 
(rapid mechanical response to the pore pressure change) can be captured by fully coupling 
the fluid flow and geomechanics. On the other hand, in the sequentially coupled model, the 
poroelasticity can also be captured according to Dana et al. (2018); Jha and Juanes (2014) 
where the geomechanical response is reflected by the Lagrangian porosity. Therefore, in 
this section, the fully coupled model will be validated against Mandel’s problem and the 
sequentially coupled model against both Terzaghi’s and Mandel’s problem.

5.3.1 Fully Coupled Model against Mandel’s Problem

The well-known Mandel-Cryer problem describes a situation when a rectangular sample 
saturated with fluid subject to a constant load (force per length), $2F$, at the top through a 
rigid plate of width $2a$ and the lateral boundary is allowed to drain. The Mandel-Cryer 
effect states that the load will be transferred to the center of the sample at the beginning 
period of drainage due to the rapid boundary pore pressure loss, resulting in a pressure 
increase before gradually reduced. The Mandel-Cryer problem assumes plane strain, that 
is, no strain is allowed in the y-direction (perpendicular to x and z), as shown in 
Figure 5.11.
Abousleiman et al. (1996) provided analytical solutions to this problem, which will be adopted here for model validation:

\[ p(x, t) = p_0 \sum_{n=1}^{\infty} \frac{2\sin\alpha_n}{\alpha_n - \sin\alpha_n \cos\alpha_n} \left( \cos \left( \frac{\alpha_n x}{a} \right) - \cos\alpha_n \right) \exp \left( -\frac{\alpha_n^2 ct}{a^2} \right) \]  

(5.1)

in which \( x \) is the horizontal distance away from the center of the sample, \( t \) is the time after the start of drainage and \( \alpha_n \) is the root of:

\[ \frac{\tan(\alpha_n)}{\alpha_n} = \frac{1 - \nu}{\nu_u - \nu} \]  

(5.2)

in which parameters, undrained Poisson’s ratio \( \nu_u \), Skempton coefficient, \( B \), undrained bulk modulus, \( K_u \), Biot’s modulus, \( M \) can be expressed as:

\[ \nu_u = \frac{3\nu + \alpha B(1 - 2\nu)}{3 - \alpha B(1 - 2\nu)} \]

\[ B = \alpha M / K_u \]

\[ K_u = \lambda + \frac{2}{3} G + \alpha^2 M \]

\[ \frac{1}{M} = C_f \phi + \frac{(1 - \alpha)(\alpha - \phi)}{K} \]  

(5.3)

in which \( C_f \) is fluid compressibility, \( \phi \) is the porosity, \( \lambda \) and \( G \) are Lamé parameters, \( \nu \) is Poisson’s ratio, \( \alpha \) is Biot’s coefficient. \( c \) in Equation 5.1 can be written as:

\[ c = \frac{k}{\mu \left( \frac{1}{M} + \frac{\alpha^2}{\lambda + 2G} \right)} \]  

(5.4)
where \( \mu \) is the fluid viscosity and \( k \) is permeability. The instantaneous pressure \( p_0 \) in Equation 5.1 after applying the top load rises to:

\[
p_0 = \frac{F}{3a}B(1 + \nu_u)
\]  

(5.5)

According to the model setup, a TOUGH-THM model is built with 51 by 51 by 1 grids, with dimensions of 1001 m \( \times \) 1001 m \( \times \) 10 m, indicating that \( a = 500 \) m. All grids are set to be constant stress boundaries with 15 MPa of y-direction normal stress and 5 MPa of mean stress at the stress load stage while all grids except top grids are boundary grids without stress flux at the drainage stage. The first simulation stage is modeling the pore pressure increase after the instant load was applied and the pore pressure increased to about 2.086 MPa which is consistent with the analytical solution. The second simulation stage models the fluid drainage out of the sample and pore pressure in the center (the blue triangle in Figure 5.11) is observed. Input parameters are shown in Table 5.4. The comparison of results is shown in Figure 5.12.

Table 5.4: Input parameters of Mandel-Cryer problem for the analytical solution and TOUGH2-THM

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix Permeability</td>
<td>1( \times )10(^{-13} )</td>
<td>m(^2)</td>
</tr>
<tr>
<td>Initial Pore Pressure</td>
<td>1( \times )10(^5)</td>
<td>Pa</td>
</tr>
<tr>
<td>Initial Sample Temperature</td>
<td>60</td>
<td>°C</td>
</tr>
<tr>
<td>Production (constant pressure)</td>
<td>1( \times )10(^5)</td>
<td>Pa</td>
</tr>
<tr>
<td>Rock/Fracture Porosity</td>
<td>0.094</td>
<td>Unitless</td>
</tr>
<tr>
<td>Production Well Index</td>
<td>1.7( \times )10(^{-9} )</td>
<td>m(^3)</td>
</tr>
<tr>
<td>Rock Expansion Coefficient</td>
<td>4( \times )10(^{-6} )</td>
<td>1/°C</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>0.25</td>
<td>Unitless</td>
</tr>
<tr>
<td>Young’s Modulus</td>
<td>5</td>
<td>GPa</td>
</tr>
<tr>
<td>Biot’s Coefficient</td>
<td>1</td>
<td>Unitless</td>
</tr>
<tr>
<td>Water Viscosity (60 °C, analytical)</td>
<td>0.00046</td>
<td>Pa \cdot s</td>
</tr>
<tr>
<td>Water Compressibility (analytical)</td>
<td>4.04( \times )10(^{-10} )</td>
<td>1/Pa</td>
</tr>
<tr>
<td>Water Density (analytical)</td>
<td>983</td>
<td>kg/m(^3)</td>
</tr>
</tbody>
</table>
5.3.2 Sequentially Coupled Model against Terzaghi’s Problem

In Terzaghi’s problem, a laterally constrained specimen is subjected to a uniform compressive load at the top surface. All sides are no-flux boundaries except the top surface which is open to flow, as illustrated by Figure 5.13. At time $t = 0^+$, the specimen is compressed and the pore pressure rises to the undrained value, known as the Skempton effect. The undrained value of pressure serves as the initial condition for the drained
consolidation simulation.

Table 5.5: Input parameters of Terzaghi’s problem for analytical solutions and the sequentially coupled model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permeability</td>
<td>$1 \times 10^{-12}$</td>
<td>m$^2$</td>
</tr>
<tr>
<td>Initial Pore Pressure</td>
<td>$1.0786 \times 10^6$</td>
<td>Pa</td>
</tr>
<tr>
<td>Initial Sample Temperature</td>
<td>20</td>
<td>°C</td>
</tr>
<tr>
<td>Production (constant pressure)</td>
<td>0</td>
<td>Pa</td>
</tr>
<tr>
<td>Rock/Fracture Porosity</td>
<td>0.375</td>
<td>Unitless</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>0.25</td>
<td>Unitless</td>
</tr>
<tr>
<td>Young’s Modulus</td>
<td>0.1</td>
<td>GPa</td>
</tr>
<tr>
<td>Biot’s Coefficient</td>
<td>1</td>
<td>Unitless</td>
</tr>
<tr>
<td>Water viscosity (60 °C, analytical)</td>
<td>0.001</td>
<td>Pa $\cdot$ s</td>
</tr>
<tr>
<td>Water compressibility (analytical)</td>
<td>$4.4 \times 10^{-10}$</td>
<td>1/Pa</td>
</tr>
<tr>
<td>Water density (analytical)</td>
<td>1000</td>
<td>kg/m$^3$</td>
</tr>
</tbody>
</table>

Figure 5.14: Comparison between numerical and analytical solutions of Terzaghi’s problem

Under this condition, as time increases, the specimen consolidates vertically as the fluid leaks out from the top surface. Diffusion of pore pressure decouples from the stress and satisfies a homogeneous diffusion equation. Strain due to compaction is proportional to the pressure drop. The analytical solution of pore pressure and vertical displacement can be expressed by:

$$p(z, t) = \frac{4}{\pi} p_0 \sum_{m=0}^{\infty} \frac{1}{2m+1} \exp \left[ \frac{-(2m + 1)^2 \pi^2 ct}{4L^2} \right] \sin \left[ \frac{(2m + 1) \pi z}{2L} \right]$$  \hspace{1cm} (5.6)
\[ u(z, t) = c_M p_0 \left\{ (L - z) - \frac{8L}{\pi^2} \sum_{m=0}^{\infty} \frac{1}{(2m + 1)^2} \exp \left[ -\left(\frac{(2m + 1)\pi^2}{4L^2} \right) \right] \cos \left[ \frac{(2m + 1)\pi z}{2L} \right] \right\} + u_0 \]  

(5.7)

where

\[ p_0(z) = \frac{\alpha M}{K_u + 4G/3} p_L \]  

(5.8)

\[ u_0(z) = \frac{1}{K_u + 4G/3} p_L (L - z) \]  

(5.9)

in which \( M = [\phi C_f + (\alpha - \phi)/K_s]^{-1} \) the Biot Modulus, \( c_{br} \) the solid grain compressibility, \( K_u = \lambda + 2G/3 + \alpha^2 M \) the undrained bulk modulus, \( c_M = (\lambda + G)^{-1} \) the vertical uniaxial compressibility, and \( c = k/[\rho g (M^{-1} + \alpha^2 c_M)] \) the consolidation coefficient.

Using the sequentially coupled model, Terzaghi’s consolidation problem has been validated. The specimen is set to have a dimension of 1.8 m × 1.8 m × 15 m. The load applied on the top surface is 1.1 MPa, resulting in an initial undrained pressure of 1.078 MPa. Input parameters are summarized in Table 5.5. The comparison results between numerical and analytical solutions are shown in Figure 5.14, displaying a good match.

5.3.3 Sequentially Coupled Model against Mandel’s Problem

The background and analytical solutions of Mandel’s problem have been introduced in section 5.3.1 for the fully coupled model. The boundary conditions of the sequentially coupled model, however, is closer to the original Mandel’s problem: the top boundary has been computed analytically and the top in the numerical model settles uniformly according to the analytical solution:

\[ u_z(b, t) = -\frac{F}{a \lambda} \frac{\nu(1 - \nu)}{1 - 2\nu} \left( 1 + 2 \left( \frac{\nu - \nu}{1 - \nu} - 1 \right) \right) \sum_{n=1}^{\infty} \frac{\sin \alpha_n \cos \alpha_n}{\alpha_n - \sin \alpha_n \cos \alpha_n} \exp \left( -\frac{\alpha_n^2 ct}{a^2} \right) b \]  

(5.10)

which is equivalent to applying a certain traction on a rigid plate.

The model in Figure 5.11 can be divided into four equal parts by \( x = 0 \) and \( z = 0 \). Only one of the four parts is used in the modeling: the \( x = 0 \) face is fixed with zero displacement in the X-direction and the \( z = 0 \) face is fixed with zero displacement in the
Z-direction. The top force applied is $6 \times 10^7$ Pa/m and the dimension of the sample is $6 \times 6$ m (a quarter of the sample). The input parameters for the sequentially coupled model of Mandel’s problem are listed in Table 5.6.

Table 5.6: Input parameters of Mandel-Cryer problem for analytical solution and the sequentially coupled model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix Permeability</td>
<td>$1 \times 10^{-14}$</td>
<td>m$^2$</td>
</tr>
<tr>
<td>Initial Sample Temperature</td>
<td>20</td>
<td>°C</td>
</tr>
<tr>
<td>Production (constant pressure)</td>
<td>$1 \times 10^5$</td>
<td>Pa</td>
</tr>
<tr>
<td>Matrix Porosity</td>
<td>0.375</td>
<td>Unitless</td>
</tr>
<tr>
<td>Production Well Index</td>
<td>$4 \times 10^{-15}$</td>
<td>m$^3$</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>0.25</td>
<td>Unitless</td>
</tr>
<tr>
<td>Young’s Modulus</td>
<td>1</td>
<td>GPa</td>
</tr>
<tr>
<td>Biot’s Coefficient</td>
<td>1/0.8</td>
<td>Unitless</td>
</tr>
<tr>
<td>Water Viscosity (20 °C, analytical)</td>
<td>0.001</td>
<td>Pa · s</td>
</tr>
<tr>
<td>Water Compressibility (analytical)</td>
<td>$4.4 \times 10^{-10}$</td>
<td>1/Pa</td>
</tr>
<tr>
<td>Water Density (analytical)</td>
<td>1000</td>
<td>kg/m$^3$</td>
</tr>
</tbody>
</table>

Two Biot coefficients have been used: 1 and 0.8. The comparison between numerical and analytical solutions is shown in Figure 5.15. The good match between analytical solutions and numerical simulations validate the sequentially coupled model implementation.

Figure 5.15: Comparison between numerical and analytical solution of the Mandel’s problem for sequentially coupled model. Left: log scale in time; Right: normal scale in time.
In this chapter, the fully coupled model is applied to conduct numerical studies on both synthetic and field EGS models. The synthetic model contains six connected/intersected hydraulic fractures with injection from one end of the reservoir to the other. Numerical studies are conducted by changing key input parameters such as matrix thermal expansion coefficient, matrix permeability, and thermal conductivity. The field EGS model is established based on the national geothermal research field experiments, EGS-Collab, with a hydraulically stimulated fracture and two detected natural fractures. Different models are built to match the measured output temperatures and flow rates. Both models are run on a High Performance Computing (HPC) cluster sponsored by the Energy Modeling Group at Colorado School of Mines. The synthetic model has 1076 grids and uses 8 cores on two nodes and the field model has 36436 grids and runs with 80 cores on 10 nodes.
6.1 Applications on a Synthetic EGS Model

Firstly, the fully coupled model is applied onto a synthetic EGS model with an intermediate scale and multiple fracture embedded. Multiple parameters are varied for sensitivity analyses. The comparison of stress, pressure, temperature and porosity/permeability is performed at the end of the simulation. Production behaviours are compared dynamically with respect to time. Several key findings are summarized at the end of this section for the synthetic EGS model.

Table 6.1: Input parameters of synthetic EGS model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix Permeability</td>
<td>2×10^{-18}</td>
<td>m(^2)</td>
</tr>
<tr>
<td>Fracture Permeability</td>
<td>2×10^{-11}</td>
<td>m(^2)</td>
</tr>
<tr>
<td>Initial Reservoir Pressure</td>
<td>4.53×10^7</td>
<td>Pa</td>
</tr>
<tr>
<td>Initial Reservoir Temperature</td>
<td>300</td>
<td>°C</td>
</tr>
<tr>
<td>Production (constant pressure)</td>
<td>1×10^7</td>
<td>Pa</td>
</tr>
<tr>
<td>Injection (constant rate)</td>
<td>2</td>
<td>kg/s</td>
</tr>
<tr>
<td>Injection Specific Enthalpy</td>
<td>4.6×10^5</td>
<td>J/kg</td>
</tr>
<tr>
<td>Rock/Fracture Porosity</td>
<td>0.05</td>
<td>Unitless</td>
</tr>
<tr>
<td>Rock/Fracture Heat Conductivity</td>
<td>5</td>
<td>W/(m · °C)</td>
</tr>
<tr>
<td>Rock/Fracture Specific Heat</td>
<td>1000</td>
<td>J/(kg · °C)</td>
</tr>
<tr>
<td>Production Well Index</td>
<td>4×10^{-11}</td>
<td>m(^3)</td>
</tr>
<tr>
<td>Rock Expansion Coefficient</td>
<td>4×10^{-6}</td>
<td>1/°C</td>
</tr>
<tr>
<td>Matrix/Fracture Poisson’s Ratio</td>
<td>0.25</td>
<td>Unitless</td>
</tr>
<tr>
<td>Matrix Young’s Modulus</td>
<td>30</td>
<td>GPa</td>
</tr>
<tr>
<td>Biot Coefficient</td>
<td>0.8</td>
<td>Unitless</td>
</tr>
<tr>
<td>Initial Fracture Aperture</td>
<td>5×10^{-5}</td>
<td>m</td>
</tr>
<tr>
<td>Maximum Mechanical Fracture Aperture</td>
<td>2.5×10^{-4}</td>
<td>m</td>
</tr>
<tr>
<td>Fracture Permeability Correlation Coefficient, (d) (in Equation 3.50)</td>
<td>4e×10^{-8}</td>
<td>1/Pa</td>
</tr>
</tbody>
</table>

Figure 6.2: Stress XX, YY and ZZ distribution of the synthetic EGS model.
6.1.1 Base Case and Effects of Mechanics Coupling

Combining EDFM and TOUGH2-THM, a synthetic geothermal reservoir model has been established and cold water is injected for 500 days. The model has dimensions of 100 m × 100 m × 24 m and six long embedded discrete fractures. The geometrical mesh is shown in Figure 6.1, with injection and production both at fracture elements.

The first step was to produce the pore pressure equilibrium state. TOUGH2-THM is used without geomechanical coupling for fluid/heat flow modeling and the initial state is saved for further simulation. Afterward, the initial state is input into the program for stress field initialization. Reference stress state is on the top of the reservoir:

Figure 6.3: Temperature and pressure distribution of the fractured geothermal reservoir simulation (a) coupling geomechanics; (b) only fluid/heat flow without coupling geomechanics.

The first step was to produce the pore pressure equilibrium state. TOUGH2-THM is used without geomechanical coupling for fluid/heat flow modeling and the initial state is saved for further simulation. Afterward, the initial state is input into the program for stress field initialization. Reference stress state is on the top of the reservoir:
\( \tau_{xx} = 1.27 \times 10^8 \text{ Pa}, \tau_{yy} = 0.96 \times 10^8 \text{ Pa}, \tau_{zz} = 1.21 \times 10^8 \text{ Pa} \), and initial shear stresses are all zero. Reservoir boundaries are set to be constant stress boundaries (using initial stress). Other reservoir input parameters are shown in Table 6.1.

The results of effective stress (Equation 3.30) distribution are shown in Figure 6.2 in which the negative stress is compressive so that the visualization can be consistent for both fully and sequentially coupled models. The fractured volume has an effective stress contour conforming to the fracture strikes and shapes, which indicates the contribution from thermal stresses. The tensile deformation of the reservoir is more likely to occur in the temperature reduction area.

![Production Rate under Different Coupling Conditions](image1)

![Production Temperature under Different Coupling Conditions](image2)

Figure 6.4: Comparison of flow rate and production temperature for three cases, TH only (no geomechanics coupling) and THM (coupled model) and THM with larger thermal expansion coefficient.

The simulation results of pressure and temperature distribution are shown in Figure 6.3. Two cases were compared: (1) fluid/heat flow model without coupling geomechanics; (2) coupled fluid/heat flow and geomechanics model. As can be seen, cold temperature isothermal contour covered slightly larger area close to fractures in the geomechanical model, but reverse phenomena were observed for pressure isobar contour. This is due to the elliptic nature of the pressure equation, such that the distribution of pressure change across the space is globally coupling and basically dependent on permeability: the geomechanical case had a lower matrix permeability due to compression and thermal strain but the fracture permeability was enhanced by one or two orders. The pressure was
retained to be high in no coupling case. On the other hand, temperature change relies on both conductive and convective heat flow: fluid traveled a longer distance around fracture due to enhanced matrix permeability while heat transferred faster from the highly (mass) conductive fracture path in the geomechanical case. The reservoir heats the injected fluid mainly by the conduction processes.

![Stress XX Distribution](image1)
![Stress YY Distribution](image2)
![Stress ZZ Distribution](image3)

(a)

![Stress XX Distribution](image4)
![Stress YY Distribution](image5)
![Stress ZZ Distribution](image6)

(b)

![Stress XX Distribution](image7)
![Stress YY Distribution](image8)
![Stress ZZ Distribution](image9)

(c)

Figure 6.5: Stress XX, YY and ZZ distribution of the synthetic EGS model at the end of production: sensitivity analysis for thermal conductivity. (a) $\lambda = 8 \text{ W/(m}^\circ\text{C)}$; (b) $\lambda = 5 \text{ W/(m}^\circ\text{C)}$; (c) $\lambda = 2 \text{ W/(m}^\circ\text{C)}$

The flow rate and production temperature out of the production well were compared in Figure 6.4 with blue and green colors: it is shown that the flow rate of the THM coupling case is lower than the TH no-coupling case in the very early transient stage and gradually
becomes higher; the production temperature of the THM coupling case is lower than the TH no-coupling case by 1-2 °C due to the enhanced permeability of the fracture and the matrix around the injection area.

Figure 6.6: Pressure, temperature and matrix permeability distribution of the synthetic EGS model at the end of production: sensitivity analysis for thermal conductivity. (a) $\lambda = 8$ W/(m°C); (b) $\lambda = 5$ W/(m°C); (c) $\lambda = 2$ W/(m°C)

6.1.2 Effects of Thermal Conductivity

The injected cold fluid is heated by the high-temperature geothermal reservoir. The matrix of EGS should have low permeability so that the injected fluid can be circulated with a high recovery factor, resulting in a less dominating convective process. Heat conduction, consequently, is the major heat source of fluid circulation. It is intuitive that
the higher thermal conductivity would facilitate the heat mining process by providing more heat to the fluid. The sensitivity analysis of thermal conductivity has been performed in this section, by increasing the matrix conductivity from $2 \text{ W/(m} \cdot \text{°C)}$ to $5 \text{ W/(m} \cdot \text{°C)}$ and to $8 \text{ W/(m} \cdot \text{°C)}$ and keeping other parameters in Table 6.1 unchanged.

Figure 6.7: First row: comparison of production temperature, flow rate and permeability: sensitivity analysis for thermal conductivity; second row: flow rates of the early stage, middle stage and late stage.

The stress distributions for the three cases are shown in Figure 6.5. The pressure and temperature distributions of the three cases are shown in Figure 6.6. The temperatures show that the higher thermal conductivity causes the cold water to cool down the reservoir, reaching a lower reservoir temperature. Intuitively, the lower temperature generates a higher absolute thermal stress so that the effective stress becomes more tensile and less compressive under an approximately constant strain (not including thermal strain). This intuition agrees with the stress distribution. The tensile stress effect is the least in the z-direction since the fractures are all vertical and exert a stronger impact on XX and YY stresses. The higher conductivity also increases the fracture permeability more than in the other two cases, which reduces the pressure near the injection area. Although the lower temperature near the fracture in the higher conductivity scenario increases the
water viscosity, the reduced pressure infers that the permeability is the dominant factor of pressure distribution.

Figure 6.8: Stress XX, YY and ZZ distribution of the synthetic EGS model at the end of production: sensitivity analysis for matrix permeability. (a) $2 \times 10^{-18}$ m$^2$; (b) $1 \times 10^{-17}$ m$^2$; (c) $5 \times 10^{-17}$ m$^2$

The comparisons of flow rates, production temperatures, and observation point permeability are shown in Figure 6.7. It can be seen that the high conductivity reservoir matrix tends to heat the circulation fluid with a higher temperature. The flow rate at the early stage is larger in the higher thermal conductivity reservoir because of the rapid temperature reduction near the fracture. However, at the late stage, it is a different scenario in which the matrix permeability enhancement also spreads more widely in the
high conductivity reservoir and the mass transfer process loses more circulation fluid. Nevertheless, the above intuitive analysis may not reflect the complex interaction between thermal, hydrologic and mechanics processes.

6.1.3 Effects of Matrix Permeability

Matrix permeability is an important factor for examining the contribution of convective processes to the coupled THM modeling. A more permeable matrix could consume more injected fluid and release hot fluid (if the reservoir is not considered as a hot dry rock) into the production well. Injected fluid may penetrate the reservoir matrix and decrease the reservoir temperature by convection and produced fluid from the matrix may help improve the heat recovery efficiency if the reservoir is saturated with hot fluid.

The matrix permeability has been increased from the base case, $2 \times 10^{-18}$ m$^2$, to $1 \times 10^{-17}$ m$^2$ and $5 \times 10^{-17}$ m$^2$ for sensitivity analysis. Stress distributions are shown in Figure 6.8 and pressure/temperature distribution in Figure 6.9. It can be seen that when matrix permeability is increased, the injection area has a slightly lower temperature and the production volume has a slightly higher temperature, both due to the stronger convection. The stress fields are almost identical for the three scenarios.

Comparisons of production temperatures, flow rates, and observation point permeability are shown in Figure 6.10. Convective flow from the matrix to the fractures enhances the production rate in the early stage. Due to the heating and convection from reservoir to the injected fluid, the higher matrix permeability case results in a slightly lower fracture permeability at the observation point and a higher production temperature in the late stage.

6.1.4 Effects of Fracture Aperture Model

Fracture aperture models, described by Equation 3.50, determine the responses of fracture aperture to the normal stresses. In this section, the coefficient, $d$, is varied for a sensitivity analysis.
Figure 6.9: Pressure and temperature distribution of the synthetic EGS model at the end of production: sensitivity analysis for matrix permeability. (a) $2 \times 10^{-18}$ m$^2$; (b) $1 \times 10^{-17}$ m$^2$; (c) $5 \times 10^{-17}$ m$^2$. 
The base case has a coefficient of $4 \times 10^{-8}$ 1/Pa which is increased to $1 \times 10^{-7}$ 1/Pa and $2.5 \times 10^{-7}$ 1/Pa. The aperture is plotted against normal effective stresses in Figure 6.11. The base case ($4 \times 10^{-8}$ 1/Pa) is equivalent to a low fracture stiffness and subject to a higher deformation when the normal stress increases or reduces.

The stresses distributions are shown in Figure 6.12 and the pressure/temperature distributions are shown in Figure 6.13. It can be observed that the temperature field is not disturbed much comparing the three models but the average reservoir pressure has been enhanced due to the stiff fracture response. Accordingly, the effective stresses reflect a
stronger tensile trend when the pressure is raised, although the stress contours still conform more toward the fracture shapes and distributions. Higher pressure essentially reduces the effective stresses of the reservoir.

Figure 6.12: Stress XX, YY and ZZ distribution of the synthetic EGS model at the end of production: sensitivity analysis for fracture aperture models. (a) $4 \times 10^{-8}$ 1/Pa; (b) $1 \times 10^{-7}$ 1/Pa; (c) $2.5 \times 10^{-7}$ 1/Pa
Figure 6.13: Pressure and temperature distribution of the synthetic EGS model at the end of production: sensitivity analysis for fracture aperture models. (a) $4 \times 10^{-8}$ 1/Pa; (b) $1 \times 10^{-7}$ 1/Pa; (c) $2.5 \times 10^{-7}$ 1/Pa
Figure 6.14: Comparison of production temperature, flow rate and permeability: sensitivity analysis for fracture aperture models

Figure 6.15: Stress XX, YY and ZZ distribution of the synthetic EGS model at the end of production: sensitivity analysis for injection temperatures. (a) 105 °C; (b) 86 °C; (c) 68 °C
Figure 6.16: Pressure and temperature distribution of the synthetic EGS model at the end of production: sensitivity analysis for injection temperatures. (a) 105 °C; (b) 86 °C; (c) 68 °C
The comparisons of production temperatures, flow rates, and observation permeabilities are plotted in Figure 6.14. The larger fracture aperture in the base case enhances the fracture permeability which in turn lowers the production temperature by about 2 °C but the overall production temperature has almost the same trend. The larger fracture aperture also results in a slightly higher flow rate in the middle plot, especially in the late stage when temperature reduction takes effect. The stable flow rates in the late stage of the three scenarios are almost identical, which may be caused by the similar matrix porosity fields since the temperature fields are not subject to drastic change. The observation permeability reflects that the higher coefficient makes the fracture less stiff: initially fracture permeability is lower and rapidly rises over the other two cases in the late stage.

![Figure 6.17: Comparison of production temperature, flow rate and permeability: sensitivity analysis for injection temperatures.](image)

6.1.5 Effects of Injection Temperature

The injection temperature is varied to observe how temperature would affect the simulation results. The injection temperature is decreased from 105 °C to 86 °C and 68 °C. Injection temperature may become a key engineering parameter since produced fluid may need to be reinjected for circulation. Injection temperature is a factor worth to be investigated in EGS development. The stress distributions and pressure/temperature distributions are shown in Figure 6.15 and Figure 6.16 respectively. The lower injection temperature amplifies the contribution of thermal stresses and the stronger tensile stresses are reflected by the warmer color contour in Figure 6.15, especially around the fracture volume. On the other hand, due to the enhanced fracture permeability by the lower
injection temperature, the injection area tends to have a lower injection pressure. The lower temperature should have increased the water viscosity which in turn raises the injection pressure but the enhanced fracture permeability mitigates this effect.

The comparisons of flow rate, production temperature, and observation permeability are plotted in Figure 6.17. Obviously, the production temperature is decreased when the injection temperature is reduced. Flow rates are not affected by the injection temperature although in the late stage, higher fracture permeability (68 ° C) reaches a slightly lower production rate due to the enhanced mass transfer between fractures and matrix.

Figure 6.18: Stress XX, YY and ZZ distribution of the synthetic EGS model at the end of production: sensitivity analysis for thermal expansion coefficient. (a) $4 \times 10^{-6}$ $1/\degree C$; (b) $8 \times 10^{-6}$ $1/\degree C$; (c) $16 \times 10^{-6}$ $1/\degree C$
Comparing the temperature reduction evolution in Figure 6.17 and Figure 6.7, it is demonstrated that when the thermal conductivity is high enough, even the cold fluid can be heated sufficiently. This observation also infers that heat conduction plays an essential role in the heat transfer processes.

6.1.6 Effects of Skeleton Linear Thermal Coefficient

The thermal strain is dependent on the skeleton linear thermal coefficient which induces the matrix porosity change. Besides, the normal effective stress acting on the fracture faces is computed by this coefficient as well. It is intuitive to expect the increased thermal coefficient to enhance fracture permeability, so that cold fluid has a faster path to reach the production well with earlier thermal breakthrough. However, the results are not in the expected trend which will be shown in this section. In order to accommodate the effect of the tensile thermal stresses, a new fracture aperture model in Table 6.2 is used in this section for all three cases: (1) thermal expansion coefficient $= 4 \times 10^{-6} \, {1/}^{\circ}C$; (2) thermal expansion coefficient $= 8 \times 10^{-6} \, {1/}^{\circ}C$; (3) thermal expansion coefficient $= 16 \times 10^{-6} \, {1/}^{\circ}C$.

Table 6.2: Input parameters of fracture aperture model for sensitivity analysis of expansion coefficient

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Fracture Aperture</td>
<td>$5 \times 10^{-5}$</td>
<td>m</td>
</tr>
<tr>
<td>Maximum Mechanical Fracture Aperture</td>
<td>$2 \times 10^{-4}$</td>
<td>m</td>
</tr>
<tr>
<td>Fracture Permeability Correlation Coefficient, $d$ (in Equation 3.50)</td>
<td>$8 \times 10^{-9}$</td>
<td>1/Pa</td>
</tr>
</tbody>
</table>

The stress distributions of the three cases are shown in Figure 6.18. The thermal stresses near the fractures with higher coefficients dominate the effective stresses, forming a stronger tensile area. The temperature impact around the fracture area is not as strong as thermal stresses, as shown in the second column in Figure 6.19. The colder color contour indicates the stronger cooling effect of the larger thermal expansion coefficient which enhances both fracture and matrix permeability and hence the convective flow from fracture to matrix.
Figure 6.20 shows that the higher coefficient results in a higher production temperature, which is mainly due to the contribution from the enhanced permeability matrix. If the matrix is not saturated with fluid, the result might differ from the current production temperature. The matrix permeability distribution is plotted in the third column in Figure 6.19. Moreover, the injected fluid loses mass into the matrix and causes a lower stable flow rate in the second plot in Figure 6.20.

Figure 6.19: Pressure and temperature distribution of the synthetic EGS model at the end of production: sensitivity analysis for thermal expansion coefficient. (a) \(4 \times 10^{-6} \, ^{\circ}C\); (b) \(8 \times 10^{-6} \, ^{\circ}C\); (c) \(16 \times 10^{-6} \, ^{\circ}C\)
6.1.7 Discussion and Summary of Synthetic Model Results

The synthetic model has a dimension of 100 m × 100 m × 24 m, which is smaller than the reservoir model in some relevant literature. Models in the literature could have a dimension up to 500 m to 1000 m in width and length, with hydraulic fracture of length 200 m to 500 m. The consideration of a smaller model focus is twofold: (1) there is no research or technology of ensuring a consistent 500 m hydraulic fracture in an EGS field and a 100 m fracture might be more realistic; (2) the computational cost might increase if a larger model is built without losing the accurate grid size. The small dimension of the model results in a rapid temperature decline at the production well and may diminish the effect of the geomechanics. In such an intermediate scale EGS reservoir, according to the results illustrated in this section, the following inference can be drawn:

- Geomechanical effects impact the hydraulic properties such as porosity and permeability and in turn the temperature distribution. The temperature difference is around 1-2 °C but may be larger in a large-scale reservoir and a long-term circulation;
- Thermally-induced stress/strain dominates the deformation of the reservoir, and is hence sensitive to all thermal parameters such as thermal conductivity, expansion coefficient, and injection temperature;
- Pressure is strongly dependent on permeability as a global coupling variable, when permeability is dynamically influenced by thermal or mechanical processes, the pressure field could be disturbed. The enhancement of fracture permeability could lower the reservoir average pressure significantly;
• Temperature distribution or variation is less sensitive to the change of fracture/matrix permeability than the pressure field;
• The permeability of fracture could be enhanced to 10 times the initial permeability due to the cold injection while the matrix permeability is also increased due to the temperature reduction;
• Heat recovery factor depends on both flow rate and production temperature. The injected fluid could flow into the reservoir especially when matrix permeability is enhanced. The flow rate out of production well might be less than injected rate and all injected fluid might not be recycled;
• The heat conduction process contributes more to the thermal processes due to the low matrix permeability, while heat convective plays a more important role only when the matrix permeability rises to a certain degree under the mechanical effect.

6.2 Applications on the EGS-Collab Field Experimental Model

Secondly, the fully coupled model is applied onto an experimental field where the EGS development was investigated by geological and geophysical monitoring. The data measured in this project is directly used to set up the modeling. However, uncertainty is still an inevitable factor in the numerical simulation, in terms of both fracture geometry and properties. Different models are established to incorporate such uncertainty and results are analyzed in comparison to other modeling works.

6.2.1 EGS-Collab Project Background

EGS Collab project was initiated by the Department of Energy (DOE) to facilitate the other field-scale EGS project, Frontier Observatory for Research in Geothermal Energy (FORGE). In the EGS Collab project, geological characterization, geophysical imaging, and monitoring, stimulation, flow tests, and numerical simulations are all conducted to fully assist in understanding the stimulation process, fracture initiation and propagation, fluid flow, and heat transfer in fractures, at Sanford Underground Research Facility (SURF) in Lead, South Dakota (Chai et al. 2019; Chen et al. 2019; Frash et al. 2019; Fu
et al. 2019; Lu and Ghassemi 2019; Mattson et al. 2019; White et al. 2019; Wu et al. 2021, 2019). The first experiment, Experiment 1, was performed to create a hydraulic fracture that connects an injection and production well, with a scale around 10-20 m. The temperature of the testbed is around 30 °C, which is not an ideal temperature for developing a geothermal system but this field site has been well studied before with a substantial source of data ready to be used for scientific research.

Figure 6.21: Schematics of the tunnel drift, hydraulic fracture, natural fracture and wells of EGS-Collab Experiment 1: E1-I is the injection well while E1-P is the production well; OP-T connector is a natural fracture detected by geophysical characterization; E1-OT, E1-OB, E1-PST, E1-PSB, E1-PDT and E1-PDB are monitor wells; there are two production points on well E1-P, denoted as E1-PI and E1-PB (Wu et al. 2021).

Experiment 1 was conducted at 4850 level (4850 ft deep) and eight wells of around 60 m length were drilled sub-horizontally. One of the wells (E1-I) was used to stimulate the reservoir and create hydraulic fractures, another was the production well (E1-P) and all the other six wells (E1-OT, E1-OB, E1-PST, E1-PSB, E1-PDT and E1-PDB) were monitor wells. The schematic illustration of the wells and fractures is shown in Figure 6.21. Each well was cored and characterized using various tools. Pressure and flow rate are also measured and logged at different locations on the wellbores of the injection, production, and observation wells. After the fracture was stimulated by injecting water at different flow
rates in three phases, it was confirmed that the fracture had been driven to the production well (E1-P). Tracer and flow tests were conducted and the outflow from several monitoring wells (E1-OT, E1-PST, and E1-PSB) proved that stimulated fracture is connected to an reactivated natural fracture system.

![Conceptual model of EGS-Collab fracture system](image)

Figure 6.22: Conceptual model of EGS-Collab fracture system: purple plane represents the hydraulic fracture and the yellow and blue planes are natural fractures. Green and red lines are injection and production wells. Yellow lines are all monitoring wells.

### 6.2.2 Fracture Models

There are quite a few studies that contributed to characterizing the geometry of the fracture system including both hydraulic and natural fractures, using micro-seismicity (Chen et al. 2019; Schoenball et al. 2019) or discrete fracture network (Lu and Ghassemi 2019; Neupane et al. 2019; White et al. 2019). It can be seen that a lot of uncertainty is associated with the interpretation of the fracture geometry, aperture, permeability, and distribution. The conceptual model for fracture geometry used in this study, referred from Dr. Beckers of the EGS-Collab Modeling and Simulation team, is shown in Figure 6.22. The fracture geometry, due to the uncertainty, has been simplified to a rectangular shape without losing the match with the micro-seismicity cloud. PDT-OT natural fracture (yellow fracture in Figure 6.22) is added based on the observation of Distributed Temperature Sensing (DTS) data. OT-P connector (blue fracture in Figure 6.22) is included as an interpreted deep fracture zone (Neupane et al. 2019) interconnected with
the hydraulic fracture and the E1-OT well. The fracture model for this EGS-Collab Experiment 1 testbed is established as Figure 6.23 where the hydraulic fracture has been partitioned into seven symmetrical layers. The heterogeneous properties can be assigned to the seven layers in this model. The potential heterogeneity is introduced to model the larger fracture aperture and less stiffness in the middle where injection and production occur.

Figure 6.23: Three-fracture models for the EGS-Collab Experiment 1: the layered fracture is hydraulically stimulated fracture and the pink and orange fractures are natural fractures.

Two production locations E1-PB and E1-PI represent two intersections of E1-P with hydraulic fracture and natural fracture. One injection well and five production wells are included in the model: the intersection between E1-I and the hydraulic fracture is the injection well; the two blue dots and two red dots are the production intersections with two natural fractures; the intersection between E1-P and the hydraulic fracture is the E1-PB production point. The injection temperature was 30 °C initially for 30 days and dropped down to 12 °C for the rest of the circulation.
The reservoir temperature is measured as around 31 °C which is stable in the deep reservoir. However, the artificially dug drift disturbs the temperature field, leading to a temperature gradient that cools down towards the tunnel. In this dissertation, two models have been established: one with a constant temperature and the other with a temperature gradient. The initial pressure is measured as $8.3 \times 10^6$ Pa. The reference stress state on the top of the reservoir is $\sigma_{xx} = 3.55 \times 10^7$ Pa, $\sigma_{yy} = 2.17 \times 10^7$ Pa, $\sigma_{zz} = 4.15 \times 10^7$ Pa.

Table 6.3: Input parameters of the EGS-Collab model (homogeneous hydraulic fracture properties)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix Permeability</td>
<td>1e-18×10^-18</td>
<td>m²</td>
</tr>
<tr>
<td>Hydraulic Fracture Permeability</td>
<td>8×10^-11</td>
<td>m²</td>
</tr>
<tr>
<td>Hydraulic Fracture Porosity (First 4 Layers)</td>
<td>0.01/0.04/0.1/0.2</td>
<td>Unitless</td>
</tr>
<tr>
<td>OT-P Natural Fracture Porosity</td>
<td>0.5</td>
<td>Unitless</td>
</tr>
<tr>
<td>OT-P Natural Fracture Permeability</td>
<td>8×10^-13</td>
<td>m²</td>
</tr>
<tr>
<td>PDT-OT Natural Fracture Permeability</td>
<td>0.5</td>
<td>Unitless</td>
</tr>
<tr>
<td>PDT-OT Natural Fracture Porosity</td>
<td>8×10^-13</td>
<td>m²</td>
</tr>
<tr>
<td>Initial Reservoir Pressure</td>
<td>8.3×10^6</td>
<td>Pa</td>
</tr>
<tr>
<td>Initial Reservoir Temperature</td>
<td>31</td>
<td>°C</td>
</tr>
<tr>
<td>Production (constant pressure)</td>
<td>1×10^6</td>
<td>Pa</td>
</tr>
<tr>
<td>Injection (constant rate)</td>
<td>0.00672</td>
<td>kg/s</td>
</tr>
<tr>
<td>Injection Specific Enthalpy</td>
<td>1.41×10^5/0.79×10^5</td>
<td>J/kg</td>
</tr>
<tr>
<td>Matrix Heat Conductivity</td>
<td>5</td>
<td>W/(m·°C)</td>
</tr>
<tr>
<td>Matrix Specific Heat</td>
<td>805</td>
<td>J/(kg·°C)</td>
</tr>
<tr>
<td>Production Well Index</td>
<td>1×10^-13</td>
<td>m³</td>
</tr>
<tr>
<td>Rock Expansion Coefficient</td>
<td>8×10^-6</td>
<td>1/°C</td>
</tr>
<tr>
<td>Matrix/Fracture Poisson’s Ratio</td>
<td>0.25</td>
<td>Unitless</td>
</tr>
<tr>
<td>Matrix Young’s Modulus</td>
<td>70</td>
<td>GPa</td>
</tr>
<tr>
<td>Matrix Biot’s Coefficient</td>
<td>0.8</td>
<td>Unitless</td>
</tr>
<tr>
<td>Initial Hydraulic Fracture Aperture</td>
<td>1.5×10^-5</td>
<td>m</td>
</tr>
<tr>
<td>Maximum Mechanical</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hydraulic Fracture Aperture</td>
<td>5×10^-5</td>
<td>m</td>
</tr>
<tr>
<td>Hydraulic Fracture Permeability Correlation Coefficient, d</td>
<td>6×10^-8</td>
<td>1/Pa</td>
</tr>
<tr>
<td>Initial Natural Fracture Aperture</td>
<td>1×10^-5</td>
<td>m</td>
</tr>
<tr>
<td>Maximum Mechanical</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Natural Fracture Aperture</td>
<td>2.5×10^-5</td>
<td>m</td>
</tr>
<tr>
<td>Natural Fracture Permeability Correlation Coefficient, d</td>
<td>3×10^-8</td>
<td>1/Pa</td>
</tr>
</tbody>
</table>
The hydraulic fracture can be modeled by two scenarios: (1) the permeability and aperture model of the fracture vary with layers; (2) the permeability and aperture model of the fracture is homogeneous for all layers. The other input parameters are shown in Table 6.3 in which it can be seen that the natural fracture is less sensitive to the stress compared to the hydraulic fractures. The fracture aperture model of different layers for the heterogeneous case is listed in Table 6.4: the middle layer could be potentially subject to larger aperture enhancement. In addition to the hydraulic fracture heterogeneity and temperature gradient, flow rate measurement added a layer of complexity since the constant pressure boundary cannot provide results matching the measured results. The controlled rate according to the measurement is set to be the boundary and the corresponding results are illustrated in the following sections.

Table 6.4: Input hydraulic fracture parameters of the EGS-Collab model (heterogeneous hydraulic fracture properties for the top 4 layers, the other three are symmetrical)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydraulic Fracture Permeability</td>
<td>8e-11/9e-11/9e-11/10e-11</td>
<td>m²</td>
</tr>
<tr>
<td>Hydraulic Fracture Porosity</td>
<td>0.01/0.04/0.1/0.2</td>
<td>Unitless</td>
</tr>
<tr>
<td>Initial Fracture Aperture</td>
<td>1.5e-5/1.5e-5/2e-5/2e-5</td>
<td>m</td>
</tr>
<tr>
<td>Maximum Mechanical Fracture Aperture</td>
<td>4e-5/5e-5/6e-5/6.5e-5</td>
<td>m</td>
</tr>
<tr>
<td>Fracture Permeability Correlation Coefficient, d</td>
<td>5e-8/6e-8/6e-8/6e-8</td>
<td>1/Pa</td>
</tr>
</tbody>
</table>

6.2.3 Effects of Mechanics Coupling and Fracture Heterogeneity

Based on the inference in Section 6.1, the permeability of hydraulic fractures could have a strong impact on the production rate and temperature, and pressure/temperature distribution. Uncertainty still exists in the stimulated fracture, especially in its fracture aperture and permeability. The model of heterogeneous fracture aperture/permeability fulfills the general fracture shape: large width in the middle and small width near the edge. Mechanics facilitates the permeability enhancement by imposing tensile stresses on the fracture when the temperature decreases along the cold fluid flowing path. In this field experiment, the maximum temperature reduction is around 20 °C which applies $6 \times 10^6$ Pa thermal stress on the fracture face.
The coupled THM modeling and TH modeling without mechanics coupling for both heterogeneous and homogeneous hydraulic fracture cases are compared. The pressure drop is dominated by the fracture in the early stage of production: the pressure distribution after one day of production is shown in the first plot of Figure 6.24 for a reference. The pressure distribution after one year of circulation tends to be hydrostatic except the peak value around the injection well. Temperature and stress distributions for the heterogeneous and homogeneous case (mechanics coupled or no-coupling) cannot be differentiated by eyeballing so only the heterogeneous case is shown in Figure 6.24. The thermal stresses is still an important factor that can be observed by the comparison between temperature distribution and stress distribution. Apparent stronger tensile stresses are observed around the temperature reduction zone.

![Figure 6.24: Pressure, temperature, and stress distributions at the end of long-term water circulation: heterogeneous hydraulic fracture permeability](image)

The production temperature and the flow rate out of E1-PB, and the production point permeability for the four cases: (1) coupled THM modeling for heterogeneous hydraulic fracture; (2) coupled THM modeling for homogeneous hydraulic fracture; (3) TH modeling for heterogeneous hydraulic fracture; (4) TH modeling for homogeneous hydraulic fracture; are shown in Figure 6.26.
Figure 6.25: Injection rate/temperature, outflow rate and temperature of the long-term water circulation at EGS-Collab Experiment 1 testbed. The dotted temperature data denotes questionable temperature measurements with damaged thermistors (Wu et al. 2021).

It can be seen that mechanics coupling could reduce the production temperature, probably due to the enhanced fracture permeability, for both heterogeneous and homogeneous cases. Flow rates are strongly impacted by homogeneity and heterogeneity. The average hydraulic fracture permeability of the heterogeneous case is higher than the homogeneous case so the overall flow rates are higher. Comparing the flow rates impacted by mechanics coupling recognizes the permeability enhancement effect. In the late stage when flow rates are all stable, the same trend is observed: heterogeneous cases are higher
than homogeneous ones and mechanics coupling cases are higher than non-coupling ones. The production fracture grid permeability reflects how the permeability responds to flow and heat: the permeability initially decreases due to the compaction effect (the pore pressure drop increases the effective stress) of open flow but increases gradually when cooling temperature and thermal breakthrough affects the reservoir.

Despite the results obtained above, the outflow rate (303 ml/min) of well E1-PB does not match the measured flow rate (≈ 100 ml/min) in Figure 6.25. Moreover, the production temperature measured by the thermistors is stable and constant through the last three months (around $2.3 \times 10^7$ sec and later) while the simulated temperature decreases from $30.6 \degree C$ to $30.5 \degree C$ despite the difference is quite negligible.

Figure 6.26: Comparison of production temperature, flow rate out of E1-PB, and production fracture grid permeability of EGS-Collab model: coupling THM vs. TH without mechanics, and homogeneous fracture permeability/aperture vs. heterogeneous fracture permeability/aperture.

6.2.4 Temperature Gradient and Controlled Outflow Rate Models

The underground drift was built as a research facility that connects to the surface directly. Therefore, compared to the far-field stable temperature, the temperature around the drift is much lower and close to $22 \degree C$. Wu et al. (2021) and White et al. (2019) established a model with a temperature gradient from the drift towards the testbed based on the DTS measured data. An approximate temperature gradient is also set up for initial conditions in this work, as shown in Figure 6.27 for both reservoir and fractures. After 1 year of water circulation, the stresses and pressure/temperature distribution are shown in Figure 6.28 and Figure 6.29 respectively by comparing the heterogeneous case in Section
6.2.3.

**Figure 6.27:** Initial temperature distribution considering the cooling effect towards the drift for both reservoir and fractures.

**Figure 6.28:** Comparison of stress XX, YY and ZZ distribution of EGS-Collab model: (a) heterogeneous fracture properties with constant temperature; (b) heterogeneous fracture properties with temperature gradient.

The warmer color of the temperature gradient case in stress XX and YY reflects the extra tensile stresses created by the lower temperature. The pressure fields of these two cases are almost identical but note that the color scales of the two cases are different. If
the same color scale is used, the visualization of the temperature gradient case will be all dark blue due to the cooling towards the drift.

Table 6.5: Outflow rates (kg/s) out of all wells: comparison between measured data (approximate) and simulation results

<table>
<thead>
<tr>
<th>Well Name</th>
<th>Measured Outflow Rate</th>
<th>Simulated Outflow Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1-PB</td>
<td>0.00233</td>
<td>0.00522</td>
</tr>
<tr>
<td>E1-PI</td>
<td>0.00367</td>
<td>0.000463</td>
</tr>
<tr>
<td>E1-OTB</td>
<td>0.000417</td>
<td>0.000385</td>
</tr>
<tr>
<td>E1-PDT</td>
<td>0.000500</td>
<td>0.000343</td>
</tr>
<tr>
<td>E1-OTI</td>
<td>0.000333</td>
<td>0.000315</td>
</tr>
</tbody>
</table>

Figure 6.29: Comparison of pressure and temperature distribution of EGS-Collab model: (a) heterogeneous fracture properties with constant temperature; (b) heterogeneous fracture properties with temperature gradient.
The comparison of production temperature, flow rates, and injection/production permeability are plotted and compared in Figure 6.30. Due to the initial temperature difference, there is a gap between the production temperature of the two cases but the cooling trend is the same. The constant temperature case has a 0.43 °C drop down and 0.38 °C for the temperature gradient case. In terms of the flow rates, all five production wells are plotted in Figure 6.30 and compared with the measured data in Table 6.5. The mismatch between the measurement and the simulation indicates that there is still missing information or data in the reservoir/fracture model. The dynamic permeability of the production fracture overlaps with the previous heterogeneous fracture aperture case since the thermal stresses are not the dominating factor here. On the other hand, the injection fracture permeability firstly responds to the pressure change and then starting from the cold water injection, the permeability rapidly rises to a high level.

![Production Temperature, Flow Rate, Injection/Production Permeability](image)

Figure 6.30: Comparison of production temperature, flow rate and injection/production permeability for the constant initial temperature and temperature gradient cases.

The second object in this section is to control the flow rate in the experiments where a constant production pressure was specified in the above case. Using the input data in the temperature gradient case above, the injection rate is kept unchanged but the production rates out of E1-PB, E1-PI, E1-OTB, E1-PDT and E1-OTI are set as 0.00167, 0.00333, 0.000840, 0.000440 and 0.000440 kg/s.

The stress, pressure/temperature and E1-PB production temperature are shown in Figure 6.31. The stress and pressure are not plotted in the same scale as Figure 6.29 and 6.28. Due to the rate controls, the pressure of the whole reservoir rises to a high level close to initial reservoir pressure, disturbing the stress field to reach a distinct state from
previous cases. The production temperature decreases 0.4 °C over the whole circulation process.

6.2.5 Discussion and Summary of the EGS-Collab Testbed Modeling

The EGS-Collab testbed Experiment 1 is modeled using the fully coupled THM model with EDFM in this section. Mechanics coupling for both homogeneous and heterogeneous fracture aperture models is compared with no coupling scenario. The results demonstrate a similar trend to the synthetic model results in which flow rates are increased while production temperature is reduced with mechanics coupling. The intuitive heterogeneous fracture aperture model is compared with the homogeneity case to examine how the flow rates and thermal breakthrough are affected. Finally, the temperature gradient due to the man-made drift has been added into the initial condition of the reservoir model and the controlled outflow rates matching the field measurement are also considered.

The outflow rates of the five wells in the model cannot match the field measurement with a higher rate out of E1-PB than E1-PI. In terms of the production temperature, the modeling results are relatively close to the field measurement except that no thermal breakthrough has been observed in the testbed. The controlled flow rate model includes
consistent rate parameters. The modeling and simulation team of EGS-Collab is not able to accurately match the field measurement either due to the uncertainty in fracture aperture, natural fracture characterization, and measurement errors. Wu et al. (2021) applied a coupled THM model on EGS-Collab Experiment 1 and produced results similar to this work, as shown in Figure 6.32. Their study considers the cooling effect of the wellbore as well due to the intermediate reservoir scale. The temperature drop is around 0.2-0.4 °C without considering the wellbore cooling effect at well E1-PB which is qualitatively close to this study. The heterogeneous fracture aperture models illustrates that the smaller fracture aperture delays the thermal breakthrough, which is also similar to the heterogeneity case in this work.

Other input parameters are tested using the current model, which shows that when natural fractures are more conductive, the flow rates can be matched. These results are not included in this dissertation but indicate that there might be unrevealed uncertainty of fracture/reservoir characterization in the testbed.

Figure 6.32: Production temperature of E1-PB and E1-PI by Wu et al. (2021) using a coupled THM model.
APPLICATIONS OF THE SEQUENTIALLY COUPLED MODEL ON EGS MODELING

The fully coupled model has been used on a synthetic intermediate scale reservoir for coupled THM modeling. As mentioned earlier, although the fully coupled model is able to capture the fracture aperture change based on the normal effective stress, the assumption that the stress state is the same for fracture and its containing grid is loose. In order to develop a physically rigorous model, the sequentially coupled model adopts XFEM as the mechanical approach to handle the discontinuity brought by the fractures. The fluid flow/heat transfer is solved by TOUGH2-EGS which is programmed using the same methodology as TOUGH2-CSM. TOUGH2-EGS is coupled with XFEM to complete the sequentially coupled model. This chapter focuses on applying this sequentially coupled model to an intermediate scale geothermal reservoir with a dominating single hydraulic fracture for long-term water circulation simulation.

Firstly, the convergence performance is briefly illustrated by varying the fracture stiffness. The mechanical coupling is then compared with no-mechanics coupling to examine the impact of mechanics. The sensitivity analysis is performed for the thermal conductivity, matrix permeability, injection temperature and rate in this single fracture model. Secondly, the single fracture model is extended to incorporate MINC as a model of induced/reactivated natural fractures. The number of MINC partitions and natural fracture spacing are varied to investigate their effects on mass/heat transfer. Finally, the single fracture case has been set up for the fully coupled modeling so that the fully coupled model can be benchmarked and examined by the sequentially coupled model.

7.1 Single Dominating Fracture EGS Model

The sequentially coupled model is firstly applied on to an intermediate scale EGS model with a single hydraulic fractures. Similarly to the fully coupled model, multiple parameters are investigated for their impacts on the modeling results, which are summarized at the end of this section.
7.1.1 Base Case and Mechanics Coupling Effect

The sequentially coupled model was applied to an intermediate-scale synthetic EGS injection/production case where a single hydraulic fracture dominates the fluid flow/heat transfer. The model has a dimension of 120 m × 60 m × 40 m with a uniform grid block size 4 m × 4 m × 4 m. Hence there are 4500 grids in total. The hydraulic fracture is modeled as an elliptic shape with a length of around 60 m and a height of 20 m as shown in Figure 7.1. The injection well is connected with one end of the fracture and the production well connected with the other end. The injection/production is operated for 1000 days. The input data has been summarized in Table 7.1. In this subsection, the mechanics is coupled and decoupled to investigate its impact on the production behavior.

Figure 7.1: EDFM model for the hydraulic fracture in EGS: injection and production points have been marked.
Table 7.1: Input parameters of the synthetic EGS model (sequentially coupled model)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix Permeability</td>
<td>$2 \times 10^{-17}$</td>
<td>m$^2$</td>
</tr>
<tr>
<td>Fracture Permeability</td>
<td>$2 \times 10^{-11}$</td>
<td>m$^2$</td>
</tr>
<tr>
<td>Initial Reservoir Pressure</td>
<td>$3.593 \times 10^7$</td>
<td>Pa</td>
</tr>
<tr>
<td>Initial Reservoir Temperature</td>
<td>200</td>
<td>°C</td>
</tr>
<tr>
<td>Production (constant pressure)</td>
<td>$1.5 \times 10^7$</td>
<td>Pa</td>
</tr>
<tr>
<td>Injection (constant rate)</td>
<td>0.5</td>
<td>kg/s</td>
</tr>
<tr>
<td>Injection Specific Enthalpy</td>
<td>$3.35 \times 10^5$</td>
<td>J/kg</td>
</tr>
<tr>
<td>Rock/Fracture Porosity</td>
<td>0.05/0.4</td>
<td>Unitless</td>
</tr>
<tr>
<td>Rock/Fracture Heat Conductivity</td>
<td>5</td>
<td>W/(m$^2$C)</td>
</tr>
<tr>
<td>Rock/Fracture Specific Heat</td>
<td>1000</td>
<td>J/(kg$^o$C)</td>
</tr>
<tr>
<td>Production Well Index</td>
<td>$4 \times 10^{-13}$</td>
<td>m$^3$</td>
</tr>
<tr>
<td>Rock Expansion Coefficient</td>
<td>$4 \times 10^{-5}$</td>
<td>1/°C</td>
</tr>
<tr>
<td>Matrix/Fracture Poisson’s Ratio</td>
<td>0.3</td>
<td>Unitless</td>
</tr>
<tr>
<td>Matrix Young’s Modulus</td>
<td>30</td>
<td>GPa</td>
</tr>
<tr>
<td>Biot’s Coefficient</td>
<td>0.5</td>
<td>Unitless</td>
</tr>
<tr>
<td>Minimum Fracture Residual Aperture</td>
<td>$2 \times 10^{-4}$</td>
<td>m</td>
</tr>
<tr>
<td>Initial Fracture Aperture</td>
<td>$6 \times 10^{-3}$</td>
<td>m</td>
</tr>
<tr>
<td>Maximum Mechanical Fracture Aperture</td>
<td>$1.2 \times 10^{-2}$</td>
<td>m</td>
</tr>
<tr>
<td>Stress on Top</td>
<td>$-1 \times 10^8$</td>
<td>Pa</td>
</tr>
<tr>
<td>Stress on Face XZ Top</td>
<td>$-6 \times 10^7$</td>
<td>Pa</td>
</tr>
<tr>
<td>Stress Gradient on Face XZ</td>
<td>$1.75 \times 10^5$</td>
<td>Pa/m</td>
</tr>
<tr>
<td>Stress on Face YZ Top</td>
<td>$-7 \times 10^7$</td>
<td>Pa</td>
</tr>
<tr>
<td>Stress Gradient on Face YZ</td>
<td>$2 \times 10^5$</td>
<td>Pa/m</td>
</tr>
</tbody>
</table>

Note that the important assumption in the sequentially coupled model is that: the fracture aperture has a constant initial value and the relative displacement computed from XFEM is the change of fracture aperture, as defined in Equation 3.46. XFEM may produce the results in which discontinuity faces interpenetrate each other, therefore the minimum aperture residual is set as $2 \times 10^{-4}$ m, which is the lowest aperture that can be reached. A maximum mechanical aperture is also predefined as $1.2 \times 10^{-2}$ m, which is the upper limit for fracture opening. In this section, the role of mechanics is first examined. The displacement, stress, pressure, temperature and porosity distribution of the base case is shown in Figure 7.2. The thermal expansion coefficient in this case is $4 \times 10^{-5}$ 1/°C, which is relatively large and generates large tensile thermal strains around the fracture, dominating the x, y, and z effective stresses. Displacement of the XFEM is continuous and
shows the kink contour where the fracture is located in the first row. Lagrangian porosity is used as the porosity correlation so that the porosity has been enhanced around the fractured area.

The comparison of pressure and temperature, flow rates, production temperature of the mechanics coupling and no-coupling cases is shown in Figure 7.3 and 7.4. It can be seen that the pressure field of no coupling scenario is higher than the coupling case due to the permeability enhancement, which is in a similar trend to the fully coupled model. Temperature, also similar to the fully coupled model, is not sensitive to the mechanical response, with two almost identical distributions. The production temperature of the coupling case is around 2 °C lower than the no-coupling case, which is attributed again to the permeability enhancement of fractures. The flow rate of the coupling case in the late
stage is initially higher and then lower than the no-coupling case. It can be inferred that the matrix permeability enhancement also strengthens the convective process and circulated water could lose mass during this process.

Figure 7.3: Comparison of pressure and temperature distribution of (a) coupled THM modeling; (b) TH modeling for the single-fracture EGS model (sequentially coupled model).

Figure 7.4: Comparison of production temperature, flow rate of coupled THM modeling and no mechanics coupling, the second figure is plotted for the 115-1000 days.
7.1.2 Effects of Fracture Stiffness on Convergence Performance

In the flow step of each coupling iteration, fluid flow and heat transfer are solved implicitly by Newton’s method in which the primary variables, pressure, and temperature are kept updated until the residual diminishes to a negligible value. When pressure is updated, fracture porosity needs to dynamically respond to the pressure change, without which the coupling iterations will not converge based on our numerical experiments (when fracture porosity is kept constant in the flow step). This is mainly because the fracture is the only highly conductive path for the slightly compressible fluid within the reservoir and if the fracture does not respond to the pore pressure change, it’s hard for the fluid flow to penetrate the matrix and adjust the pressure by itself.

In the flow step, the hydraulic fracture porosity is updated by Equation 4.69 while in the coupling iteration, the hydraulic fracture porosity obeys the correlation in Equation 4.43. The fracture stiffness, $K_{HF}$ in Equation 4.69 has a strong influence on the convergence performance. The sensitivity analysis is performed for this parameter using the input data of the base case. The coupling iteration number of each time step is accumulated and plotted in the first one of Figure 7.5. This iteration accumulation demonstrates that when the stiffness is lower than $9.6 \times 10^8$ Pa, the accumulated coupling iteration is much fewer than the $1.9 \times 10^9$ Pa and $2.0 \times 10^9$ Pa. When the stiffness is higher than $2.2 \times 10^9$ Pa or lower than $2.4 \times 10^8$ Pa, the convergence performance starts to degrade and may not converge eventually in some time step. When the stiffness is different, the simulation result is the same as shown in the last two plots of Figure 7.5. The oscillation during the first 1000 seconds may be due to the numerical oscillation of the coupling algorithm.

7.1.3 Effects of Thermal Conductivity

The thermal conductivity effect on fractured geothermal reservoir circulation has been discussed for the fully coupled model. The same discussion is necessary to be conducted for the sequentially coupled model. The input parameters of the base case in Table 7.1 are
kept unchanged except the thermal conductivity is increased and decreased to 8 W/(m°C) and 2 W/(m°C) from 5 W/(m°C). The distributions of stress, displacement and pressure/temperature/porosity are shown in Figure 7.6, 7.7 and 7.8 respectively.

Figure 7.5: Comparison of accumulated coupling iteration number. Left: accumulative iteration number when stiffness is varied; middle: result accuracy benchmark for the different stiffness; right: result accuracy benchmark or the different stiffness.

The results illustrate that the reservoir matrix temperature is lower when the thermal conductivity is higher and the cooling effect from the cold injection spreads more widely. This cooling effect generates the negative thermal strain and raises x, y and z displacement (making them more negative). The effective stresses are dominated by the thermal stress/strain, with contours conforming to the temperature distribution. However, compared to the fully coupled model in Figure 6.5, the lower average temperature in the higher conductivity scenario results in lower tensile stresses as shown in Figure 7.7. In Figure 7.8, it is apparent that the lower average temperature in higher conductivity scenario has a higher temperature around the fracture area, due to the inefficient heating process from the reservoir to the fracture fluid, which generates the higher tensile stresses. Additionally, when the pressure is compared in the three conductivity scenarios, the average pressure is obviously higher in the high conductivity case which is also distinct from the fully coupled model. The fracture locations and distributions are different in the fully and sequentially coupled case, which may be the reason of the difference. The porosity distribution could potentially give a reasonable explanation: the porosity/permeability near fractured volume is enhanced more in the low conductivity case and facilitate the fluid flow processes which reduce the overall pressure.
Figure 7.6: Comparison of x, y, and z displacement, sensitivity analysis of thermal conductivity: (a) $\lambda = 8\text{W/(m}^\circ\text{C)}$; (b) $\lambda = 5\text{W/(m}^\circ\text{C)}$; (c) $\lambda = 2\text{W/(m}^\circ\text{C)}$.

The relative fracture face displacement for the three scenarios is plotted in Figure 7.9. The lower conductivity case has a less heating effect from the reservoir and consequently gives a larger fracture aperture (the larger positive value indicates larger apertures) due to the contraction of the matrix discontinuous faces.

The production temperature, flow rate, and injection fracture permeability are compared for the three thermal conductivity in Figure 7.10. Intuitively, when the thermal conductivity is higher, heat can be transferred efficiently from the reservoir matrix to the fluid in the fracture, raising the production temperature. The late stage flow rate is the highest when thermal conductivity is the lowest and the matrix permeability is also the
lowest, which is mainly because of the stronger fluid loss to the enhanced matrix permeability. The injection permeability shows the effect of lower average reservoir temperature: the matrix tends to contract and increases the relative displacement of the fracture faces initially. When the temperature field becomes more stable, the matrix tends to be compressed by the external traction which will gradually decrease the fracture aperture. At the end of the circulation, the low conductivity model has the highest fracture permeability/aperture, which matches the observation in Figure 7.9.

Figure 7.7: Comparison of x, y, and z stresses, sensitivity analysis of thermal conductivity: (a) $\lambda = 8\text{W/(m$^\circ$C)}$; (b) $\lambda = 5\text{W/(m$^\circ$C)}$; (c) $\lambda = 2\text{W/(m$^\circ$C)}$. 
Figure 7.8: Comparison of pressure/temperature/matrix porosity, sensitivity analysis of thermal conductivity: (a) $\lambda = 8\text{W/(m}^\circ\text{C)}$; (b) $\lambda = 5\text{W/(m}^\circ\text{C)}$; (c) $\lambda = 2\text{W/(m}^\circ\text{C)}$.

Figure 7.9: Comparison of relative fracture face displacement, sensitivity analysis of thermal conductivity: (a) $\lambda = 8\text{W/(m}^\circ\text{C)}$; (b) $\lambda = 5\text{W/(m}^\circ\text{C)}$; (c) $\lambda = 2\text{W/(m}^\circ\text{C)}$
7.1.4 Effects of Matrix Permeability

Matrix permeability is an intrinsic property of the reservoir, which is the major factor controlling fluid flow and the coupled heat transfer. In this section, the sensitivity analysis has been conducted for the matrix permeability using the same input parameters of the base case, except the matrix permeability is reduced from $2 \times 10^{-17}$ m$^2$ to $5 \times 10^{-18}$ m$^2$ and $4 \times 10^{-19}$ m$^2$. There is no observable difference in the displacement in x, y and z directions among the three matrix permeability scenarios, all of which are the same as the base case in (b) of Figure 7.6. The differences in x, y, and z effective stresses are also slight, as shown in Figure 7.11. As the matrix permeability decreases from the top row to the bottom, effective stresses tend to be more compressive. This can be explained by referring to Figure 7.12 in which the temperature field is not strongly disturbed or changed by the matrix permeability. However, matrix permeability strongly affects the pressure distribution: the lower matrix permeability prevents the fluid penetrating the reservoir and the average pressure is relatively lower in this case, resulting in more compressive effective stresses. The matrix permeability impact is observed to be similar to the fully coupled model where the effective stress is also not influenced much under the reduction of the matrix permeability. It can be inferred that matrix permeability has a strong impact on fluid flow through pore pressure distribution but not on the heat transfer process. The heat convection contribution is rather limited.
Figure 7.11: Comparison of x, y, and z stresses, sensitivity analysis of matrix permeability: (a) $2 \times 10^{-17}$ m$^2$; (b) $5 \times 10^{-18}$ m$^2$; (c) $4 \times 10^{-19}$ m$^2$.

The comparison of production temperature, flow rate, and injection fracture permeability is shown in Figure 7.13. The production temperature increases with the matrix permeability, over the entire circulation, since high matrix permeability could supply hot water to the fracture and also increases the steady flow rate in the late stage. The injection fracture permeability increases with a similar rate for the three cases and finally due to the fact that stable pressure is more compressive in the low matrix permeability scenario, the fracture permeability is lower in the late stage.
Figure 7.12: Comparison of pressure, temperature and porosity, sensitivity analysis of matrix permeability: (a) $2 \times 10^{-17}$ m$^2$; (b) $5 \times 10^{-18}$ m$^2$; (c) $4 \times 10^{-19}$ m$^2$.

Figure 7.13: Comparison of production temperature, flow rate and injection fracture permeability, sensitivity analysis of matrix permeability.
7.1.5 Effect of Injection Rate

The injection rate is another factor that is worth investigating since this rate impacts both the fluid flow and heat transfer processes. The injection rate is raised from the base case, 0.5 kg/s to 1 kg/s and 2 kg/s for sensitivity analysis in this section. The displacement and stress distributions for the three rate scenarios are shown in Figure 7.14 and 7.15. The overall displacement distributions of the three cases are in the same trend. The fracture location marks the discontinuous displacement change and the higher rate lowers the reservoir temperature which contracts the reservoir in the x, y and z directions in the negative direction (same as the boundary traction). The dominating thermal

Figure 7.14: Comparison of displacement in x, y and z directions, sensitivity analysis of injection rate: (a) 0.5 kg/s; (b) 1 kg/s; (c) 2 kg/s.
stress/strain is still obvious in the effective stress distribution. Similarly, the higher rate lowers the volume close to fracture where the tensile stresses are stronger.

Figure 7.15: Comparison of stress in x, y and z directions, sensitivity analysis of injection rate: (a) 0.5 kg/s; (b) 1 kg/s; (c) 2 kg/s.

The comparison of pressure, temperature, and porosity for the three cases is plotted in Figure 7.16. The higher injection rate obviously decreases the reservoir temperature and increases the porosity, especially near the fracture volume. The higher injection rate also causes the reservoir pressure to rise rapidly due to both more injected mass and increased fluid viscosity (temperature decreases) and stay at a high level which also facilitates the fluid flow into the production well.
Figure 7.16: Comparison of pressure, temperature and porosity, sensitivity analysis of injection rate: (a) 0.5 kg/s; (b) 1 kg/s; (c) 2 kg/s.

The fracture width distributions of the three cases are shown in Figure 7.17. The comparison of production temperature, flow rate and injection fracture permeability is also presented in Figure 7.18. The higher injection rate opens the fracture aperture as soon as the injection starts, and accelerates the thermal breakthrough much sooner than lowering the injection temperature in the next section, Figure 7.23 but similar to the low thermal conductivity scenario in Figure 7.10. The low thermal conductivity may not necessarily open the fracture but fail to heat the fluid fast enough. It indicates that the reservoir is able to heat the fluid within an appropriate flow rate even when the injection temperature is low. If the injection rate is too fast, the aperture enhancement would further accelerate
the thermal breakthrough. The stable production flow rates are all slightly lower than the injection rate, indicating fluid loss to the reservoir.

Figure 7.17: Comparison of fracture width, sensitivity analysis of injection rate: (a) 0.5 kg/s; (b) 1 kg/s; (c) 2 kg/s.

Figure 7.18: Comparison of production temperature, flow rate and injection fracture permeability, sensitivity analysis of injection rate.

7.1.6 Effects of Injection Temperature

The injection temperature is an important operational parameter as mentioned previously, especially when recycled water is reinjected into the reservoir. In this section, the sequentially coupled THM model is applied to investigate the sensitivity of the reservoir to the injection temperature. Temperature is the dominating factor in EGS fluid flow/heat transfer and deformation, which can be observed from the previous results of stresses conforming to the temperature contours.

Injection temperatures are increased to 105 °C or decreased to 45 °C from the base case, 75 °C. The displacements in the x, y and z direction for the three cases are shown in Figure 7.19. The effective stresses are compared among the three scenarios in Figure 7.20.
Figure 7.19: Comparison of displacement in x, y and z directions, sensitivity analysis of injection temperature: (a) 105 °C; (b) 75 °C; (c) 45 °C.
Figure 7.20: Comparison of stress in x, y and z directions, sensitivity analysis of injection temperature: (a) 105 °C; (b) 75 °C; (c) 45 °C.
It can be observed that increasing the temperature significantly reduces the negative displacement in all x, y and z directions due to the smaller thermal strains. Accordingly, tensile stresses are also weakened in all three directions around the fracture volume, but compressive stresses become stronger outside.

Figure 7.21: Comparison of pressure, temperature and porosity, sensitivity analysis of injection temperature: (a) 105 °C; (b) 75 °C; (c) 45 °C.

The pressure, temperature and matrix porosity are compared in Figure 7.21. Intuitively, the reservoir volume close to the fracture has a higher temperature in the higher injection temperature case, which consequently impedes the increase of the matrix porosity/permeability. The overall lower average permeability results in higher average reservoir pressure, especially near the injection area. The fracture width grows to a larger
aperture when injection temperature is reduced due to the relative contraction of the rock matrix/fracture face, as shown in Figure 7.22.

![Fracture Width Comparison](image1.png)

(a) 105 °C; (b) 75 °C; (c) 45 °C.

Figure 7.22: Comparison of fracture width, sensitivity analysis of injection temperature.

The comparison for production temperature, flow rate, and injection fracture permeability is conducted in Figure 7.23. As discussed in section 7.1.5, the reservoir is able to heat the injected fluid within a certain injection rate. When injection temperature is lowered, the reservoir is capable of heating the fluid in the early stage when the thermal breakthrough occurs later than the high injection rate or low thermal conductivity cases. The injection temperature does not have a significant impact on the flow rate. The stable rate in the late stage is higher in the high injection temperature scenario, indicating a less fluid loss due to the lower average matrix permeability.

![Production Temperature Comparison](image2.png)

Production Temperature under Various Injection Temperatures

![Production Rate Comparison](image3.png)

Production Rate under Various Injection Temperatures

![Injection Point Permeability Comparison](image4.png)

Injection Point Permeability under Various Injection Temperatures

Figure 7.23: Comparison of production temperature, flow rate and injection fracture permeability, sensitivity analysis of injection temperature.
7.1.7 Effects of Skeleton Linear Thermal Expansion Coefficient

The skeleton linear thermal expansion coefficient determines directly the thermal strain/stresses and hence impacts the deformation prominently. The coefficient in the literature ranges from the scale of $10^{-6} \, {1/{^\circ C}}$ to $10^{-5} \, {1/{^\circ C}}$. In the base case, $4 \times 10^{-5} \, {1/{^\circ C}}$ is used to emphasize the impact of thermal stresses/strains. The sensitivity analysis in this section reduces this value to $2 \times 10^{-5} \, {1/{^\circ C}}$ and $8 \times 10^{-6} \, {1/{^\circ C}}$.

![Comparison of displacement in x, y and z directions, sensitivity analysis of thermal expansion coefficient: (a) $8 \times 10^{-6} \, {1/{^\circ C}}$; (b) $2 \times 10^{-5} \, {1/{^\circ C}}$; (c) $4 \times 10^{-5} \, {1/{^\circ C}}$.](image)

In the exploration phase of an EGS project, reservoir rocks are sampled to analyze their mechanical properties, including the thermal expansion coefficient. The comparison of displacement, effective stress for the three cases is shown in Figure 7.24 and 7.25. The
significant displacement variation demonstrates the role of thermal stress/strain: with
decreasing thermal expansion coefficient, the thermal strain is reduced (much less
contraction in x, y and z directions). The same trend is observed in the effective stress
distribution, tensile stresses are apparently weakened, comparing the low expansion
coefficient and the high expansion coefficient. The compressive stresses outside the
fractured volume are also weakened as the coefficient decreases.

Figure 7.25: Comparison of stress in x, y and z directions, sensitivity analysis of thermal
expansion coefficient: (a) $8 \times 10^{-6}$ 1/°C; (b) $2 \times 10^{-5}$ 1/°C; (c) $4 \times 10^{-5}$ 1/°C.

The pressure, temperature and porosity distribution are compared in Figure 7.26. The
lower thermal expansion coefficient prevents the increase of the matrix permeability and
cools the reservoir a bit. The average reduced matrix porosity/permeability raises the
injection pressure but might reduce the convective flow to transfer heat to the fractured volume. The fracture widths of the three cases are shown in Figure 7.27 which illustrates the larger fracture aperture due to the stronger rock matrix contraction.

Figure 7.26: Comparison of pressure, temperature and porosity, sensitivity analysis of thermal expansion coefficient: (a) $8 \times 10^{-6}$ °C$^{-1}$; (b) $2 \times 10^{-5}$ °C$^{-1}$; (c) $4 \times 10^{-5}$ °C$^{-1}$.

The production temperature, flow rate, and injection permeability are compared in Figure 7.28. It can be seen that the production temperature of the lower thermal expansion is higher due to the low fracture permeability. The highly permeable fractures resulting from the high thermal expansion coefficient enhances the flow rate in the early stage, as shown in the middle plot, and reaches a lower stable outflow rate due to more fluid loss to the more permeable matrix.
A notable comparison of the sensitivity analysis of the thermal expansion coefficient between the fully coupled model and the sequentially model shows the distinct trend of the production temperature when this coefficient is increased: the fully coupled model provides an enhanced temperature while the sequentially coupled model gives the opposite. This is mainly due to the fact that fracture density is higher in the fully coupled model where the fluid from the matrix can supply the production while a single fracture cannot benefit from this supply.

7.1.8 Discussion and Summary of the Sequentially Coupled Model on the Synthetic EGS Reservoir

The sequentially coupled model is applied on an intermediate-scale synthetic EGS reservoir with a dominating single fracture. The convergence performance of this type of EGS reservoir highly depends on the fracture porosity/aperture response to the pore pressure. Sensitivity analyses have been performed for key parameters such as thermal
conductivity, matrix permeability, injection rate and temperature, and thermal expansion coefficient. The displacement, effective stresses, and pressure/temperature/matrix porosity distributions are compared in these sensitivity analyses. The following inference and conclusion can be reached in this section:

- When geomechanics is coupled, the fracture aperture is enlarged due to the rock matrix contraction, and hence accelerates the fluid flow in the fracture. Consequently, production temperature is reduced and thermal breakthrough is accelerated;
- Thermal stresses/strains dominate the deformation which can be seen from the displacement and effective stress distribution, especially when the thermal expansion coefficient is reduced;
- Fracture aperture can be directly computed in the sequentially coupled model, and directly compared in the sensitivity analysis. The aperture indicates the relative displacement between the rock faces of the discontinuity;
- The fluid heating is dependent strongly on the heat conduction, which can be seen from the sensitivity analysis of the thermal conductivity;
- The convection may not be a dominant factor based on the sensitivity analysis of the matrix permeability, but still makes contributions towards the fluid mass recovery;
- The injection rate and temperature are key operating parameters in EGS development since the reservoir could heat the fluid efficiently when the injection rate is appropriate even when the injection temperature is not high;
- The increased matrix porosity/permeability due to the thermal strain usually strengthens the fluid mass loss to the reservoir and induce prominent pressure distribution but may not cause significant variation in the temperature field;
- The fully coupled model and the sequentially coupled model may not give consistent results due to the complex fracture systems in the fully coupled model, as shown in the sensitivity analysis of thermal expansion coefficient.
7.2 Extension of EDFM EGS Model to Combined EDFM and MINC Model

The stimulation process may induce new fractures or reactivate existing natural fractures as reviewed in section 1.3. The methodology of extending the current EDFM model to a combined EDFM and MINC model has also been discussed in section 4.3. In this section, this extension will be modeled and how the natural fractures affect the injection/production process will be investigated. Introducing natural fractures into the system potentially facilitates the fluid flow process by these highly conductive paths. The initial natural fracture permeability is the first factor to be discussed. The volume fraction of the natural fracture serves as a buffer to take in the high-rate cold water so the volume fraction is also considered for sensitivity analysis. A multi-porosity model in which the matrix is partitioned into multiple continua with different mechanical properties is also discussed. Finally, the MINC model has been studied widely in the literature on its capability of accurately capturing the transient mass/heat transfer process between the matrix and the fracture. Increasing MINC partitioned number usually improves the model accuracy. This effect will also be studied in this section by changing the number of MINC continuums.

Table 7.2: Input parameters of the synthetic EGS model with MINC (sequentially coupled model)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix Permeability</td>
<td>$2 \times 10^{-19}$</td>
<td>m²</td>
</tr>
<tr>
<td>Hydraulic Fracture Permeability</td>
<td>$2 \times 10^{-11}$</td>
<td>m²</td>
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<td>Rock/Hydraulic Fracture Porosity</td>
<td>0.05/0.4</td>
<td>Unitless</td>
</tr>
<tr>
<td>MINC Matrix/Natural Fracture Permeability</td>
<td>$1 \times 10^{-18}/4 \times 10^{-15}$</td>
<td>m²</td>
</tr>
<tr>
<td>MINC Matrix/Natural Fracture Porosity</td>
<td>0.08/0.5</td>
<td>Unitless</td>
</tr>
<tr>
<td>Matrix/Fracture Poisson’s Ratio</td>
<td>0.3</td>
<td>Unitless</td>
</tr>
<tr>
<td>MINC Matrix/Natural Fracture Poisson’s Ratio</td>
<td>0.3</td>
<td>Unitless</td>
</tr>
<tr>
<td>Matrix Young’s Modulus</td>
<td>40</td>
<td>GPa</td>
</tr>
<tr>
<td>MINC Matrix/Natural Fracture Young’s Modulus</td>
<td>30/5</td>
<td>GPa</td>
</tr>
<tr>
<td>Biot’s Coefficient</td>
<td>0.7</td>
<td>Unitless</td>
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<tr>
<td>MINC Matrix Biot’s Coefficient</td>
<td>0.6</td>
<td>Unitless</td>
</tr>
<tr>
<td>MINC Matrix/Matrix Expansion Coefficient</td>
<td>$2 \times 10^{-5}/4 \times 10^{-5}$</td>
<td>1/°C</td>
</tr>
<tr>
<td>MINC Partition Volume Fraction</td>
<td>0.02, 0.15, 0.3, 0.3, 0.23</td>
<td>Unitless</td>
</tr>
<tr>
<td>Natural Fracture Spacing</td>
<td>0.5, 0.4, 0.2</td>
<td>m</td>
</tr>
</tbody>
</table>
Figure 7.29: EDFM-MINC model: the blue box represents the SRV in which EDFM (hydraulic fractures) and MINC (induced/natural fractures) coexist.
Figure 7.30: Modeling results of the base case for EDFM-MINC: displacement in x, y and z directions (first row); effective xx, yy and zz stresses (second row); pressure, temperature and fracture width (third row).
7.2.1 Base Case of EDFM-MINC Model

The EDFM MINC model is partitioned into two types of volumes: (1) the volume contains only matrix; (2) the SRV contains both the hydraulic fracture and induced/reactivated natural fractures. The SRV is marked as the blue box in Figure 7.29. The input data is shown in Table 7.2. The other parameters such as injection rate and production pressure, are all the same as those in Table 7.1. The key parameters added are: (1) the volume fraction of MINC partition and the fracture spacing which control the MINC mesh; (2) MINC matrix and natural fracture mechanical properties which correlate the porosity with stress/strain, pressure and temperature in Equation 3.45.

The modeling results are shown in Figure 7.30. Although the hydrological and mechanical properties in this base case are modified compared to the single-fracture EGS model in Table 7.1. The two results (the other shown in Figure 7.2) can be compared qualitatively:

1. The temperature distribution of the MINC case shows that the pressure drop volume expands due to the highly conductive natural fractures around the hydraulic fracture;
2. The temperature reduction increases the contraction of the whole reservoir (thermal strain), which is reflected from the x and y displacement;
3. The effective stresses are distributed more uniformly within the SRV in which tensile stresses are still dominating the volume close to the hydraulic fracture;
4. The fracture width (compared to (b) of Figure 7.9) is reduced to a low level, due to both the contraction and the low thermal expansion coefficient of the MINC matrix continuums.

7.2.2 Effects of Natural Fracture Permeability

Induced/natural fractures act as conductive paths for the fluid flow in the low-permeable EGS reservoirs. The injected fluid can flow through the induced/natural fractures and contact more areas of EGS for the heat conduction process. This type of fracture may be common in EGS sites, either generated in the tectonic processes or induced by the stimulation, but their characterization remains challenging. A lot of uncertainty is
associated with induced/natural fracture permeability/aperture so in this section the
sensitivity analysis is conducted using the sequentially coupled THM model. The natural
permeability is decreased from base case $4 \times 10^{-15}$ m$^2$ to $4 \times 10^{-16}$ m$^2$ and $4 \times 10^{-17}$ m$^2$.

![Image](77x535 to 221x644)

![Image](233x535 to 378x644)

![Image](390x535 to 535x644)

(a)

![Image](77x406 to 221x515)

![Image](233x406 to 378x515)

![Image](390x406 to 535x515)

(b)

![Image](77x277 to 221x386)

![Image](233x277 to 378x386)

![Image](390x277 to 535x386)

(c)

Figure 7.31: Comparison of displacement in x, y and z directions, sensitivity analysis of
natural/induced fracture permeability: (a) $4 \times 10^{-15}$ m$^2 = 4$ md; (b) $4 \times 10^{-16}$ m$^2 = 0.4$
md; (c) $4 \times 10^{-17}$ m$^2 = 0.04$ md.

The comparison of displacement, effective stress and pressure/temperature for the three
cases are shown in Figure 7.31, 7.32 and 7.33. Obviously, the higher natural fracture
permeability leads the cold fluid towards all directions of SRV in Figure 7.33, reducing the
temperature of the SRV. When the natural fracture permeability is lower, the temperature
distribution appears more similarly to the single-fracture case. Besides, the highly
conductive natural fractures lower the average pressure in the whole reservoir. The lower temperature induces higher thermal stresses/strains which leads to a higher displacement in x, y, and z directions in (a) of Figure 7.31 and also higher tensile stresses within the SRV. The lower natural fracture permeability results in an effective stress distribution similar to the single-fracture case in (b) and (c) of Figure 7.32, which is also observed in the trend of temperature distribution.

Figure 7.32: Comparison of stress in x, y and z directions, sensitivity analysis of natural/induced fracture permeability: (a) $4 \times 10^{-15} \text{ m}^2 = 4 \text{ md}$; (b) $4 \times 10^{-16} \text{ m}^2 = 0.4 \text{ md}$; (c) $4 \times 10^{-17} \text{ m}^2 = 0.04 \text{ md}$. 
Figure 7.33: Comparison of pressure, temperature and porosity, sensitivity analysis of natural/induced fracture permeability: (a) $4 \times 10^{-15}$ m$^2 = 4$ md; (b) $4 \times 10^{-16}$ m$^2 = 0.4$ md; (c) $4 \times 10^{-17}$ m$^2 = 0.04$ md.
The fracture width of the three scenarios is shown in Figure 7.34. The fracture aperture is smaller in the higher natural fracture permeability case, probably due to the relatively uniform contraction and large displacement in the y-direction. The injection fracture permeability is plotted in the rightmost of Figure 7.35 and it shows that the fracture permeability/aperture for the higher natural fracture permeability case increases to the highest level and then drops below the lower natural fracture permeability case. The higher permeability reached may be due to the lower temperature and strong contraction between the hydraulic fracture continuity and the lower in the late stage may be attributed to the lower pore pressure and more steady temperature field. The thermal breakthrough of the high natural fracture permeability is significantly delayed, probably because the cold water flow through the natural fracture and get heated by the SRV and reservoir matrix.

Figure 7.34: Comparison of fracture width, sensitivity analysis of natural/induced fracture permeability: (a) \(4 \times 10^{-15} \text{ m}^2 = 4 \text{ md}\); (b) \(4 \times 10^{-16} \text{ m}^2 = 0.4 \text{ md}\); (c) \(4 \times 10^{-17} \text{ m}^2 = 0.04 \text{ md}\).

Figure 7.35: Comparison of production temperature, flow rate and injection fracture permeability, sensitivity analysis of natural/induced fracture permeability.
7.2.3 Effects of Natural Fracture Volume Fraction

The volume fraction of the natural fracture also has the potential to influence the fluid flow/heat transfer processes since more fluid can flow and be stored into the highly conductive natural fracture. In this section, the sensitivity analysis is performed for the natural fracture volume fraction. The fraction is decreased from 0.02 to 0.002 and 0.0004. The displacement, effective stress, and temperature distributions for the three cases are compared, but no observable difference can be seen. Therefore, the distributions are not plotted in this section but can be referred to in (b) rows of Figure 7.31, 7.32, 7.33. Pressure distribution and fracture aperture change is shown in Figure 7.36. The fracture aperture is larger in the higher natural fracture fraction volume, hence the pressure is lower near the injection point.

![Pressure Distribution](image1)
![Pressure Distribution](image2)
![Pressure Distribution](image3)

Figure 7.36: Comparison of production temperature and fracture width, sensitivity analysis of natural/induced fracture volume fraction: (a) 0.02; (b) 0.002; (c) 0.0004.

Production temperature, flow rate and injection fracture permeability are compared in Figure 7.37. The results agree with the fracture width profile since the permeability of higher natural fracture permeability is enhanced to a higher level as well, which results in a lower production temperature.
Figure 7.37: Comparison of production temperature, flow rate and injection fracture permeability, sensitivity analysis of natural/induced fracture volume fraction.

Table 7.3: Input parameters of the synthetic EGS model with MINC and different properties for the multiple continua (sequentially coupled model)

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<tr>
<th>Parameters</th>
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<tr>
<td>Matrix Permeability</td>
<td>2e-20</td>
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<td>MINC 4 continuums/</td>
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</tr>
<tr>
<td>Natural Fracture Permeability</td>
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<td>m²</td>
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<tr>
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<tr>
<td>Natural Fracture Porosity</td>
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<td>Unitless</td>
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<tr>
<td>MINC 4 continuums/</td>
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</tr>
<tr>
<td>Natural Fracture Young’s Modulus</td>
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<td>GPa</td>
</tr>
<tr>
<td>MINC Partition Volume Fraction</td>
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<td>Unitless</td>
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</tbody>
</table>
Figure 7.38: Comparison of displacement in x, y and z directions: (a) base case; (b) multiple continua.

Figure 7.39: Comparison of stress in x, y and z directions: (a) base case; (b) multiple continua.
7.2.4 Multiporosity Model

The multiple continuum matrix in the MINC model has the same mechanical properties as the above sensitivity analysis. In this section, the multiple continua have their own properties, such as porosity, permeability and Young’s modulus, as shown in Table 7.3. The other parameters that are not listed are kept unchanged in Table 7.2. This multiporosity model is compared with its base case which uses the base case data in Table 7.2 except the matrix permeability and volume fraction of Table 7.3.

The displacement, effective stress, pressure/temperature and fracture width of the base and multiporosity cases are shown in Figure 7.38, 7.39, 7.40 and 7.41. The displacements in the x and y directions of the two cases are mostly identical. A slight difference can be observed in the z-displacement. The effective stresses are also similar in the distribution but due to the different Young’s modulus which is used to compute effective stresses, stress discontinuity can be observed, especially on the SRV boundaries.

Pressure distribution in the base case is slightly higher due to the low hydraulic fracture permeability/aperture, which can be illustrated in Figure 7.41. The fracture aperture is also reflected in Figure 7.42: the thermal breakthrough is slower in the base case and the injection fracture permeability is lower as well.

The multiple continua in the MINC mainly enhances the Young’s modulus by upscaling or weighted average, which in turn increases the thermal strain/stress and results in a stronger matrix contraction across the fracture discontinuity.

7.2.5 Investigation of MINC Partition Number and Natural Fracture Spacing in EDFM-MINC Model

Increasing the number of MINC partition number usually improves the modeling result accuracy in the transient state. A lot of studies draw this conclusion, such as modeling tight gas or oil reservoir where the matrix permeability is extremely low. The transient mass transfer between highly permeable fractures and non-permeable matrix can be better captured by increasing the MINC number.
Figure 7.40: Comparison of pressure, temperature: (a) base case; (b) multiple continua.

Figure 7.41: Comparison of fracture width: (a) base case; (b) multiple continua.
In the EGS project, the granite matrix is usually non-permeable, which is favorable for heat and mass recovery. In order to investigate the partition number effect on the mass/heat transfer, the modeling is conducted for 2 MINC, 5 MINC and 10 MINC, and the results are compared.

The TH model without mechanics coupling is investigated first as a basis of coupled THM modeling, with different fracture spacing. When the fracture spacing is smaller, pressure/temperature in the fracture tends to reach rapid equilibrium with the matrix and the large partition number might not be necessary. Input data in Table 7.2 is used, except the natural fracture permeability is $4 \times 10^{-16}$ m$^2$ and fracture spacing is 0.4-4-40 m, uniformly in the three orthogonal sets. The TH modeling results of production rate and temperature are shown in Figure 7.43. The comparison demonstrates the essential role of fracture spacing: when fracture spacing is large (40 m), 5 and 10 MINC are able to capture the rate and temperature accurately while the double porosity is not.

Since TH modeling results of the 0.4 m fracture spacing are all the same for 2, 5 and 10 MINC cases, there is no need to compare the coupled THM modeling among these three scenarios using this fracture spacing value. In the 4 m and 40 m spacing scenarios, the results of 5 and 10 MINC are reasonably close, indicating that 5 partitions of the MINC grid are accurate enough for fluid flow and heat transfer. Hence, only 5 MINC and DP (2 MINC) models are to be compared for 4 and 40 m fracture spacing.
Figure 7.43: Comparison of production flow rate and temperature using different fracture spacing: (a) 0.4 m; (b) 4 m; (c) 40 m.
The comparison shows that the displacement, stress, pressure/temperature and fracture width are all the same for 2 and 5 MINC without observable difference with the 40 m fracture spacing. The result for 5 MINC is shown in Figure 7.44. This comparison indicates that the state (pressure, temperature and fracture width) of the EGS model at the end of the circulation might not be sensitive to the MINC number. Comparing the different spacing between 40 m and 4 m for the 5 MINC case demonstrates that the displacement and stresses are the same for the two spacing cases as well. The only slight difference is the pressure distribution as shown in Figure 7.45 which is mainly attributed to the fracture density. The temperature profiles show that the 40 m spacing case in (a) has a slightly lower temperature around the fracture volume. The fracture width in (a) is also slightly smaller than that in (b).

Figure 7.44: Coupled THM modeling results for 5 MINC partition with 40 m natural fracture spacing.
Figure 7.45: Comparison of pressure/temperature/fracture width for 5 MINC case: (a) 40 m spacing; (b) 4 m spacing.

Moreover, it is necessary to compare the overall dynamic production rate and temperature for various scenarios: number of partitions, coupling or not coupling mechanics, and fracture spacing. The results are illustrated in Figure 7.46. Comparing the lines with and without markers, it can be seen that coupled THM modeling raises the production temperature by 10 °C. The flow rates of the coupled THM model is lower than the no-coupling case, even in the late stage of the circulation. This is probably attributed to the higher natural fracture permeability opened by the thermal strain. Comparing the dotted and broken lines, or the triangle and square markers, the impact of fracturing spacing can be observed although the final temperature and rate are not affected significantly. The transient stage is under the most influence which might impact the heat recovery efficiency. Comparing the red and blue curves, the DP and 5 MINC models produce distinct production behaviors in the larger fracture spacing case.
Figure 7.46: Comparison of flow rate and production temperature for 2/5 MINC, TH/THM modeling, and 4/40 m fracture spacing: lines without markers represent no mechanics coupling; red lines represent 5 MINC; dotted lines represent 4 m spacing.

7.2.6 Discussion and Summary of the EDFM-MINC Model

In this section, the original single hydraulic fracture EGS model has been extended to a combined single hydraulic fracture and induced/natural fractures model. The hydraulic fracture is contained in the SRV where uniformly distributed fractures are modeled by the MINC approach. In this approach, the outermost layer is the natural fracture and the inner layers are matrix or multiple continua with various properties. The sensitivity analysis is performed for natural fracture permeability, and natural fracture volume fraction. The multiple continua with various properties are set as the inner layers and computed as a new model. This model is also compared with the base EDFM-MINC case. Effects of MINC partition number and natural fracture spacing are investigated afterward. The coupled THM modeling results are also compared with the TH modeling without mechanics coupling. The conclusions of this section can be summarized as:

1. Natural fracture permeability is an important factor in the EDFM-MINC modeling. The higher natural fracture permeability allows the cold injected fluid to contact more reservoirs in addition to the hydraulic fracture and hence absorb more energy to enhance the production temperature;

2. When natural fracture permeability is low, the THM processes tend to be similar to the single fracture dominating case where the temperature reduction concentrates...
around the hydraulic fracture and the pressure field has a much higher average value than the high permeability case;

3. The higher natural fracture permeability allows more cold fluid to flow within the SRV, enhances the rock matrix contraction and the hydraulic fracture aperture is opened more in the early stage. When the heating process is weakened in the late stage and heat transfer processes reach the model boundary, pressure could drop fast and gradually close the hydraulic fracture;

4. The coupled THM process is not as sensitive to the natural fracture volume fraction as to the permeability. The higher fraction allows more fluid flow within the SRV, decreases the production temperature and opens hydraulic fracture to reach a higher aperture;

5. Multiple continua with various properties could also affect the contraction across the fracture discontinuity by the changed mechanical properties;

6. For the smaller natural fracture spacing, DP and 5-MINC are both capable of capturing the transient mass/heat transfer between fractures and matrix. But for larger spacing, more MINC partition number improves the modeling accuracy in both mass and heat transfer processes;

7. The coupling of mechanics makes a more notable difference in the EDFM-MINC scenario than the EDFM single-fracture scenario. Without the mechanics coupling, production temperature is underestimated;

8. The fracture spacing/density has an apparent impact on the transient behavior of mass/heat transfer and needs to be well characterized;

9. The large spacing usually requires more MINC partition numbers to achieve accurate modeling of transient behaviors.

7.3 Comparison of the Fully and Sequentially Coupled Model on the Same Input Data Set

Although the sequentially coupled model is more rigorous in fracture mechanics, it is desired to compare the fully coupled model with the sequentially coupled model for
benchmarking purposes. The input data of the base case in Table 7.1 is used for both models. The constitutive relations of matrix porosity and permeability in both models are Lagrangian porosity and Kozeny-Carman permeability. The boundary tractions for the sequentially coupled model are unchanged while the reference point stress state is set for the fully coupled model to be consistent with the sequentially coupled model. However, due to the different boundary conditions and the fracture mechanics solver, the simulation results of the two models do not match each other completely although they have the same trends in general.

Figure 7.47: Comparison of fracture width/permeability and matrix porosity for the fully and sequentially coupled models: (a) the fully coupled model; (b) the sequentially coupled model.
The fracture permeability of the fully coupled model and the relative displacement of the fracture discontinuity are shown in Figure 7.47. Although they are different variables but indicate the same physical meaning: the ratio between the current fracture aperture and the initial aperture. It can be seen that the two plots have a similar contour. The matrix porosity is shown in the other column. The porosity distribution around the fractured volume in the fully coupled model is apparently lower than in the sequentially coupled model but higher than in the non-fractured volume.

Pressure and temperature distribution comparisons are presented in Figure 7.48. The temperature fields of the two models are almost identical, which might indicate insensitivity of the heat conduction process to the mechanics compared to the fluid flow process. The pressure distribution of the fully coupled model is more uniform and lower than the sequentially coupled model, which may also explain why the matrix porosity is lower around fractured volume (note that the color scales of the two models are different). But the pressure contours of the two models have the same trend.

Effective stresses of the two models at the end of the circulation are shown in Figure 7.49. Due to the dominant thermal stresses generated by the temperature field, the results of both models have the same trend: tensile stresses concentrate around the fractured volume. Tensile stresses are stronger in the fully coupled model.

The flow rate, production temperature, and injection fracture permeability are compared in Figure 7.50. The production temperature matches each other, indicating that the heat conduction process is not strongly sensitive to the mechanics. The late stage production rate is comparable but the apparent discrepancy is observed in the transient early stage. Injection fracture permeability of the fully coupled model keeps increasing due to the stronger tensile effect. In the sequentially coupled model, the fracture permeability at the injection point increases to the highest level and then drops: it might be due to the fact that the rock contraction has been suppressed by the compressive boundary tractions and fractures start to close.
Figure 7.48: Comparison of pressure and temperature distribution for the fully and sequentially coupled models: (a) the fully coupled model; (b) the sequentially coupled model.
Figure 7.49: Comparison of \( xx \), \( yy \) and \( zz \) effective stresses for the fully and sequentially coupled models: (a) the fully coupled model; (b) the sequentially coupled model.

Figure 7.50: Comparison of production temperature, flow rate, and injection fracture permeability for the fully and sequentially coupled models.
CHAPTER 8
CONCLUSIONS AND RECOMMENDATIONS

In this chapter, the work that has been done for the dissertation is summarized and conclusions drawn from the studies are also listed and discussed after the summary. In the end, recommendations for the future or related work are made.

8.1 Summary and Contributions

In this dissertation, two coupled THM models are developed for EGS reservoirs using two coupling schemes: fully coupled and sequentially/iteratively coupled approaches. The stimulated fracture for fluid flow circulation is modeled by EDFM which has been improved to explicitly handle multiple intersected fractures within a grid block. The fully coupled model is based on the TOUGH2-CSM parallel framework and extended to incorporate all stress tensor components as the primary variables. This extension is made to fully implicitly simulate the dynamic fracture aperture in response to the stress, pressure, and temperature under the complex fluid flow, heat transfer, and mechanical deformation induced by the cold water injection into the hot reservoir. The fully coupled model discretizes the stress equations using an IFD approach and the assumption is made that the discrete fracture has the same stress state with the containing grid block. This assumption is physically loose although the model is capable of capturing the physics of the fractured EGS reservoir. Hence a rigorous model using EDFM and XFEM is developed based on the sequential coupling scheme where TOUGH2-EGS and an in-house XFEM simulator are coupled. The XFEM simulator has been improved to solve a general shaped embedded discrete fracture with arbitrary strikes, using the newly implemented numerical discretization. In addition, the body force, boundary traction, internal fluid traction and thermal stress/strain are all incorporated into the XFEM simulator. The fully and sequentially coupled models are applied to synthetic EGS models and sensitivity analyses have been performed on the key parameters. Moreover, the fully coupled model is applied to a DOE field experiment and the results are compared with the literature. The
sequentially coupled model is extended to incorporate induced/natural fractions based on the MINC approach so that both hydraulic fractures and natural fractures can be considered in an EGS reservoir. Sensitivity analyses are also conducted for the EDFM-MINC model with respect to the natural fracture permeability and volume fraction. The contributions made from the development and numerical studies are:

1. EDFM has been improved from the original version to explicitly handle multiple fractures within each grid. This capability can be used to model the intersection of hydraulic fractures and natural fractures. In this dissertation, this feature is needed to compute the normal stress states on the fracture faces;

2. The fracture aperture in response to effective stresses, pressure, and temperature is captured in the fully coupled model which can simulate the complex fracture behavior in EGS circulation. The methodology can be used in any non-isothermal coupled fluid flow and geomechanics model;

3. XFEM has been improved to handle an arbitrary hydraulic fracture in 3D space with efficient numerical discretization approaches and accurate internal/external traction computation;

4. XFEM is sequentially coupled with a fluid flow/heat transfer simulator for a 3D model which is also novel based on the literature review. Fracture mechanics in response to the fluid flow and heat transfer can be fully captured by the coupled XFEM-TOUGH2 program;

5. The EDFM has been combined with MINC to model both hydraulic and natural fractures and the impact of these fractures on a 3D nonisothermal model is evaluated which has not been found in the current existing research works;

6. The influence of key parameters of thermal conductivity, fracture permeability, thermal expansion coefficient, injection temperature, natural fracture spacing, and MINC partition number are all investigated for an intermediate-scale EGS reservoir, using the stress, pressure, temperature, flow rate, and production temperature as
indicators. The results provide a better understanding of the physics behind the EGS injection/production.

8.2 Conclusions

Numerical studies and sensitivity analyses performed have inferred several key conclusions in the coupled THM modeling of the EGS reservoir. Note that these conclusions are only applicable to the intermediate-scale EGS reservoir and may not be general to all the scales. Key parameters of EGS projects include thermal conductivity, fracture permeability, injection rate and temperature, and natural fracture characterizations. Based on the studies performed in this dissertation, the following conclusions can be drawn:

1. The fluid flow, heat transfer and mechanical deformation processes are tightly coupled in EGS reservoirs. Thermally induced stress/strain usually dominates the deformation so the coupled process is sensitive to the thermal parameters such as thermal conductivity, expansion coefficient, and injection temperature;

2. The hydraulic fracture aperture is enlarged due to the cold fluid injection in both fully and sequentially coupled models although the mechanism behind is different: the fully coupled model opens the fracture by thermal stresses and the sequentially coupled model by thermal strain/contraction of the reservoir matrix;

3. The increase of fracture permeability could allow more cold fluid to pass and accelerate the thermal breakthrough;

4. The interaction between fracture and matrix is complex: the matrix heats the fracture fluid by heat conduction, the higher conductivity leads to efficient heating of the fluid and delays the thermal breakthrough but also reduces the reservoir temperature more rapidly. The fully and sequentially coupled model have different effects on the production behaviors in this scenario;

5. The temperature reduction near the fracture area increases the porosity and permeability of the rock matrix, hence enhances the mass transfer or convective flow between them. The fluid loss from injection to the reservoir is increased;
6. Due to the dominant role of thermal stress/strain, the expansion coefficient makes a significant impact on the EGS reservoir, changing both heat transfer and fluid flow processes by modifying the reservoir porosity/permeability. The higher thermal expansion coefficient could lower (or enhance) the production temperature (based on fracture patterns) but also affect the fluid mass recycling;

7. The injection rate also affects the EGS heat recovery process, and needs to be specially designed for the heat recovery efficiency;

8. Natural fractures are important conductive paths for the fluid flow to contact more reservoir and recover more heat and the accurate modeling of the natural fractures using MINC are dependent on the MINC partition especially when the fracture spacing is large. Hence, the fracture characterization is essential;

9. The fully and sequentially coupled model are generally giving similar results or trends but not completely the same; They have their own advantages or disadvantages that are needed to be considered in different applications.

8.3 Recommendations

There are still potential works to be done in the future for this dissertation and they are listed as below:

1. The current EGS model is based on a water-saturated EGS reservoir which can be modified to a multiphase simulator easily since TOUGH2-EGS and TOUGH2-CSM are both able to model water-air two-phase flow in the simulator. Work needs to be done to set up and conduct computations for such two-phase flow models;

2. The fracture aperture correlation used in the fully coupled model is usually applicable to natural fractures. A more common choice is the Barton-Bandis fracture model. This correlation can be adopted to investigate any potential effect on the modeling results;

3. The current XFEM solver is only capable of handling a single fracture reservoir. The fracture intersection, especially fracture cut-fracture tip intersection in 3D space, cannot be handled and no such research or implementation is found. This issue is
more of a mathematical research but could significantly improve the capability of the coupled THM model if successfully solved;

4. TOUGH2-EGS is written in Fortran and XFEM solver is written in MATLAB. The current program calls MATLAB batch from Fortran and wastes time on starting and closing MATLAB batch mode. It may be desired to rewrite XFEM in Fortran and save time spent on the program interface;

5. XFEM can be fully coupled with TOUGH2-EGS but efforts are needed on the XFEM solver integration;

6. The sequentially coupled THM model can be validated or benchmarked using commercial multi-physics software such as COMSOL;

7. The coupled model has oscillations numerically in the very early stage of the simulation, especially in the single-fracture case. An effort may be necessary to investigate the reason behind;

8. A large-scale EGS reservoir simulation is desired since more literature published focus on the large-scale modeling, with a 1 km reservoir dimension and 200-500 m long hydraulic fractures. The larger scale will also diminish the deformation effect and keep the fractured volume far away from the boundary, which are more realistic. Results of such a large-scale model can be compared with the existing literature for benchmark purposes;

9. The conclusions based on current sensitivity analysis are qualitative. In order to investigate the mass and heat transfer process, such as, where the cold water flows into the reservoir or how much reservoir fluid has been produced can be studied by incorporating tracer into the model. To explore the dynamic stress/strain, pressure and temperature field, a 4D data interpretation and analysis is necessary to observer how THM processes interact with each other in the EGS development.
REFERENCES


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Figure A.1: Email permission from the author of the paper