Reflection Moveout and Parameter Estimation

for

Horizontal Transverse Isotropy

by

AbdulFattah A. Al-Dajani
A thesis submitted to the Faculty and the Board of Trustees of the Colorado School of Mines in partial fulfillment of the requirements for the degree of Master of Science, Geophysics.

Golden, Colorado
Date 12/09/1996

Signed: AbdulFattah A. Al-Dajani

Approved: Dr. Ilya Tsvankin
Dr. Phillip R. Romig
Associate Professor of Geophysics
Professor and Head
Thesis Advisor
Department of Geophysics

Golden, Colorado
Date 13 December 1996
**ABSTRACT**

The transversely isotropic model with a horizontal axis of symmetry (HTI) has been used extensively in studies of shear-wave splitting to describe fractured formations with a single system of parallel, vertical, penny-shaped cracks. Here, I present an analytic description of long-spread reflection moveout in horizontally-layered HTI media with arbitrary strength of anisotropy. Also, I discuss P-wave reflection-moveout inversion to estimate the parameters for horizontal transverse isotropy.

The hyperbolic moveout equation parameterized by the exact normal-moveout (NMO) velocity provides sufficient accuracy for P-waves on conventional spreadlengths (close to the reflector depth). The influence of anisotropy, however, leads to the deviation of the moveout curve from a hyperbola as spreadlength increases, even in a single-layer model. To account for nonhyperbolic moveout, I have derived an exact expression for the azimuthally-dependent quartic term of the Taylor series traveltime expansion $[t^2(x^2)]$ valid for any pure mode in an HTI layer. The quartic moveout coefficient and the normal-moveout velocity are then substituted into a nonhyperbolic moveout equation originally designed for vertical transverse isotropy (VTI media). Numerical examples for media with both moderate and uncommonly strong nonhyperbolic moveout show that this equation accurately describes azimuthally-dependent P-wave reflection traveltimes in an HTI layer, even for spreadlengths twice as large as the reflector depth.

In multilayered HTI media, the NMO velocity and the quartic moveout coefficient reflect the combined influence of layering and azimuthal anisotropy. The conventional Dix equation for NMO velocity remains entirely valid for any azimuth in HTI media.
if the *group-velocity* vectors (rays) for data in a common-midpoint (CMP) gather do not deviate from the vertical incidence plane. Although this condition is not exactly satisfied in realistic HTI media, rms averaging of the interval NMO velocities provides a good approximation for moveout across conventional spreads.

Furthermore, the quartic moveout coefficient for multilayered HTI media can also be calculated with good accuracy using the known averaging equations for vertical transverse isotropy. This allows me to extend the nonhyperbolic moveout equation to horizontally stratified media composed of any combination of isotropic, VTI, and HTI layers. In addition to providing analytic insight into the behavior of reflection moveout, these results can be used in modeling of reflection traveltimes in TI media with a vertical and/or horizontal symmetry axis. Also, this suggests that the same approach can be used to describe reflection moveout in more complicated anisotropic media such as orthorhombic.

Using the azimuthal dependence of $P$-wave normal-moveout (NMO) velocity, measured in three different source-to-receiver orientations, we can obtain the vertical velocity $V_{\text{P,vert}}$, anisotropy parameter $\delta^{(V)}$, and the azimuth $\alpha$ of the symmetry axis. The nonhyperbolic portion of the moveout curve, on the other hand, offers the possibility of estimating the anisotropic parameter $\epsilon^{(V)}$ needed to find the crack density.

Parameter estimation is quite sensitive to the angular separation between the survey lines, and to the set of azimuths used in the inversion procedure. The accuracy in estimating the parameter $\alpha$, in particular, is also sensitive to the strength of anisotropy (the reduction of error in $\alpha$ estimates is linearly proportional to the strength of anisotropy measured by $\delta^{(V)}$). The accuracy in resolving $\delta^{(V)}$ is about the same for any strength of the anisotropy (there is a slight improvement in the
accuracy with increasing $|\delta^{(V)}|$. This implies that we should expect about the same absolute error in $\delta^{(V)}$ for a wide range of $\delta^{(V)}$, and the relative error in $\delta^{(V)}$ will be smaller for stronger anisotropy. The accuracy in estimating $V_{p\text{vert}}$, however, increases as anisotropy becomes weaker.

In order to maximize the accuracy and stability in parameter estimation, it is best to have the azimuths for the three source-to-receiver directions 60° apart. However, an angular separation of 45° between each of the three source-to-receiver directions also provides adequate accuracy and stability. The accuracy in resolving the parameters for an angular separation of 60° is consistent at all azimuths, and for most ranges of azimuths the associated errors are the least.

In layered HTI media, applying Dix differentiation to obtain interval moveout velocity provides sufficient accuracy in the inversion for the medium parameters, especially if the symmetry-axis direction is uniform throughout the media. An HTI layer overlain by an azimuthally isotropic overburden (as might happen for fractured reservoirs), however, should have a relative thickness (in time) to the total thickness of at least 0.2 in order to obtain acceptable estimates of the medium parameters, provided that the azimuthal variation in the interval NMO velocity within the layer is about 10% or higher.

Some of the conclusions drawn here for HTI media, such as the optimal spread of azimuths to be used in the inversion and the minimum thickness of the layer of interest, should be also valid for pure $S$-wave reflections and for more complicated horizontally-stratified azimuthally anisotropic media (e.g., orthorhombic).
### TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>iii</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>viii</td>
</tr>
<tr>
<td>ACKNOWLEDGMENTS</td>
<td>xvii</td>
</tr>
<tr>
<td>Chapter 1 INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>Chapter 2 REFLECTION MOVEOUT FOR HORIZONTAL TRANSVERSE ISOTROPY</td>
<td>6</td>
</tr>
<tr>
<td>2.1 Description of the HTI model and notation</td>
<td>6</td>
</tr>
<tr>
<td>2.2 Analytic approximations of reflection moveout</td>
<td>10</td>
</tr>
<tr>
<td>2.3 Moveout in a single HTI layer</td>
<td>12</td>
</tr>
<tr>
<td>2.3.1 Normal-moveout and horizontal velocity</td>
<td>12</td>
</tr>
<tr>
<td>2.3.2 Quartic moveout coefficient</td>
<td>14</td>
</tr>
<tr>
<td>2.4 Moveout in multilayered media</td>
<td>17</td>
</tr>
<tr>
<td>Chapter 3 NUMERICAL STUDY OF ANALYTIC APPROXIMATIONS FOR P-WAVE REFLECTION MOVEOUT</td>
<td>22</td>
</tr>
<tr>
<td>3.1 Single HTI layer</td>
<td>23</td>
</tr>
<tr>
<td>3.2 Multilayered media</td>
<td>28</td>
</tr>
<tr>
<td>3.2.1 Multilayered HTI media</td>
<td>28</td>
</tr>
<tr>
<td>3.2.2 HTI-VTI and isotropic-HTI layered media</td>
<td>39</td>
</tr>
<tr>
<td>Chapter 4 P-WAVE REFLECTION-MOVEOUT INVERSION FOR HORIZONTAL TRANSVERSE ISOTROPY</td>
<td>43</td>
</tr>
<tr>
<td>4.1 The inverse problem</td>
<td>43</td>
</tr>
<tr>
<td>4.2 Error analysis</td>
<td>46</td>
</tr>
<tr>
<td>4.2.1 Conditioning of the problem</td>
<td>47</td>
</tr>
<tr>
<td>4.2.2 Error propagation (covariance matrix)</td>
<td>51</td>
</tr>
<tr>
<td>4.2.3 Numerical inversion</td>
<td>57</td>
</tr>
<tr>
<td>4.3 The inverse problem in layered media</td>
<td>60</td>
</tr>
<tr>
<td>4.3.1 Error analysis</td>
<td>62</td>
</tr>
<tr>
<td>4.3.2 Numerical inversion in layered media</td>
<td>65</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

2.1 Sketch of the transversely isotropic model with a horizontal symmetry axis caused by a system of parallel vertical cracks. HTI media contain two vertical planes of mirror symmetry defined by the crack orientation (after Rüger, 1996). .......................................................... 7

2.2 Schematic 3-D picture showing a reflection raypath for a source and receiver along a seismic line that makes the azimuth angle $\alpha$ with respect to the symmetry-axis plane of an HTI layer. Since the model has a horizontal symmetry plane, the incident and reflected rays of pure modes lie in the vertical incidence (sagittal) plane. Here, the CMP line is taken as a 2D seismic line. Later, we shall see implications for 3D survey data. .......................................................... 13

3.1 Orientation of the hypothetical 2-D survey lines over a horizontal HTI layer used in Figures 3.2 and 3.3. .............................. 24

3.2 Comparison between the analytic $P$-wave normal-moveout velocity [equation (2.10)] and the moveout velocity estimated on a finite spread in a single HTI layer. The solid curve is the moveout velocity as a function of azimuth determined by fitting a hyperbola to the exact $t-x$ curves [equation (3.1)]; the dashed curve is the normal-moveout (zero-spread) velocity from equation (2.10). The curves are calculated for four different HTI models and three spreadlengths ($X/D = .5, X/D = 1,$ and $X/D = 2$ in three columns from left to right). The model parameters are given in Table 3.1. .......................................................... 25

3.3 Comparison between the exact traveltimes and moveout approximations in the four single-layer HTI models whose parameters are given in Table 3.1. The gray curves are the exact reflection traveltimes as functions of the offset-to-depth ratio for survey-line azimuths $\alpha$ of $0^\circ$, $30^\circ$, $45^\circ$, $60^\circ$, and $90^\circ$. The black curves in (a), (c), (e), and (g) are the time residuals after conventional hyperbolic moveout correction using equation (2.3), while the black curves in (b), (d), (f), and (h) are the residuals after the nonhyperbolic moveout correction using equation (2.5). .......................................................... 27
3.4 P-wave group-velocity (ray) deviation from the incidence plane in HTI media (Model 5 in Table 3.2) for a receiver at a horizontal distance equal to the target depth (1.5 km) for five different source-to-receiver directions: 0°, 30°, 45°, 60°, and 90°. The HTI layers have a uniform symmetry-axis orientation. (a) A ray that propagates in the first layer only is confined to the incidence plane at all azimuths. (b) A ray propagating through all layers is confined to the incidence plane only in the two symmetry-plane directions (azimuths 0° and 90°).

3.5 Accuracy of the rms-averaging equation for P-wave normal-moveout velocity in layered HTI media with a uniform orientation of the symmetry-axis. The solid curve is the moveout velocity as a function of azimuth determined by fitting a hyperbola to the exact \( t - x \) curves [equation (3.1)] on two different spreadlengths: (a) \( X/D = 1 \), and (b) \( X/D = 2 \). The dashed curve is the normal-moveout (zero-spread) velocity from the Dix equation (2.16). The velocities are calculated for the reflection from the bottom of the third layer of Model 6 whose parameters are given in Table 3.2.

3.6 Accuracy of the Dix rms-averaging equation for P-wave normal-moveout velocity in layered HTI media with depth-varying symmetry-axis direction. The solid curve is the moveout velocity as a function of azimuth determined by fitting a hyperbola to the exact \( t - x \) curves [equation (3.1)] on two different spreadlengths: (a) \( X/D = 1 \), and (b) \( X/D = 2 \). The dashed curve is the normal-moveout (zero-spread) velocity from the Dix equation (2.16). Again, the velocities are calculated for the reflection from the bottom of the third layer, this time for Model 5 described in Table 3.2. The symmetry-axis in the second layer, however, is rotated by 60° with respect to symmetry-axis direction in the first and third layer.
3.7 Comparison between the exact traveltimes and the moveout approximations for a three-layer HTI model (Model 5 in Table 3.2) with a uniform symmetry-axis direction in all layers. The effective gradient in the vertical velocity is about 0.24 s$^{-1}$. The gray curves are the exact reflection traveltimes from all three interfaces for azimuths $\alpha$ of 0°, 30°, 45°, 60°, and 90°. The black curves in (a) are the time residuals after the conventional hyperbolic moveout correction using equation (2.3), with the NMO velocity from equation (2.16), while the black curves in (b), (c) and (d) are the residuals after nonhyperbolic moveout correction using equation (2.5) with the effective NMO velocity from equation (2.16), the effective quartic coefficient from equation (2.17), and three different ways to compute the effective horizontal velocity: equation (2.19) in (b), equation (2.20) in (c), and equation (2.18) in (d). The nonhyperbolic moveout equation provides the highest accuracy if the horizontal velocity is calculated using either rms (b) or fourth-power (c) averaging.

3.8 Comparison between the exact traveltimes and the moveout approximations for a three-layer HTI model with a uniform symmetry-axis direction in all layers. The description of the plots is the same as in Figure 3.7, but this time for Model 6 in Table 3.2. The effective gradient in the vertical velocity is about 0.67 s$^{-1}$. The most accurate representation of reflection moveout is given in (c).

3.9 Comparison between the exact traveltimes and the moveout approximations for a three-layer HTI model with a uniform symmetry-axis direction in all layers. The description of the plots is the same as in Figures 3.7 and 3.8, but this time for Model 7 in Table 3.2. The effective gradient in the vertical velocity is about 1.0 s$^{-1}$. The most accurate representation of reflection moveout is given in (d).
3.10 Comparison between the exact traveltimes and the moveout approximations for two three-layer HTI models with depth-varying orientation of the symmetry axis. The parameters of the model in (a) and (b) are the same as for Model 5 in Table 3.2, but the symmetry-axis of the second layer is rotated by 60° with respect to the symmetry-axis in the first and third layer. The parameters of the model in (c) and (d) are the same as for Model 6 in Table 3.2, but the symmetry-axis of the second layer is rotated by 45° with respect to the symmetry-axis in the first layer, while the symmetry-axis of the third layer is rotated 90° with respect to the symmetry-axis in the first layer. The gray curves are the exact reflection traveltimes from all three interfaces for azimuths $\alpha$ of 0°, 30°, 45°, 60°, and 90°. The black curves in (a) and (c) are the time residuals after the conventional hyperbolic moveout correction using equation (2.3), with the NMO velocity from equation (2.16), while the black curves in (b) and (d) are the residuals after nonhyperbolic moveout correction using equation (2.5) with the effective coefficients from equations (2.16), (2.17), and (2.20).

3.11 Comparison between the exact traveltimes and the moveout approximations for a three-layer model consisting of two HTI layers with a uniform symmetry-axis direction and a VTI layer in between. The first and third layers are identical to the first and third layers of Model 6 in Table 3.2, while the second layer has the VTI symmetry (Dog Creek shale) with $\epsilon = 0.225$, $\delta = 0.1$, $V_{P0} = 2.25$ km/s, and $V_{S0} = 1.4$ km/s, where $\epsilon$, $\delta$, $V_{S0}$, and $V_{P0}$ are Thomsen's (1986) parameters. The gray curves are the exact reflection traveltimes from all three interfaces for azimuths $\alpha$ of 0°, 30°, 45°, 60°, and 90°. The black curves in (a) are the time residuals after conventional hyperbolic moveout correction using equation (2.3), with the NMO velocity from equation (2.16), while the black curves in (b) are the residuals after nonhyperbolic moveout correction using equation (2.5) with the effective coefficients from equations (2.16), (2.17), and (2.20). The NMO velocity and the quartic moveout coefficient in the VTI layer are given by the known expressions discussed by Thomsen (1986) and Tsvankin and Thomsen (1994).
3.12 Comparison between the exact traveltimes and the moveout approximations for a model that includes a stack of five homogeneous isotropic layers on top of an HTI layer. The gray curves are the exact reflection traveltimes from all six interfaces for azimuths $\alpha$ of 0°, 30°, 45°, 60°, and 90°. The black curves in (a) are the time residuals after the conventional hyperbolic moveout correction using equation (2.3) with the NMO velocity from equation (2.16), while the black curves in (b) are the residuals after the nonhyperbolic moveout correction using equation (2.5) and the effective $V_{nmo}$ and $A_4$ from equations (2.16) and (2.17). The effective horizontal velocity for the isotropic layers is taken to be equal to the maximum horizontal velocity above the reflector; for the reflection from the bottom of the HTI layer, the horizontal velocity is calculated from equation (2.20). For layer 1 $V_p = 2.0$ km/s, and the depth $d_1 = 0.2$ km; for layer 2 $V_p = 2.5$ km/s, and $d_2 = 0.4$ km; for layer 3 $V_p = 3.0$ km/s, and $d_3 = 0.6$ km; for layer 4 $V_p = 3.5$ km/s, and $d_4 = 0.8$ km; for layer 5 $V_p = 2.5$ km/s, and $d_5 = 1.0$ km; for layer 6 $\epsilon^{(V)} = -0.143$ ($\epsilon = 0.2$), $\delta^{(V)} = -0.318$ ($\delta = -0.2$), $V_{p\text{vert}} = 2.958$ km/s ($V_{p0} = 2.5$ km/s), and $d_6 = 1.5$ km.

4.1 (a) Plan view of 2D survey lines (source-to-receiver azimuths in 3D) over a horizontal HTI layer with $V_{p\text{vert}} = 2.0$ km/s, and $\delta^{(V)} = -0.2$, with the symmetry-axis in the $x$-direction. Two different sets of solutions for the symmetry-axis direction (dashed lines) provide the same NMO-velocity variation, as shown in (b). The correct solution (Sol. 1, horizontal dashed line) has $V_{p\text{vert}} = 2.0$ km/s and $\delta^{(V)} = -0.2$, while Sol. 2 has $V_{p\text{vert}} = 1.549$ km/s and $\delta^{(V)} = 0.333$.

4.2 The reciprocal of the condition number ($\kappa^{-1}$) as a function of $\alpha_2$ and $\delta^{(V)}$; $\alpha_1 = 0°$ and $\alpha_3 = 90°$.

4.3 The reciprocal of the condition number ($\kappa^{-1}$) as a function of $\alpha_2$ and $\delta^{(V)}$; $\alpha_1 = 0°$ and $\alpha_3 = 120°$.

4.4 The reciprocal of the condition number ($\kappa^{-1}$) as a function of azimuth, $\alpha_2$, for five different angular separations between three survey lines. Each set of azimuths is rotated so that the middle direction, $\alpha_2$, spans the azimuths from 0° to 180° measured from the symmetry-axis direction. The five curves correspond to azimuth separations of 7.5° (a), 15° (b), 30° (c), 45° (d), and 60° (e); $\delta^{(V)} = -0.2$.

4.5 The reciprocal of the condition number $\kappa^{-1}$ as a function of azimuth $\alpha_2$ (same as in Figure 4.4, but for $\delta^{(V)} = -0.1$).
4.6 Magnification factor in the absolute error in $\alpha$ measured in radians ($\alpha$ component in the diagonal of $[JTJ]^{-1/2}$) as a function of azimuth ($\alpha_2$) for three different angular separations between survey direction. The three sets of azimuth combinations are rotated so that the central azimuth $\alpha_2$ spans azimuths from 0° to 180° measured from the symmetry-axis direction. The three curves in (a) correspond to angular separations of 30° (gray), 45° (dashed black), and 60° (solid black); $\delta^{(V)}=-0.2$. The three curves in (b) correspond to the same test as in (a), but for $\delta^{(V)}=-0.1$. .......................................................... 54

4.7 The same as Figure 4.6, but for the absolute error in $\delta^{(V)}$ ($\delta^{(V)}$ component in the diagonal of $[JTJ]^{-1/2}$). The vertical axis is dimensionless. 55

4.8 The same as Figures 4.6 and 4.7, but for the relative error in $V_{P_{\text{vert}}}$ ($V_{P_{\text{vert}}}$ component in the diagonal of $[JTJ]^{-1/2}$). The vertical axis is dimensionless. .......................................................... 55

4.9 Schematic time section showing a model that contains an azimuthally isotropic overburden over an HTI layer. $\Delta t_N$ is the two-way vertical traveltime in the HTI layer. The total two-way traveltime to the bottom of the HTI layer is $T_N$, while the NMO (stacking) velocity at the top and bottom of the HTI layer is denoted as $V_{N-1}$ and $V_N$, respectively. The ratio $\Delta t_N/T_N = \rho$. .......................................................... 61

4.10 Plot of $\kappa^{-1}$ as a function of $\rho$ for $V_{N-1} = 2.0$ km/s, $V_{P_{\text{vert}}} = 3.0$ km/s. (a) correspond to $\delta^{(V)} = -0.2$, while (b) correspond to $\delta^{(V)} = -0.1$. The azimuths are $\alpha_1 = 0^\circ$, $\alpha_2 = 60^\circ$, and $\alpha_3 = 120^\circ$. .......................... 63

4.11 Magnification factors in the absolute error in $\alpha$ (a) measured in radians, the absolute error in $\delta^{(V)}$ (b), and the relative error in $V_{P_{\text{vert}}}$ (c) (diagonal elements of $[JTJ]^{-1/2}$) as a function of the layer-thickness ratio, $\rho$. The selected survey-line azimuths are: $\alpha_1 = 0^\circ$, $\alpha_2 = 60^\circ$, and $\alpha_3 = 120^\circ$. The model parameters are $V_{N-1} = 2.0$ km/s, $V_{P_{\text{vert}}} = 3.0$ km/s, and $\delta^{(V)} = -0.2$. .......................................................... 64
4.12 Estimated $V_{p\text{vert}}, \delta^V$, and the azimuth of the symmetry-axis ($\alpha_1$), as well as the associated error bars as functions of $\rho$. The plots in the left column correspond to $\delta^V = -0.2$, while those in the right correspond to $\delta^V = -0.1$. $V_{N-1} = 2.0 \text{ km/s}$, $V_{p\text{vert}} = 3.0 \text{ km/s}$. The azimuths are $\alpha_1 = 0^\circ$, $\alpha_2 = 60^\circ$, and $\alpha_3 = 120^\circ$. The black dots and error bars represent the computed mean and standard deviation, respectively. The solutions for $V_{p\text{vert}}$ are normalized by the true vertical velocity (3.0 km/s).

4.13 Survey lines over a three-HTI-layer model with a uniform symmetry-axis direction in all layers.

4.14 Estimation of interval NMO velocities from synthetic data. The plots on the left show the azimuthal dependence of the effective NMO (stacking) velocity obtained from the conventional-spread reflection moveout for each reflector on the spreadlength equal to the target depth (1.5 km). The plots on the right show the interval NMO velocity (solid curves) as a function of azimuth computed for each layer using Dix differentiation, and the exact interval NMO velocity (dashed curves) from equation (2.10). The model geometry and parameters are shown in Figure 4.13. The maximum interval NMO velocity corresponds to the isotropy-plane direction (crack orientation), while the minimum corresponds to the symmetry-axis direction.

4.15 Estimation of interval NMO velocities from synthetic data. (a) shows the azimuthal dependence of the effective NMO (stacking) velocity obtained from the conventional-spread reflection moveout from the bottom of the HTI layer on the spreadlength equal to the target depth (1.5 km). (b) shows the interval NMO velocity (solid curves) as a function of azimuth computed for the HTI layer using Dix differentiation, and the exact interval NMO velocity (dashed curves) from equation (2.10). The model consists of five isotropic layers overlaying an HTI layer. The isotropic layers have an equal thickness of 0.2 km with $P$-wave velocities of 2.0, 2.5, 3.0, 3.5, and 2.5 km/s, respectively. The parameters of the HTI layer are the same as those of the third layer in Figure 4.13. The effective NMO velocity at the top of the HTI layer, estimated from reflection moveout, is 2.67 km/s.
4.16 Estimation of interval NMO velocities from synthetic data for a model with depth-varying symmetry-axis direction. The plots on the left show the azimuthal dependence of the effective NMO (stacking) velocity obtained from the conventional-spread reflection moveout for each reflector on the spreadlength equal to the target depth (1.5 km). The plots on the right show the interval NMO velocity (solid curves) as a function of azimuth computed for each layer using Dix differentiation, and the exact interval NMO velocity (dashed curves) from equation (2.10). The model geometry and parameters are the same as in Figure 4.13; however, the symmetry-axis of the second layer is rotated by 60° with respect to the axis direction in the first layer.

4.17 Estimation of interval NMO velocities from synthetic data for a model with depth-varying symmetry-axis direction. (a) shows the azimuthal dependence of the effective NMO (stacking) velocity obtained from the conventional-spread reflection moveout for the second reflector on a spreadlength equal to the target depth (1.5 km). (b) shows the interval NMO velocity (solid curves) as a function of azimuth computed for the second layer using Dix differentiation, and the exact interval NMO velocity (dashed curves) from equation (2.10). The model geometry and parameters are the same as in Figure 4.13; however, the symmetry-axis of the second layer is rotated by 30° with respect to the axis direction in the first layer.

B.1 For a homogeneous HTI layer, the specular reflection point for any offset coincides with the zero-offset reflection point, and there is no reflection-point dispersal on CMP gathers. \( y \) denotes the CMP location and \( h \) is half the source-receiver offset.

B.2 The group- and phase-velocity vectors for the reflected waves in a homogeneous HTI layer. The incident (SO) and reflected (OR) group-velocity vectors (rays) lie in the vertical incidence plane and are symmetric with respect to the horizontal plane. The phase-velocity vector (direction OD) of the reflected ray OR is confined to the plane formed by OR and the axis of symmetry. Triangle RCB defines a plane normal to the symmetry axis (after Tsvankin, 1996b).
C.1 Reflection from a dipping interface overlain by a sequence of horizontal, homogeneous, azimuthally anisotropic layers. We assume that the rays (group-velocity vectors) from the zero-offset reflection point are confined to the incidence plane but do not put any restrictions on the orientation of the corresponding slowness (phase-velocity) vectors. This implies that the normal to the reflector may deviate from the incidence plane.
ACKNOWLEDGMENTS

I am very grateful to my thesis advisor Dr. Ilya Tsvankin for his guidance, encouragement and enthusiasm for this work. I am also grateful to Dr. Ken Larner for his support and for many positive critical comments. Many thanks to Dr. John Scales for his help and for interesting discussions.

I would like to thank Dr. Jack Cohen, Dr. Tom Davis, and Dr. Tom Boyd for their kindness and support. I am very grateful to Dr. Dirk Gajewski from University of Hamburg for providing his 3D ray-tracing code. Many thanks to my colleagues at the Center for Wave Phenomena, especially Tariq Alkhalifah and Andreas Rüger for their friendship and support.

I would like to thank the Center for Wave Phenomena and the Department of Geophysics for the technical support. Special thanks to the Saudi Arabian Oil Company (Saudi Aramco) for the financial support. I am grateful to Mahmoud Abdul-Baqi and John Ward of Saudi Aramco for making the scholarship at Colorado School of Mines (CSM) possible.

Finally, I would like to thank my wife Dr. Raidah Al-Baradie for her continuous encouragement and support throughout the years. I would like to thank my children: Omar, Ahmed, Saleem, and Mohammed for making my life so rewarding. Special thanks to my brothers and sisters: Baker, Jamal, Aladdin, Amal, Wafa, Maha, and Mona, for their support. I would like to thank my parents in law: Saleem Al-Baradie and Waleedah Qaddourah, for their blessing and support. Finally, I would like to dedicate this work to the people who taught me the great values and the necessary lessons to be successful, my late parents: Ahmed Al-Dajani and Maryam Dawood.
Recent experimental studies (Lynn et al., 1995; Mallick et al., 1996) have shown that P-wave reflection moveout and amplitude-variation-with-offset (AVO) response may be strongly influenced by the presence of azimuthal anisotropy. However, the current understanding of seismic signatures in azimuthally anisotropic media is hardly sufficient for the inversion and processing of seismic data, even if the medium is horizontally homogeneous. This work is devoted to an analytic and numerical study of reflection moveout in transversely isotropic models with horizontal axis of symmetry (HTI media) – the simplest type of azimuthally anisotropic media, such as that associated with a system of parallel vertical cracks embedded in an isotropic matrix (Crampin, 1985; Thomsen, 1988).

Weak-anisotropy approximations for reflection moveout in HTI media were discussed by Thomsen (1988) and Li and Crampin (1993); the latter paper also treats reflection traveltimes in an orthorhombic layer. Using an approximate “skewed” hyperbolic moveout formula of Byun et al. (1989), Sena (1991) derived analytical expressions for long-spread travelt ime-offset curves in multilayered, weakly anisotropic VTI and HTI media.

Recently, Tsvankin (1996b) presented an exact equation for azimuthally-dependent normal-moveout velocity valid for pure modes in a single HTI layer. He also showed that all kinematic signatures including normal moveout (as well as plane-wave polarizations) in the symmetry plane of HTI media that contains the symmetry axis (the
"symmetry-axis" plane) are given by the same equations as for transversely isotropic media with a *vertical* symmetry axis (VTI). The analogy between HTI and VTI media allowed Tsvankin (1996b) and Rüger (1996) to introduce Thomsen's (1986) parameters for HTI media using exactly the same expressions as for vertical transverse isotropy. Thus, an HTI medium can be characterized in terms of the Thomsen parameters $\delta^{(V)}$ and $\epsilon^{(V)}$ for an *equivalent* VTI medium. This notation proved to be much more convenient in describing reflection signatures than are the generic Thomsen coefficients defined with respect to the symmetry axis. For instance, the $P$-wave NMO velocity for reflection from a horizontal interface in HTI media depends on the vertical velocity, orientation of the symmetry axis, and a single anisotropic coefficient – the parameter $\delta^{(V)}$ expressed through the stiffnesses in the same way as is Thomsen's coefficient $\delta$ for VTI media (Tsvankin, 1996b).

Despite all these developments, some important issues pertaining to moveout analysis for horizontal transverse isotropy remained unresolved. Among them is the behavior of long-spread (nonhyperbolic) moveout in the presence of azimuthal anisotropy and the feasibility of obtaining interval NMO velocities for vertically heterogeneous HTI models with strong velocity anisotropy. Both problems need to be examined *outside* the vertical symmetry planes of HTI media since reflection moveout within the symmetry planes is identical to that in VTI media.

It is well known that reflection moveout in anisotropic media is generally nonhyperbolic, unless the anisotropy is elliptical. Hake et al. (1984) derived the coefficient $A_4$ of the quartic term in the Taylor series approximation of $t^2 - x^2$ reflection-moveout curves for pure modes in TI media with a vertical axis of symmetry. Tsvankin and Thomsen (1994) obtained the coefficient $A_4$ for converted $P - SV$ waves and represented the quartic terms of the pure modes in a more compact form using Thomsen's
(1986) notation. They also developed a nonhyperbolic moveout equation for layered VTI media based on the exact quadratic (NMO velocity) and quartic moveout coefficients that converges at infinitely large horizontal offsets as well as at zero offset. In TI media with a vertical symmetry axis, this equation remains close to the exact P-wave moveout for uncommonly long spreads that may be three times as large as the reflector depth. The moveout expression of Tsvankin and Thomsen (1994) will serve as a basis for my study of nonhyperbolic reflection moveout in HTI media.

Here, I derive a concise expression for the quartic moveout coefficient valid for all pure modes in an HTI layer with any strength of the anisotropy. This azimuthally-dependent quartic moveout term is used to extend the nonhyperbolic equation of Tsvankin and Thomsen (1994) to horizontal transverse isotropy. The quadratic and quartic moveout coefficients in multilayered HTI media are obtained by the same averaging equations as for vertical transverse isotropy. Extensive numerical testing demonstrates excellent accuracy of this nonhyperbolic moveout equation on long common-midpoint (CMP) spreads, even for media with significant depth-varying azimuthal anisotropy and pronounced nonhyperbolic moveout.

The influence of nonhyperbolic moveout can hamper the estimation of normal-moveout velocity using conventional hyperbolic semblance analysis (e.g., Gidlow and Fatti, 1990). Even if the analytic normal-moveout velocity for a stack of layers has been extracted from finite-spread moveout, it is not clear whether or not interval NMO velocities in azimuthally anisotropic media can be obtained from the Dix equation, which is no longer strictly valid outside the symmetry planes.

My numerical analysis on ray-traced synthetic data shows that in a single HTI layer the hyperbolic moveout equation parameterized by the exact NMO velocity (given by Tsvankin, 1996b) provides sufficient accuracy for conventional spreadlengths
(equal to the reflector depth). I demonstrate that the accuracy of the Dix equation in layered HTI media depends on the deviation of the group-velocity vector from the incidence plane, which is generally not large enough to cause significant errors in the rms averaging of interval NMO velocities. Therefore, conventional-spread moveout can be accurately described by the normal-moveout velocity obtained through rms averaging of the exact interval NMO velocities.

The last part of the thesis deals with the inversion of reflection moveout in HTI media. Reflection moveout in anisotropic media can provide valuable information for estimation of HTI parameters. Anisotropic coefficients of HTI media can be related to the crack orientation and crack density, which are of great interest in characterization of fractured reservoirs. Anisotropy parameter estimation has been studied in detail for VTI media. White et al. (1983) measured the anisotropy parameters through phase-velocity analysis of vertical seismic profiling (VSP) data. Byun and Corrigan (1990) presented a numerical technique based on semblance analysis to estimate iteratively the five elastic coefficients in a multilayered VTI medium. Tsvankin and Thomsen (1995) studied the traveltime inversion in VTI media and suggested combining $P$ and $SV$ data to invert for the vertical velocities. Alkhalifah and Tsvankin (1995) showed that all time-related processing in VTI media is governed by only two parameters: the short-spread NMO velocity and the anisotropy parameter $\eta$ [where $\eta = (\epsilon - \delta)/(1 + 2\delta)$]. They also showed how to obtain $\eta$ using the dip dependence of NMO velocity. Later, Alkhalifah (1996) presented a 3-D semblance analysis technique based on Tsvankin and Thomsen’s (1994) moveout equation to estimate the anisotropic parameter $\eta$ using nonhyperbolic reflection moveout.

The inversion for the parameters of azimuthally anisotropic media has been limited mostly to shear-wave splitting analysis, with the goal of estimating the crack
orientation and the crack density. One of the few parameter-estimation algorithms based on moveout analysis of $P$-wave data was presented by Sena (1991); however, Sena’s method is limited to weak anisotropy and requires knowledge of the vertical velocity.

Here, I discuss estimation of anisotropic parameters and detection of fracture orientation from $P$-wave moveout data in homogeneous and horizontally-layered HTI media. Error analysis provides insight into the stability and accuracy of the inversion, as well as the optimal survey design. Numerical applications and synthetic data examples illustrate the accuracy of the inversion procedure.
Chapter 2

REFLECTION MOVEOUT FOR HORIZONTAL TRANSVERSE ISOTROPY

In this chapter I derive a concise expression for the quartic moveout coefficient valid for all pure modes in an HTI layer with any strength of the anisotropy. This azimuthally-dependent quartic moveout term is used to extend the nonhyperbolic equation of Tsvankin and Thomsen (1994) to horizontal transverse isotropy. The quadratic and quartic moveout coefficients in multilayered HTI media are obtained by the same averaging equations as for vertical transverse isotropy. Extensive numerical testing demonstrates excellent accuracy of this nonhyperbolic moveout equation, even for media with significant depth-varying azimuthal anisotropy and pronounced nonhyperbolic moveout.

2.1 Description of the HTI model and notation

Here, I describe the main features of the HTI model and a convenient Thomsen-style notation for horizontal transverse isotropy introduced by Tsvankin (1996b) and Rüger (1996). Proper understanding of wave propagation in the two mutually orthogonal vertical symmetry planes (Figure 2.1) is important in the analysis of seismic signatures in HTI media. In the plane normal to the symmetry axis (so-called isotropy plane, which would be parallel to fracture orientation), body-wave velocities are independent of direction, and the influence of anisotropy manifests itself only through the different velocities of the $S$-waves having the two different polarizations (the split
Fig. 2.1. Sketch of the transversely isotropic model with a horizontal symmetry axis caused by a system of parallel vertical cracks. HTI media contain two vertical planes of mirror symmetry defined by the crack orientation (after Rüger, 1996).

Shear waves in HTI media will be denoted as "$S^{\|}$" and "$S^{\perp}$", with the $S^{\|}$-wave polarized within the isotropy plane, and the $S^{\perp}$-wave polarization vector confined to the plane formed by the symmetry axis and the slowness vector). In the second vertical symmetry plane, which contains the axis of symmetry (the "symmetry-axis plane"), the velocities do change with propagation angle, but the Christoffel equation has exactly the same form as that for transversely isotropic media with a vertical symmetry axis (Rüger, 1996). This means that the phase velocity and polarization vector are the same functions of the stiffness coefficients and phase angle with vertical as in VTI media. Since phase velocity determines group (ray) velocity and group angle, all kinematic signatures in the symmetry-axis plane, including normal-moveout velocity and long-spread (nonhyperbolic) reflection moveout, are given by the known VTI equations.
Taking advantage of this limited equivalence, Tsvankin (1996b) and Rüger (1996) introduced the Thomsen parameters of the “equivalent” VTI model through the same equations as those used by Thomsen (1986) for actual VTI media. For the HTI model with the symmetry axis in the $x_1$-direction, these parameters are defined through the stiffness coefficients $c_{ij}$ and density $\rho$ as

\begin{align*}
V_{P\text{vert}} & \equiv \sqrt{\frac{c_{33}}{\rho}}, \\
V_{S^{\perp}\text{vert}} & \equiv \sqrt{\frac{c_{55}}{\rho}}, \\
\epsilon^{(V)} & \equiv \frac{c_{11} - c_{33}}{2c_{33}}, \\
\delta^{(V)} & \equiv \frac{(c_{13} + c_{55})^2 - (c_{33} - c_{55})^2}{2c_{33}(c_{33} - c_{55})}, \\
\gamma^{(V)} & \equiv \frac{c_{66} - c_{44}}{2c_{44}},
\end{align*}

(2.1)

where $V_{P\text{vert}}$ and $V_{S^{\perp}\text{vert}}$ are the vertical velocities of the $P$- and $S^{\perp}$-wave, respectively (note that in the HTI model $c_{55} = c_{66}$). The vertical velocity of the (fast) shear wave $S^{\parallel}$ is determined as

\[ V_{S^{\parallel}\text{vert}} \equiv \sqrt{\frac{c_{44}}{\rho}} = \frac{V_{S^{\perp}\text{vert}}}{\sqrt{1 + 2\gamma^{(V)}}}. \]

This notation makes it possible to obtain the kinematic signatures and polarizations in the symmetry-axis plane of HTI media just by adapting the corresponding equations for vertical transverse isotropy expressed through Thomsen parameters. The convenient features of Thomsen notation in the analytic description of seismic wavefields in VTI media were summarized by Tsvankin (1996a). Furthermore, as
discussed in more detail below, the Thomsen coefficients of the equivalent VTI model control the moveout _outside_ the symmetry planes of HTI media, where the analogy with VTI media is no longer valid.

The exact expressions for the phase velocity in HTI media in terms of the parameters $\epsilon^{(V)}$, $\delta^{(V)}$, and $\gamma^{(V)}$ were presented in Tsvankin (1996b). For $P$- and $S^\perp$-waves, the phase velocity is given by

$$
\frac{V^2(\theta)}{V_{P\text{vert}}^2} = 1 + \epsilon^{(V)} \cos^2 \theta - \frac{f^{(V)}}{2} \pm \frac{f^{(V)}}{2} \sqrt{\left(1 + \frac{2\epsilon^{(V)} \cos^2 \theta}{f^{(V)}}\right)^2 - \frac{2(\epsilon^{(V)} - \delta^{(V)}) \sin^2 2\theta}{f^{(V)}}},
$$

(2.2)

where the plus sign corresponds to the $P$-wave, and the minus to the $S^\perp$-wave, $f^{(V)} \equiv 1 - (V_{S^\perp\text{vert}}/V_{P\text{vert}})^2$, and $\theta$ is the phase angle with the horizontal symmetry axis. For $P$-waves, phase velocity and all other kinematic signatures in HTI media depend largely on the vertical velocity $V_{P\text{vert}}$ and the coefficients $\epsilon^{(V)}$ and $\delta^{(V)}$, while the influence of the $S^\perp$-wave vertical velocity (and, therefore, $f^{(V)}$) is practically negligible (Tsvankin, 1996b).

Alternatively, HTI media could be characterized by the "generic" Thomsen parameters defined with respect to the symmetry axis. However, since the symmetry axis is horizontal, these parameters (especially the coefficient $\delta$) are not well-suited to describing reflection seismic signatures, which are largely dependent on _near-vertical_ velocity variations. For reference, the relationships between the two sets of Thomsen parameters, described by Tsvankin (1996b) and Rüger (1996), are reproduced in Appendix A.

Note that the numerical values of the parameters $\epsilon^{(V)}$, $\delta^{(V)}$, and $\gamma^{(V)}$ are quite different from those typical for actual VTI media. While the coefficients $\epsilon$ and $\gamma$ for vertical transverse isotropy are non-negative, in HTI media $\epsilon^{(V)} \leq 0$ and $\gamma^{(V)} \leq 0$. 
Also, for typical ratios of the vertical velocities \( V_{S_{\perp}} / V_{P_{\perp}} \leq 0.707 \), the parameter \( \delta^{(V)} \leq 0 \) (see Appendix D), which is possible but not typical for such VTI formations as shales. Similar to actual VTI media, however, the difference \( \epsilon^{(V)} - \delta^{(V)} \) that characterizes the "anellipticity" of a medium is usually positive for horizontal transverse isotropy.

2.2 Analytic approximations of reflection moveout

In the practice of seismic data processing, reflection moveout in CMP gathers is conventionally approximated by the hyperbolic equation

\[
t^2 = t_0^2 + \frac{x^2}{V_{nmo}^2},
\]

where \( t \) is the reflection traveltime at source-receiver offset \( x \), \( t_0 \) is the two-way zero-offset traveltime, and \( V_{nmo} \) is the normal-moveout velocity defined in the zero-spread limit.

Equation (2.3) is strictly valid only for a homogeneous isotropic (or elliptically anisotropic) layer. The presence of layering and/or anisotropy leads to deviation of the traveltime curve from the hyperbola (2.3) with increasing offset, and the finite-spread moveout (stacking) velocity, usually obtained from hyperbolic semblance analysis, no longer coincides with the analytic NMO velocity. However, for vertical transverse isotropy the hyperbolic moveout equation for \( P \)-waves usually provides sufficient accuracy on conventional-length spreads (i.e., spreadlength close to the reflector depth – see Tsvankin and Thomsen, 1994).

Nonhyperbolic moveout on longer spreads can be described by a three-term Tay-
lor series expansion (Taner and Koehler, 1969),

\[ t^2 = t_0^2 + A_2 x^2 + A_4 x^4, \]  

(2.4)

where \( A_2 = 1/V_{nmo}^2 \), and \( A_4 \) is the quartic moveout coefficient. The expression for the parameter \( A_4 \) for pure modes in horizontally layered VTI media was given by Hake et al. (1984) and represented in a more compact form by Tsvankin and Thomsen (1994).

Due to the influence of the \( x^4 \) term, however, the quartic equation (2.4) becomes divergent with increasing offset and can be replaced by a more accurate nonhyperbolic moveout equation developed by Tsvankin and Thomsen (1994),

\[ t^2 = t_0^2 + A_2 x^2 + \frac{A_4 x^4}{1 + A x^2}, \]  

(2.5)

where \( A = A_4/(1/V_{hor}^2 - 1/V_{nmo}^2) \); \( V_{hor} \) is the horizontal velocity. The denominator of the nonhyperbolic term ensures the convergence of this approximation at infinitely large horizontal offsets. As a result, equation (2.5) provides an accurate description of \( P \)-wave traveltimes on long CMP spreads (2-3 times as large as the reflector depth), even for models with pronounced nonhyperbolic moveout.

Although equation (2.5) was originally designed for vertical transverse isotropy, it could be used in arbitrary anisotropic media if the appropriate coefficients \( A_2, A_4, \) and \( A \) were found. My goal is to extend this nonhyperbolic moveout approximation to single- and multi-layered HTI media. As discussed in the previous section, for a CMP line parallel to the symmetry axis no generalization is necessary since the moveout in the symmetry-axis plane can be obtained directly from the original VTI equation (2.5) by substituting the Thomsen coefficients of the equivalent VTI model.
Clearly, in the isotropy plane long-spread reflection moveout of any given mode is not influenced by the anisotropy at all. The analogy with VTI media also holds for multilayered models containing VTI and HTI layers, but all having a common symmetry planes.

For CMP lines outside the vertical symmetry planes of HTI media, however, it is necessary to obtain the azimuthally-dependent parameters of equation (2.5). Below, we accomplish this task for horizontally layered HTI media with arbitrary strength of anisotropy.

2.3 Moveout in a single HTI layer

2.3.1 Normal-moveout and horizontal velocity

An analytic expression for the quadratic moveout coefficient \( A_2 \) (or the NMO velocity) in a single HTI layer was developed by Tsvankin (1996b). The exact normal-moveout velocity of any pure mode on a CMP line in the azimuth direction \( \alpha \) with respect to the symmetry-axis direction (Figure 2.2) is given by

\[
V_{\text{nmo}}^2 = V_{\text{vert}}^2 \frac{1 + \frac{1}{V} \left. \frac{d^2 V}{d \theta^2} \right|_{\theta=90^\circ}}{1 + \sin^2 \alpha \left. \frac{1}{V} \frac{d^2 V}{d \theta^2} \right|_{\theta=90^\circ}},
\]

(2.6)

where \( V_{\text{vert}} \) is the vertical velocity, and \( V \) is the phase velocity as a function of the phase angle \( \theta \) with the symmetry axis; the phase velocity and its second derivative are evaluated in the vertical phase (and group) direction. For a fixed orientation of the symmetry axis, the influence of anisotropy in equation (2.6) is concentrated in a single velocity term \( \left. \frac{1}{V} \frac{d^2 V}{d \theta^2} \right|_{\theta=90^\circ} \), which represents the anisotropic correction to the
Fig. 2.2. Schematic 3-D picture showing a reflection raypath for a source and receiver along a seismic line that makes the azimuth angle $\alpha$ with respect to the symmetry-axis plane of an HTI layer. Since the model has a horizontal symmetry plane, the incident and reflected rays of pure modes lie in the vertical incidence (sagittal) plane. Here, the CMP line is taken as a 2D seismic line. Later, we shall see implications for 3D survey data.

NMO velocity in the symmetry-axis plane:

$$\left. \frac{1}{V} \frac{d^2 V}{d \theta^2} \right|_{\theta=90^\circ} = \frac{V_{\text{nmo}}^2 (\alpha = 0)}{V_{\text{vert}}^2} - 1.$$  

This term can be expressed through the anisotropic parameters in the following way (Tsvankin, 1996b):

$$\left. \frac{1}{V} \frac{d^2 V}{d \theta^2} \right|_{\theta=90^\circ} \quad [P-\text{wave}] = 2\delta^{(V)} , \quad (2.7)$$

$$\left. \frac{1}{V} \frac{d^2 V}{d \theta^2} \right|_{\theta=90^\circ} \quad [S^\perp-\text{wave}] = 2\sigma^{(V)} , \quad (2.8)$$

$$\left. \frac{1}{V} \frac{d^2 V}{d \theta^2} \right|_{\theta=90^\circ} \quad [S^\parallel-\text{wave}] = 2\gamma^{(V)} , \quad (2.9)$$
where

\[ \sigma^{(V)} = \left( \frac{V_{p,\text{vert}}}{V_{s,\text{vert}}} \right)^2 (\epsilon^{(V)} - \delta^{(V)}) , \]

and the anisotropic coefficients \( \delta^{(V)} \), \( \epsilon^{(V)} \) and \( \gamma^{(V)} \) were introduced in equation (2.1). Therefore, the azimuthal dependence of NMO velocity for horizontal transverse isotropy is governed by the Thomsen parameters of the equivalent VTI medium in a rather simple fashion.

Normal-moveout velocity of the \( P \)-wave thus depends on the parameter \( \delta^{(V)} \) as follows:

\[ V_{nmo}^2(\alpha) = V_{p,\text{vert}}^2 \frac{1 + 2\delta^{(V)}}{1 + 2\delta^{(V)} \sin^2 \alpha} . \] (2.10)

This remarkably simple expression for \( V_{nmo}(\alpha) \) shows, as I will discuss in detail in Chapter 4, that normal-moveout velocity has elliptical variation with azimuths.

To obtain the quantity \( A \) in the nonhyperbolic moveout equation (2.5), we also have to find the azimuthally-dependent horizontal group velocity \( (V_{\text{hor}}) \), which controls reflection moveout at offsets approaching infinity. Since the influence of small errors in \( V_{\text{hor}} \) is not significant for spreadlengths typically used in reflection surveys, we will ignore the difference between phase and group velocity and calculate \( V_{\text{hor}} \) as the phase velocity evaluated in the azimuthal direction of the CMP line. Therefore, for the \( P \)- and \( S_{\perp} \)-waves we find the horizontal velocity by substituting \( \theta = \alpha \) into equation (2.2). For \( S_{\parallel} \)-waves, as we will demonstrate next, the nonhyperbolic moveout term vanishes, and computing the horizontal velocity is not necessary.

### 2.3.2 Quartic moveout coefficient

Application of the nonhyperbolic moveout equation (2.5) also requires knowledge of the quartic moveout coefficient \( A_4 \). An exact expression for \( A_4 \) in a single HTI
layer, derived in Appendix B, is

\[
A_4(\alpha) = \cos^4 \alpha \left[ -\frac{4}{V} \frac{d^2V}{d\theta^2} + 3 \left( \frac{1}{V} \frac{d^2V}{d\theta^2} \right)^2 + \frac{1}{V} \frac{d^4V}{d\theta^4} \right]_{\theta=90^\circ} = \cos^4 \alpha \cdot A_4(\alpha = 0). \quad (2.11)
\]

As before, \( \alpha \) is the azimuth angle between the CMP line (i.e., the direction between source and receiver) and the symmetry axis. Equation (2.11) is valid for HTI models with arbitrary strength of anisotropy and can be used for any pure-mode reflection \( (P, S^\perp, S^\parallel) \). The most interesting feature of equation (2.11) is the simplicity of the azimuthal dependence of the quartic moveout coefficient. The maximum absolute value of \( A_4 \) corresponds to the symmetry-axis plane \( (\alpha = 0^\circ) \), while in the isotropy plane \( A_4(\alpha = 90^\circ) = 0 \), so moveout is purely hyperbolic. The form of the angular dependence \( (\cos^4 \alpha) \) implies that the quartic coefficient rapidly decreases with azimuth away from the symmetry-axis plane.

The term \( A_4(\alpha = 0) \) represents the quartic coefficient in the symmetry-axis plane, where all moveout parameters are given by the same equations as for vertical transverse isotropy. Although an equivalent expression was obtained by Hake et al. (1984), here \( A_4(\alpha = 0) \) is represented for the first time as a simple function of phase velocity and its derivatives. In addition to the axis orientation, the influence of anisotropy on the quartic moveout coefficient is absorbed by just two velocity terms:

\[
\frac{1}{V} \frac{d^2V}{d\theta^2} \bigg|_{\theta=90^\circ} \quad \text{and} \quad \frac{1}{V} \frac{d^4V}{d\theta^4} \bigg|_{\theta=90^\circ}.
\]

To find the quartic coefficient for the \( P \)-wave, we evaluate the fourth derivative of phase velocity using equation (2.2):

\[
\frac{1}{V} \frac{d^4V}{d\theta^4} \bigg|_{\theta=90^\circ} = 24(\epsilon^{(V)} - \delta^{(V)})(1 + \frac{2\delta^{(V)}}{f^{(V)}}) - 2\delta^{(V)}(4 + 6\delta^{(V)}). \quad (2.12)
\]
Substituting equations (2.7) and (2.12) into equation (2.11), we obtain an explicit expression for the \(P\)-wave quartic moveout coefficient:

\[
A_4^{(P)}(\alpha) = \cos^4 \alpha \left[ \frac{-2(\epsilon^{(V)} - \delta^{(V)})(1 + 2\delta^{(V)}/f^{(V)})}{t_0^2 V_{\text{vert}}^4 (1 + 2\delta^{(V)})^4} \right].
\] (2.13)

As expected from the analogy between the symmetry-axis plane of HTI media and vertical transverse isotropy, the expression in brackets \([A_4(\alpha = 0^\circ)]\) in equation (2.13) is identical to the \(P\)-wave quartic coefficient for VTI media given in Tsvankin and Thomsen (1994). Alkhalifah and Tsvankin (1995) showed that the contribution of the shear-wave vertical velocity to the \(P\)-wave quartic coefficient in VTI media can be ignored, so \(A_4\) can be represented as a function of just two effective parameters: normal-moveout velocity \((V_{\text{nmo}})\) and the anisotropic parameter \(\eta = (\epsilon - \delta)/(1 + 2\delta)\). Clearly, this conclusion remains valid for the \(P\)-wave quartic moveout coefficient in HTI media, with \(V_{\text{nmo}}\) calculated in the symmetry-axis plane and \(\eta = \eta^{(V)} = (\epsilon^{(V)} - \delta^{(V)})/(1 + 2\delta^{(V)})\).

Similarly, for the \(S^\perp\)-wave, using equation (2.2) yields

\[
\frac{1}{V} \frac{d^4V}{d\theta^4} \bigg|_{\theta=90^\circ} = -24\sigma^{(V)}(1 + \frac{2\delta^{(V)}}{f^{(V)}}) - 2\sigma^{(V)}(4 + 6\sigma^{(V)}),
\] (2.14)

and equation (2.11) takes the form

\[
A_4^{(S^\perp)}(\alpha) = \cos^4 \alpha \left[ \frac{2\sigma^{(V)}(1 + 2\delta^{(V)}/f^{(V)})}{t_0^2 V_{S^\perp,\text{vert}}^4 (1 + 2\sigma^{(V)})^4} \right].
\] (2.15)

Again, the expression in brackets in equation (2.15) is identical to the \(S^\perp\)-wave quartic coefficient in VTI media given in Tsvankin and Thomsen (1994).

Similar to the quadratic moveout coefficient (NMO velocity), the quartic moveout coefficient for horizontal transverse isotropy is much simpler to describe by the
Thomsen parameters of the equivalent VTI medium than by the generic coefficients.

For the shear wave $S^{\parallel}$, the anisotropy is elliptical, and the quartic coefficient in the symmetry-axis plane is equal to zero\(^1\). According to equation (2.11), this means that the quartic moveout term for the $S^{\parallel}$-wave vanishes in all other azimuthal directions as well, and $S^{\parallel}$-wave moveout in a single HTI layer is purely hyperbolic.

Thus, these last two sections provide the expressions for the NMO velocity, the quartic moveout coefficient, and the horizontal velocity needed to construct the non-hyperbolic moveout equation (2.5) for a single HTI layer.

### 2.4 Moveout in multilayered media

In multilayered anisotropic media, both the quadratic and quartic moveout coefficients reflect the combined influence of layering as well as anisotropy. On conventional-length spreads, hyperbolic moveout equation (2.3) can be expected to provide an adequate description of moveout, but the NMO velocity should be averaged over the stack of layers. For vertical transverse isotropy, this averaging is performed by means of the conventional isotropic Dix (1955) equation (Hake et al., 1984). Furthermore, Alkhalifah and Tsvankin (1995) showed that the Dix equation remains valid in symmetry planes of any horizontally layered anisotropic media (even if the reflector beneath the horizontal layering is dipping), provided the interval NMO velocities are evaluated at the ray-parameter value for the zero-offset ray. In Appendix C this generalized Dix equation is extended to arbitrary directions of the CMP line in azimuthally anisotropic media under the assumption that the group-velocity vector does not deviate from the vertical incidence plane (the orientation of the phase-velocity vector in this case has no influence on the results). For horizontal

\(^1\)This can be checked by substituting the phase-velocity equation (A.4) into equation (2.11).
interfaces, the ray parameter of the zero-offset ray is always zero, and the generalized
NMO equation (C.4) reduces to the conventional rms formula of Dix,

\[ V_{nmo}^2 = \frac{1}{t_0} \sum_{i=1}^{N} V_{nmo}^2 \Delta t_i, \]  

(2.16)

where \( t_0 \) is the two-way zero-offset time to reflector \( N \), \( V_{nmo} \) is the NMO velocity for
layer \( i \), and \( \Delta t_i \) is the two-way zero-offset time in layer \( i \). The interval NMO velocity
\( V_{nmo} \) for any wave type in HTI media is given by equation (2.6).

Thus, rms averaging of the interval velocities remains valid in azimuthally anisotropic
media as long as the *group-velocity* vector (ray) is confined to the incidence plane for
the whole raypath. This assumption is strictly satisfied only in a single HTI layer (as
well as in any other homogeneous layer with a horizontal symmetry plane), and in the
two symmetry planes of multilayered HTI media with common symmetry-axis direc­
tion in all layers. For off-symmetry azimuthal directions in multilayered HTI media,
both group- and phase-velocity vectors deviate from the incidence plane in order to
satisfy Snell’s law at each interface. However, incident and reflected rays usually lie
closer to the incidence plane than do the corresponding phase vectors because each
ray has to return to the CMP line at the receiver location. Deviation of rays from
the incidence plane is especially small in stratified models with a similar character of
azimuthal velocity variations in all layers (e.g., in HTI media with a uniform crack
orientation). In principle, the Dix equation in layered azimuthally anisotropic media
can be replaced with an exact (but in general more involved) expression to account
for arbitrary ray trajectories in azimuthally anisotropic media given in Grechka and
Tsvankin (1996a). Here, I restrict myself to implementing equation (2.16) and study­
ing its accuracy in multilayered HTI models.

To use the nonhyperbolic moveout equation (2.5) in multilayered media, we also
need to account for the influence of layering on the quartic moveout term. The exact coefficient $A_4$ for pure modes in VTI media was presented by Hake et al. (1984):

$$A_4 = \frac{\sum_{i=1}^{N} V_{nmo}^2 \Delta t_i}{4(\sum_{i=1}^{N} V_{nmo}^2 \Delta t_i)^4} - t_0 \sum_{i=1}^{N} A_{4i} V_{nmo}^8 \Delta t_i^3, \quad (2.17)$$

where $A_{4i}$ is the quartic moveout coefficient for layer $i$.

The first term in the right-hand side of equation (2.17) has the same form as for isotropic media (Al-Chalabi, 1974), but it contains the interval NMO velocities, which differ from the true vertical velocities in the presence of anisotropy. The second term goes to zero in isotropic or elliptically anisotropic media and, therefore, represents a purely anisotropic contribution to the quartic moveout coefficient. Tsvankin and Thomsen (1994) showed that the nonhyperbolic moveout equation (2.5) with the quartic term given by equation (2.17) accurately describes $P$-wave reflection moveout in multilayered VTI media.

Outside the symmetry planes of stratified HTI media, both phase- and group-velocity vectors deviate from the incidence plane, violating the main assumptions behind the VTI averaging equation (2.17). Nonetheless, I apply equation (2.17) to horizontal transverse isotropy, but with the exact expressions for the interval values $V_{nmo}$ [equation (2.6)] and $A_{4i}$ [equation (2.11)], which honor the azimuthal dependence of the moveout coefficients. Therefore, in the numerical examples below both the quadratic and quartic moveout coefficients in layered HTI media are calculated using the same averaging equations as for VTI media, but with the exact interval values derived for horizontal transverse isotropy.

For layered media, it is not clear just how we should define the effective horizontal velocity ($V_{hor}$) contained in the quantity $A$ of the nonhyperbolic moveout
equation (2.5). One might think of several different ways to compute this quantity (e.g., Tsvankin and Thomsen, 1994; Alkhalifah, 1996). In theory, $V_{\text{hor}}$ should be equal to the maximum horizontal velocity of the medium above the reflector (Tsvankin and Thomsen, 1994):

$$V_{\text{hor}} = \max [V_{\text{hor}}],$$

where $V_{\text{hor}}$ is the interval horizontal velocity in layer $i$. As discussed above, the interval horizontal velocity $V_{\text{hor}}$ in HTI media can be well approximated by the phase velocity [equation (2.2)] evaluated at the azimuth of the CMP line.

While this choice of horizontal velocity makes equation (2.5) converge at infinite source-receiver offsets $x$ (Tsvankin and Thomsen, 1994), it may generate somewhat inaccurate results at the intermediate offsets typical of reflection surveys, especially in the presence of thin high-velocity layers (Alkhalifah, 1996).

Another possible alternative is to compute the effective horizontal velocity as the rms average of the interval values of $V_{\text{hor}}$ given as

$$V_{\text{hor}}^2 = \frac{1}{t_0} \sum_{i=1}^{N} V_{\text{hor}}^2 \Delta t_i.$$

It is important to mention, however, that the rms horizontal velocity determined by equation (2.19) is not well-suited for moveout modeling in layered isotropic media and, consequently, in a certain vicinity of the isotropy plane in HTI media (if the symmetry axis has the same orientation in all layers). In isotropic or elliptically anisotropic media, the interval values of $V_{\text{anno}}$ and $V_{\text{hor}}$ are identical, and the effective NMO velocity [equation (2.16)] becomes equal to the effective horizontal velocity [equation (2.19)]. As a result, the coefficient $A$ in the denominator of the nonhyperbolic moveout term goes to infinity, and equation (2.5) reduces to the hy-
perbolic equation (2.3). This means that the rms averaging in equation (2.19) does not allow us to account for nonhyperbolic moveout in layered isotropic (or elliptically anisotropic) media. This problem, which may lead to traveltime errors in the presence of significant vertical gradients in velocity ($\geq 0.5$ s$^{-1}$), can be avoided by using an averaging equation that makes the effective horizontal velocity closer to the maximum $V_{\text{hor}}$,

$$V_{\text{hor}}^4 = \frac{1}{t_0} \sum_{i=1}^{N} V_{\text{hor},i}^4 \Delta t_i.$$  

(2.20)

Numerical tests on synthetic-modeled data described in the coming chapter, show that rms averaging [equation (2.19)] works well in HTI media with relatively small gradients in vertical velocity ($\leq 0.3$ s$^{-1}$), but is less suitable for media with more pronounced vertical heterogeneity. Higher accuracy for HTI models with typical gradients in the vertical velocity (0.5-0.8 s$^{-1}$) is provided by fourth-power averaging [equation (2.20)].

The approximations made in this chapter make it possible to apply the concise averaging equations developed for vertical transverse isotropy at the expense of partly ignoring out-of-plane phenomena in multilayered azimuthally anisotropic HTI media.
NUMERICAL STUDY OF ANALYTIC APPROXIMATIONS FOR
P-WAVE REFLECTION MOVEOUT

In this chapter I present a numerical study of P-wave reflection moveout designed to test the accuracy of the hyperbolic [equation (2.3)] and nonhyperbolic [equation (2.5)] moveout equations in HTI media. For single-layer models, it is necessary to find out whether or not the exact NMO velocity [equation (2.10)] makes the hyperbolic moveout equation sufficiently accurate on conventional-length spreads. Another important question is whether the new equation (2.13) for the quartic moveout coefficient allows the nonhyperbolic equation (2.5) to match the reflection traveltimes at the large offsets increasingly used in reflection surveys. In addition, it is necessary to verify the validity of the VTI averaging expressions for the NMO velocity [equation (2.16)] and for the quartic moveout coefficient [equation (2.17)] outside the vertical symmetry planes of horizontally-stratified HTI media. It is also important to find out how to define the averaging expression for the horizontal velocity in layered media [a quantity necessary in evaluating the nonhyperbolic moveout equation (2.5)].

The exact traveltimes were computed using a 3-D anisotropic ray-tracing code developed by Gajewski and Pšenčík (1987). The moveout velocity on finite spreads $V_{mo}$ was obtained by least-squares fitting of a hyperbolic moveout equation to the ray-traced traveltimes, i.e.,
Table 3.1. Parameters of single-layer HTI models used to generate the synthetic data. The depth to the bottom of the HTI layer (reflector) in all models is 1.5 km.  $e^{(V)}$, $\delta^{(V)}$, $V_{S_{\perp} \text{vert}}$, and $V_{\text{vert}}$ are the HTI parameters of the equivalent VTI medium, while $\varepsilon$, $\delta$, $V_{S_{\perp 0}}$, and $V_{P 0}$ are the generic Thomsen parameters (see Appendix A). 

\[
V_{mo}^2 = \frac{\sum_{j=1}^{N} x_j^2}{\sum_{j=1}^{N} t_j^2 - N t_0^2},
\]

where $x_j$ is the offset of the $j$-th trace, $t_j$ is the corresponding two-way reflection traveltime, $t_0$ is the two-way vertical traveltime, and $N$ is the number of traces. In the least-squares fitting procedure, the vertical time $t_0$ was fixed at the correct value.

### 3.1 Single HTI layer

Due to the presence of two orthogonal vertical symmetry planes in HTI media, it is sufficient to study reflection moveout in a single quadrant of azimuths (Figure 3.1). The parameters of four models with different combinations of anisotropic parameters and different strength of azimuthal anisotropy used in this study are listed in Table 3.1.

First, I test the accuracy of the $P$-wave hyperbolic moveout equation parameterized by the exact NMO velocity [equation (2.10)]. Figure 3.2 compares the best-fit values of stacking velocity $V_{mo}$ (solid curves) with values computed from equation (2.10) (dashed curves) for three spreadlengths, for the four models. The plots in the three columns (from left to right) correspond to spreadlength-to-depth ratio
Fig. 3.1. Orientation of the hypothetical 2-D survey lines over a horizontal HTI layer used in Figures 3.2 and 3.3.

$X/D$ of 0.5, 1.0, and 2.0. Those in the four rows pertain to the four models. As seen in Figures 3.2a,d,g, and j, for short spreadlengths (e.g., $X/D \leq 0.5$) the moveout velocity obtained from the exact traveltimes using equation (3.1) well approximates the analytic NMO value [equation (2.10)]. Predictably, the difference between the two velocities increases on longer spreads due to the increase of the anisotropy-induced deviations of the moveout curve from a hyperbola. This fact is demonstrated in Figures 3.2b,e,h, and k for spreadlength equal to the reflector depth ($X/D = 1$), and it is further illustrated in Figures 3.2c,f,i, and l for $X/D = 2$. It should be mentioned that Model 2 is characterized by an uncommonly high magnitude of nonhyperbolic moveout, which leads to a sizable difference between the two velocities even for $X/D = 1$. For typical HTI models, the moveout velocity obtained from the exact traveltimes using equation (3.1) is generally close to the analytic NMO value [equation (2.10)] for conventional-length spreads ($X/D \leq 1$).
Fig. 3.2. Comparison between the analytic P-wave normal-moveout velocity [equation (2.10)] and the moveout velocity estimated on a finite spread in a single HTI layer. The solid curve is the moveout velocity as a function of azimuth determined by fitting a hyperbola to the exact \( t - x \) curves [equation (3.1)]; the dashed curve is the normal-moveout (zero-spread) velocity from equation (2.10). The curves are calculated for four different HTI models and three spreadlengths (\( X/D = .5, X/D = 1, \) and \( X/D = 2 \) in three columns from left to right). The model parameters are given in Table 3.1.
Figure 3.2 also shows that the difference between the finite-spread and NMO velocity reaches its maximum in the symmetry-axis plane (azimuth $\alpha = 0^\circ$) and goes smoothly to zero in the isotropy plane ($\alpha = 90^\circ$). Clearly, despite the influence of out-of-plane phenomena, the magnitude of nonhyperbolic moveout outside the symmetry planes is smaller than in the direction of the symmetry-axis. This result could be expected from the simple azimuthal dependence of the quartic moveout coefficient (2.13). As discussed previously, reflection moveout in the symmetry-axis plane is described by the VTI equations extensively studied in the literature (Hake et al., 1984; Tsvankin and Thomsen, 1994; Alkhalifah, 1996).

Figure 3.3, on the other hand, shows the application of the hyperbolic moveout equation (2.3) (left column) as well as the nonhyperbolic moveout equation (2.5) (right column) as a function of $X/D$ for the same four models. The gray curves correspond to the exact (ray-traced) reflection traveltimes for survey-line azimuths (or equivalently, source-to-receiver azimuths) $\alpha$ of $0^\circ$, $30^\circ$, $45^\circ$, $60^\circ$, and $90^\circ$ measured from the symmetry-axis direction. The inadequacy of the hyperbolic moveout equation (2.3), parameterized by the exact NMO velocity, for long spreads is illustrated in detail by Figures 3.3a,c,e, and g showing the time residuals (black curves) after the conventional hyperbolic moveout correction. Deviation of the hyperbolic curve from the exact traveltimes is more pronounced for Model 2, which has an extremely large quartic moveout term (Figure 3.3c). For such unusually strong nonhyperbolic moveout, conventional hyperbolic velocity analysis does not work well even on spreadlengths close to the reflector depth. In contrast, the nonhyperbolic moveout equation (2.5), which includes the exact quadratic and quartic moveout coefficients, provides excellent accuracy for all HTI models and for the whole range of offsets, as demonstrated from the time residuals (black curves) in Figures 3.3b,d,f, and h. Application of the
Fig. 3.3. Comparison between the exact travel times and moveout approximations in the four single-layer HTI models whose parameters are given in Table 3.1. The gray curves are the exact reflection travel times as functions of the offset-to-depth ratio for survey-line azimuths $\alpha$ of $0^\circ$, $30^\circ$, $45^\circ$, $60^\circ$, and $90^\circ$. The black curves in (a), (c), (e), and (g) are the time residuals after conventional hyperbolic moveout correction using equation (2.3), while the black curves in (b), (d), (f), and (h) are the residuals after the nonhyperbolic moveout correction using equation (2.5).
nonhyperbolic moveout equation reduces the residual moveout at large offsets (i.e., twice as large as the reflector depth) by a factor of ten compared with the residuals after the hyperbolic correction. Note that the maximum error of both approximations occurs in the symmetry-axis plane (azimuth 0°), while in the isotropy plane (azimuth 90°) the reflection moveout is purely hyperbolic, and the error is zero.

3.2 Multilayered media

3.2.1 Multilayered HTI media

![Graph](image)

**Fig. 3.4.** P-wave group-velocity (ray) deviation from the incidence plane in HTI media (Model 5 in Table 3.2) for a receiver at a horizontal distance equal to the target depth (1.5 km) for five different source-to-receiver directions: 0°, 30°, 45°, 60°, and 90°. The HTI layers have a uniform symmetry-axis orientation. (a) A ray that propagates in the first layer only is confined to the incidence plane at all azimuths. (b) A ray propagating through all layers is confined to the incidence plane only in the two symmetry-plane directions (azimuths 0° and 90°).

As discussed in the previous chapter, the conventional Dix equation is no longer strictly valid outside the symmetry planes of multilayered HTI media because the group-velocity vector does not lie in the incidence plane for the whole raypath. Let me illustrate this fact by a numerical example whose results are shown in Figure 3.4.
Table 3.2. Parameters of three HTI models used to generate synthetic data. For each model, the symmetry-axis direction in all layers is parallel to the x-axis.

<table>
<thead>
<tr>
<th></th>
<th>Model 5</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Layer 1</td>
<td>Layer 2</td>
<td>Layer 3</td>
</tr>
<tr>
<td>$\varepsilon^V (\epsilon)$</td>
<td>-1.43 (.2)</td>
<td>-0.045 (.05)</td>
<td>-1.43 (.2)</td>
</tr>
<tr>
<td>$\delta^V (\delta)$</td>
<td>-1.84 (.1)</td>
<td>-0.203 (.15)</td>
<td>-0.318 (.2)</td>
</tr>
<tr>
<td>$V_{p\text{vert}} (V_{p0})$ (km/s)</td>
<td>2.662 (2.25)</td>
<td>2.622 (2.5)</td>
<td>2.958 (2.5)</td>
</tr>
<tr>
<td>$V_{s\perp\text{vert}} (V_{s\perp0})$ (km/s)</td>
<td>1.5 (1.5)</td>
<td>1.5 (1.5)</td>
<td>1.5 (1.5)</td>
</tr>
<tr>
<td>Depth (km)</td>
<td>0.7</td>
<td>1.0</td>
<td>1.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Model 6</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Layer 1</td>
<td>Layer 2</td>
<td>Layer 3</td>
</tr>
<tr>
<td>$\varepsilon^V (\epsilon)$</td>
<td>-1.43 (.2)</td>
<td>-0.045 (.05)</td>
<td>-1.43 (.2)</td>
</tr>
<tr>
<td>$\delta^V (\delta)$</td>
<td>-1.84 (.1)</td>
<td>-0.203 (.15)</td>
<td>-0.318 (.2)</td>
</tr>
<tr>
<td>$V_{p\text{vert}} (V_{p0})$ (km/s)</td>
<td>2.0 (1.69)</td>
<td>2.5 (2.384)</td>
<td>3.0 (2.535)</td>
</tr>
<tr>
<td>$V_{s\perp\text{vert}} (V_{s\perp0})$ (km/s)</td>
<td>1.15 (1.15)</td>
<td>1.4 (1.4)</td>
<td>1.525 (1.525)</td>
</tr>
<tr>
<td>Depth (km)</td>
<td>0.5</td>
<td>1.0</td>
<td>1.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Model 7</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Layer 1</td>
<td>Layer 2</td>
<td>Layer 3</td>
</tr>
<tr>
<td>$\varepsilon^V (\epsilon)$</td>
<td>-0.083 (.1)</td>
<td>-0.115 (.15)</td>
<td>-0.083 (.1)</td>
</tr>
<tr>
<td>$\delta^V (\delta)$</td>
<td>-1.86 (-1)</td>
<td>-0.257 (-1.15)</td>
<td>-0.271 (-0.2)</td>
</tr>
<tr>
<td>$V_{p\text{vert}} (V_{p0})$ (km/s)</td>
<td>2.0 (1.826)</td>
<td>2.75 (2.412)</td>
<td>3.5 (3.195)</td>
</tr>
<tr>
<td>$V_{s\perp\text{vert}} (V_{s\perp0})$ (km/s)</td>
<td>1.5 (1.5)</td>
<td>1.75 (1.75)</td>
<td>2.0 (2.0)</td>
</tr>
<tr>
<td>Depth (km)</td>
<td>0.5</td>
<td>1.0</td>
<td>1.5</td>
</tr>
</tbody>
</table>

for a three-layer HTI model that has a common symmetry-axis direction in all layers. Figure 3.4a, for example, shows the deviation of the P-wave group-velocity vector (ray) from the incidence plane for a ray which propagates in the first layer only from a source to the first reflector and back to a receiver at a horizontal distance of 1.5 km. The computation has been conducted for five different source-to-receiver directions (azimuths 0°, 30°, 45°, 60°, and 90°, measured from the symmetry-axis direction). In this case, the ray is confined to the incidence plane at all azimuths (see Chapter 2); hence, the deviation is zero and all curves coincide (Figure 3.4a). Figure 3.4b, on the
other hand, shows the deviation of a $P$-wave ray that propagates from a source to the third reflector and back to a receiver at a horizontal distance of 1.5 km for the same five source-to-receiver directions. Note that the ray is confined to the incidence plane only for source-to-receiver directions along the symmetry-plane directions (azimuths 0° and 90°). Between the symmetry planes, however, the ray does deviate from the incidence plane to satisfy Snell's law at each reflector. Nevertheless, despite the out-of-plane phenomenon, as seen in Figures 3.5 and 3.6, the effective normal-moveout velocity calculated by rms averaging of the exact interval values [equation (2.16)] is still sufficiently close to the moveout velocity determined from hyperbolic moveout analysis on conventional-length spreads. For HTI models with a uniform orientation of the symmetry-axis throughout the layers, the maximum deviation of the NMO velocity from the finite-spread value is observed for the CMP line in the throughgoing symmetry-axis plane of the medium (α = 0°), as shown in Figure 3.5.

Recall that in HTI media with a uniform symmetry-axis orientation, the Dix equation (2.16) is valid for the NMO velocity in the symmetry-axis and isotropy planes. In both symmetry planes, the difference between the finite-spread and rms-averaged-NMO velocity is caused entirely by the influence of nonhyperbolic moveout, which is much more pronounced in the symmetry-axis plane. In the isotropy plane, nonhyperbolic moveout is due to vertical velocity variation only. Outside the vertical symmetry planes in HTI media, equation (2.16) itself may become inaccurate since the group-velocity vector does deviate from the incidence plane. Figure 3.5, however, shows that the rms averaging provides a good approximation for NMO velocity at all azimuths.

Where the symmetry-axis direction varies with depth, the azimuth of the survey line with the maximum deviation of the NMO velocity from the finite-spread value
Fig. 3.5. Accuracy of the rms-averaging equation for P-wave normal-moveout velocity in layered HTI media with a uniform orientation of the symmetry-axis. The solid curve is the moveout velocity as a function of azimuth determined by fitting a hyperbola to the exact $t - x$ curves [equation (3.1)] on two different spreadlengths: (a) $X/D = 1$, and (b) $X/D = 2$. The dashed curve is the normal-moveout (zero-spread) velocity from the Dix equation (2.16). The velocities are calculated for the reflection from the bottom of the third layer of Model 6 whose parameters are given in Table 3.2.

depends on the variation of the symmetry-axis direction from one layer to another and on the strength of azimuthal anisotropy for each layer, as demonstrated in Figure 3.6. Note that the difference between the two velocities in Figure 3.6 does not reach its maximum along azimuth $0^\circ$ which corresponds to the symmetry-axis direction in the first and third layer. Although equation (2.16) in this model becomes somewhat inaccurate (since the group-velocity vector does deviate from the incidence plane), the rms averaging provides a good approximation for NMO velocity at all azimuths on conventional spreads.

Detailed comparisons of the exact traveltimes with the moveout equations [the hyperbolic (2.3) and the nonhyperbolic (2.5)] in three-layer HTI models are shown in Figures 3.7, 3.8, and 3.9. All three models have a common symmetry-axis direction throughout the layers and a similar magnitude of nonhyperbolic moveout. The only major difference between the three models is the value of the $P$-wave vertical velocity
Fig. 3.6. Accuracy of the Dix rms-averaging equation for $P$-wave normal-moveout velocity in layered HTI media with depth-varying symmetry-axis direction. The solid curve is the moveout velocity as a function of azimuth determined by fitting a hyperbola to the exact $t - x$ curves [equation (3.1)] on two different spreadlengths: (a) $X/D = 1$, and (b) $X/D = 2$. The dashed curve is the normal-moveout (zero-spread) velocity from the Dix equation (2.16). Again, the velocities are calculated for the reflection from the bottom of the third layer, this time for Model 5 described in Table 3.2. The symmetry-axis in the second layer, however, is rotated by $60^\circ$ with respect to symmetry-axis direction in the first and third layer.

gradient. Model 5 in Table 3.2 (Figure 3.7) has an effective $P$-wave vertical velocity gradient of about $0.24 \text{ s}^{-1}$, while it is close to a more typical value of $0.67 \text{ s}^{-1}$ for Model 6 (Figure 3.8), and it reaches $1.0 \text{ s}^{-1}$ for Model 7 (Figure 3.9). The importance of the velocity gradient will become clear when I discuss the accuracy and the implementation of the nonhyperbolic moveout equation. The gray curves are the exact reflection traveltimes from all three interfaces for azimuths $\alpha$ of $0^\circ$, $30^\circ$, $45^\circ$, $60^\circ$, and $90^\circ$, measured with respect to the symmetry axis direction. Plot (a) is each of Figures 3.7–3.9 shows the time residuals (black curves) after a conventional moveout correction using equation (2.3), while plots (b), (c), and (d) show the time residuals after nonhyperbolic moveout correction using equation (2.5) and three different ways to compute the effective horizontal velocity [equations (2.18)–(2.20)].

Evidently, the hyperbolic moveout equation (2.3) based on the exact interval
NMO velocities averaged by formula (2.16) provides a good approximation to the traveltimes on spreadlengths that do not exceed the reflector depth (Figures 3.7a, 3.8a, 3.9a). As in the homogeneous model, the error of the hyperbolic moveout equation increases with offset, this time due to the combined influence of anisotropy and layering.

Long-spread moveout in layered HTI media is described here by equation (2.5) with the moveout coefficients given by equations (2.16) for the effective NMO velocity and (2.17) for the effective quartic coefficient. As mentioned in the previous chapter, it is not clear how to compute the effective horizontal velocity, which can be obtained in several ways [equations (2.18)–(2.20)]. The comparison between the accuracy of applying the nonhyperbolic moveout equation for the three models, each with different ways to compute the effective horizontal velocity, allows us to understand and to choose the best way of averaging the horizontal velocities. The rms averaging equation (2.19) works well in HTI media with relatively small gradients in vertical velocity \( \leq 0.3 \text{ s}^{-1} \), as demonstrated in Figure 3.7b. Note that the fourth-power averaging for the effective horizontal velocity [equation (2.20)] provides a comparable accuracy (Figure 3.7c), while using the maximum horizontal velocity [equation (2.18)] leads to inferior results (Figure 3.7d). For media with a typical vertical velocity gradient of 0.5-0.8 \text{ s}^{-1}, the highest accuracy is provided by the fourth-power averaging [equation (2.20)], as demonstrated by Figure 3.8c as opposed to Figures 3.8b and 3.8d. For media with an uncommonly large vertical velocity gradient, neither averaging equation for the horizontal velocity (especially rms averaging) adequately approximates reflection moveout at large offsets. The highest accuracy in this case is obtained by using the maximum horizontal velocity above the reflector [equation (2.18)] (compare Figure 3.9d with Figures 3.9b and 3.9c). Application of the maximum horizontal
velocity, however, might lead to erroneous reflection moveout approximation in the presence of a thin high-velocity layer in the medium (Alkhalifah, 1996).

Despite the approximate character of the averaging expressions, the nonhyperbolic moveout equation (2.5) provides excellent accuracy for multilayered HTI models with throughgoing symmetry planes, as demonstrated in Figures 3.7b, 3.8c, and 3.9d. Similar to the result for single-layer models, the residual moveout at large offsets (twice the reflector depth) after nonhyperbolic moveout correction is about an order of magnitude lower than the residual after the hyperbolic correction.

These conclusions hold for models with layer-dependent orientation of the symmetry-axis as well. This is illustrated in Figure 3.10 which shows a detailed comparison of the exact traveltimes with the moveout equations [the hyperbolic (2.3) and the nonhyperbolic (2.5)] for two different three-layer HTI models. Different character of the azimuthal velocity variations from layer to layer (due to different strengths of azimuthal anisotropy and different symmetry-axis orientation) causes a more significant deviation of the group-velocity vector from the incidence plane compared to the case where the media has a uniform-symmetry-axis direction. Nonetheless, this deviation in typical HTI models is not large enough to cause measurable errors in the use of equation (2.16), as demonstrated by Figures 3.10a and c.

It is important to mention that the hyperbolic moveout equation breaks down if we disregard the azimuthal dependence of the interval NMO velocities described by equation (2.10). Application of any single value of NMO velocity would lead to misalignment of reflection events and poor stacking quality in certain ranges of azimuthal angles.
Fig. 3.7. Comparison between the exact traveltimes and the moveout approximations for a three-layer HTI model (Model 5 in Table 3.2) with a uniform symmetry-axis direction in all layers. The effective gradient in the vertical velocity is about 0.24 s\(^{-1}\). The gray curves are the exact reflection traveltimes from all three interfaces for azimuths \(\alpha\) of 0°, 30°, 45°, 60°, and 90°. The black curves in (a) are the time residuals after the conventional hyperbolic moveout correction using equation (2.3), with the NMO velocity from equation (2.16), while the black curves in (b), (c) and (d) are the residuals after nonhyperbolic moveout correction using equation (2.5) with the effective NMO velocity from equation (2.16), the effective quartic coefficient from equation (2.17), and three different ways to compute the effective horizontal velocity: equation (2.19) in (b), equation (2.20) in (c), and equation (2.18) in (d). The nonhyperbolic moveout equation provides the highest accuracy if the horizontal velocity is calculated using either rms (b) or fourth-power (c) averaging.
**Fig. 3.8.** Comparison between the exact traveltimes and the moveout approximations for a three-layer HTI model with a uniform symmetry-axis direction in all layers. The description of the plots is the same as in Figure 3.7, but this time for Model 6 in Table 3.2. The effective gradient in the vertical velocity is about 0.67 s\(^{-1}\). The most accurate representation of reflection moveout is given in (c).
Fig. 3.9. Comparison between the exact traveltimes and the moveout approximations for a three-layer HTI model with a uniform symmetry-axis direction in all layers. The description of the plots is the same as in Figures 3.7 and 3.8, but this time for Model 7 in Table 3.2. The effective gradient in the vertical velocity is about 1.0 s⁻¹. The most accurate representation of reflection moveout is given in (d).
The nonhyperbolic moveout equation (2.5) provides excellent accuracy for models with depth-varying orientation of the symmetry-axis (Figure 3.10b and d), comparable to the accuracy achieved for media with a uniform symmetry-axis direction.

Fig. 3.10. Comparison between the exact traveltimes and the moveout approximations for two three-layer HTI models with depth-varying orientation of the symmetry axis. The parameters of the model in (a) and (b) are the same as for Model 5 in Table 3.2, but the symmetry-axis of the second layer is rotated by 60° with respect to the symmetry-axis in the first and third layer. The parameters of the model in (c) and (d) are the same as for Model 6 in Table 3.2, but the symmetry-axis of the second layer is rotated by 45° with respect to the symmetry-axis in the first layer, while the symmetry-axis of the third layer is rotated 90° with respect to the symmetry-axis in the first layer. The gray curves are the exact reflection traveltimes from all three interfaces for azimuths $\alpha$ of 0°, 30°, 45°, 60°, and 90°. The black curves in (a) and (c) are the time residuals after the conventional hyperbolic moveout correction using equation (2.3), with the NMO velocity from equation (2.16), while the black curves in (b) and (d) are the residuals after nonhyperbolic moveout correction using equation (2.5) with the effective coefficients from equations (2.16), (2.17), and (2.20).
3.2.2 HTI–VTI and isotropic–HTI layered media

In the previous examples, I examined reflection moveout in models consisting only of azimuthally anisotropic layers with the HTI symmetry. However, fractured reservoirs are often overlain by a nearly isotropic overburden, and are possibly interbedded with TI layers (e.g., shales) which have vertical axis of symmetry (VTI).

Consider first the case where we have a combination of both orientations of TI model (i.e., HTI layers interbedded with VTI layers). The model used to generate the results in Figure 3.11 consists of a homogeneous VTI layer sandwiched between two
HTI layers. Figure 3.11 shows that the hyperbolic moveout equation parameterized by the NMO velocity (2.16) gives, as before, an adequate description of the moveout on conventional spreads (the spreadlength equals the target depth, 1.5 km), while the nonhyperbolic moveout equation (2.5) provides an excellent approximation to the reflection traveltimes at large as well as small offsets. The \( P \)-wave interval NMO velocity for the VTI layer is given by \( V_{\text{NMO}} = V_{p0} \sqrt{1 + 2\delta} \) (Thomsen, 1986), where \( \delta \) and \( V_{p0} \) are Thomsen’s (1986) parameters for VTI medium, and the interval quartic coefficient \( (A_4) \) is given by the known VTI expression (Hake et al., 1984; Tsvankin and Thomsen, 1994).

Finally, consider an isotropic overburden with vertical velocity gradient overlaying an HTI layer. Although it seems that the presence of isotropic layers should help to mitigate out-of-plane phenomena since phase- and group-velocity vectors in isotropic media coincide with each other, deviation of the group-velocity vector from the incidence plane may even increase if we replace some of the HTI layers in the model by isotropic ones. Indeed, to keep the group-velocity vector close to the incidence plane, the character of azimuthal velocity variations should be similar throughout the section. In the limit of a single HTI layer (a completely uniform character of azimuthal anisotropy), the group-velocity vector is strictly confined to the incidence plane. During the transmission from an HTI layer into a purely isotropic medium, the phase-velocity vector stays in the same vertical plane, while the group-velocity vector must coincide with the phase-velocity vector after the transmission. Since in the HTI layer the azimuths of the group- and phase-velocity vectors may be substantially different, this leads to an azimuthal rotation of the group vector at the boundaries and potential errors in the averaging equations (2.16) and (2.17).

Nevertheless, as demonstrated by Figure 3.12, the conclusions drawn for lay-
ered HTI models remain essentially valid in this case as well. The model used to generate the results in Figure 3.12 includes five isotropic layers above a layer with HTI symmetry. The hyperbolic moveout equation parameterized by the NMO velocity (2.16) gives an adequate description of the moveout on conventional spreads, while the nonhyperbolic moveout equation (2.5) is close to the reflection traveltime at all horizontal offsets. The improvement in accuracy gained by using the nonhyperbolic moveout equation is especially significant for the reflection from the bottom of the HTI layer. The accurate result for this (deepest) event was ensured by using the exact interval expressions (2.10) and (2.13) for the azimuthally-dependent quadratic and quartic moveout coefficients in the HTI layer. As before, the effective horizontal velocity for the reflection from the bottom of the HTI layer was obtained by fourth-power averaging [equation (2.20)]. However, for the purely isotropic overburden with a pronounced vertical-velocity gradient in Figure 3.12, equation (2.5) turns out to be somewhat more accurate with the effective $V_{\text{hor}}$ chosen to be equal to the maximum horizontal velocity above the reflector.
Fig. 3.12. Comparison between the exact traveltimes and the moveout approximations for a model that includes a stack of five homogeneous isotropic layers on top of an HTI layer. The gray curves are the exact reflection traveltimes from all six interfaces for azimuths $\alpha$ of $0^\circ$, $30^\circ$, $45^\circ$, $60^\circ$, and $90^\circ$. The black curves in (a) are the time residuals after the conventional hyperbolic moveout correction using equation (2.3) with the NMO velocity from equation (2.16), while the black curves in (b) are the residuals after the nonhyperbolic moveout correction using equation (2.5) and the effective $V_{nmo}$ and $A_4$ from equations (2.16) and (2.17). The effective horizontal velocity for the isotropic layers is taken to be equal to the maximum horizontal velocity above the reflector; for the reflection from the bottom of the HTI layer, the horizontal velocity is calculated from equation (2.20). For layer 1 $V_p = 2.0$ km/s, and the depth $d_1 = 0.2$ km; for layer 2 $V_p = 2.5$ km/s, and $d_2 = 0.4$ km; for layer $3$ $V_p = 3.0$ km/s, and $d_3 = 0.6$ km; for layer 4 $V_p = 3.5$ km/s, and $d_4 = 0.8$ km; for layer 5 $V_p = 2.5$ km/s, and $d_5 = 1.0$ km; for layer 6 $\epsilon^{(V)} = -0.143$ ($\epsilon = 0.2$), $\delta^{(V)} = -0.318$ ($\delta = -0.2$), $V_{p \text{ vert}} = 2.958$ km/s ($V_{p 0} = 2.5$ km/s), and $d_6 = 1.5$ km.
Chapter 4

P-WAVE REFLECTION-MOVEOUT INVERSION FOR HORIZONTAL TRANSVERSE ISOTROPY

In this chapter I discuss estimation of anisotropic parameters and detection of fracture orientation from P-wave moveout data in homogeneous and horizontally-layered HTI media. Estimating the anisotropy parameters allows the possibility of estimating the crack density, a quantity (in addition to crack orientation) of great interest in the characterization of fractured reservoirs. Error study, including analysis of the condition number, provides insight into both the properties of the inverse problem and the optimal survey design. Finally, I show results of numerical applications and synthetic data examples, and discuss the accuracy and stability of the inversion procedure.

4.1 The inverse problem

The azimuthal dependence of NMO velocity in an HTI layer is elliptical, as seen by recasting equation (2.10) as

\[ V_{\text{nmo}}^2(\alpha) = \frac{V_{\text{nmo}}^2}{V_{\text{nmosym}}^2} \frac{V_{\text{nmoiso}}^2}{\sin^2 \alpha + V_{\text{nmoiso}}^2 \cos^2 \alpha}, \]  

(4.1)

where \( V_{\text{nmoiso}} \) is the NMO velocity in the isotropy plane \( (V_{\text{vert}}) \), \( V_{\text{nmosym}} \) is the NMO velocity in the symmetry-axis plane \( (V_{\text{vert}} \sqrt{1 + 2\delta^{(W)}}) \), and \( \alpha \) is the angle between the symmetry axis and the survey line in 2D acquisition (or, equivalently, the source-
to-receiver orientation in 3D acquisition). Clearly, from equation (4.1), $V_{nmo_{iso}}$ and $V_{nmo_{sym}}$ are the semi-axes of the NMO ellipse.

The elliptical variation of NMO-velocity with azimuth is well known for the case of an isotropic layer above a dipping interface (Levin, 1971). There, the NMO velocities in the dip and strike directions determine the semi-axes of the ellipse. It turns out that the elliptical behavior of the NMO-velocity is a general property of inhomogeneous, azimuthally anisotropic media of any complexity (e.g., orthorhombic media), with interfaces of arbitrary strike and dip (Grechka and Tsvankin, 1996a).

NMO velocities at three distinct survey-line azimuths are thus sufficient, as well as necessary, to reconstruct the elliptical distribution of the NMO velocity [equation (4.1)] and obtain the semi-axes and the orientation of the NMO ellipse (the parameters $V_{nmo_{iso}}$, $V_{nmo_{sym}}$, and $\alpha$). If the symmetry-axis direction is known, then NMO-velocity measurements in two different azimuthal directions are sufficient to reconstruct the NMO velocity ellipse.

Instead of inverting for the parameters of the ellipse [equation (4.1)], namely its orientation and the semi-axes, we can estimate the medium parameters directly. For any number of input moveout velocity measurements, equation (2.10) yields two different sets of solutions corresponding to two orthogonal symmetry axes, each with different combinations of $V_{p_{vert}}$ and $\delta^{(V)}$. One solution has a positive $\delta^{(V)}$ with a low $V_{p_{vert}}$, while the other has a negative $\delta^{(V)}$ with a high $V_{p_{vert}}$. Both solutions provide the same values of NMO velocity in all azimuthal directions; hence, we cannot distinguish between the symmetry-axis plane and isotropy plane from these measurements alone.

To illustrate this conclusion, consider an HTI layer with $V_{p_{vert}} = 2.0 \text{ km/s}$, $\delta^{(V)} = -0.2$, and the symmetry axis pointing in the $x$-axis direction (Figure 4.1a). Note that a $\delta^{(V)}$ value of -0.2 corresponds to approximately a 20% azimuthal variation in
the NMO velocity between the two symmetry-plane directions. Suppose we compute the NMO velocities along three different source-to-receiver azimuths measured from the symmetry-axis direction ($\alpha_1$, $\alpha_2$, and $\alpha_3$). As shown in Figure 4.1b, there are two sets of solutions corresponding to orthogonal directions of the symmetry axis that satisfy the three NMO velocities and produce the same elliptical variation of NMO velocity with azimuth. However, for typical ratios of the vertical velocities ($V_{S\perp vert}/V_{P\text{vert}} \leq .707$), the parameter $\delta^{(V)}$ is negative (see Appendix D); therefore, the NMO velocity reaches its maximum in the isotropy plane and minimum in the symmetry-axis plane. Thus, if we assume that $\delta^{(V)} < 0$, we can unambiguously identify the symmetry-axis direction. Also, additional information such as nonhyperbolic reflection moveout whose magnitude reaches maximum in the symmetry-axis plane, can help to distinguish the symmetry-axis plane from the isotropy one and obtain the correct $V_{P\text{vert}}$ and $\delta^{(V)}$. 

**Fig. 4.1.** (a) Plan view of 2D survey lines (source-to-receiver azimuths in 3D) over a horizontal HTI layer with $V_{P\text{vert}} = 2.0$ km/s, and $\delta^{(V)} = -0.2$, with the symmetry-axis in the $x$-direction. Two different sets of solutions for the symmetry-axis direction (dashed lines) provide the same NMO-velocity variation, as shown in (b). The correct solution (Sol. 1, horizontal dashed line) has $V_{P\text{vert}} = 2.0$ km/s and $\delta^{(V)} = -0.2$, while Sol. 2 has $V_{P\text{vert}} = 1.549$ km/s and $\delta^{(V)} = 0.333$. 
4.2 Error analysis

To estimate the sensitivity of the NMO velocity to the anisotropic parameters, I evaluate the Jacobian of equation (2.10). The Jacobian is obtained by calculating the derivatives of NMO velocity with respect to the model parameters $V_{pvert}$, $\delta^{(V)}$, and $\alpha$. Although the NMO-velocity equation (2.10) is nonlinear, its dependence on the anisotropy parameters is smooth enough to use the Jacobian for developing insight into the inverse problem. The derivatives used to form the Jacobian are as follows:

\[
d_1(\alpha) = \frac{V_{pvert}}{V_{nmo}(\alpha)} \frac{\partial V_{nmo}(\alpha)}{\partial V_{pvert}} = 1,
\]

\[
d_2(\alpha) = \frac{1}{V_{nmo}(\alpha)} \frac{\partial V_{nmo}(\alpha)}{\partial \delta^{(V)}} = \frac{\cos^2 \alpha}{(1 + 2\delta^{(V)})(1 + 2\delta^{(V)} \sin^2 \alpha)},
\]

\[
d_3(\alpha) = \frac{1}{V_{nmo}(\alpha)} \frac{\partial V_{nmo}(\alpha)}{\partial \alpha} = \frac{2\delta^{(V)} \sin \alpha \cos \alpha}{1 + 2\delta^{(V)} \sin^2 \alpha}.
\]

The normalization of the derivatives chosen here simplifies the comparison of the contribution of each parameter to the NMO velocity. As a result, the information provided by these derivatives consists of relative values for $V_{pvert}$, and absolute values for $\delta^{(V)}$ and $\alpha$ (the latter measured in radian).

The sensitivity of this inversion to errors in the input data (NMO velocities) can be estimated using the Jacobian matrix

\[
J = \begin{pmatrix}
  d_1(\alpha_1) & d_2(\alpha_1) & d_3(\alpha_1) \\
  d_1(\alpha_2) & d_2(\alpha_2) & d_3(\alpha_2) \\
  d_1(\alpha_3) & d_2(\alpha_3) & d_3(\alpha_3)
\end{pmatrix},
\]

where $\alpha_1$, $\alpha_2$, and $\alpha_3$ are the azimuths of the survey lines (CMP gathers) with respect to the symmetry axis of the HTI model.
The condition number for the Jacobian matrix provides an approximate overall estimate of the quality (stability) of the inversion for all three parameters. I will use the condition number as a criterion to design the best experimental setup. After that, I will quantify propagation of errors from the input measurements (NMO velocity) into the medium parameters via a covariance matrix study and a numerical error analysis.

4.2.1 Conditioning of the problem

The reciprocal of the condition number, \( \kappa^{-1} \), for the Jacobian matrix \( J \) is given by

\[
\kappa^{-1} = \sqrt{\frac{\lambda_{\text{min}}}{\lambda_{\text{max}}}},
\]

(4.2)

where \( \lambda_{\text{max}} \) and \( \lambda_{\text{min}} \) are the maximum and minimum eigenvalues, respectively, of the matrix \( A = J^T J \) (\( J^T \) is the transpose of \( J \)).

A small \( \kappa^{-1} \) number (\( \approx 0 \)) implies an ill-conditioned (i.e., nearly singular) problem, while a large \( \kappa^{-1} \) number usually implies a well-conditioned problem.

In seismic acquisition, the best offset coverage usually corresponds to the dip and strike directions of the subsurface structure. Therefore, setting two azimuths perpendicular to each other in the following investigation allows us to simulate the most common acquisition design. Intuitively, a maximum azimuthal separation between the survey lines (or, equivalently, between the source-to-receiver azimuths) can be expected to increase both the stability and resolution; hence, setting two azimuths 120° apart is another combination of particular interest considered here. In 3-D land acquisition where the azimuthal (offset) coverage is flexible to some extent, it is necessary to investigate the selection of azimuth ranges from the full range obtained in a survey which provide the best inversion results. Also, in marine surveys the azimuths
are generally quite limited. Hence, knowing the optimal survey orientations to obtain the best inversion results is crucial to improve the acquisition design.

**Fig. 4.2.** The reciprocal of the condition number ($\kappa^{-1}$) as a function of $\alpha_2$ and $\delta^{(V)}$; $\alpha_1=0^\circ$ and $\alpha_3=90^\circ$.

**Fig. 4.3.** The reciprocal of the condition number ($\kappa^{-1}$) as a function of $\alpha_2$ and $\delta^{(V)}$; $\alpha_1=0^\circ$ and $\alpha_3=120^\circ$.

Figure 4.2 shows the reciprocal of the condition number, $\kappa^{-1}$, as a function of $\alpha_2$ and $\delta^{(V)}$, where $\alpha_1 = 0^\circ$ and $\alpha_3 = 90^\circ$. For comparison, Figure 4.3 shows a similar plot of $\kappa^{-1}$ as in Figure 4.2, but for $\alpha_1$ and $\alpha_3$ equal to $0^\circ$ and $120^\circ$, respectively. Notice from both Figures (4.2 and 4.3) that when the third-line direction $\alpha_2$ coincides with either of the other two azimuths (only two azimuths are available), the problem is
clearly singular, with $\kappa^{-1}=0$. Note also that for $\delta^{(V)}=0$ (no azimuthal variation in NMO velocity, as in isotropic media), we obviously cannot resolve the symmetry-axis direction, and again $\kappa^{-1}=0$. Since the ellipticity of the NMO-velocity function increases with increasing $|\delta^{(V)}|$ (i.e., the azimuthal variation in NMO velocity becomes larger), the stability improves with an increase in the absolute value of $\delta^{(V)}$. Not surprisingly, for typical values of $\delta^{(V)}$, as shown by Figures 4.2 and 4.3, the maximum of $\kappa^{-1}$ (the highest stability) corresponds to the third-line direction $\alpha_2$ being midway in between the other two lines. When the third azimuth orientation is close to any of the other two (intuitively a poor choice), the problem again becomes ill-conditioned. Note that, for $\alpha_2$ midway between $\alpha_1$ and $\alpha_3$, the difference in the maximum $\kappa^{-1}$ for the two cases shown in Figures 4.2 and 4.3 is small, which means that we should expect a comparable and adequate quality (stability) in the inversion procedure using these two sets of azimuths. It should be emphasized that $\kappa^{-1}$ is largely a qualitative measure of the stability of the inversion procedure, and the difference in the condition number of less than one order of magnitude should not be considered too quantitatively. As we will see below, however, there is strong reason to prefer the case of three azimuths each separated by 60°.

Let us study the azimuthal variation of the computed reciprocal of the condition number $\kappa^{-1}$ for a set of three survey-line orientations (or source-to-receiver azimuths) with a fixed angular separation (the angle between two survey lines, $\alpha_1$ and $\alpha_3$, is fixed, and the third one, $\alpha_2$, is midway in between). The survey lines are simultaneously rotated so that the middle line (angle $\alpha_2$) spans the azimuthal range from 0° to 180° measured from the symmetry-axis direction.

Figure 4.4 shows the results of this study for five different angular separations between the lines: (a) 7.5°, (b) 15°, (c) 30°, (d) 45°, and (e) 60°. $\kappa^{-1}$ shows the
Fig. 4.4. The reciprocal of the condition number ($\kappa^{-1}$) as a function of azimuth, $\alpha_2$, for five different angular separations between three survey lines. Each set of azimuths is rotated so that the middle direction, $\alpha_2$, spans the azimuths from $0^\circ$ to $180^\circ$ measured from the symmetry-axis direction. The five curves correspond to azimuth separations of $7.5^\circ$ (a), $15^\circ$ (b), $30^\circ$ (c), $45^\circ$ (d), and $60^\circ$ (e); $\delta^{(V)} = -0.2$.

least variation with azimuth for the maximum angular separation ($60^\circ$) between the survey lines (curve e in Figure 4.4). Even though the global maximum for $\kappa^{-1}$ is not associated with curve e, we should choose a survey design that has a higher overall stability for the whole range of azimuths, since we usually do not know the symmetry-axis direction in advance. The angular separation of $45^\circ$ (curve d in Figure 4.4), provides a higher value of $\kappa^{-1}$ for a limited range of azimuths ($75^\circ \leq \alpha_2 \leq 105^\circ$) than that for the $60^\circ$ angular separation; however, at other azimuths, $\kappa^{-1}$ drops by about 50% (e.g., for $\alpha_2 = 30^\circ$). This variation in $\kappa^{-1}$ makes the $45^\circ$ angular separation less desirable compared to the $60^\circ$. $\kappa^{-1}$, however, is not too small for any value of $\alpha_2$ for curves d and e. Therefore, we should not expect any stability problem when those angular separations are used.

Clearly, as we should expect, narrower angular separations ($< 45^\circ$) do not provide stability comparable to that for $45^\circ$ or $60^\circ$. Interestingly, curve c generates the global maximum for $\kappa^{-1}$ for $\alpha_2 = 90^\circ$. This is, however, a local phenomenon for this
particular case ($\delta^{(V)}=-0.2$) that does not appear either in other azimuthal directions or for smaller values of $\delta^{(V)}$ (Figure 4.5). Away from $\alpha_2 = 90^\circ$ the value of $\kappa^{-1}$ for curve c is small compared to that for curve d and especially curve e.

![Graph](image)

**Fig. 4.5.** The reciprocal of the condition number $\kappa^{-1}$ as a function of azimuth $\alpha_2$ (same as in Figure 4.4, but for $\delta^{(V)}=-0.1$).

The same conclusions can be drawn for the model with weaker anisotropy (smaller $\delta^{(V)}$), shown in Figure 4.5. Notice that corresponding values for $\kappa^{-1}$ in Figure 4.5 to those in Figure 4.4 are smaller, linearly proportional to $\delta^{(V)}$.

Overall, the condition-number analysis shows that wide angular separations between the survey-line azimuths ($\geq 45^\circ$), as we might have expected, provide a well-conditioned (behaved) inverse problem for any orientation of the three lines with respect to the symmetry axis.

### 4.2.2 Error propagation (covariance matrix)

The propagation of errors from the input measurements (NMO velocity) to the medium parameters ($V_{\text{vert}}, \delta^{(V)}$, and $\alpha$) could be analyzed by calculating the covariance matrix of this inverse problem.
For the special case of a perfect forward modeling operator, a perfect parameterization of the model (i.e., no discretization errors), no a priori information, no uncertainties other than those associated with the input NMO-velocity data, and data uncertainties that are normally distributed with a known covariance, the covariance for the least-squares estimates of the model parameters is given by (Tarantola, 1987)

\[ C_M = J^*C_D(J^*)^T \] (4.3)

where \( J^* \) is the pseudo-inverse of the Jacobian matrix \( J \) of this inverse problem and \( C_D \) is the data covariance matrix.

Let us further simplify the calculation by considering the fact that the Jacobian has the full-column rank [three NMO velocity measurements along distinct source-to-receiver azimuths, for the range of angular separations that I am considering, do not produce zero eigenvalues (i.e., no null eigenvectors)] and that the data have independent, identically distributed errors with a known variance \( c_d \). In this case, equation (4.3) reduces to

\[ C_m = c_d[J^TJ]^{-1}, \] (4.4)

where \( c_d \) is a single measurement representing the variance of the input data (NMO velocity).

Thus, in the following analysis, I am only estimating the portion of the model covariance that comes from the forward modeling operator.

I will use the square-root of the diagonal elements of \( C_m \) to approximately estimate the expected error (standard deviation) for each parameter: the absolute error in \( \alpha \) \((\Delta \alpha)\) measured in radians, the absolute error in \( \delta^{(V)} \) \((\Delta \delta^{(V)})\), and the percentage
error in $V_{P,\text{vert}} (\Delta V_{P,\text{vert}}/V_{P,\text{vert}})$. If we set $c_d$ to unity, the diagonal elements of $[J^T J]^{-1/2}$ simply measure the magnification factors of the error in each parameter for any given error in the input NMO-velocity measurements (given in percent). The magnification factors of the errors for the three parameters are denoted as $M_\alpha$ (measured in radians), $M_{\delta(V)}$ (dimensionless), and $M_{V_{\text{vert}}}$ (dimensionless).

Similar to the analysis of the previous section, let us study the square-root of the covariance matrix (error propagation) as a function of azimuth ($\alpha_2$) for three angular separations between the survey lines: $30^\circ$, $45^\circ$, and $60^\circ$. The central azimuth, $\alpha_2$, spans the angular range from $0^\circ$ to $180^\circ$ measured from the symmetry-axis direction. The variance of the input data (NMO-velocity) measurements $c_d$ in equation (4.4), as described above, is set to unity. The results of this study for the magnification factors of the errors for each parameter are shown in Figures 4.6-4.8.

As demonstrated by Figure 4.6, the accuracy in estimating the parameter $\alpha$ improves by a factor of about 3 by using an angular separation of $60^\circ$, as opposed to $30^\circ$, for most ranges of azimuths. For $\alpha_2$ near the symmetry-plane directions, the error in $\alpha$ estimates, however, is about the same for the three angular separations. Where the symmetry-axis direction is not known in advance, however, the behavior of the $30^\circ$ angular separation is inappropriate. This indicates that the parameter $\alpha$ is quite sensitive to both the angular separation between the survey lines and the set of azimuths used in the inversion procedure. The accuracy in resolving $\alpha$ for an angular separation of $60^\circ$ is highly consistent; for most ranges of azimuths; the associated absolute error in $\alpha$ is the least. Notice again that the error in $\alpha$ is inversely proportional to the absolute value of $\delta(V)$ (Figure 4.6a compared to Figure 4.6b).

From the ratio between results for the $60^\circ$ and $30^\circ$ angular separations shown in Figure 4.7, the reduction of the absolute errors in $\delta(V)$ with increasing angular
Fig. 4.6. Magnification factor in the absolute error in $\alpha$ measured in radians ($\alpha$ component in the diagonal of $[J^T J]^{-1/2}$) as a function of azimuth ($\alpha_2$) for three different angular separations between survey direction. The three sets of azimuth combinations are rotated so that the central azimuth $\alpha_2$ spans azimuths from 0° to 180° measured from the symmetry-axis direction. The three curves in (a) correspond to angular separations of 30° (gray), 45° (dashed black), and 60° (solid black); $\delta^{(V)}=-0.2$. The three curves in (b) correspond to the same test as in (a), but for $\delta^{(V)}=-0.1$.

separation is about 4. Therefore, the parameter $\delta^{(V)}$ is also sensitive to the angular separation between the survey lines and the set of azimuths used in the inversion. Still, the accuracy for angular separations of 45° and especially of 60° is less variable at all azimuths. Interestingly, the propagation of errors into $\delta^{(V)}$ is about the same for different strength of the anisotropy (compare Figure 4.7a with Figure 4.7b). This implies that we should expect the same absolute error in $\delta^{(V)}$ for a wide range of $\delta^{(V)}$, and the relative error in $\delta^{(V)}$ will be smaller for stronger anisotropy.

Figure 4.8 demonstrates that the best accuracy in $V_{\text{vert}}$ for all azimuths is again achieved for an angular separation of 60°, with slightly poorer results for the 45°. As the case with the other two parameters, the accuracy in inverting for $V_{\text{vert}}$ depends on both the angular separation between the survey lines and the set of azimuths used in the inversion procedure (the maximum ratio between the results for the 60° and 30° angular separations shown in Figure 4.8 is about 5). The accuracy in resolving
Fig. 4.7. The same as Figure 4.6, but for the absolute error in $\delta^{(V)}$ component in the diagonal of $[J^T J]^{-1/2})$. The vertical axis is dimensionless.

Fig. 4.8. The same as Figures 4.6 and 4.7, but for the relative error in $V_{\text{vert}}$ component in the diagonal of $[J^T J]^{-1/2})$. The vertical axis is dimensionless.
$V_{\text{Pvert}}$, however, is about the same for the three angular separations when the central line ($\alpha_2$) is close to the isotropy-plane direction. Considering again the fact that the symmetry-axis direction is not known in advance, the choice 30° (or any smaller angular separation) is inadequate. It is interesting to observe that the accuracy in estimating $V_{\text{Pvert}}$ increases as anisotropy becomes weaker (compare Figure 4.8b with Figure 4.8a). This is not surprising since $V_{\text{Pvert}}$ is essentially an "isotropic" quantity, and it can be expected to be better resolved as the medium becomes closer to the isotropic one ($\delta^{(V)} = 0$).

To compute the expected absolute error (standard deviation) in the estimated symmetry-axis direction $\alpha$ ($\Delta \alpha$) for a particular angular separation between the survey lines, we carry out the following steps:

- Pick the magnification factor for a given set of azimuths from Figure 4.6.
- Multiply this factor by the error (standard deviation) in the input NMO-velocity measurements to get the absolute error in $\alpha$ (in radians).

Similarly, to estimate the absolute error in $\delta^{(V)}$ ($\Delta \delta^{(V)}$), we have to apply the same procedure but using Figure 4.7. For $V_{\text{Pvert}}$, Figure 4.8 should be used to find the percentage error in this parameter ($\Delta V_{\text{Pvert}}/V_{\text{Pvert}}$). For example, consider an azimuth combination of $\alpha_1 = 0^\circ$, $\alpha_2 = 60^\circ$, and $\alpha_3 = 120^\circ$ and a model with $V_{\text{Pvert}} = 2.0$ km/s and $\delta^{(V)} = -0.2$. From Figures 4.6a, 4.7a, and 4.8a, an error in the input NMO-velocity measurements of ±1.6% is expected to cause the following errors in the medium parameters: ±0.048 radians in $\alpha$ (±2.75°), ±0.02 in $\delta^{(V)}$, and ±2.2% in $V_{\text{Pvert}}$ (±0.044 km/s).

Overall, from Figures 4.6-4.8, we conclude that the best angular separation for the inversion procedure is 60°. All three parameters are sensitive to the angular separation between the survey lines, as well as to the set of azimuths used in the
Table 4.1. Sets of azimuth combinations used to generate the numerical results in Tables 4.2 and 4.3. The azimuths are measured from the symmetry-axis direction. An azimuth direction of 150° is the same as −30°.

### 4.2.3 Numerical inversion

The above analysis based on the Jacobian matrix is approximate since the NMO-velocity equation (2.10) is nonlinear. In this section, I perform nonlinear inversion by means of the Newton-Raphson method and study the sensitivity of the results to errors in the input information. I consider the same HTI models studied earlier, with $\delta^{(V)} = −0.1$ and $\delta^{(V)} = −0.2$, both having $V_{P_{\text{vert}}} = 2.0$ km/s. The test was performed for twelve different sets of three source-to-receiver azimuths (Table 4.1). After 100 trials with ±3 % range of random errors introduced into the exact NMO velocities, I obtain the results displayed in Table 4.2 for $\delta^{(V)} = −0.2$, and in Table 4.3...
for $\delta^{(V)} = -0.1$. It is important to mention, as discussed earlier, for each trial two different sets of solutions are obtained that satisfy the input NMO velocities. The correct solution is selected under the assumption that the parameter $\delta^{(V)}$ is typically negative. Note that a $\pm 3\%$ range of random errors (uniformly distributed) introduced into the exact NMO velocities corresponds to a standard deviation of about $1.6\%$.

Supporting the conclusions of the error analysis, the inversion results in Tables 4.2 and 4.3 demonstrate that the parameter $\alpha_1$ is better estimated as the absolute value of $\delta^{(V)}$ increases (the size of error reduction is almost linearly proportional to $\delta^{(V)}$). Comparing the inversion results for $\delta^{(V)}$ in Table 4.2 with their counterparts in Table 4.3, we observe that the improvement in the estimation is not significant with increase in $\delta^{(V)}$. Also, the errors in estimating $V_{Pvert}$ are somewhat smaller in the case of weaker anisotropy [e.g., compare set 9 in both tables (4.2, 4.3)].

As predicted by the covariance study, the smallest errors, measured by the mean and the standard deviation, are associated with sets of azimuths with maximum separation between the survey lines ($60^\circ$ apart), especially for the symmetry-axis direction $\alpha_1$ (sets 9-12 in Tables 4.2 and sets 9-10 in Table 4.3). An angular separation of $45^\circ$ provides an accuracy (e.g., sets 5-8) comparable to the one for the $60^\circ$ separation, especially for $\delta^{(V)}$ and $V_{Pvert}$. Note that the accuracy for $\alpha_1$ deteriorates when two of the selected azimuths happen to coincide with the symmetry planes of the medium (set 5 as opposed to set 7). As expected, the worst results correspond to narrow azimuthal separations (sets 1 and 2).

The variations in the mean and the size of standard deviation of the three parameters in Tables 4.2 and 4.3 indicate that all three parameters are sensitive to the angular separation between the survey lines and to the set of azimuths used in the
Table 4.2. Inversion results using twelve different sets of source-to-receiver azimuths given in Table 4.1. Here, $\delta^{(V)} = -0.2$ and $V_{P\text{vert}} = 2.0$ km/s.
Table 4.3. Inversion results using four different sets of source-to-receiver azimuths given in Table 4.1. Here, $\delta^{(V)} = -0.1$ and $V_{\text{vert}} = 2.0 \text{ km/s}$.

inversion (as was the case in the covariance matrix analysis).

4.3 The inverse problem in layered media

So far I have considered the inverse problem for a single homogeneous HTI layer. Generally, however, a fractured zone that may be characterized by the HTI symmetry is overlaid by an overburden that may be inhomogeneous and anisotropic. In this section the inverse problem is studied for a model with an azimuthally isotropic overburden (e.g., purely isotropic, VTI, or both) above the HTI layer. Once the interval NMO velocities in the HTI layer have been found by Dix differentiation of the moveout velocity from the top and bottom of the HTI layer, we can apply the single-layer inversion discussed above. The additional question to be addressed, however, is associated with the relative thickness of the HTI layer (compared with the total thickness) and its influence on the stability and accuracy of the parameter estimation.
Fig. 4.9. Schematic time section showing a model that contains an azimuthally isotropic overburden over an HTI layer. $\Delta t_N$ is the two-way vertical traveltime in the HTI layer. The total two-way traveltime to the bottom of the HTI layer is $T_N$, while the NMO (stacking) velocity at the top and bottom of the HTI layer is denoted as $V_{N-1}$ and $V_N$, respectively. The ratio $\Delta t_N/T_N = \rho$.

To set up the inverse problem, consider the model shown in Figure 4.9 where the NMO velocity from the bottom of the HTI layer is given by the Dix equation:

\[
V_N^2 = V_{N-1}^2(1 - \rho) + V_{nmo}^2\rho, \tag{4.5}
\]

with $V_{N-1}$ being the NMO velocity for a reflection from the top of the HTI layer, $V_{nmo}$ is the interval NMO velocity of the HTI layer given by equations (2.10) or (4.1), and $\rho (\Delta t_N/T_N)$ is the ratio of the two-way interval traveltime in the HTI layer ($\Delta t_N$) to the two-way total traveltime to the bottom of the HTI layer ($T_N$).

From equation (4.5), the interval NMO velocity for the HTI layer can be represented as

\[
V_{nmo}^2 = \frac{V_N^2 - V_{N-1}^2(1 - \rho)}{\rho}.
\]

Therefore, the interval NMO velocity in the HTI layer that will be estimated in the inversion process is dependent on the relative thickness of the layer $\rho$. This fact, which is well known from isotropic interval velocity analysis, influences the accuracy of the parameter estimation in the HTI layer. In order to gain more insight into this
inverse problem, I conduct the following error analysis.

4.3.1 Error analysis

To study the sensitivity of the effective NMO velocity to the anisotropic parameters of the HTI layer and the layer thickness, we evaluate the Jacobian of equation (4.5). The derivatives used to form the Jacobian are

\[ d_1(\alpha) = \frac{V_{N-1}}{V_N(\alpha)} \frac{\partial V_N(\alpha)}{\partial V_{N-1}}, \]

\[ d_2(\alpha) = \frac{V_{\text{vert}}}{V_N(\alpha)} \frac{\partial V_N(\alpha)}{\partial V_{\text{vert}}}, \]

\[ d_3(\alpha) = \frac{1}{V_N(\alpha)} \frac{\partial V_N(\alpha)}{\partial \delta^{(V)}}, \]

and

\[ d_4(\alpha) = \frac{1}{V_N(\alpha)} \frac{\partial V_N(\alpha)}{\partial \alpha}. \]

Again, the normalization of the derivatives simplifies the assessment of the relative importance of each parameter. Hence, the information provided by these derivatives consists of relative values for \( V_{N-1} \), and \( V_{\text{vert}} \), and absolute values for \( \delta^{(V)} \) and \( \alpha \) (measured in radian). Note that \( V_{N-1} \) is not an unknown (it is one of our measurements), but it is included in the Jacobian to simplify the analysis of the inverse problem.

The sensitivity of this inversion to errors in the input data (i.e., NMO velocity
at the top and bottom of the HTI layer) can be assessed from the Jacobian matrix

\[
J = \begin{pmatrix}
1 & 0 & 0 & 0 \\
d_1(\alpha_1) & d_2(\alpha_1) & d_3(\alpha_1) & d_4(\alpha_1) \\
d_1(\alpha_2) & d_2(\alpha_2) & d_3(\alpha_2) & d_4(\alpha_2) \\
d_1(\alpha_3) & d_2(\alpha_3) & d_3(\alpha_3) & d_4(\alpha_3)
\end{pmatrix},
\]

where \(\alpha_1, \alpha_2,\) and \(\alpha_3\) are the azimuths of the CMP gathers measured from the symmetry-axis of the HTI layer.

As shown above, to obtain maximum stability and accuracy, the best set of azimuths for use in the inversion process corresponds to the maximum angular separation (e.g., \(\alpha_1 = 0^\circ, \alpha_2 = 60^\circ,\) and \(\alpha_3 = 120^\circ\)). This conclusion remains valid here as well. Hence, in the upcoming tests I set the azimuths to \(\alpha_1 = 0^\circ, \alpha_2 = 60^\circ,\) and \(\alpha_3 = 120^\circ\) to concentrate on the dependence of the inversion results on \(\rho\) and \(\delta^{(V)}\).

\[
\begin{array}{c}
\text{Fig. 4.10. Plot of } k^{-1} \text{ as a function of } \rho \text{ for } V_{N-1} = 2.0 \text{ km/s, } V_{\text{vert}} = 3.0 \text{ km/s.} \\
\text{(a) correspond to } \delta^{(V)} = -0.2, \text{ while (b) correspond to } \delta^{(V)} = -0.1. \text{ The azimuths are } \\
\alpha_1 = 0^\circ, \alpha_2 = 60^\circ, \text{ and } \alpha_3 = 120^\circ.
\end{array}
\]

The stability of the inverse problem for this azimuthal separation, measured using the reciprocal of the condition number [equation (4.2)], is linearly proportional to the layer thickness ratio (\(\rho\)) for \(\rho < 0.4\) (Figure 4.10). For \(\rho > 0.4, k^{-1}\) flattens out, as seen in Figure 4.10. Also, as in the homogeneous case, the stability increases
Fig. 4.11. Magnification factors in the absolute error in $\alpha$ (a) measured in radians, the absolute error in $\delta^{(V)}$ (b), and the relative error in $V_{P\text{vert}}$ (c) (diagonal elements of $[J^TJ]^{-1/2}$) as a function of the layer-thickness ratio, $\rho$. The selected survey-line azimuths are: $\alpha_1 = 0^\circ$, $\alpha_2 = 60^\circ$, and $\alpha_3 = 120^\circ$. The model parameters are $V_{N-1} = 2.0$ km/s, $V_{P\text{vert}} = 3.0$ km/s, and $\delta^{(V)} = -0.2$.

(approximately linearly) with an increase in the absolute value of $\delta^{(V)}$ (compare Figure 4.10a with Figure 4.10b). As we expect, for small thickness ratios (e.g., $\rho < 0.1$) the inverse problem is ill-conditioned.

To study the propagation of error (standard deviation) into the parameters of the HTI layer as a function of the thickness ratio $\rho$ for a given error in the input measurements (NMO velocity for reflections from the top and bottom of the HTI layer), I compute the covariance matrix [equation (4.4)] for the Jacobian matrix $J$ of this inverse problem. The assumptions here are the same as that for the homogeneous case discussed earlier. Setting the variance of the input NMO-velocity measurements to unity means that the square-root of the covariance represents the magnification of error (standard deviation) in each parameter for any given error (standard deviation) in the input NMO-velocity. Figure 4.11 shows the error magnification factors (the square-root of the diagonal elements of $[J^TJ]^{-1/2}$) as a function of $\rho$: (a) magnification factor in the absolute error in $\alpha$ ($M_\alpha$) measured in radians, (b) magnification factor in the absolute error in $\delta^{(V)}$ ($M_{\delta^{(V)}}$), and (c) magnification factor in the percentage error in $V_{P\text{vert}}$ ($M_{V_{P\text{vert}}}$). As seen in Figure 4.11, the magnification of error in each parameter is proportional to $1/\rho$. Thus, for $\rho < 0.4$ the resolution, measured by
the square-root of the variance (i.e., standard deviation) in Figure 4.11, improves significantly (linearly) as \( p \) increases. For \( p > 0.4 \) the resolution remains almost the same, which is consistent with the results obtained from the reciprocal of the condition number.

### 4.3.2 Numerical inversion in layered media

In this section I perform the nonlinear inversion of NMO velocity in layered media by means of the Newton-Raphson method (as for the single-layer model) and study the sensitivity of the results to errors in the input information (NMO velocity at the top and bottom of the HTI layer) as a function of \( p \). Consider two HTI models, with \( \delta^{(V)} = -0.1 \) and \( \delta^{(V)} = -0.2 \), both of which have \( V_{\text{vert}} = 3.0 \) km/s; the NMO velocity at the top of the HTI layer \( V_{N-1} = 2.0 \) km/s. From the study of the single-layer model, azimuthal separation of 60° between the survey azimuths, in general, produces the best inversion results. Let us select here \( \alpha_1 = 0°, \alpha_2 = 60°, \) and \( \alpha_3 = 120° \) to be the three source-to-receiver azimuths used in the inversion process. From 100 trials with \( \pm 3 \% \) range of random error introduced into the exact NMO velocities at the top and the bottom of the HTI layer, we obtain the perturbed interval NMO velocities (using Dix differentiation), which are then used to estimate the parameters of the HTI layer. The solutions (the mean and standard deviation) as a function of \( p \) are displayed in Figure 4.12 for \( \delta^{(V)} = -0.2 \) (a, b, and c) and for \( \delta^{(V)} = -0.1 \) (d, e, and f). As in the homogeneous case, the solutions are obtained under the assumption that the parameter \( \delta^{(V)} \) is negative.

Consistent with the study of the covariance matrix, Figure 4.12 shows that the errors in the estimates do not change much for \( p > 0.4 \), and vary significantly (almost linearly) with \( p \) for small \( p \). In other words, for small thickness ratios (\( p < 0.4 \), the
FIG. 4.12. Estimated $V_{P\text{vert}}$, $\delta^V$, and the azimuth of the symmetry-axis ($\alpha_1$), as well as the associated error bars as functions of $\rho$. The plots in the left column correspond to $\delta^{(V)} = -0.2$, while those in the right correspond to $\delta^{(V)} = -0.1$. $V_{N-1} = 2.0$ km/s, $V_{P\text{vert}} = 3.0$ km/s. The azimuths are $\alpha_1 = 0^\circ$, $\alpha_2 = 60^\circ$, and $\alpha_3 = 120^\circ$. The black dots and error bars represent the computed mean and standard deviation, respectively. The solutions for $V_{P\text{vert}}$ are normalized by the true vertical velocity (3.0 km/s).
error in the parameter estimation is magnified by the factor $1/\rho$ compared to the error in the effective NMO velocity. This fact may also be directly extracted from the Dix equation.

The parameter $\alpha_1$ is better resolved for higher absolute values of $\delta^{(V)}$ (Figure 4.12a,d) (e.g., the error bars for $\alpha_1$ become twice as large when $\delta^{(V)}$ changes from -0.2 to -0.1). The improvement in resolution for the parameter $\delta^{(V)}$ with increasing $\delta^{(V)}$ is not as dramatic as that in $\alpha_1$ (Figure 4.12b,e). As in the single-layer model, the absolute error measured by the error bars is almost the same for both values of $\delta^{(V)}$; therefore, the relative error is smaller for stronger anisotropy. The resolution for $V_{p_{\text{vert}}}$, as in the single-layer case, slightly improves for weaker anisotropy (Figure 4.12c,f).

For typical errors in the estimated NMO (stacking) velocity ($\leq 2\%$), as is the case in this numerical study, the minimum relative thickness (in time) of the HTI layer to the total thickness ($\rho$) needed to resolve the three parameters with acceptable accuracy is at least about 0.2 (Figure 4.12). As the strength of anisotropy in the HTI layer increases or the velocity contrast between the overburden and the HTI layer becomes larger, we may obtain acceptable medium parameter estimations for smaller values of $\rho$. It is important to mention, however, that parameter estimation may become unstable for an HTI layer with weak azimuthal anisotropy ($|\delta^{(V)}| < 0.1$ which corresponds to azimuthal interval-NMO-velocity variation $< 10\%$), especially for small values of $\rho$ ($\rho < 0.2$). The significant deviation of the mean from the true solution in Figure (4.12b,c,e, and f) for $\rho < 0.2$ could be interpreted as a direct indication of instability in the inversion process. The apparent bias in the solutions (the mean values) of $\delta^{(V)}$ and $V_{p_{\text{vert}}}$ for $\rho < 0.2$ is caused by the combined influence of the assumption that the solutions should have negative $\delta^{(V)}$ and the presence of
highly inaccurate estimates of the perturbed interval NMO velocities for small $\rho$ (from Dix differentiation). Actually, as a result of Dix differentiation, the perturbations in the interval NMO velocities for small $\rho$ (e.g., $\leq 0.1$) exceed the absolute differences between the exact interval NMO velocities. In this case, even a single erroneous value of the interval NMO velocity that is much higher than the actual value makes the inverted vertical velocity to be larger than the exact $V_{\text{vert}}$. Consequently, the computation of the mean value will be influenced. Notice that improper estimates of $\delta^{(V)}$ cause $V_{\text{vert}}$ estimates to be erroneous as well, since the combined solutions should satisfy the input interval NMO velocities.

4.4 Synthetic data examples

In horizontally stratified HTI media, Dix differentiation is sufficiently accurate for obtaining the interval NMO velocity. Still, we have to bear in mind that the layer not only has to be thick enough to ensure an acceptable accuracy in the interval velocity evaluation and parameter estimation (i.e., $\rho \geq 0.2$), but also the azimuthal variation of the interval velocity should be pronounced as well ($\geq 10\%$).

To illustrate this conclusion on synthetic data, consider a three-layer HTI model with a uniform symmetry-axis direction (that coincides with the $x$-axis) shown in Figure 4.13 (Model 5 in Chapter 3). Conventionally, stacking velocity is estimated using semblance (coherency) analysis. Here, however, NMO velocity is estimated by fitting a hyperbola to the traveltime curve in a least-squares sense. Thus, NMO (stacking) velocity is obtained from equation (3.1):

$$V_{\text{mo}}^2 = \frac{\sum_{j=1}^{N} x_j^2}{\sum_{j=1}^{N} t_j^2 - N t_0^2},$$

where $x_j$ is the offset of the $j$-th trace, $t_j$ is the corresponding two-way reflection
FIG. 4.13. Survey lines over a three-HTI-layer model with a uniform symmetry-axis
direction in all layers.

traveltime, $t_0$ is the two-way vertical traveltime, and $N$ is the number of traces. In
the least-squares fitting procedure, the vertical time $t_0$ again was fixed at the correct
value.

Applying Dix differentiation, we obtain an estimate of the interval $V_{nmo}$ for each
layer as a function of azimuth (Figure 4.14). Even though estimating the effective
NMO velocity on finite spreads, in general, introduces errors due to the influence of
nonhyperbolic moveout, we obtain relatively accurate values for both the effective as
well as the interval NMO velocity for all azimuths (Figure 4.14). Since $\delta^{(V)} < 0$, the
maximum interval NMO velocity in Figure 4.14 corresponds to the fracture orientation
(isotropy plane), while the minimum is observed in the symmetry-axis direction. If
$\delta^{(V)} > 0$ (which is not likely in HTI media), the minimum interval NMO velocity
would correspond to the isotropy plane.
Fig. 4.14. Estimation of interval NMO velocities from synthetic data. The plots on the left show the azimuthal dependence of the effective NMO (stacking) velocity obtained from the conventional-spread reflection moveout for each reflector on the spreadlength equal to the target depth (1.5 km). The plots on the right show the interval NMO velocity (solid curves) as a function of azimuth computed for each layer using Dix differentiation, and the exact interval NMO velocity (dashed curves) from equation (2.10). The model geometry and parameters are shown in Figure 4.13. The maximum interval NMO velocity corresponds to the isotropy-plane direction (crack orientation), while the minimum corresponds to the symmetry-axis direction.
From equations (2.10) or (4.1), the interval $\delta^{(V)}$ can be obtained from the interval NMO velocities in the two symmetry planes as

$$\delta^{(V)} = \frac{1}{2} \left( \frac{V_{nmo_{min}}}{V_{nmo_{max}}} \right)^2 - \frac{1}{2},$$

where $V_{nmo_{min}}$ and $V_{nmo_{max}}$ are the estimated interval NMO velocities along the symmetry-axis direction and in the isotropy plane, respectively, and $\delta^{(V)}$ is assumed to be negative. Note that for positive values of $\delta^{(V)}$, $\delta^{(V)} = \frac{1}{2} \left( \frac{V_{nmo_{max}}}{V_{nmo_{min}}} \right)^2 - \frac{1}{2}$; in this case, the minimum NMO velocity, as mentioned earlier, corresponds to the isotropy plane.

Using equation (4.6) and the interval NMO velocities for the three layers in Figure 4.14, we obtain the following interval $\delta^{(V)}$ estimates for each layer: -0.17, -0.18, and -0.32, respectively, as compared with the true values of -0.184, -0.203, and -0.318.

Figure 4.15 shows the results of the same test for a different model, common for fractured reservoirs, which contains an isotropic overburden with a large velocity gradient ($1 \text{ s}^{-1}$) overlaying an HTI layer (the same model used to generate Figure 3.12). As in the previous test, the azimuthal NMO-velocity variation for reflections from the bottom of the HTI layer allows a direct detection of the symmetry planes of the HTI medium. The estimated $\delta^{(V)}$ for the HTI layer using equation (4.6) and the interval NMO velocities in Figure 4.15 is -0.31 as compared with the correct value of -0.32.

It is important to mention that the NMO-velocity curves in Figures 4.14 and 4.15 are ellipses in the horizontal plane, with the semi-axes coinciding with the NMO velocities in the symmetry planes (this latter feature is valid only if the symmetry-axis direction is uniform throughout the media or is rotated by $\pm 90^\circ$ from layer to layer). Although Dix differentiation outside the symmetry planes is approximate, it provides sufficient accuracy in obtaining NMO velocity for any azimuth. The
Fig. 4.15. Estimation of interval NMO velocities from synthetic data. (a) shows the azimuthal dependence of the effective NMO (stacking) velocity obtained from the conventional-spread reflection moveout from the bottom of the HTI layer on the spreadlength equal to the target depth (1.5 km). (b) shows the interval NMO velocity (solid curves) as a function of azimuth computed for the HTI layer using Dix differentiation, and the exact interval NMO velocity (dashed curves) from equation (2.10). The model consists of five isotropic layers overlaying an HTI layer. The isotropic layers have an equal thickness of 0.2 km with P-wave velocities of 2.0, 2.5, 3.0, 3.5, and 2.5 km/s, respectively. The parameters of the HTI layer are the same as those of the third layer in Figure 4.13. The effective NMO velocity at the top of the HTI layer, estimated from reflection moveout, is 2.67 km/s.

difference between the exact NMO velocity (constructed from the NMO velocities along the symmetry planes directions) and the results of the Dix averaging at each azimuth for typical HTI models is small (≤ 1%).

Even if the symmetry-axis direction varies with depth, rms averaging of the interval NMO velocity provides acceptable accuracy in calculating the effective NMO velocity (see Chapter 3). Due to the amplification of errors in Dix differentiation, however, interval-velocity estimation in such media is less accurate than in the previous examples (where the symmetry-axis direction is uniform), especially if the variation in the symmetry-axis direction from one layer to another is large (e.g., layer 2 in Figure 4.16). Notice that for the second layer in Figure 4.16, the estimated interval NMO velocity has a minimum at about 70° azimuth, which is 10° off from the direction of the symmetry axis. Considering the abrupt and significant symmetry-axis
Fig. 4.16. Estimation of interval NMO velocities from synthetic data for a model with depth-varying symmetry-axis direction. The plots on the left show the azimuthal dependence of the effective NMO (stacking) velocity obtained from the conventional-spread reflection moveout for each reflector on the spreadlength equal to the target depth (1.5 km). The plots on the right show the interval NMO velocity (solid curves) as a function of azimuth computed for each layer using Dix differentiation, and the exact interval NMO velocity (dashed curves) from equation (2.10). The model geometry and parameters are the same as in Figure 4.13; however, the symmetry-axis of the second layer is rotated by 60° with respect to the axis direction in the first layer.
rotation from one layer to another, such an error could be expected. It is important to mention that the large difference between the exact and the estimated interval velocity in the second layer in Figure 4.16 (about 8%) is due to the influence of the nonhyperbolic moveout ($X/D = 1.5$) in addition to the error in the Dix differentiation. Muting the nonhyperbolic portion of the moveout curve reduces this error in the estimated interval velocity to a more acceptable value. For a smoother variation in the symmetry-axis direction with depth, the estimated interval NMO velocity is closer to the exact value (Figure 4.17).

![Graphs showing effective NMO and interval NMO velocities](image)

**Fig. 4.17.** Estimation of interval NMO velocities from synthetic data for a model with depth-varying symmetry-axis direction. (a) shows the azimuthal dependence of the effective NMO (stacking) velocity obtained from the conventional-spread reflection moveout for the second reflector on a spreadlength equal to the target depth (1.5 km). (b) shows the interval NMO velocity (solid curves) as a function of azimuth computed for the second layer using Dix differentiation, and the exact interval NMO velocity (dashed curves) from equation (2.10). The model geometry and parameters are the same as in Figure 4.13; however, the symmetry-axis of the second layer is rotated by 30° with respect to the axis direction in the first layer.

It is important to emphasize again that, as we have seen in the previous sections, three NMO velocity measurements along three distinct survey lines (in 2-D case or, equivalently, azimuth orientations in 3-D acquisition) with angular separation of 60° or even 45°, are sufficient to perform the inversion procedure. Coverage along other directions adds some redundancy which may become useful in enhancing the quality
(reduce ambiguity) of the inversion process.

Issues such as the presence of coherent noise, the noise level in the seismic data, structure, and fold influence the estimation of NMO velocity from the reflection move-out. Moreover, constructing a CMP gather with any specific azimuthal direction in 3-D acquisition survey generally requires collecting (sorting) traces from a range of azimuths (offsets), and the residual differences between the seismic traces influence the accuracy in the NMO-velocity estimation (it is similar to the influence of residual statics). Consequently, the accuracy in estimating the medium parameters will be affected as well. Those issues, however, are quite common in 3-D surface seismic surveys and can be handled in the processing stage.

4.5 $\epsilon^{(V)}$ and crack density estimation using $P$-wave nonhyperbolic reflection moveout

An important parameter in the characterization of fractured reservoirs is the crack density $\zeta$, which is proportional to the product of the number of cracks per unit volume and their mean cubed diameter (Tsvankin, 1996b). Although all three generic Thomsen anisotropic coefficients ($\epsilon$, $\delta$, $\gamma$) are dependent on $\zeta$, the parameter most directly related to the crack density is $\gamma$, which governs the degree of shear-wave splitting at vertical incidence. For parallel, circular ellipsoidal (penny-shaped) cracks distributed in a porous isotropic rock, $\gamma$ is given by (Thomsen, 1995; Tsvankin, 1996b)

$$
\gamma = \frac{8}{3} \frac{1 - P}{2 - P} \zeta,
$$

where $P$ is the Poisson's ratio of the dry isotropic porous medium.
For typical values of the Poisson's ratio the coefficient \(8(1 - P)/[3(2 - P)]\) is close to unity, and \(\gamma \approx \zeta\). Therefore, for penny-shaped cracks, measurements of \(\gamma\) provide a good direct estimate of the crack density (Tsvankin, 1996b).

The parameter \(\gamma\) in HTI media is usually obtained directly from the fractional difference between the vertical \(S^\perp\) and \(S^\parallel\) velocities using the traveltimes of split shear waves at vertical incidence (e.g., Crampin, 1985; Thomsen, 1988; Tsvankin, 1996b).

Using the constraint on the elastic constants for HTI media due to thin-parallel cracks, the shear wave splitting parameter \(\gamma\) can be represented as (Tsvankin, 1996b)

\[
\gamma = \frac{V_{p,\text{vert}}^2}{2V_{S^\perp,\text{vert}}^2} \left[ \frac{\epsilon^{(V)}[2 - 1/f^{(V)}] - \delta^{(V)}}{1 + 2\epsilon^{(V)}/f^{(V)} + \sqrt{1 + 2\delta^{(V)}/f^{(V)}}} \right].
\]

(4.7)

The \(P\)-wave NMO velocity from horizontal reflectors can provide us with the parameters \(V_{P,\text{vert}}, \delta^{(V)}\), and the orientations of the symmetry planes. However, equation (4.7) also requires the knowledge of the anisotropy parameter \(\epsilon^{(V)}\), as well as the ratio \((V_{S^\perp,\text{vert}}/V_{P,\text{vert}})\). In general, we may obtain \(V_{S^\perp,\text{vert}}\) from other information (e.g., shear data, well logs, etc.), or simply, as suggested by Tsvankin (1996b), we may set the ratio to a typical value (i.e., 0.6) without losing much accuracy.

\(\epsilon^{(V)}\), on the other hand, may be obtained from \(P\)-wave reflection moveout data and horizontal reflectors by applying nonhyperbolic moveout analysis. Therefore, estimation of \(\epsilon^{(V)}\) is expected to be subject to more uncertainty than are \(V_{P,\text{vert}}, \delta^{(V)}\), and \(\alpha\). Since the kinematics of wave propagation in the symmetry-axis plane is identical to that in VTI media, we can use known VTI moveout analysis methods (e.g., Alkhalifah, 1996; Bougeant and Pratt, 1996) in HTI media as well. Hence, the effective coefficients of the long-spread reflection moveout equation (2.5) (i.e., the effective NMO velocity, the effective horizontal velocity, and the quartic coefficient \(A_4\)) could be estimated. After applying Dix-type differentiation to extract the interval
values of the moveout parameters, the interval anisotropic parameter $\epsilon^{(V)}$ can be found either from the quartic coefficient [equation (2.13)] or simply from the ratio of the interval horizontal velocity to the interval vertical velocity

$$
\epsilon^{(V)} = \frac{1}{2}[(V_{P\text{vert}}/V_{\text{hor}})^2 - 1].
$$  \tag{4.8}

As an example, synthetic traveltime data for the model in Figure 4.13 were used to estimate the effective coefficients of the long-spread reflection moveout equation (2.5). The method used here is a three-parameter fitting (in a least-squares sense) to fit the traveltime curve from each reflector. Even though three-parameter fitting is not a practical method to apply in data processing, it is sufficient for the purpose of this illustration. A more practical and stable method to use is to apply a nonhyperbolic semblance analysis to estimate the NMO velocity and the horizontal velocity simultaneously, as demonstrated by Alkhalifah (1996). The interval $V_{P\text{vert}}$ is then estimated from the azimuthal variation of the interval NMO velocity, as explained previously. The interval horizontal velocity in the symmetry-axis plane direction is obtained by Dix differentiation. Using equation (4.8), the estimated $\epsilon^{(V)}$ values for layer 1, 2, and 3 are $-0.143$, $-0.033$, and $-0.142$, respectively, as compared with the true values of $-0.143$, $-0.045$, and $-0.143$.

It is important to emphasize that equation (4.8) is accurate only in the symmetry-axis plane. In principle, outside the symmetry-axis plane direction, the azimuthal dependence of the quartic moveout coefficient $A_4$ [equation (2.13)] may be used to estimate $\epsilon^{(V)}$. The stability and accuracy of such inversion is beyond the scope of this thesis. The issues involving the feasibility of using the nonhyperbolic portion of the reflection moveout along the symmetry-axis plane in HTI media are the same as those for VTI and have been discussed in detail by Alkhalifah (1996) and Grechka.
Estimating $\varepsilon^{(V)}$ in addition to $\delta^{(V)}$ and $V_{P\text{vert}}$ from $P$-wave reflection moveout data allows a qualitative estimate of the crack density ($\zeta$) [equation (4.7)]. Therefore, the azimuthal dependence of $P$-wave reflection moveout not only provides a tool to detect the crack orientation but it also allows the estimation of the crack density.
Chapter 5

CONCLUSIONS

I have presented an analytic description of nonhyperbolic (long-spread) reflection moveout in layered transversely isotropic media with a horizontal symmetry axis. For a single-layer HTI model, the hyperbolic moveout equation parameterized by the exact NMO velocity given in Tsvankin (1996b) provides a good approximation for $P$-wave traveltimes on conventional-length CMP spreads (close to the reflector depth). However, the accuracy of the hyperbolic equation rapidly decreases with offset due to the influence of anisotropy-induced nonhyperbolic moveout.

The treatment of long-spread moveout is based on an exact expression for the azimuthally-dependent quartic moveout coefficient $A_4$, which has been derived for any pure mode in a homogeneous HTI layer with arbitrary strength of anisotropy. The expression for $A_4$ has an extremely simple form, with a single trigonometric function ($\cos^4 \alpha$, where $\alpha$ is the angle with the symmetry axis) multiplied with the quartic coefficient in the symmetry-axis plane. Therefore, the magnitude of nonhyperbolic moveout rapidly decreases with azimuth away from the symmetry-axis plane, where it can be obtained by analogy with VTI media. To account for deviations from hyperbolic moveout on long spreads (2-3 times as large as the reflector depth), I have substituted the exact azimuthally-dependent values of the NMO velocity and the quartic moveout coefficient into the nonhyperbolic moveout equation originally developed by Tsvankin and Thomsen (1994) for VTI media. Synthetic examples show that this equation provides excellent accuracy for $P$-waves recorded in all azimuthal
directions over an HTI layer, even for models with significant velocity anisotropy and pronounced nonhyperbolic moveout.

In multilayered HTI media, the moveout coefficients reflect the combined influence of layering, and azimuthal anisotropy. By extending the generalized Dix (1955) equation of Alkhalifah and Tsvankin (1995) to off-symmetry directions in azimuthally anisotropic media, I show that the NMO velocity in a stack of horizontal HTI layers is given by the conventional rms averaging procedure if the group-velocity vector (ray) is confined to the incidence plane. Although in vertically inhomogeneous HTI media the rays do diverge from the incidence plane on off-symmetry CMP lines, the magnitude of these deviations usually is not sufficient to cause measurable errors with use of the Dix equation, especially for models with a similar character of the azimuthal velocity variations in all layers (e.g., media with uniform orientation of cracks). To determine the quartic moveout coefficient $A_4$ in stratified HTI media, I use the same averaging equations as for vertical transverse isotropy (Hake et al., 1984; Tsvankin and Thomsen, 1994), but with the exact interval values of $V_{nmo}$ and $A_4$ in each HTI layer. For layered HTI media with a typical vertical velocity gradient (0.5–0.8 s$^{-1}$), the effective horizontal velocity should be computed using fourth-power averaging of the interval values. The NMO velocity, the quartic moveout coefficient, and the horizontal velocity, all averaged over the stack of layers above the reflector, are used in the same nonhyperbolic moveout equation as that for the single-layer model. Extensive numerical testing for stratified HTI media with both uniform and depth-varying orientation of the symmetry axis, as well as for models composed of HTI and VTI, isotropic and HTI layers, demonstrates high accuracy of this nonhyperbolic approximation in the description of long-spread reflection moveout. In addition to providing analytic insight into the behavior of reflection moveout, these results can be used in modeling
of reflection traveltimes in general TI (azimuthally isotropic and anisotropic) media. Also, we can expect that the same approach can be used to describe reflection moveout (conventional and on long spreads) in more complicated anisotropic media such as orthorhombic.

Also, I have discussed moveout inversion of P-wave reflection moveout for horizontal transverse isotropy. NMO-velocity measurements obtained for three distinct survey azimuths are sufficient to invert for the three medium parameters ($V_{P_{\text{vert}}}$, $\delta^{(V)}$, and $\alpha$), under the assumption that $\delta^{(V)}$ is typically negative. Without this assumption, additional information, such as the azimuthal variation in nonhyperbolic moveout, is needed to distinguish between the symmetry-axis and the isotropy planes. For HTI models due to parallel penny-shaped cracks, the symmetry axis is normal to the crack plane. Hence, the azimuthal dependence of reflection moveout in HTI media makes it possible to detect the crack orientation. Furthermore, the nonhyperbolic portion of the reflection moveout also allows estimation of the coefficient $c^{(V)}$ needed to find the crack density.

The accuracy in estimating the parameter $\alpha$ is sensitive to the strength of anisotropy (the reduction of error in $\alpha$ estimates is linearly proportional to the strength of anisotropy measured by $\delta^{(V)}$). The accuracy in resolving $\delta^{(V)}$ is about the same for any strength of the anisotropy (there is a slight improvement in the accuracy with increasing $|\delta^{(V)}|$). This implies that we should expect almost the same absolute error in $\delta^{(V)}$ for a wide range of $\delta^{(V)}$, and the relative error in $\delta^{(V)}$ will be smaller for stronger anisotropy. In contrast, the accuracy in estimating $V_{P_{\text{vert}}}$ increases as anisotropy becomes weaker.

Parameter estimation is quite sensitive to the angular separation between the survey lines, and to the set of azimuths used in the inversion procedure. In order to
maximize resolution and stability in estimating the medium parameters, especially the orientation of the symmetry-axis ($\alpha$), it is best to have the three azimuths 60° apart. However, an angular separation of 45° between the azimuths also provides adequate inversion results. For an angular separation of 60°, the accuracy in resolving the parameters is consistent at all azimuths and yields minimum errors for most ranges of azimuths.

Therefore, in 3-D land acquisition surveys where the acquisition is relatively flexible, it is necessary to have azimuthal coverage along those directions (60° apart). Coverage along other directions will add redundancy which may become useful in enhancing the quality (reduce ambiguity) of the inversion process. Constructing a CMP gather for a specific azimuthal direction in 3-D acquisition survey, however, requires collecting (sorting) traces from a range of azimuths (sectors). The residual differences between the seismic traces within the sector influence the accuracy in NMO-velocity estimation (it is similar to the influence of residual statics). Those issues, however, are quite common in 3-D surface seismic surveys and can be handled in the processing stage. In conventional marine surveys, the azimuthal coverage is quite limited. Therefore, in order to obtain the required coverage along the optimal azimuth directions (60° apart), the receiver lines (streamers) should be carried along those directions. Other acquisition issues such as the bin size, line spacing, receiver interval, fold, etc, are the same as those in any typical seismic survey (see Stone, 1994). Moreover, issues such as the presence of coherent noise, signal-to-noise (S/N) ratio, lateral velocity heterogeneity, and structure influence the estimation of NMO velocity from reflection moveout and the inversion for medium parameters.

If the orientation of the symmetry axis is known, then two NMO-velocity measurements (two survey azimuths) are sufficient to invert for $V_{P_{\text{vert}}}$ and $\delta^{(V)}$. Since
normal-moveout velocity has a nonlinear dependence on the azimuthal angle $\alpha$ ($\sin^2 \alpha$), at least two NMO velocity observations are necessary to identify the orientation of the symmetry-axis using known values of $V_{p\text{vert}}$ and $\delta^{(V)}$. In general, having more than three distinct source-to-receiver azimuths (e.g., full azimuthal coverage) provides a useful data redundancy that enhances the quality of the estimates.

In horizontally-stratified HTI media, interval NMO velocities can be obtained with sufficient accuracy using Dix differentiation, especially if the crack orientation is uniform throughout the media. In this case, we can construct the elliptical dependence of the interval NMO velocity just by finding the interval NMO velocities in the symmetry planes (the semi-axes of the ellipse). Where the symmetry-axis varies with depth, the accuracy in the interval NMO velocity and, consequently, in the interval medium parameters becomes lower.

An HTI layer overlain by an azimuthally isotropic overburden (a typical case in fractured reservoirs) should have a relative thickness (in time) with respect to the total thickness of at least 0.2 in order to obtain acceptable estimates of the medium parameters, provided that the azimuthal variation in the interval NMO velocity within the HTI layer is about 10% or higher.

Some of the conclusions made here for HTI media, such as the optimal set of azimuths to be used for the inversion, and the minimum thickness of the layer required for reliable estimates of the medium parameters, are also valid for more complicated azimuthally anisotropic media (e.g., orthorhombic), as well as for pure $S$–wave propagation. Even though the analytic description of the azimuthal dependence of the NMO velocity and of the quartic coefficient ($A_4$) have been given for horizontal reflectors, we should expect the conclusions to remain valid for gentle dips (i.e., $\leq 10^\circ$), since the dip-induced variation in moveout velocity for these dips is small (i.e., $\leq 2\%$).
Although the numerical and synthetic applications presented in this thesis concentrate on \( P \)-wave reflection moveout for horizontal transverse isotropy, the equations for the interval NMO velocity and the quartic coefficient are valid for other pure modes of wave propagation as well (i.e., shear-wave). Therefore, the next step would be to test the accuracy of the moveout equations and Dix-type averaging for \( S \)-wave propagation in horizontally-stratified HTI media.

The earth model for fractured reservoirs often is more complicated than the HTI model. Therefore, it will be important to extend the analytical description of reflection moveout (for short and long spreadlengths) to more general azimuthally anisotropic media such as orthorhombic. Ultimately, the conclusions presented here need to be tested on real data sets, which will require to study the influence of coherent noise, signal-to-noise ratio, structure, and fold on medium parameter estimation.

Finally, it is important to emphasize the advantages of integrating this methodology with other seismic exploration techniques, such as azimuthal amplitude-variation-with-offset analysis and borehole information to reduce the ambiguity in the estimation of the medium parameters.
REFERENCES


Tsvankin, I., 1996a, P-wave signatures and notation for transversely isotropic media: Geophysics, 61, 467-483.


Appendix A

TWO SETS OF THOMSEN PARAMETERS FOR HTI MEDIA

The HTI model is usually characterized by the stiffness tensor $c_{ijkl}$ corresponding to the coordinate frame in which $x_1$ represents the symmetry axis (Figure 2.1):

$$C_{HTI} = \begin{pmatrix}
    c_{11} & c_{13} & c_{13} & 0 & 0 & 0 \\
    c_{13} & c_{33} & c_{33} - 2c_{44} & 0 & 0 & 0 \\
    c_{13} & c_{33} - 2c_{44} & c_{33} & 0 & 0 & 0 \\
    0 & 0 & 0 & c_{44} & 0 & 0 \\
    0 & 0 & 0 & 0 & c_{55} & 0 \\
    0 & 0 & 0 & 0 & 0 & c_{55}
\end{pmatrix}$$

The “generic” Thomsen’s (1986) parameters defined with respect to the symmetry axis are given by (Tsvankin, 1996b; Rüger, 1996),

$$\epsilon = \frac{c_{33} - c_{11}}{2c_{11}},$$
$$\delta = \frac{(c_{13} + c_{55})^2 - (c_{11} - c_{55})^2}{2c_{11}(c_{11} - c_{55})},$$
$$\gamma = \frac{c_{44} - c_{55}}{2c_{55}},$$
$$V_{P0} = \sqrt{\frac{c_{11}}{\rho}},$$
$$V_{S\perp} = V_{S\parallel} = \sqrt{\frac{c_{55}}{\rho}},$$

(A.1)
where \( \rho \) is the medium density, and the velocities \( V_{P0} \) (\( P \)-wave), \( V_{S\perp} \), and \( V_{S\parallel} \) (\( S \)-waves) correspond to the symmetry-axis direction.

The parameters of the equivalent VTI medium introduced in equation (2.1) of the main text are related to the generic coefficients in the following way (Tsvankin, 1996b; Rüger, 1996):

\[
\begin{align*}
\epsilon^{(V)} &= -\frac{\epsilon}{1 + 2\epsilon}, \\
\delta^{(V)} &= \frac{\delta - 2\epsilon (1 + \epsilon/f)}{(1 + 2\epsilon)(1 + 2\epsilon/f)}, \\
\gamma^{(V)} &= \frac{\gamma}{1 + 2\gamma}, \\
V_{P_{\text{vert}}} &= V_{P0}\sqrt{1 + 2\epsilon}, \\
V_{S\perp_{\text{vert}}} &= V_{S\perp}, \\
V_{S\parallel_{\text{vert}}} &= V_{S\parallel}\sqrt{1 + 2\gamma},
\end{align*}
\]

(A.2)

where \( f = 1 - \left(\frac{V_{S\perp}}{V_{P0}}\right)^2 \).

The exact expression for the phase velocity of the \( P \)- and \( S\perp \)-waves in terms of the generic Thomsen parameters was presented in Tsvankin (1996a):

\[
\frac{V^2(\theta)}{V_{P0}^2} = 1 + \epsilon \sin^2 \theta - \frac{f}{2} \pm \frac{f}{2} \sqrt{\left(1 + \frac{2\epsilon \sin^2 \theta}{f}\right)^2 - \frac{2(\epsilon - \delta) \sin 2\theta}{f}},
\]

(A.3)

where the plus sign corresponds to the \( P \)-wave, and the minus sign to the \( S\perp \)-wave; \( \theta \) is the phase angle with the (horizontal) symmetry axis. Phase-velocity equation (A.3) is rewritten in terms of the parameters of the equivalent VTI model in the main text [equation (2.2)].
For the $S^{\parallel}$-wave, the anisotropy is elliptical, and the phase velocity is given by (Tsvankin 1996b),

\[ V(\theta)[S^{\parallel}-\text{wave}] = V_{S^{\parallel}_{\text{vert}}} \sqrt{1 + 2\gamma^{(V)} \cos^2 \theta}. \]  

(A.4)
Appendix B

QUARTIC MOVEOUT COEFFICIENT IN HTI MEDIA

Here, the approach suggested by Tsvankin (1996b) in his derivation of the NMO velocity in HTI media is extended to obtain the quartic coefficient $A_4$ in the Taylor series expansion of the squared traveltime $[t^2(x^2)]$. First, I find an expression for $A_4$ in terms of the one-way traveltime from the zero-offset reflection point. Since a horizontal reflector coincides with a symmetry plane in HTI media, the group-velocity (ray) vector of any pure reflected wave in an HTI layer represents the mirror image of the incident ray with respect to the horizontal plane (Tsvankin, 1996b; see Figures B.1 and B.2). This means that there is no reflection-point dispersal on CMP gathers above a homogeneous HTI layer, and I can represent the two-way traveltime along the specular raypath as the sum of the traveltimes from the zero-offset reflection point to the source and receiver (Figure B.1). The one-way traveltime from the reflection point to the source or receiver can be expanded in a Taylor series in powers of the half-offset $h$, as suggested by Hale et al. (1992) in their derivation of the normal-moveout velocity from dipping reflectors. Here, I am interested in deriving the quartic moveout coefficient, so I will keep the quartic and lower-order terms in the Taylor series,

$$
\begin{align*}
    t(y + h) &= t(y) + \frac{d t}{d x} + \frac{h^2}{2} \frac{d^2 t}{d x^2} + \frac{h^3}{6} \frac{d^3 t}{d x^3} + \frac{h^4}{24} \frac{d^4 t}{d x^4} \ldots; \\
    t(y - h) &= t(y) - \frac{d t}{d x} + \frac{h^2}{2} \frac{d^2 t}{d x^2} - \frac{h^3}{6} \frac{d^3 t}{d x^3} + \frac{h^4}{24} \frac{d^4 t}{d x^4} \ldots, \\
\end{align*}
$$

(B.1)
Fig. B.1. For a homogeneous HTI layer, the specular reflection point for any offset coincides with the zero-offset reflection point, and there is no reflection-point dispersal on CMP gathers. $y$ denotes the CMP location and $h$ is half the source-receiver offset.

where all derivatives are evaluated at CMP location $y$.

Summing the two series expansions above, I obtain

$$t_h = t(y + h) + t(y - h) = t_0 + h^2 \frac{d^2 t}{dx^2} + \frac{h^4}{12} \frac{d^4 t}{dx^4},$$

(B.2)

where $t_0$ is the two-way zero-offset traveltime. Note that for a horizontal reflector beneath an HTI medium, both the phase- and group-velocity (ray) vectors of the zero-offset reflection are vertical.

Squaring both sides of equation (B.2) and ignoring terms of higher order than $h^4$ yields

$$t_h^2 = t_0^2 + \left[2t_0 \frac{d^2 t}{dx^2}\right] h^2 + \left[\left(\frac{d^2 t}{dx^2}\right)^2 + \frac{t_0}{6} \frac{d^4 t}{dx^4}\right] h^4.$$  

(B.3)

Comparing equation (B.3) with the Taylor’s series expansion of the squared reflection traveltime $[t^2(x^2)]$

$$t_h^2 = t_0^2 + A_2(2h)^2 + A_4(2h)^4,$$

(B.4)
I find the quadratic \((A_2)\) and the quartic moveout coefficients \((A_4)\) as

\[
A_2 = \lim_{h \to 0} \left[ \frac{t_0}{2} \frac{d^2 t}{dh^2} \right], \tag{B.5}
\]

and

\[
A_4 = \frac{1}{16} \lim_{h \to 0} \left[ \left( \frac{d^2 t}{dh^2} \right)^2 + \frac{t_0}{6} \frac{d^4 t}{dh^4} \right]. \tag{B.6}
\]

Tsvankin (1996b) used equation (B.5) to derive the normal-moveout velocity \((V_{nmo}^2 = 1/A_2)\) in a single HTI layer as a function of phase velocity and the symmetry-axis azimuth. Here, I apply equation (B.6) to obtain the quartic moveout coefficient \(A_4\). Equations (B.5) and (B.6) are valid for arbitrary anisotropic media if the specular reflection point does not change with offset within CMP gathers. It turns out, however, that reflection-point dispersal does not influence the value of normal-moveout velocity (or \(A_2\)) (Hubral and Krey, 1980, Appendix D; Tsvankin, 1995), and equation (B.5) can be used for both horizontal and dipping reflection events in media with any symmetry. Equation (B.6) for the quartic term is more restrictive and can be applied only in the absence of reflection-point dispersal (which is the case for my model).

I consider a CMP line that makes the azimuthal angle \(\alpha\) with the symmetry axis (Figure B.2). Since the derivative \(dt/dh\) represents the apparent slowness within the CMP gather, it equals the projection of the slowness vector onto the CMP line,

\[
p_h = \frac{dt}{dh},
\]
Fig. B.2. The group- and phase-velocity vectors for the reflected waves in a homogeneous HTI layer. The incident (SO) and reflected (OR) group-velocity vectors (rays) lie in the vertical incidence plane and are symmetric with respect to the horizontal plane. The phase-velocity vector (direction OD) of the reflected ray OR is confined to the plane formed by OR and the axis of symmetry. Triangle RCB defines a plane normal to the symmetry axis (after Tsvankin, 1996b).

and the quartic moveout coefficient [equation (B.6)] can be rewritten as

\[
A_4 = \frac{1}{16} \lim_{h \to 0} \left[ \left( \frac{dp_h}{dh} \right) \right]^2 + \frac{t_0}{96} \lim_{h \to 0} \left[ \frac{d^3p_h}{dh^3} \right].
\] (B.7)

Introducing the group angle \( \beta \) in the incidence plane (Figure B.2) and taking into account that \( h = z_0 \tan \beta \) and \( z_0 = V_{vert} t_0 / 2 \) (\( V_{vert} \) is the vertical velocity), yields

\[
A_4 = \frac{1}{4t_0^2 V_{vert}^2} \lim_{\beta \to 0^o} \left[ \left( \frac{dp_h}{d \tan \beta} \right) \right]^2 + \frac{1}{12t_0^2 V_{vert}^3} \lim_{\beta \to 0^o} \left[ \frac{d^3p_h}{d(\tan \beta)^3} \right].
\] (B.8)

It is convenient to represent \( \beta \) and \( p_h \) as functions of the phase angle \( \theta \) with the symmetry axis (Figure B.2). Although the rays stay within the vertical incidence plane, the influence of azimuthal anisotropy moves the phase-velocity (slowness) vec-
tors of the incident and reflected waves out of plane. Still, the phase-velocity vector in transversely isotropic media always lies in the plane formed by the symmetry axis and the group-velocity vector (Figure B.2). Rewriting the derivatives in equation (B.8) in terms of \( \theta \), gives

\[
\frac{d p_h}{d \tan \beta} = \frac{d p_h}{d \theta} \frac{d \theta}{d \tan \beta},
\]

and

\[
\frac{d^3 p_h}{d (\tan \beta)^3} = \left( \frac{d \theta}{d \tan \beta} \right)^3 + 3 \frac{d^2 p_h}{d \theta^2} \frac{d \theta}{d (\tan \beta)^2} \frac{d \theta}{d \tan \beta} + \frac{d p_h}{d \theta} \frac{d^3 \theta}{d (\tan \beta)^3},
\]

where

\[
\frac{d^2 \theta}{d (\tan \beta)^2} = \frac{d}{d \tan \beta} \left( \frac{d \theta}{d \tan \beta} \right) = \left[ \frac{d}{d \theta} \left( \frac{d \theta}{d \tan \beta} \right) \right] \frac{d \theta}{d \tan \beta},
\]

and similarly

\[
\frac{d^3 \theta}{d (\tan \beta)^3} = \left[ \frac{d}{d \theta} \left( \frac{d^2 \theta}{d (\tan \beta)^2} \right) \right] \frac{d \theta}{d \tan \beta}.
\]

Substituting equations (B.9) and (B.10) into equation (B.8) yields

\[
A_4 = \lim_{\theta \to 90^\circ} \left[ \frac{1}{4 t_0^2 V_{\text{vert}}^2} \left( \frac{d p_h}{d \theta} \frac{d \theta}{d \tan \beta} \right)^2 + \frac{1}{12 t_0^2 V_{\text{vert}}^3} \frac{d^3 p_h}{d \theta^3} \left( \frac{d \theta}{d \tan \beta} \right)^3 \right] + \lim_{\theta \to 90^\circ} \left[ \frac{1}{12 t_0^2 V_{\text{vert}}^3} \frac{d p_h}{d \theta} \frac{d^3 \theta}{d (\tan \beta)^3} + \frac{1}{4 t_0^2 V_{\text{vert}}^3} \frac{d^2 p_h}{d \theta^2} \frac{d^2 \theta}{d (\tan \beta)^2} \frac{d \theta}{d \tan \beta} \right] (B.11)
\]

Next, it is necessary to compute the derivatives in equation (B.11). From simple trigonometry (Figure B.2), we can relate the angle \( \beta \) to the group angle \( \psi \) of ray OR with respect to the symmetry axis as suggested in Tsvankin (1996b):

\[
\sin \beta = \frac{\cos \psi}{\cos \alpha},
\]
\[
\tan \beta = \frac{1}{\tan \psi \sqrt{1 - \frac{\sin^2 \alpha}{\sin^2 \psi}}}
\]

Then, expressing the group angle \( \psi \) through the phase angle \( \theta \) and phase velocity \( V(\theta) \) (Thomsen, 1986), we get

\[
\tan \psi = \frac{\tan \theta + \frac{1}{V} \frac{dV}{d\theta}}{1 - \frac{\tan \theta \frac{dV}{d\theta}}{V}}. \tag{B.12}
\]

Applying the chain rule again, yields

\[
\frac{d\theta}{d \tan \beta} = \frac{d\psi}{d \tan \beta} \frac{d\theta}{d\psi}.
\]

Therefore,

\[
\frac{d\theta}{d \tan \beta} = -\frac{\sin^2 \psi (1 - \frac{\sin^2 \alpha}{\sin^2 \psi})^{\frac{3}{2}}}{\cos^2 \alpha} \left[ \frac{1 + (\frac{1}{V(\theta)} \frac{dV(\theta)}{d\theta})^2}{1 + \frac{1}{V(\theta)} \frac{d^2V(\theta)}{d\theta^2}} \right]. \tag{B.13}
\]

By representing \( \tan \beta \) as a function of \( \psi \) and \( \theta \), we can evaluate not only \( \frac{d\theta}{d \tan \beta} \), but also the higher-order derivatives in equation (B.11) as well. Evaluating the derivatives at \( \theta = \psi = 90^\circ \), we get

\[
\left. \frac{d\theta}{d \tan \beta} \right|_{\theta=\psi=90^\circ} = -\cos \alpha \left[ \frac{1}{1 + (\frac{1}{V} \frac{dV}{d\theta})|_{\theta=90^\circ}} \right], \tag{B.14}
\]

and

\[
\left. \frac{d^3\theta}{d(\tan \beta)^3} \right|_{\theta=\psi=90^\circ} = \left[ \frac{\cos \alpha (2 + \sin^2 \alpha)}{1 + \frac{1}{V} \frac{d^2V}{d\theta^2}} - \frac{\cos^3 \alpha (\frac{1}{V^2} \frac{d^2V}{d\theta^2})^2 (3 + \frac{2}{V} \frac{d^2V}{d\theta^2})}{(1 + \frac{1}{V} \frac{d^2V}{d\theta^2})^4} \right]_{\theta=90^\circ}
+ \left[ \frac{\cos^3 \alpha \frac{1}{V} \frac{dV}{d\theta}}{(1 + \frac{1}{V} \frac{d^2V}{d\theta^2})^4} \right]_{\theta=90^\circ}. \tag{B.15}
\]
The derivative \( \frac{d\theta^2}{d(\tan \beta)^2} \big|_{\theta=\psi=90^\circ} \) turns out to be unnecessary in equation (B.11) since \( \frac{d\theta^2}{d\theta^2} \big|_{\theta=\psi=90^\circ} = 0. \)

To evaluate \( A_4 \), we also need to find the derivatives of the projection of the slowness vector on the CMP line \( (p_h) \) with respect to the phase angle \( \theta \). Following Tsvankin (1996b), let us decompose the slowness vector (which is parallel to OD in Figure B.2) into two vectors parallel to sides OC and CD of triangle OCD. Then we find the horizontal projection of the slowness component parallel to CD using

\[
\cos(\angle RCB) = \frac{\tan \beta \sin \alpha}{\sqrt{1 + \tan^2 \beta \sin^2 \alpha}} = \frac{\tan \alpha}{\tan \psi}.
\]

Summing up the projections of both components onto the CMP line yields

\[
p_h = \frac{1}{V} (\cos \theta \cos \alpha + \sin \theta \sin \alpha \tan \alpha / \tan \psi), \tag{B.16}
\]

with \( \tan \psi \) determined by equation (B.12).

The derivatives of equation (B.16) with respect to \( \theta \), evaluated at \( \theta = \psi = 90^\circ \), are given by

\[
\frac{dp_h}{d\theta} \bigg|_{\theta=\psi=90^\circ} = -\frac{1}{V_{\text{vert}} \cos \alpha} - \frac{1}{V_{\text{vert}} \cos \alpha} \left[ \sin^2 \alpha \left( \frac{1}{V} \frac{d^2V}{d\psi^2} \right) \bigg|_{\theta=90^\circ} \right], \tag{B.17}
\]

\[
\frac{dp_h^2}{d\psi^2} \bigg|_{\theta=\psi=90^\circ} = 0, \tag{B.18}
\]
and

\[
\frac{dp_n^3}{d\theta^3}_{\theta=\psi=90^\circ} = \frac{\cos \alpha}{V_{\text{vert}}} \left( 1 + \frac{3}{V} \frac{d^2V}{d\theta^2} \bigg|_{\theta=90^\circ} \right) + \frac{\sin^2 \alpha}{V_{\text{vert}} \cos \alpha} \left( 1 - \frac{1}{V} \frac{d^4V}{d\theta^4} \bigg|_{\theta=90^\circ} \right). \tag{B.19}
\]

Substitution of equations (B.14), (B.15), (B.17), (B.18), and (B.19) into equation (B.11) leads, after algebraic transformations, to a concise final result:

\[
A_4 = \cos^4 \alpha \left[ -\frac{4}{V} \frac{d^2V}{d\theta^2} + 3\left( \frac{1}{V} \frac{d^2V}{d\theta^2} \right)^2 + \frac{1}{V} \frac{d^4V}{d\theta^4} \right] \bigg|_{\theta=90^\circ}. \tag{B.20}
\]

Note that the first and third derivatives of the phase velocity in the vertical direction \((\theta = 90^\circ)\) have gone to zero. Equation (B.20) is valid for any pure mode \((P, S^\perp, S^\parallel)\) in HTI media with arbitrary strength of anisotropy.
Appendix C

GENERALIZED DIX EQUATION FOR AZIMUTHALLY ANISOTROPIC MEDIA

In their derivation of the generalized Dix equation for anisotropic media, Alkhalifah and Tsvankin (1995) assumed that the phase and group-velocity vectors of incident and reflected waves are confined to the sagittal (incidence) plane. This implies that the incidence plane (i.e., the vertical plane that contains the CMP line) should be a symmetry plane of the medium, as well as the dip plane of the reflector. Here, I show that the generalized Dix equation retains the same form outside the symmetry planes of azimuthally anisotropic media, if the group-velocity vector does not deviate from the incidence plane for the whole ray path. Although this assumption cannot be satisfied exactly for multilayered azimuthally anisotropic models, it is helpful in gaining insight into the influence of azimuthal anisotropy on the accuracy of the Dix equation.

Consider CMP reflections from either a horizontal or a dipping interface overlain by a horizontally-layered arbitrary anisotropic medium (Figure C.1). Normal-moveout velocity in the CMP geometry can be represented as the following function of the one-way traveltime $t$ from the zero-offset reflection point [equation (B.5)]:

$$V_{nmo}^2 = \frac{2}{t_0} \lim_{h \to 0} \frac{d}{dh} \left( \frac{dt}{dh} \right)^{-1} = \frac{2}{t_0} \lim_{h \to 0} \frac{dh}{dp_h},$$  \hspace{1cm} (C.1)

where $h$ is half the source-receiver offset ($h$ is positive in the down-dip direction), $t_0$
is the two-way zero-offset traveltime, and \( p_h \) is the projection of the slowness vector on the CMP line.

The ray parameter \( p \) (horizontal slowness), as well as \( p_h \), remains constant along any ray above the reflector since the overburden is laterally homogeneous. In the case considered by Alkhalifah and Tsvankin (1995), the slowness vector did not deviate from the incidence plane, and \( p_h \) was equal to \( p \). However, as shown below, any difference between \( p_h \) and \( p \) has no influence on the form of the generalized Dix equation provided the group-velocity vector (ray) stays within the incidence plane.

Since we assume that the whole raypath is confined to the incidence plane, the half-offset \( h \) can be written as

\[
h = \left( \sum_{i=1}^{n} x^{(i)} - x_0 \right),
\]

where \( x^{(i)} \) is the horizontal distance traveled by the ray in layer \( i \) ("horizontal displacement"), and \( x_0 \) is the total horizontal displacement of the zero-offset ray between the reflection point and the surface (Figure C.1). Substituting \( h \) into equation (C.1) yields

\[
V_{nmo}^2 = \frac{2}{t_0} \lim_{h \to 0} \sum_{i=1}^{n} \frac{d(x^{(i)})}{dp_h}, \tag{C.2}
\]

To identify the interval values of NMO velocity in equation (C.2), let us draw an imaginary reflector through the intersection of the zero-offset ray with the bottom of layer \( i \). The normal to the reflector is chosen to coincide with the slowness (phase) vector that corresponds to the segment of the zero-offset ray in this layer. Note that since the slowness vector associated with the zero-offset ray is allowed to be out of plane, the dip plane of the imaginary reflector is generally different from the incidence
plane. Next, imagine that the intersection of the zero-offset ray with the top of layer \( i \) represents a common-midpoint location of a gather parallel to the actual CMP line. Then the segment of the zero-offset ray in layer \( i \) will coincide with the raypath of the zero-offset CMP reflection from the imaginary interface. In accordance with equation (C.1), NMO velocity from the imaginary reflector at this CMP location will be given by

\[
[V_{\text{nmo}}^{(i)}(\tilde{s}^{(i)})]^2 = \frac{2}{t_0^{(i)}} \lim_{x^{(i)} \to x_0^{(i)}} \frac{d(x^{(i)})}{dp_h},
\]

where \( t_0^{(i)} \) is the two-way traveltime along the zero-offset ray in layer \( i \), \( x_0^{(i)} \) is the horizontal displacement of the zero-offset ray in layer \( i \), and \( \tilde{s}^{(i)} \) is the slowness vector associated with the zero-offset ray. It follows that the summation in equation (C.2) is carried out over the NMO velocities from reflectors normal to the “zero-offset” slowness vectors in each layer. Substituting equation (C.3) into equation (C.2) yields

\[
V_{\text{nmo}}^2 = \frac{1}{t_0} \sum_{i=1}^{n} t_0^{(i)} [V_{\text{nmo}}^{(i)}(\tilde{s}^{(i)})]^2.
\]

Although this expression looks identical to the conventional Dix equation, interval NMO velocities in equation (C.4) correspond to reflectors with different dips determined by the orientation of the slowness vector associated with the zero-offset ray in each of the layers. In contrast with the symmetry-plane Dix equation obtained by Alkhalifah and Tsvankin (1995), equation (C.4) is influenced by the 3-D character of wave propagation since the normals to these reflectors (and the slowness vectors of the corresponding zero-offset rays) are not necessarily confined to the incidence plane (Figure C.1).
Fig. C.1. Reflection from a dipping interface overlain by a sequence of horizontal, homogenous, azimuthally anisotropic layers. We assume that the rays (group-velocity vectors) from the zero-offset reflection point are confined to the incidence plane but do not put any restrictions on the orientation of the corresponding slowness (phase-velocity) vectors. This implies that the normal to the reflector may deviate from the incidence plane.
Appendix D

SIGN OF $\delta^{(V)}$ IN HTI MEDIA

In order to determine the sign of $\delta^{(V)}$ in HTI media due to parallel cracks, we use equation (4.7) for the shear wave splitting parameter $\gamma$:

$$
\gamma = \frac{V_{P_{\text{vert}}}^2}{2V_{S_{\text{vert}}}^2} \left[ \frac{\epsilon^{(V)} [2 - 1/f^{(V)}] - \delta^{(V)}}{1 + 2\epsilon^{(V)}/f^{(V)} + \sqrt{1 + 2\delta^{(V)}/f^{(V)}}} \right].
$$

Since the denominator of the term between brackets is a positive quantity as well as the ratio of the vertical velocities, we obtain

$$
\epsilon^{(V)} [2 - 1/f^{(V)}] - \delta^{(V)} = C\gamma \geq 0,
$$

where $C$ is a positive quantity.

Therefore,

$$
\epsilon^{(V)} [2 - 1/f^{(V)}] - \delta^{(V)} - C\gamma \geq 0,
$$

or

$$
\delta^{(V)} \leq \epsilon^{(V)} [2 - 1/f^{(V)}] - C\gamma.
$$

Note that $C\gamma$ is a positive quantity.
Substituting $\epsilon^{(V)}$ in terms of the generic coefficient $\epsilon$ gives

$$\delta^{(V)} \leq \frac{\epsilon}{1 + 2\epsilon} \left[ \frac{1}{f^{(V)}} - 2 \right] - C\gamma.$$

Since $\epsilon \geq 0$ for a single system of cracks and $C\gamma$ is also a positive quantity, the sign of $\delta^{(V)}$ depends on the sign of the term

$$\frac{1}{f^{(V)}} - 2 = 2 \left( \frac{V_{S+\text{vert}}}{V_{P\text{vert}}} \right)^2 - 1.$$

Therefore, if $\frac{V_{S+\text{vert}}}{V_{P\text{vert}}} \leq 0.707$, $\delta^{(V)}$ is always negative. Even if $\frac{V_{S+\text{vert}}}{V_{P\text{vert}}} \geq 0.707$, $\delta^{(V)}$ can still be negative depending on the term $C\gamma$. 
