Quantifying limitations on signal-to-noise ratio improvement in the stacking of seismic data

by
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ABSTRACT

Stacking is the main contributor to signal-to-noise ratio (SNR) improvements in seismic data processing. The $\sqrt{N}$ SNR improvement through the stacking of an N-trace CMP gather follows the assumptions that the noise is perfectly uncorrelated, the signal is perfectly aligned and identical on every trace, and noise amplitude levels are the same on all traces. Real seismic data, however, are never ideal, therefore causing violations to these assumptions and consequently resulting in reduction of SNR improvement from the ideal. Moreover, since the noise in data necessarily includes all unwanted coherent events other than the primary reflections, SNR improvement in the stacking of seismic data becomes a two-fold issue: improvement with respect to both coherent and incoherent noises.

Here, I investigate quantitatively the influence of data imperfections and associated processing or parameter errors on the performance of stacking of seismic data. Results indicate that limitations can often be significant, especially for high-frequency components of signals in data. More importantly, since these limitations are quantifiable, one can predict the SNR of stacked output and use this prediction to assess output of processing of field data. Further, I show that one can exploit results from analysis of SNR improvement to assess the expected benefit of special efforts to optimize the stacked output, for instance, by altering the stacking parameters or choosing an alternative stacking scheme that would reduce the limitation, thereby maximizing SNR improvement.

Stacking is representative of other SNR-enhancing processes that also fail to perform ideally in the presence of data imperfections and processing errors. The
approach to analyzing SNR improvement and limitation presented in this thesis could thus serve as a pattern for similar studies with respect to other data processing techniques.
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GLOSSARY OF SYMBOLS

\( \gamma \)  
Generic notation for signal-to-noise ratio in logarithmic (db) unit. This refers to either signal-to-noise ratio (SNR) or primary-to-multiple ratio (PMR).

\( \gamma_I \)  
Input signal-to-noise ratio measured before CMP stacking.

\( \gamma_P \)  
Signal-to-noise ratio improvement attributable to CMP stacking.

\( \gamma_{P(L)} \)  
Limitation on \( \gamma_P \). This makes the value of \( \gamma_P \) smaller than that under ideal circumstances.

\( \gamma_O \)  
Output signal-to-noise ratio measured after CMP stacking.

\( \psi \)  
Generic notation for signal amplitude in logarithmic (db) unit.

\( \psi_I \)  
Input signal amplitude measured before CMP stacking.

\( \psi_P \)  
Stack signal increase. Signal improvement attributable to CMP stacking.

\( \psi_{P(L)} \)  
Limitation on \( \psi_P \). This makes the value of \( \psi_P \) smaller than that under ideal circumstances.

\( \psi_O \)  
Output signal amplitude measured after CMP stacking.

\( \eta \)  
Generic notation for noise amplitude in logarithmic (db) unit. This refers to either coherent or incoherent noise.
\( \eta_I \)  Input noise amplitude measured before CMP stacking.

\( \eta_P \)  Stack noise increase. Noise increase attributable to CMP stacking.

\( \eta_{P(L)} \)  Limitation on \( \eta_P \). This makes the value of \( \eta_P \) larger than that under ideal circumstances.

\( \eta_O \)  Output noise amplitude measured after CMP stacking.
Chapter 1

INTRODUCTION

Seismic data consist of two main constituents: signal, the useful components of the data, and noises, all components that interfere with signal. In general, signal in the reflection seismic method corresponds to the primary reflections while noises are all other events, which could be either coherent or incoherent (e.g., Weichart, 1973; Larner et al., 1983; Fulton, 1985). One of the main objectives of seismic data processing is to attenuate these noises as best possible while preserving signal; in other words, to improve signal-to-noise ratio (SNR).

Seismic data processing is dominated by three principal processes: deconvolution, common-midpoint (CMP) stacking, and migration. Of these, stacking is the one process designed with the sole purpose of improving SNR. Although there are many other additional processing techniques aimed at attenuating particular types of noises (e.g., bandpass filtering, moveout filtering), stacking remains the single most general and effective data processing technique for this purpose (Anderson and McMechan, 1990), either used by itself or in conjunction with other methods.

Figure 1.1 shows a variety of data processing methods that aim at enhancing SNR in the data, CMP stacking being one of them. Each of the methods takes advantage of some specific characteristic of the noise in the data and tries to attenuate this noise based on that special property. Bandpass filtering (e.g., in ground-roll elimination), for instance, discriminates between noise and signal frequencies where the two bands do not overlap. On the other hand, moveout filtering exploits the moveout of coherent
events (e.g., in multiple attenuation), discriminating between multiple and primary in the frequency-wavenumber domain where the two, due to difference in their moveouts, occupy different regions. It would be ideal if we could study the performance of each of these processing methods in terms of its capacity to enhance SNR, and in terms of how this performance is limited by data conditions. The results of such analysis not only could help us choose the best processing sequence adapted to the characteristics of the data but also could give us guidance as to when it is fruitless to keep trying alternative processing techniques in the hope of bringing about better SNR.

**FIG. 1.1.** Seismic data processing methods for noise attenuation.

To include detail performance analysis of every possible data processing method for SNR enhancement in a single study, however, would make the scope of the present work too large and unfocussed. For this reason, considering the principal importance of CMP stacking in SNR enhancement, notwithstanding the value of other noise
attenuation techniques, the scope of the present thesis work is limited to the study of SNR improvement by CMP stacking only. This study is intended to serve as a pattern for similar studies of SNR enhancement in other data processing methods.

Quantitative study of SNR improvement in seismic data processing has been somewhat lacking in the literature. Most studies of SNR improvement constitute a small subsection of theoretical papers discussing particular data processing methods, and results are most often presented in a qualitative manner, that is by comparing processing results visually (e.g., Naess, 1982; Beresford-Smith and Rango, 1988; Russell et al., 1990a and 1990b; Wang, 1990; Shon and Yamamoto, 1992). A number of works have incorporated some quantitative analysis of the effectiveness of a particular processing method (not necessarily CMP stacking) using examples from synthetic and field data, e.g., White (1977), Sagiroglu (1985), McFadden et al. (1986), Regone and Rethford (1990), and Anderson and McMechan (1990). Those works, however, typically focus more on the performance of the particular methods under perfect conditions and consequently place less emphasis on performance under the influence of data imperfections and processing errors. In this thesis, I concentrate on the limitations imposed by data imperfections and processing errors on the performance of data processing to arrive at realistic expectations of processing output.

In their study comparing the influences of processing limitations versus those of data acquisition errors, Shirley et al. (1985) have quantified typical performances of several data processing routines in terms of their dynamic range (i.e., SNR) contributions. Their main conclusion is that field data usually contain sufficient dynamic range for subsequent processing, and therefore it is either the processing methods that err or the data contain some imperfections other than a limitation in dynamic range. Hence, processing methods and data imperfections are suspects for limiting SNR im-
improvements from data processing. (Other studies on the importance of the dynamic range of field data include those of Musser and Dunbar, 1984; and of Ongkiehong and Huizer, 1987.) In the present study, I identify the various shortcomings in data and processing that may be responsible for limiting those SNR improvements and analyze the extent of the limitations.

Why is this knowledge of SNR improvement limitations important? Consider an example. Although frequency-wavenumber (f-k) filtering can be powerful in reducing linear noise in seismic data, its performance is limited by variation of amplitude level from trace to trace in the data set. This variation of amplitude level can be caused by, for instance, uneven coupling of geophones to the ground during field acquisition. Shirley et al. (1985) reported that they could achieve no better than 40 db of noise attenuation with their f-k filter when they simulated a data set with such channel inequality. Clearly such information can be valuable in helping a seismic data processor assess the quality to expect in processing output.

The work of Shirley et al. (1985) can be regarded as a general overview of the problem of limitations in SNR improvement in seismic data processing. Here, I take one particular subset out of the subject matter, namely CMP stacking, and conduct a detailed analysis of the influence of data imperfections, intrinsic processing errors (i.e., flaws in the processing technique used), and processing-parameter errors on the performance of CMP stacking measured in terms of signal amplitude improvement relative to that of coherent and incoherent noise. The question I address is, how many db of SNR improvement could we realistically expect from stacking in the presence of realistic shortcomings in data, processing schemes, and processing parameters. In short, in this thesis I analyze quantitatively the limitations imposed by these shortcomings on the performance of CMP stacking.
Consider, for example, the CMP gather in Figure 1.2. The NMO-corrected gather has a reflection event of unit peak amplitude with some residual static time shifts and some background incoherent noise. The noise shows change in root-mean-square (RMS) amplitude level with offset across the gather. In addition, there must have been some NMO error on the event since it is not horizontally aligned after the NMO correction. Clearly, when stacked, this CMP gather violates some of the basic assumptions in CMP stacking that are required for the \( \sqrt{N} \) SNR improvement, where \( N \) is the number of traces stacked. The stack SNR improvement in this case must necessarily be less than \( \sqrt{N} \), but how much less? Our analyses will attempt to answer such a question.

**Fig. 1.2.** A 48-trace, 4000-m CMP gather, NMO-corrected. The event has a true moveout velocity of 3000 m/s but has been erroneously corrected with a velocity of 3075 m/s. The signal wavelet is Ricker, zero-phase, with a dominant frequency of 30 Hz. The peak event amplitude is uniformly 0 db across the gather, while the noise root-mean-square amplitude is -23 db on the first trace and approximately -15 db on the mid-offset traces. Maximum residual static time shift is 5 ms from the true traveltimes.
Figures 1.3 and 1.4 show examples of such SNR analyses related to the problems that plague the CMP gather in Figure 1.2. Here, with the statics problem alone (suppose we do not have any NMO error), using uniformly-weighted averaging of traces in CMP stacking, we are likely to suffer as much as 2.5-db loss of peak signal amplitude relative to that for stacking with no static time shifts. The NMO error by itself (this time we have no static time shift) could result in about 5-db attenuation of peak signal amplitude relative to that when there is no residual moveout. These two problems combined have, in this case, produced a total amplitude attenuation of around 8 db on the stacked signal (recall that at the start the signal amplitude was 0 db). On the other hand, stacking of the background noise in the data has reduced amplitudes from about -23 db (using noise amplitude on the first trace as a reference) down to about -34 db, i.e., an 11-db attenuation. The stack SNR is simply the ratio
Fig. 1.4. Cumulative noise stack response. The horizontal axis denotes the same as in Figure 1.3; the vertical axis denotes RMS amplitude of the stack of the noise traces.

of the amplitude of the stacked signal to that of the stacked noise (their difference, in the db scale); hence, in our case, the output SNR is

\[-8 \text{ db} + 34 \text{ db} = 26 \text{ db}.

Now, if we assume trace 1 in the CMP gather represents our typical prestack trace (whose SNR is about 23 db), a stack SNR of 26 db would mean an SNR improvement of only 3 db, which is far short of the ideal $\sqrt{N}$, the improvement often taken for granted to be the case (for $N=48$, $\sqrt{N}$ is close to 17 db). Of course, we have guessed correctly that the output SNR would be less than that number, but did we think it would be that much less? An improvement of 3 db is rather modest, as we can see in Figure 1.5; we might not have stacked the data in the first place if we had known this. Of course, a key reason for the pessimistic result in this manufactured, but not unrealistic, example is that the SNR for some of the mid-offset traces is poorer than that of the shortest offset trace. Inclusion of the mid-offset traces does not help the
stack as much as we would wish. Optimum trace weighting before stacking, discussed later, gives a recipe for how much weight should be given each trace in creating the stack so as to yield the best SNR in the stack.

This present study is intended to help our understanding of why in most cases we cannot realize $\sqrt{N}$ SNR improvement in CMP stacking, to identify the various problems that could limit the effectiveness of stacking, and to quantify the extent of these limitations. The findings could serve as a guideline for setting expectations for

![Diagram](image_url)

**Figure 1.5.** Comparison between a representative prestack trace from CMP gather in Figure 1.2, trace 1 (a) and the stack trace (b) after stacking all 48 traces from the gather. Peak signal wavelet amplitudes are normalized such that the excursions in the display are equal.

stacking output. Moreover, the approach used could offer a pattern for similar analysis of SNR expectations for other data-processing steps, such as moveout filtering.

SNR improvement analyses become particularly important when the prestack SNR of the data is low to begin with. This could occur in many instances in land
data as well as in data from deep seismic targets where seismic energy is typically weakened due to geometrical spreading and earth absorption. Moreover, where a goal is to maximize resolution in the data, data that have good SNR at lower frequencies may have marginal or poor SNR at desired higher frequencies. In such cases, any modest difference in (stacking) SNR improvement could be crucial and would determine whether the data are usable or not.

In Chapter 2, I introduce the definitions related to SNR that will be used throughout this thesis. There, I present the concepts of SNR improvements and their limitations in the stacking of seismic data, describe methodologies of their measurements, and address the relevance of their analyses in practical data processing. A discussion concerning distortions of seismic wavelet due to stacking errors and their significance closes the chapter.

In Chapter 3, I discuss problems that limit the performance of stacking and introduce approaches to quantifying limitations on SNR improvements.

I quantify limitations in stacking SNR improvement in Chapter 4. The quantification employs simulations of errors using synthetic data and analytical approximations, where possible.

In Chapter 5, I demonstrate SNR improvement predictions using an example from field data: I show how well we can predict SNR improvements anticipating that we may have some limitations due to imperfections in the data and errors in the processing, and how SNR predictions can help us assess the processing output. The target zone for improvement in this marine data set from Northeast Java Sea area in Indonesia is a set of weak reflectors immediately beneath a strong platformal limestone reflector marker. This analysis also highlights the importance and applicability of analyses of stacking SNR in practical seismic data processing.
Chapter 2

SNR IMPROVEMENT AND LIMITATIONS: CONCEPTS

In this chapter, I define terms related to SNR that will be referred to throughout the thesis. I also introduce concepts of SNR improvement and its limitations, aimed at simplifying the investigations. The basic theory underlying the stacking of signal and noise is discussed to give insights on how one might estimate analytically the output of CMP stacking. In addition, here I address the relevance of SNR analysis in practical data processing. Finally, I discuss distortion of signal shape in stacking as a secondary result of stacking errors.

2.1 Definitions

Signal here is defined as a seismic event corresponding to the primary reflection energy. Noise, which refers to any event on a seismic trace other than signal, can be either coherent or incoherent (random). Examples of coherent noises are source-generated noise, such as multiple reflections, diffractions, surface waves, direct arrivals, and sideswipes in 2D data, and ambient, such as marine cable noise, and cultural noise (e.g., passing vehicles, drilling rig operations, neighboring vessels). Incoherent or random noise is most often ambient or background noises, including random noise spikes and bursts.

We shall therefore place signal-to-noise ratio in seismic data into two categories: the signal-to-noise ratio with respect to incoherent noises, which I call SNR proper, and signal-to-noise ratio with respect to coherent noises, which I designate as PMR.
(i.e., primary-to-multiple ratio; the word "multiple" here represents any coherent noises, multiple being one of the most representative of these).

Except when otherwise specified, SNR and PMR are defined as follows:

- **SNR** = the ratio between the absolute peak amplitude of a primary reflection wavelet and the root-mean-square (RMS) amplitude of incoherent noise as would be measured within a time gate that excludes the primary wavelet.

- **PMR** = the ratio between the absolute peak amplitude of a primary reflection wavelet and the absolute peak amplitude of the coherent noise wavelet.

The choice of using absolute peak amplitudes for the coherent events has a practical basis. Unlike incoherent noise, a coherent event is nonstationary; therefore its local amplitude (e.g., corresponding to the peak of the event wavelet) is more representative of its size rather than is its RMS amplitude.

In order to simplify signal-to-noise-ratio notation when referring to SNR and PMR, in general I will use the symbol \( \gamma \), which shall represent either of the signal-to-noise ratios in logarithmic (db) units.

### 2.2 \( \gamma \), its limitations and its theoretical evaluation

Given

\[
SNR_{input} \equiv \frac{\text{prestack signal amplitude}}{\text{prestack noise amplitude}} = \frac{s}{n},
\]

then

\[
SNR_{output} \equiv \frac{\text{stacked signal amplitude}}{\text{stacked noise amplitude}}
\]
Therefore,

\[
SNR_{\text{output}} \equiv SNR_{\text{input}} \cdot SNR_{\text{processing}}.
\]  

(2.1)

One would hope that the increase in amplitude of the stacked signal is typically larger than that of the stacked noise, implying an "improvement" in stack SNR.

The above formulation applies just as well to PMR, i.e., by modification of the denominators to correspond to coherent noises. Hence, for either type of noise we can rewrite expression (2.1) using the \( \gamma \) notation,

\[
\gamma_0 = \gamma_I + \gamma_P,
\]

(2.2)

where the \( \gamma \)'s represent output (O), input (I), and processing (P) signal-to-noise ratios, respectively, measured in db.

Since \( \gamma \) consists of two separate components, namely the signal component (the numerator) and the noise component (the denominator), we can rewrite expression (2.2) in terms of these.

\[
\begin{align*}
\psi_O - \eta_O &= \psi_I - \eta_I + \psi_P - \eta_P, \\
\psi_O &= \psi_I + \psi_P, \\
\eta_O &= \eta_I + \eta_P,
\end{align*}
\]

(2.3) (2.4) (2.5)

where \( \psi \) and \( \eta \) denote the signal and the noise components, respectively. A little distinction must be made now though. Whereas \( \psi_O, \eta_O, \psi_I \) and \( \eta_I \) each represents a
level of signal or noise amplitude, $\psi_P$ and $\eta_P$ are changes of signal and noise amplitudes due to processing.

To analyze signal-to-noise ratio improvement due to CMP stacking, it is convenient to separate $\gamma_P$ into its two constituents, the stack signal increase (signal improvement) $\psi_P$, and the stack noise increase $\eta_P$, measuring each independently. To study the limitations on $\gamma_P$ I therefore evaluate the action of the limiting factors (i.e., data imperfections and processing errors) on the stacking of signal and on that of noise, individually.

Let us now denote any limitation on $\gamma_p$ relative to the ideal processing SNR as $\gamma_p(L)$. Accordingly we shall also designate $\psi_p(L)$ and $\eta_p(L)$ as the limitations imposed on the signal improvement and on the noise attenuation, respectively. (Note that $\eta_p(L)$ is not a limitation on noise increase here, since what concerns us are limitations that result in the stacked noise amplitude being larger instead of becoming smaller than ideal.) Thus, if under ideal conditions CMP stacking should produce a $\gamma_p$ that is equal to $\sqrt{N}^1$, then $\gamma_p(L)$ is the difference between this ideal SNR improvement and the $\gamma_p$ that is actually realized from a certain stacking process on the data at hand. Specifically,

$$\gamma_p(L) = \gamma_p(Ideal) - \gamma_p.$$

Or equivalently,

$$\gamma_p = \gamma_p(Ideal) - \gamma_p(L). \quad (2.6)$$

In terms of $\psi_p$ and $\eta_p$, the equivalent expressions for the above are:

$$\psi_p = \psi_p(Ideal) - \psi_p(L), \quad (2.7)$$

---

$^1$More precisely, $\gamma_p(Ideal) \equiv 20 \log_{10}(\sqrt{N})$. 

and

\[ \eta_P = \eta_P(Ideal) + \eta_P(L). \quad (2.8) \]

The ideal signal improvement, \( \psi(\text{Ideal}) \), is 0 db in weighted averaging of traces in CMP stacking, i.e., signal is preserved, unchanged in amplitude. On the other hand, the ideal noise increase, \( \eta_P(Ideal) \), is \( 20 \log_{10}(1/\sqrt{N}) \) in the same stacking, i.e., noise is attenuated. Notice the (+) sign in equation (2.8) for \( \eta_P \), which signifies that \( \eta_P(L) \) here is a “limitation” that makes \( \eta_P \) larger than under ideal circumstances. \( \eta_P \) in this case corresponds to the stacking of incoherent noise.

For coherent noise, ideal attenuation can be much larger than that of the incoherent noise owing to the destructive interference of wavelet sidelobes on adjacent traces; the wavelets on adjacent traces will be misaligned due to some residual moveout so that a wavelet on one trace is out of phase relative to the wavelet on another trace; thus, they oppose each other when the traces are stacked. Clearly, this attenuation is frequency selective.

### 2.3 CMP stacking

CMP stacking involves a weighted summation of traces in a CMP gather after traces in the gather has been corrected for NMO and source/receiver static time shifts. For an N-trace CMP gather this weighted summation can be expressed as

\[ S(t) = \sum_{i=1}^{N} \alpha_i f_i(t), \quad (2.9) \]

where \( \alpha_i \) denotes the weight for the \( i \)th trace, while \( f_i(t) \) denotes the amplitude on the \( i \)th NMO-corrected trace at time \( t \). When the weights are governed by trace offset in the CMP gather, the weighting is called “offset-dependent weighting.” On the other
hand, when the weights are governed by data parameters (e.g., SNR on the traces) regardless of offset, the weighting is called “data-adaptive weighting.”

There are two sub-cases of CMP stacking following the above definition. First, when all of the weights are identical, i.e., $\alpha_i = \alpha$, equation (2.9) becomes

$$S(t) = \alpha \sum_{i=1}^{N} f_i(t).$$

(2.10)

This is called “uniformly-weighted” CMP stacking.

A second sub-case arises when we wish to preserve the average amplitude level of the original data. Then, the weights must sum to unity, i.e.,

$$\sum_{i=1}^{N} \alpha_i = 1.$$

Stacking with such weights is called “weighted averaging” of traces in CMP stacking.

A special case in CMP stacking constitutes the combination of the two sub-cases above, i.e., when we require that the weights be uniform and at the same time sum to unity. Hence,

$$\sum_{i=1}^{N} \alpha = \alpha N = 1,$$

and therefore the weights ($\alpha$) have an equal value of $1/N$ for all traces in the CMP gather. This is “uniformly-weighted averaging” of traces in CMP stacking, which is almost universally used in CMP stacking as practiced in seismic data processing. Another term often used when referring to this stacking scheme is “straight stacking.”
2.4 Stacking of coherent and incoherent events

Consider moveout-corrected CMP data at a particular zero-offset time. The stack of a coherent event in an N-trace CMP gather can be expressed simply as [from equation (2.9)]

\[ s_{\text{stack}} = \sum_{i=1}^{N} \alpha_i s_i, \]  

(2.11)

where \( s_i \) represents the amplitude of an event on the \( i \)th trace at a time corresponding to the event’s zero-offset time, and \( \alpha_i \) is the weighting coefficient for the \( i \)th trace. If all \( s_i \) are the same, e.g., when there is no residual moveout or statics variations and the signal is identical on all traces, then the expression reduces to

\[ s_{\text{stack}} = s_0 \sum_{i=1}^{N} \alpha_i, \]

(2.12)

where \( s_0 \) is a representative prestack event amplitude. In logarithmic units, \( s_{\text{stack}} = \psi_O, s_0 = \psi_I, \) and \( \sum_{i=1}^{N} \alpha_i = \psi_P; \) hence the familiar formula \( \psi_O = \psi_I + \psi_P. \)

For a CMP gather with random noise that is (1) Gaussian with zero mean, (2) spatially uncorrelated, and (3) stationary over the time window being considered, the RMS amplitude on the \( i \)th trace is just the standard deviation \( (\sigma_i) \) of the noise over the time window, i.e., the square-root of its variance, \( \sigma_i^2. \) The variance of the stacked noise is given by (e.g., Hald, 1952; Pfeiffer, 1965)

\[ \sigma_{\text{stack}}^2 = \alpha_1^2 \sigma_1^2 + \alpha_2^2 \sigma_2^2 + \ldots + \alpha_N^2 \sigma_N^2 \]

\[ = \sum_{i=1}^{N} \alpha_i^2 \sigma_i^2. \]
Therefore the RMS amplitude of the stacked noise is

$$
\sigma_{stack} = \sqrt{\sum_{i=1}^{N} \alpha_i^2 \sigma_i^2}.
$$

(2.13)

If all $\sigma_i^2$ are identical, as is the case when there is no fluctuation in expected noise level across the CMP gather, then

$$
\sigma_{stack} = \sigma_0 \sqrt{\sum_{i=1}^{N} \alpha_i^2},
$$

(2.14)

where $\sigma_0$ is the RMS amplitude of the prestack noise traces. In logarithmic units, $\sigma_{stack} = \eta_0$, $\sigma_0 = \eta_I$, and $\sqrt{\sum_{i=1}^{N} \alpha_i^2} = \eta_P$; hence again the formula $\eta_0 = \eta_I + \eta_P$.

Suppose, for example, we stack an N-trace CMP gather having perfectly-aligned and identical signal plus random noise, with uniform weights, $\alpha_i = 1$. Then

$$
\begin{align*}
S_{stack} &= s_0 \sum_{i=1}^{N} \alpha_i = s_0 \cdot N, \\
\sigma_{stack} &= \sigma_0 \sqrt{\sum_{i=1}^{N} \alpha_i^2} = \sigma_0 \cdot \sqrt{N}.
\end{align*}
$$

If $s_0/\sigma_0$ is the prestack SNR, then

$$
SNR_{improvement} = \frac{N}{\sqrt{N}} = \sqrt{N},
$$

the standard $\sqrt{N}$ SNR improvement with which we are familiar.
2.5 The ideal stacking conditions

The ideal stacking conditions refer to a set of restrictive assumptions we must make to obtain the standard $\sqrt{N}$ SNR improvement in the stacking of an N-trace moveout-corrected CMP gather using a uniformly-weighted stacking method. These assumptions are, that

1. the noise is Gaussian with zero mean,
2. the noise is spatially uncorrelated and temporally stationary,
3. there are no signal and noise amplitude variations from trace to trace,
4. the source wavelet is identical for all traces, and
5. signal is perfectly aligned on all traces.

Assuming a data set satisfies these five conditions, then $\gamma_p$ solely depends on the number of traces stacked. If we mute $M$ traces of an N-trace CMP gather (e.g., to limit wavelet stretching due to NMO correction in the shallower portion of data), then uniformly-weighted CMP stacking of the data will yield a $\gamma_p$ of $\sqrt{N - M}$ instead of $\sqrt{N}$. This loss of $\gamma_p$ owes directly to the decrease in the suppression of random background noise in the data when the number of traces stacked is reduced.

2.6 Signal-to-noise ratio analysis in practical seismic data processing

The objective of stacking is to enhance signal-to-noise ratio in the data. In practice, our main desire is for the process to have a sufficiently large $\gamma_0$ in order to facilitate ease of data interpretation; that is, the goal is for primary reflections to be clearly distinguishable from noise in the stacked output.

Consider the synthetic stack section in Figure 2.1. It contains four coherent, horizontal events, $P_1$, $P_2$, $P_3$, and $P_4$, having SNR's of 4, 2, 1.5, and 1, respectively, relative to the stationary (in time and space) background noise. Event $P_2$, with
SNR of 2 ($\approx 6$ dB), is the weakest event among the four that can still be delineated confidently across the section. This suggests that the minimum required $\gamma_O$ for interpretation purposes is about 6 dB. Therefore, a "sufficiently large" $\gamma_O$ can be defined as a $\gamma_O$ that is $\geq 6$ dB.

Fig. 2.1. Section with four coherent events, $P_1$, $P_2$, $P_3$, and $P_4$, having progressively smaller SNR's. The background noise is Gaussian and has a frequency band similar to that of the event. Event $P_2$ is the weakest event that can still be interpreted with confidence.

Now, $\gamma_O$ consists of both SNR and PMR. The 6-db minimum for $\gamma_O$ required above pertains to SNR. Coherent noise, such as a multiple, might appear indistinguishable from a primary in a stack section. Thus the objective of processing here is to attenuate the multiple, if possible, until its stacked amplitude is so low that its presence in the stack would not interfere with interpretation of the primary. Suppose, in Figure 2.1, only event $P_1$ is a primary while events $P_2$, $P_3$, and $P_4$ are multiples. Event $P_2$ can of course be mistaken as a second primary, while event $P_3$ still might be
taken for another primary, though very weak. Event $P_4$, however, will not influence our interpretation of the section since we cannot see it to begin with. Hence, when a multiple (or other coherent noise) has an amplitude equal to or lower than that of the background noise in the stack, it will no longer interfere with interpretation of any primary event in the section. In such a case, the amplitude ratio between the primary and the multiple (PMR) is necessarily equal to or better than that between this primary and the background noise (SNR). Therefore, a "sufficiently large" PMR may be adequately defined as a PMR that exceeds the SNR in a stack section. Often, despite all efforts it may be difficult to achieve such a large PMR.

Recalling the relationship between $\gamma_O$, $\gamma_I$, and $\gamma_P$ in equation (2.2), it follows that any processing method applied to the data (in our case, stacking) must be able to deliver a sufficiently large $\gamma_P$ that, when combined with $\gamma_I$, will yield an SNR $\geq 6$ db and, at the same time, a PMR at least equal to that SNR in the output. As implied in Section 2.2, however, $\gamma_P$ for either coherent or incoherent noise is limited. Hence, depending on the value for $\gamma_I$, the minimal acceptable $\gamma_O$ according to the definition above is not always attainable. Therefore, given practical limitations on processing performance, it follows that we had better have the largest possible $\gamma_I$ in field data to insure the best chance of obtaining a sufficiently large $\gamma_O$ after processing.

Unfortunately, often we do not have a choice about the field data with which we work; i.e., $\gamma_I$ is fixed. This is when it can be most useful to study and quantify the limitations of $\gamma_P$ due to data imperfections and processing errors that undermine the performance of data processing. Given a fixed $\gamma_I$ in field data, this knowledge will be valuable for assessing results of seismic data processing.
2.7 Distortion of signal shape

The amplitude loss when a poorly moveout-corrected event is stacked is just one manifestation of wavelet filtering for which amplitude and phase both change as a function of frequency. As will be shown in Chapter 3, this frequency filtering is systematic and predictable. Other problems in the data could result in less predictable wavelet-shape distortions after stacking. One such problem is the presence of random residual statics.

So, distortion of signal shape in stacking constitutes a second imperfection of stacking, amplitude attenuation being the first. Notwithstanding the importance of preserving the shape of the stacked output wavelet in seismic data processing (for instance, in seismic attributive analyses such as porosity inversion, seismic stratigraphy, and seismic reservoir characterization), however, the primary concern of seismic data processing efforts (especially stacking) is to increase the detectability of primary events in the output. That is, in the first attempt at data processing, structural identification is usually given higher priority than is the ability to extract lithologic information from the data. Hence, size is typically more important than shape, especially for data from areas where $\gamma_I$ is very low. Once the primary events have been established, we can think about improving the resolution of the wavelets (for stratigraphic purposes) and the fidelity of their shapes.

It is in this spirit that in this thesis I have concentrated attention on the amplitudes (i.e., the $\gamma$'s), while addressing wavelet-shape issues just in passing, where appropriate.
Chapter 3

CMP STACKING

Foremost among methods for enhancement of signal-to-noise ratio are the various forms of stacking, all of which exploit the redundancy of measurements of signal that is common to multiple traces while unwanted noise varies among the traces. Because it is used universally, I focus attention on CMP stacking, in which the traces in a CMP gather undergo normal-moveout correction before being summed to produce a single output trace called the stack trace.

Another stacking procedure, vertical stacking, is frequently performed as part of data preparation in preprocessing (vertical stacking is used universally in vibroseis data acquisition). The essential principle of vertical stacking and CMP stacking is the same, being nothing other than some weighted averaging of data. Vertical stacking differs in that no NMO correction is performed. Therefore, many of the results from investigation of CMP stacking apply to vertical stacking as well. For discussions concerning effectiveness of the methods commonly used in vertical stacking see, e.g., Levin (1977), Brown et al. (1977), and Kirk (1981).

One might argue that CMP stacking is not used when prestack migration or dip moveout is performed. For our purposes, however, all that we learn here is equally applicable to stacking of DMO-processed or prestack-migrated data, which also involve moveout correction, in the presence of statics and noise problems, as well as problems of variation in source or receiver coupling.

In this chapter, I first introduce the most prominent of problems limiting the
performance of CMP stacking, NMO error, and develop approaches to quantify the resulting loss of signal-to-noise ratio. This discussion is intended to give an idea of the quantification of signal-to-noise ratio improvement and degradation in seismic data processing (i.e., $\gamma_P$ analysis). Next, I discuss qualitatively other data imperfections and stacking errors that could undermine the effectiveness of CMP stacking, leaving their quantification for Chapter 4. In short, the present chapter is a direct introduction to the more quantitative Chapter 4.

3.1 Normal moveout errors and CMP stacking

The averaging of data to increase the reliability of signal measurements is a statistical process that relies on signal being common to all measurements while noise differs. Only when the moveout velocity used in the NMO correction is accurate can the stacking process yield a stacked signal of maximum amplitude. Incorrect moveout velocities cause errors in the moveout corrections and consequently a misalignment of signal wavelets on the traces across the gather, resulting in attenuation (relative to the ideal) of the amplitude of the stacked signal, which often can be significant.

3.1.1 Accuracy of velocity analysis in the presence of interfering events

Consider the CMP gathers in Figures 3.1a-c. On these CMP gathers, the amplitude of the multiple event is four times that of the primary event, and the two events cross one another. The velocity spectra shown on the right-hand side of the CMP gathers have resolved the true moveout velocity of the multiple event well, but it yields some error for the primary (in velocity and time). The ability to separate the two events with accuracy in the velocity spectra depends on the general extent
Fig. 3.1. CMP gathers with two events and their velocity analysis spectra. The multiple (M) has four times the amplitude of the smaller-moveout primary (P). The zero-offset time of P is 2.5 s in all cases, while that of M is 2.47 s in (a), 2.485 s in (b), and 2.5 s in (c). The true moveout velocity of P is 3000 m/s, and that of M is 2600 m/s. The wavelet of the events is Ricker with 30-Hz dominant frequency. The velocity analysis was performed using normalized cross-correlation for the coherency measure (Yilmaz, 1987), with a 26-ms window at 13-ms time increments.
to which the two events overlap on the traces in the gather. Interference of events in Figure 3.1b, for instance, is the most extensive among the three examples in Figure 3.1: here the errors in interpreted velocity and time for the primary are seen to be largest.

In general, the larger the amplitude of the interfering multiple relative to that of the primary and the smaller the difference in moveout velocity for the multiple and the primary, the more extensive are the interference problems and the larger will be errors in the interpreted moveout-velocity spectrum. In such situations, the interfering multiple can mask the weaker primary to the extent that no moveout velocity corresponding to this primary can be identified (Figure 3.2), the primary becomes hidden and thus uninterpretable in the velocity analysis.

The above examples consider only one interfering multiple. In field data, interference of many noise events (not only multiples but also other coherent events such as neighboring primaries, diffractions, and ground-roll) with a single primary event is common; hence, we should expect even more difficulty in resolving moveout velocities.

Errors in velocity interpretation usually include errors in both velocity and time. Since moveout velocities are interpolated in time between picked velocity points down the length of the data, the size of the error for a particular event at a certain zero-offset time will depend on the locations of picks above and below it (Figure 3.3).

Thus, moveout error due to mis-picking of stacking velocities is perhaps much more common than often realized, especially for weak primary events whose velocities tend to be unresolved among interfering stronger noises. Moreover, interpreters usually pick stronger events, ignoring others. Unfortunately, the stronger events are not always the primaries.

In addition to interference by other coherent events, other factors that adversely
Fig. 3.2. Velocity spectra resulting from performing velocity analyses on CMP data similar to that in Figure 3.1b, but with the multiple having a moveout velocity of 2800 m/s and zero-offset time of 2.485 s. The multiple-to-primary amplitude ratio is (a) 0.5, (b) 1, (c) 2, and (d) 4. The moveout velocity of the primary (3000 m/s at 2.5 s) in (c) and (d) is uninterpretable.
influence the performance of velocity analysis are subsurface complexity and inhomogeneity, as well as near-surface time anomalies (Schneider, 1971).

### 3.1.2 Uniformly-weighted CMP stacking with moveout velocity errors

Signal-to-noise ratio improvement, $\gamma_P$, in the stacking of seismic data depends on both signal improvement ($\psi_P$) and noise increase ($\eta_P$). Since NMO error limits signal improvement while leaving random noise unaltered, in this section I give examples quantifying the limitation on signal improvement when signal is misaligned in the stack.

**Fig. 3.3.** In a velocity analysis, an event associated with a true moveout velocity of $v_0$ at a zero-offset time $t_0$ may be interpreted as having a moveout velocity of $v_2$ at a zero-offset time $t_2$. The velocity error associated with this event, $\Delta v$, depends on a time-velocity pick for an earlier event $(t_1, v_1)$.

For the present analysis, straight stacking is the stacking method used through-
out the experiments. Since in straight stacking, when there is no NMO error, the amplitude of the stacked signal is identical to that of the signal on individual (identical) unstacked traces, stacking with NMO error must necessarily result in attenuation (i.e., "negative" improvement) of signal amplitude. The magnitude of this attenuation is the limitation on signal improvement, i.e., $\psi_{P(L)}$. In turn, a limitation on signal improvement contributes to a limitation on SNR improvement.

To analyze the sensitivity of stacking output to NMO errors (resulting from errors in moveout velocity), I first conduct experiments with simulated CMP gathers and then give analytical formulations that approximate the results.

**The synthetic gathers.** The CMP gathers used in the following experiments consist of 48 traces with a 4000-m spreadlength and zero nearest-trace offset. The 4000-m spreadlength is chosen to simulate the common cable length used nowadays, while the 48-traces is selected for display purposes (we shall see later that in straight stacking, for typical trace spacing the number of traces in a CMP gather does not influence the stacked output). Unless otherwise specified, the primary event has a zero-offset time of 2.5 s and a true moveout velocity of 3000 m/s. The signal wavelet is a Ricker wavelet, zero-phase, with 30-Hz dominant frequency. The prestack signal amplitude is standardized at unit amplitude (0 db) on all traces (all amplitudes in the experiments are given by the absolute peak amplitude of wavelets). Straight stacking of perfectly aligned signal yields an output signal amplitude of 0 db; hence, this is the reference amplitude against which relative change in stacked signal amplitudes are measured.

The synthetic gather in Figure 3.4 simulates data that have been properly corrected for trace-to-trace amplitude variation, such as that due to geometrical spreading, the earth's absorption, and source and receiver coupling variations. Also, it is...
assumed that there is no AVO (amplitude versus offset) influence in the data.

**Fig. 3.4.** This prototypical CMP gather consists of 48 traces with a 4000-m spread length. The primary event has a moveout velocity of 3000 m/s, and the signal wavelet is Ricker zero-phase with 30-Hz dominant frequency and peak amplitude of 0 dB on all traces.

**Moveout undercorrection and overcorrection.** Figure 3.5a shows a CMP gather moveout-corrected with a moveout velocity \( v_{NMO} \) 10 percent above the true moveout velocity for the event. Figure 3.5b pictures the cumulative stacking results, i.e., from left to right, the stacking of an increasing number of traces in the moveout-corrected gather. For example, trace 20 is the stack of traces 1 through 20, etc. Notice the decrease in amplitude as the number of traces stacked increases, signifying attenuation. Also note the broadening of the wavelet in the stack, compared to its prestack shape, signifying loss of resolution due to the high-cut filtering action of stacking the misaligned event.

The curve in Figure 3.5c plots the peak wavelet amplitude corresponding to
Fig. 3.5. (a) CMP gather in Figure 3.4 moveout-corrected with a moveout velocity ($v_{NMO}$) that is 10 percent higher than the correct value; (b) cumulative stack; (c) peak amplitude of the stack traces; and (d) reflection time for these peaks.
the cumulative results in Figure 3.5b. Because the signal is misaligned, the mean amplitude after stacking all 48 traces in the gather is about 12 db lower than that of a perfect stack (0 db). This means that having a 10-percent error in $v_{NMO}$ results in a $\psi_P$ of -12 db. Since $\eta_P$ associated with the random background noise is not influenced by NMO error, it follows that the change in signal-to-noise ratio due to stacking, $\gamma_P$, is 12-db worse than if there had been no NMO error.

Figure 3.5d tracks the movement of the peak of the stacked wavelet due to the NMO error. Even after stacking all 48 traces, the wavelet peak is delayed by only 4 ms. The shift of the signal peak due to NMO error apparently does not constitute a significant problem.

Next, I extend the experiment by varying the $v_{NMO}$ error from 0 percent up to 100 percent. Amplitudes for the cumulative stacks are shown in Figure 3.6. The trace number is now expressed in terms of offset since misalignment of the signal, and thus the stack amplitude, depend more fundamentally on spreadlength than on the number of traces stacked. The cumulative stack curves suggest that it is better to use shorter spreadlengths to minimize amplitude losses where moveout velocities for the primaries are not accurately known. This would mean that we might wish not to include some of the far-offset traces in the stack, as is certainly the practice for reflections from shallow reflectors. Since this also means that we would decrease the number of traces to be stacked, however, we would consequently lose some of the power of the stack to suppress noise, our primary reason for employing CMP stacking.

The discrete stacking formula for this experiment is

$$S(t) = \frac{1}{N} \sum_{i=1}^{N} f(t - \Delta t_i),$$

(3.1)

where $f(t)$ is the signal wavelet as a function of time, and $\Delta t_i$, the residual moveout
or NMO error on the $i$th trace, is given as

$$\Delta t_i = \left( t^2 + \frac{x_i^2}{v_{\text{correct}}^2} \right)^{1/2} - \left( t^2 + \frac{x_i^2}{v_{\text{wrong}}^2} \right)^{1/2}$$  \hspace{1cm} (3.2)

$$\approx \frac{x_i^2}{2t} \left( \frac{1}{v_{\text{correct}}^2} - \frac{1}{v_{\text{wrong}}^2} \right).$$  \hspace{1cm} (3.3)

Here, $x_i$ is the offset for the $i$th trace, $v_{\text{correct}}$ is the true $v_{NMO}$, and $v_{\text{wrong}}$ is the velocity used for NMO correction. Equation (3.3) is valid for $x_i << t v$, where $v$ represents the smaller of the correct and incorrect velocity.

![Figure 3.6](image)

**Figure 3.6.** Cumulative stack amplitudes when the CMP gather in Figure 3.4 is moveout corrected with (a) $v_{NMO}$ varying from +0 percent up to +100 percent, in 5-percent increments, from the true moveout velocity of event, and (b) $v_{NMO}$ varying from +0 percent up to +10 percent, in 0.5-percent increments. A 3-percent velocity undercorrection (not an uncommonly large error) results in a 6-db amplitude attenuation for the full spread of 4000 m.

The amplitude curves in Figure 3.6 approach a common curve as NMO error increases. This behavior is easily understood from equation (3.3), since, as stacking
velocity \( (v_{\text{wrong}}) \) tends to infinity the situation approaches that of stacking the original CMP gather with no NMO correction. Of course in practice, data are not stacked with such extreme moveout velocities. Even with the full range of \( v_{NMO} \) encountered in practice (1500 m/s - 5000 m/s), it is still hard to think that one would stack, for instance, a primary event associated with a moveout velocity of 3000 m/s, using a velocity of 1500 m/s or 5000 m/s. Therefore, only the smaller \( v_{NMO} \) errors (such as up to \( \pm 10 \) percent) are an issue for the stacking of primary events. The results for larger \( v_{NMO} \) errors nevertheless will be of interest when we deal with the stacking of multiple events, which we purposely choose to mis-stack.

Schneider (1971) listed limits of acceptable NMO velocity errors for various applications employing conventional stacking. His observations are repeated in Table 3.1. An upper limit of 10-percent velocity error was suggested for general moveout corrections and CMP stacking purposes. However, the attenuation of stacked signal associated with a 10-percent \( v_{NMO} \) error, such as pictured in Figure 3.7, suggests that this size error can lead to sizable loss of signal. For instance, the loss of stacked amplitude can be as much as 15 db for a primary event at 2-s zero-offset time. A

<table>
<thead>
<tr>
<th>Use of velocity</th>
<th>Acceptable ( v_{RMS} ) error</th>
</tr>
</thead>
<tbody>
<tr>
<td>NMO corrections for CDP stack as currently practiced.</td>
<td>2-10 percent</td>
</tr>
<tr>
<td>Structural anomaly detection: 100 ft anomaly at 10,000 ft depth.</td>
<td>0.5 percent</td>
</tr>
<tr>
<td>Gross lithologic identification: 1000 ft interval at 10,000 ft depth.</td>
<td>0.7 percent</td>
</tr>
<tr>
<td>Stratigraphic detailing: 400 ft interval at 10,000 ft depth.</td>
<td>0.1 percent</td>
</tr>
</tbody>
</table>

Table 3.1. Acceptable moveout velocity errors. After Schneider, 1971.
high SNR will be required in the input data to guarantee an acceptable SNR in the stacked output (e.g., in order to produce an output SNR of at least 6 db) if we wish to tolerate such a moveout error in the data processing. Otherwise a more accurate stacking velocity is required.

Fig. 3.7. Loss of amplitude due to ± 10-percent error in the moveout velocity used in stacking. The correct \( v_{NMO} \) function in this experiment is \( v(t) = 1500 + 600t \) m/s. A 30-percent stretch muting (i.e., muting of part of a trace where wavelets undergo stretching of 30 percent or more due to moveout correction) to limit stretching of wavelet is applied. This muting results in reduction of effective spreadlength of the CMP gather for \( t \leq 1.9 \) s.

3.1.3 Analysis of the stacking filter

To gain understanding of the behavior of peak stack amplitude for erroneously NMO-corrected data, let us examine Figures 3.8 and 3.9. Figure 3.8a is a CMP
gather showing a multiple event with a true $v_{NMO}$ of 2400 m/s, moveout-corrected with a $v_{NMO}$ of 3000 m/s; the stack trace is displayed on the right hand side of the CMP gather (Figure 3.8b). The curves for the cumulative amplitude response from stacking of this gather are plotted in Figure 3.9. Notice that the sum curve (upper curve in the figure; this is the result of summation, rather than averaging, of the CMP traces) becomes stationary beyond a certain offset. Thus it appears that only a limited number of the near traces in the CMP gather contribute to the peak amplitude of the stacked output. This is supported when we look at the stack trace in Figure 3.8b. This trace consists of a main wavelet and a small tail, with near-zero amplitude in between: for a range of intermediate offsets, wavelets on the input traces interfere destructively. I shall henceforth refer to the minimum offset for which such
cancellation occurs as the "asymptotic offset." The asymptotic offset is the offset that, for a given \( v_{NMO} \) error, yields a residual NMO of approximately the dominant period \( T \) of the multiple wavelet. In our case, \( T \) is around 26 ms, which corresponds to an offset of approximately 1500 m. As the trace spacing goes to zero, the ratio of the amplitude of the tail in the stacked trace to the peak wavelet amplitude goes to zero. For finite trace spacing, however, this "tail wavelet" constitutes a processing noise that can interfere with weak true reflections.

Let us now seek an analytic expression for the asymptotic peak value of the stacked wavelet as the trace spacing goes to zero. Equation (3.1) can then be rewritten in a continuous form

\[
S(t) = \frac{1}{x_{\text{max}}} \int_0^{x_{\text{max}}} f(t - \tau(x))dx, \tag{3.4}
\]
with
\[ \tau(x) = \frac{x^2}{2t} \left( \frac{1}{v_{\text{correct}}} - \frac{1}{v_{\text{wrong}}} \right). \quad (3.5) \]

The trace offset \( x \) is now continuous, ranging from 0 to \( x_{\text{max}} \) (maximum offset or spreadlength), and as in equation (3.3), \( x \) must be \( \ll t \) \( v \) for equation (3.5) to hold. Since the stacked wavelet is unchanged for \( x > x_{\text{max}} \) (we assume that \( x_{\text{max}} \) exceeds the asymptotic offset), equation (3.4) may be rewritten as (for the moment, let us leave out the averaging factor \( 1/x_{\text{max}} \) to reflect a sum- rather than a mean-amplitude of the stack)
\[ S(t) = \int_0^\infty f(t - \tau(x))dx. \quad (3.6) \]

Also, since, from equation (3.5), \( S(t) \) is approximately symmetrical with respect to undercorrection and overcorrection,
\[ S(t) = \frac{1}{2} \int_{-\infty}^{\infty} f(t - \tau(x))dx. \quad (3.7) \]

Taking the Fourier transform of \( S(t) \) and using the shift theorem,
\[ S(s) = F(s) \frac{1}{2} \int_{-\infty}^{\infty} e^{-i2\pi s \tau(x)}dx, \quad (3.8) \]

where \( F(s) \) is the Fourier transform of \( f(t) \), \( s \) being the temporal frequency. Hence stacking in the time domain is a filtering operation in the frequency domain with a stacking filter whose transfer function is
\[ S_f(s) = \frac{S(s)}{F(s)} = \frac{1}{2} \int_{-\infty}^{\infty} e^{-i2\pi s \tau(x)}dx. \quad (3.9) \]
Substituting $\tau(x)$ from equation (3.5) into equation (3.9), we get

$$S_f(s) = \frac{1}{2} \int_{-\infty}^{\infty} e^{-\frac{i \pi}{2} \left( \frac{1}{v_{\text{correct}}} - \frac{1}{v_{\text{wrong}}} \right) x^2} \, dx. \quad (3.10)$$

The expression for $\tau(x)$ used here is the approximation for $x << t v$, but since we know that contribution to the integral comes from near traces only, we may comfortably use it in the infinite integral.

By the method of stationary phase (Bleistein, 1984), the approximate solution to equation (3.10) is

$$S_f(s) = \frac{1}{2} \sqrt{\frac{t}{s(v_{\text{correct}}^{-1} - v_{\text{wrong}}^{-1})}} e^{it}, \quad (3.11)$$

the stationary-phase approximation for the (summation) stack process, independent of spreadlength $x_{\text{max}}$ and, of course, of data multiplicity N. Thus, the amplitude spectrum of the stacking filter shows $1/\sqrt{s}$ behavior, and the phase spectrum is either $e^{i\pi}$ or $e^{-i\pi}$ depending on whether the data are undercorrected or overcorrected, respectively. If the traces are averaged, rather than summed, an additional factor $1/x_{\text{max}}$ is required.

The amplitude spectrum in equation (3.11) in db units is just $\psi_P$ (i.e., signal improvement due to stacking) as a function of the frequency $s$. This asymptotic result is a handy tool, for now it is not necessary to perform numerical experiments in order to predict the stacked amplitudes due to moveout velocity errors, as long as spreadlengths are greater than the asymptotic offsets.

Figure 3.10 compares the cumulative amplitude response curve generated from the numerical experiment (the "Mean" curve in Figure 3.9) against the one produced from the stationary-phase approximation. The two response curves are practically
FIG. 3.10. Cumulative stacked-amplitude response from numerical experiments and that from the stationary-phase approximation. The two curves are virtually identical beyond the asymptotic offset.

identical for offset larger than the defined asymptotic offset.

Since the amplitude response of the stacking filter has the form of $\kappa s^{-1/2}$, $\kappa$ being a proportionality constant (a function of zero-offset time and $v_{NMO}$ error), the dominant frequency of the output wavelet is shifted towards a lower frequency when data are mis-stacked. Note that this frequency shift does not depend on $\kappa$; thus, for instance, a zero-phase Ricker wavelet with a dominant frequency of 30 Hz when stacked with some $v_{NMO}$ error will produce a wavelet with a dominant frequency of about 26 Hz regardless of the moveout error, as long as the spreadlength exceeds the asymptotic offset. The peak amplitudes of wavelets we measure in the experiments are represented primarily by these dominant-frequency components.

This seemingly curious result that the reduction in dominant frequency is independent of the error in NMO velocity results from the stipulation that the NMO
error exceeds the dominant period $T$. For relatively small velocity error, a relatively large number of offsets (out to the asymptotic offset) contribute to the main wavelet in the stack trace.

Since the asymptotic formula is developed from continuous summation, it is inaccurate for sparse trace spacing; however, it is adequate for trace spacings commonly used in practice (see additional discussion on the uniform-weighting method in Appendix A). For coarser trace spacing, the amplitude and phase spectra would oscillate about the asymptotic behavior.

Also, as mentioned above, the asymptotic formula fails for maximum offsets that are less than the asymptotic offset for a given $v_{NMO}$ error, that is, for maximum NMO errors less than the dominant period $T$ of the wavelet to be stacked. Hence, to supplement the asymptotic formula, we would like to develop an empirical approximation to estimate amplitude attenuation for stacking with such small NMO error. For this purpose, let us express this NMO error (or the $v_{NMO}$ error) in terms of $T$ at a certain spreadlength of the CMP gather. Amplitude response in stacking should depend only on the ratio of the NMO error to the dominant period of the input wavelet, as long as residual moveout is approximately parabolic.

To demonstrate this contention, I performed a number of numerical simulations of stacking of wavelets associated with various dominant periods. Figure 3.11 plots the peak amplitude for stacking of events with NMO error ranging from 0 to $T$. The third-order polynomial that gives least-square error fit for the amplitude response curve in Figure 3.11 is

$$A(\tau) = -0.082 + 2.0\tau - 24.5\tau^2 + 14.4\tau^3,$$  \hspace{1cm} (3.12)

where $\tau$ denotes the maximum NMO error (i.e., NMO error at the maximum spreadlength)
Fig. 3.11. Amplitude response curve for stacking an event with NMO error from 0 to \( T \), where \( T \) is the dominant period of the input wavelet. The curve is averaged from numerical experiments using zero-phase Ricker wavelets with dominant frequencies ranging from 10 to 80 Hz. The small asterisks surrounding the curve are the data points.
expressed in terms of dominant period of the input wavelet \( T \) \( (0 \leq \tau \leq 1) \). Equation (3.12) is our empirical formula for stacking with a small NMO error; this is to supplement the asymptotic formula of equation (3.11), which applies to spreadlengths longer than the asymptotic offset. Figure 3.12 compares the cumulative stacked-amplitude responses generated from the numerical experiment (the "numerical" curve displayed in Figure 3.10) against that generated from combination of equations (3.11) and (3.12). The response curves are practically identical for all maximum offsets.

Figure 3.12. Comparison of cumulative stacked-amplitude response from numerical experiment with that from combination of empirical and stationary-phase approximation. A slight discontinuity occurs at the transition between the empirical and stationary-phase portions of the combination curve (in the vicinity of asymptotic offset).

Figure 3.13 shows an example prediction chart for amplitude in straight stacking (i.e., \( \psi_P \), if for a primary, or \( \eta_P \), if for a multiple) due to \( v_{NMO} \) errors; this contour plot is computed using a combination of equations (3.11) and (3.12). In this experiment,
the CMP gather is stacked using a $v_{NMO}$ function $v_{stack}(t) = 1500 + 600t$ m/s, where reflection time $t$ is given in s. Contoured is $\psi_P$ or $\eta_P$ as a function of true moveout velocity and zero-offset time for stacking velocity $v_{stack}$. This example suggests that an amplitude attenuation (of multiples or diffractions, for example) of as much as 24 db could occur in straight stacking.

From equations (3.11) and (3.12) we find that the magnitude of amplitude attenuation increases with

1. increase of error in moveout velocity,
2. increase in spreadlength $x_{max}$ (if the traces are averaged),
3. increase in the dominant frequency $s$ of the wavelet,
4. decrease in zero-offset time $t$, and
5. decrease in moveout velocity $v_{correct}$.

Equations (3.11) and (3.12) assume a CMP gather with a near-trace offset of zero. In practice, our near-trace offset is often greater than zero. Having a CMP gather with a nonzero near-trace offset is the same as muting some of the near traces in a CMP gather for which near-trace offset is zero, a stacking scheme that is called "inner-trace muting." Inner-trace muting is discussed in Chapter 4 and Appendix A.

### 3.2 What could go wrong in CMP stacking?

In Section 3.1.1, I demonstrated how one type of data imperfection, interfering events, could cause errors in moveout velocity estimation, which, in turn, impose limitations on the performance of CMP stacking. This constitutes one example from a long list of problems that can undermine the performance of CMP stacking. Here, I introduce these limiting factors qualitatively, deferring their quantitative analyses to Chapter 4.
FIG. 3.13. Example prediction chart for stacking amplitude ($\psi_P$ or $\eta_P$) due to $v_{NMO}$ errors. The stacking velocity function is indicated by the dashed line. Contour lines denotes $\psi_P$ or $\eta_P$ in db. A 30-percent stretch muting to limit NMO-stretch has been applied during the stacking. The jagged 8-db contours mark the transition between empirical and stationary-phase approximations.
As we have seen, the $v_{NMO}$ error limitation on signal-to-noise ratio improvement ($\gamma_P$), or more accurately on signal improvement ($\psi_P$), is readily predictable given the error in NMO velocity. This is because the parameters contributing to the error can be easily estimated with sufficient accuracy from the data. (It is true that the problem of event interference exemplified above is only qualitative, but once we decide on the $v_{NMO}$ error the rest of the computation is straightforward.) We will also see that some problems in stacking are actually intrinsic to the processing schemes themselves; for instance, the choice of using a small-spread hyperbola for the traveltime equation instead of the true nonhyperbolic behavior naturally leads to some inaccuracy in the traveltime estimates. However, these limitations are somewhat systematic, and thereby can be estimated to a useful degree. On the other hand, a limitation induced by subsurface inhomogeneity, for instance, is much less systematic, since subsurface complexity is not easily characterized.

Hence, I shall call the limiting factors of the first type discussed in the preceding paragraph “well-estimated limitations on $\gamma_P$,” while the latter type shall be called “poorly-estimated limitations on $\gamma_P$.” Table 3.2 lists these problems grouped according to these two rather loose categories. The letters “s” and “n” within the parenthesis following the name of a limiting factor indicate whether it is the stacked signal (therefore $\psi_P$) that is degraded or the stacked noise (therefore $\eta_P$) that is in-

<table>
<thead>
<tr>
<th>Well-estimated limitations</th>
<th>Poorly-estimated limitations</th>
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<tbody>
<tr>
<td>NMO velocity error (s,n)</td>
<td>residual statics (s)</td>
</tr>
<tr>
<td>nonhyperbolic moveout (s)</td>
<td>subsurface inhomogeneity (s)</td>
</tr>
<tr>
<td>geometrical spreading (s)</td>
<td>SNR variation (s,n)</td>
</tr>
<tr>
<td>anelastic attenuation (s)</td>
<td>channel inequality (s,n)</td>
</tr>
<tr>
<td>random noise (n)</td>
<td>coherent noise (s,n)</td>
</tr>
<tr>
<td>trace muting (n)</td>
<td>other noise (n)</td>
</tr>
</tbody>
</table>

Table 3.2. Limitations on $\gamma_P$ in the stacking of seismic data.
effectively attenuated, or both. The listing encompasses types of data imperfection and intrinsic processing or parameter errors that are particularly representative of problems in CMP stacking. Each of the entries is discussed in some detail below.

3.2.1 Well-estimated limitations on $\gamma_P$

**NMO velocity error.** At first glance one might consider moveout velocity error just as a processing-parameter flaw. Yet considering that this parameter error usually results from difficulty in picking an accurate stacking velocity due to data problems, we might also say this is a manifestation of seismic data imperfections.

Moveout velocity error results in NMO error (i.e., a residual moveout) in the NMO-corrected CMP gather before stacking. If we consider how much this NMO error can deteriorate the primary amplitude upon stacking, then we are quantifying signal attenuation ($\psi_P$). At the same time, however, NMO error also governs multiple attenuation in stacking. Since multiples are noise, here we would be quantifying noise attenuation ($\eta_P$).

**Nonhyperbolic moveout.** This problem constitutes an intrinsic data-processing error. Its basic consequence is NMO error in stacking, albeit usually on a much smaller scale than that caused by $v_{NMO}$ problems.

The moveout equation customarily used in CMP stacking is given as

$$t^2(x) = t^2(0) + \frac{x^2}{v_{NMO}^2},$$  \hspace{1cm} (3.13)

$$\Delta t_{NMO} = (t^2(0) + \frac{x^2}{v_{NMO}^2})^{1/2} - t(0),$$  \hspace{1cm} (3.14)

where $x$ is the trace offset, $v_{NMO}$ the moveout velocity, $t(0)$ the zero-offset time, $t(x)$ the two-way traveltime of the event at offset $x$, and $\Delta t_{NMO}$ the moveout correction.
Equation (3.13) constitutes the first two terms of the Taylor series expansion of the reflection time series; hence it is called the small-spread hyperbola (Hubral and Krey, 1980); this is the equation usually used to approximate the true traveltime curve in seismic data processing. Furthermore, the stacking velocity obtained from velocity analysis is actually only the best fitting hyperbola, which does not correspond to the hyperbola of equation (3.13), especially when there is inhomogeneity or anisotropy in the subsurface.

Since this error is systematic, some authors have proposed correction factors to mitigate its deteriorative influence in traveltime and velocity estimations (e.g., Brown, 1969).

**Geometrical spreading.** The decay of signal amplitude as a function of traveltime due to geometrical spreading results in the variation of signal amplitude across a CMP gather. Specifically, signal amplitude decreases systematically from trace to trace as offset increases. Since this is a violation of assumption (3) of the ideal stacking conditions described in Section 2.5, we cannot expect a $\sqrt{N}$ SNR improvement from the stacking of data, i.e., geometrical spreading results in a loss of $\gamma$.

Alternatively, we often apply a gain correction to data to equalize signal amplitudes across the gather, e.g., for deconvolution purposes. Unfortunately, this process makes the random background noise nonstationary, both temporally and spatially. Spatial nonstationarity means violation of the requirement for optimum noise attenuation in the uniformly-weighted stack, therefore limiting noise attenuation ($\eta$). However, as we shall see later, gain application to compensate for loss of signal amplitude due to geometrical spreading only slightly augments the loss of $\gamma$ from the geometrical spreading alone.
Anelastic attenuation. As a seismic pulse traverses its path through earth layers it is subjected to anelastic attenuation due to conversion of a fraction of its elastic energy into heat. This attenuation is frequency selective and depends on the path traversed by the pulse.

As does geometrical spreading, anelastic attenuation causes signal amplitude to vary from trace to trace across a CMP gather and thus, when the data are stacked, results in SNR improvement that is less than $\sqrt{N}$.

In data processing, this type of attenuation is sometimes corrected to an extent together with the geometrical spreading during gain recovery. Unlike geometrical spreading, however, augmentation of the loss of $\gamma_p$ from the application of anelastic correction can be significant, especially for higher frequency components of the signal when the earth section under consideration is highly attenuative.

In addition, anelastic attenuation also causes signal wavelet to vary from trace to trace, i.e., it introduces dispersion of waveforms that makes wavelets that have traveled greater distances (e.g., those on farther-offset traces) appear somewhat broader and delayed in phase (relative to those on near-offset traces). This delay might cause some misalignment of wavelets across an NMO-corrected CMP gather, which would result in a limitation when the traces are stacked. For the range of frequencies typically encountered in exploration seismology, however, the existence of such delay is generally insignificant (Strick, 1970), therefore the resulting limitation in CMP stacking is expected to be relatively small.

Random noise. The background noise in a CMP gather is often modeled as (1) Gaussian with zero mean, (2) spatially uncorrelated, and (3) temporally and spatially stationary. Straight stacking of this noise yields an $\eta_P$ that is equal to $1/\sqrt{N}$, where $N$ is the number of traces stacked in the CMP gather. Further, if we
mute M traces from the gather, then $\eta_P$ will be $1/\sqrt{N-M}$ instead. Likewise, if we apply trace-dependent weights before stacking, $\eta_P$ will also exceed $1/\sqrt{N}$, due to a reduction in the effective number of traces stacked.

Now, if trace muting or trace weighting is the only cause for stacking limitation, then SNR improvement $\gamma_P$ will be reduced by as much as the limitation on the noise suppression due to the reduction of the effective number of traces stacked (see Section 2.5).

**Trace muting.** In CMP stacking we often reduce the number of the CMP traces stacked, for instance, when we wish to exclude bad traces or ground-roll from the stack. The most common cause for reduction of the number of traces stacked, however, is the practice of trace muting to avoid direct and refraction arrivals or to limit stretching of wavelets due to NMO correction.

The stretching of wavelets in normal moveout correction, an intrinsic processing problem, has been studied in some detail by Dunkin and Levin (1973). During NMO correction, although the signal wavelet undergoes stretching (shift of signal energy towards lower frequencies), the maximum amplitude of the wavelet is maintained. This means that if one does not care about signal-shape degradation, we could still stack all the traces in a CMP gather and obtain a perfect signal amplitude, as long as $v_{NMO}$ is correct. However, excessive stretching is usually not desirable, and one customarily mutes portions of the traces that exhibit more than a certain amount of stretching (e.g., 10-30 percent). This necessarily results in some loss of the power of stacking in suppressing random noise.

Wavelet stretching due to moveout correction is quantified as (Yilmaz, 1987;
where Δt_{NMO}, x, t, and v(t), denote normal moveout, offset, time, and moveout velocity, respectively. The stretch is expressed in terms of percentage of wavelet stretching. From equation (3.15), limiting the NMO-stretch to 30 percent thus means muting the trace having an offset x at times less than that which would produce a stretch value, Stretch, greater than or equal to 0.3.

For a specified stretch limit, the number of traces that must be muted increases towards the shallower part of the data and thus imposes a more severe limitation on random-noise suppression. Prestack SNR, however, tends to be higher at early times; therefore, here the stacking limitation might not cause too much concern.

### 3.2.2 Poorly-estimated limitations on γ_p

Following are those limitations whose quantitative analysis is not as simple as that for the limitations discussed above. Here, quantification of their influence on γ_p depends on our ability to estimate various characteristics of the data, often in a statistical sense.

**Residual statics.** After field static correction, remaining trace-to-trace time distortions often exist in an NMO-corrected CMP gather. These “residual static time shifts” cause misalignments of signal wavelets across the gather that are unsystematic (recall that NMO error also causes misalignments of wavelets, albeit systematic). Hence, unlike the case of NMO error, it is not easy to predict the output of stacking an event in a CMP gather that has residual timing misalignment of signal. We can estimate an expected influence (in a statistical sense) of such errors, however, if we can...
assume a probability density function for their distribution across the CMP gather.

**Subsurface inhomogeneity.** Subsurface inhomogeneity, such as when the overburden has lateral velocity variation, results in traveltime curves that can be highly nonhyperbolic. The nonhyperbolic moveout here differs from that considered in the "well-estimated limitations" section in the sense that the traveltime curve here is so complicated (highly nonhyperbolic) that an approximation is difficult to find since the subsurface parameters are virtually unknown.

If we ignore subsurface inhomogeneity when we NMO-correct the CMP gather, the discrepancy in traveltimes manifests itself in the form of time shifts that are similar to those of residual statics. Lacking precise information about the distribution of time misalignment in the data makes the limitation poorly estimated. If the statistical distribution can be estimated, however, the limitation can be estimated using the approach that will be discussed for the residual statics problem.

**SNR variation.** Here input SNR ($\gamma_I$) is not constant from trace to trace in the CMP gather. It could be that only the signal amplitudes vary (fluctuating source energy from one shot to the next, AVO influence on some reflecting horizons) while the random background noise is constant, or, the signal amplitudes are constant while the noise varies (e.g., interference from another vessel, varying wind and wave noises), or both signal and noise amplitudes fluctuate in some unpredictable fashion.

Quantifying $\psi_P$ and $\eta_P$ is perhaps best done by a statistical analysis involving a modeling of the distribution of the variation of amplitudes of the signal and noise. In this way one can estimate the general influence of SNR variation on the stacked signal and the stacked noise, and thereby on $\gamma_P$. 
**Channel inequality.** Channel inequality results from unequal sensitivity of geophone/hydrophone sensors employed in the data acquisition. Variations could be due to malfunctioning of the devices. Channel inequality shows in the form of fluctuation of signal and background-noise amplitude levels from trace to trace, while their ratios are constant.

As with SNR-variation problem, channel inequality may be best characterized by the statistical distribution of the amplitude variation across the CMP gather. Then one can estimate the general influence of channel inequality on $\gamma_p$.

**Coherent noise.** Coherent noises are usually source-related noises. These include linear noises such as ground-roll, air waves, direct arrivals, and refractions, and nonlinear noises such as multiples, diffractions, side scatters, offline reflections (sideswipes), and side lobes of strong primary wavelets.

Their main corruptive influence on CMP stacking arises from their interference with primary events. Their interference in velocity analysis causes inaccuracy in moveout velocity estimation as discussed in Section 3.1.1. Moreover, strong multiples can obscure primaries in the stacked output. Side lobes of the wavelet of a strong primary event can also interfere with (and hide) a nearby weak primary in a stack section, therefore acting as a noise.

Quantifying the attenuation of coherent noise (e.g., a multiple) in stacking corresponds to analysis of $\eta_p$. Although one can rather easily model the stacking response of a coherent noise, the extent of its existence in a data set is hard to measure (e.g., how many multiples do we have). In this respect, this limitation is considered poorly estimated.
**Other noise.** Referred to here are those noises that show a limited spatial coherency but occur at random across the CMP gather. Examples include random spikes or noise bursts that stand out sporadically over the background-noise amplitude level (these are stray noises that usually afflict a few traces within a short time duration).

These noises are troublesome when their amplitudes are anomalously high relative to that of the average background noise in the data. Traces afflicted thus bias the stacked output, resulting in a stack SNR that can be less than that of the average input SNR in the rest of the CMP traces. Suppose, for example, there is only one trace in an N-trace CMP gather that is corrupted by such noise at a certain zero-offset time. Then, when the RMS amplitude of this noise exceeds

\[ n_{\text{spike}} = \sqrt{N^2 - N + 1} n, \]

where \( n \) is the average RMS amplitude of the background noise in the rest of the traces, CMP stacking with uniform weights will no longer attenuate the average background noise; specifically, for that threshold level of noise spike, \( \eta_P \) is just equal to \( \psi_P \) (assuming signal amplitude is uniform across the gather and there is no moveout error) such that the resulting \( \gamma_P \) is just 0 db. This anomalously high amplitude of a noise spike is by no means uncommon. This is why a trace corrupted by such noise must not be included in the stacking.

Since the amplitudes and occurrence of such noise in a CMP gather are not predictable, the extent of their limitation on the performance of CMP stacking is considered poorly estimated. If the statistical distribution of the anomalous amplitudes could be determined, however, the limitation could be estimated using the approach discussed in the SNR variation problem.
Chapter 4

QUANTIFYING THE LIMITATIONS

Let us now quantify some of the limitations on signal-to-noise ratio improvement in CMP stacking listed above. I focus attention on the influence of individual limiting factors introduced in Section 3.2 and finish with a discussion of limitations due to compounded errors in CMP stacking.

4.1 Generic formulas

Let us assume our data satisfy assumptions (1), (2), and (4) of the five assumptions listed in Section 2.5. Then, the essential reasons that stacking SNR improvement differs from \( \sqrt{N} \) are that (1) signal wavelets are misaligned across the gather, and (2) signal and/or noise amplitudes vary from trace to trace. Restating this in a different way, limitations on signal-to-noise improvement in stacking of seismic data are essentially caused by (1) time misalignments, and (2) amplitude variations. (Phase distortion from trace to trace is considered to be of secondary importance.) Hence, quantification of stacking limitations requires only two basic sets of formulas that will be applicable to any limitation.

4.1.1 Time misalignment

When there is no amplitude variation, departures from alignment of wavelets across a CMP gather determine the amplitude (and shape) of stacked signal \( \psi_0 \). In addition, they also determine the amplitude of stacked, misaligned coherent noise.
(\eta_0), such as a multiple. For an N-trace CMP gather, output of the straight stacking of a misaligned coherent event may be expressed [following equation (3.1)] as

\[ S(t) = \frac{1}{N} \sum_{i=1}^{N} f(t - \tau_i), \quad (4.1) \]

where \( \tau_i \) represents the time misalignment on the \( i \)th trace.

Time misalignment problems can be of two kinds: systematic and nonsystematic or erratic (pseudo random). The systematic kind is exemplified by the problem of NMO velocity error, introduced quantitatively in Chapter 3. In this case \( \tau_i \) is the residual moveout [equations (3.2) and (3.3)], and, consequently \( \psi_P \) or \( \eta_P \) can be estimated via equations (3.11) and (3.12).

Now, when the time misalignment is nonsystematic, such as in the problem of residual statics, \( \tau_i \) may be usefully estimated statistically from the data. In this case it is convenient to express \( \tau_i \) in terms of a hypothesized distribution of the time misalignments. Rewriting equation (4.1) we have

\[ S(t) \equiv \int_{-\tau_{max}}^{\tau_{max}} f(t - \tau) \tilde{p}(\tau) \, d\tau. \quad (4.2) \]

This is to say that the time misalignment \( \tau \) is characterized by a range of values \(-\tau_{max} \leq \tau \leq \tau_{max}\) that are distributed with a probability density function \( \tilde{p}(\tau) \). This statistical expression is convenient for estimating the stacked output without having to measure \( \tau_i \) individually from trace to trace. Since \( S(t) \) in equation (4.2) can represent amplitudes of both the stacked signal (\( \psi_0 \)) and the stacked noise (\( \eta_0 \)), then \( \psi_P \) and \( \eta_P \) can be obtained by subtracting input amplitudes \( \psi_I \) and \( \eta_I \) from the stacked output, respectively.
4.1.2 Amplitude variation

Let us instead assume there is no time-misalignment of wavelets across the CMP gather. Using results from Section 2.4, for an N-trace CMP gather, straight stacking of signal having trace-to-trace amplitude variation yields a stacked output of [from equation (2.11)]

$$
\psi_O = \frac{1}{N} \sum_{i=1}^{N} s_i,
$$

whereas that of random background noise yields [from equation (2.13)]

$$
\eta_O = \frac{1}{N} \sqrt{\sum_{i=1}^{N} n_i^2},
$$

where \( s_i \) is the amplitude of the signal on the \( i \)th trace and \( n_i \) is the RMS amplitude of the background noise on the \( i \)th trace. \( \psi_P \) and \( \eta_P \) can then be obtained by subtracting input amplitudes \( \psi_I \) and \( \eta_I \) from stacked amplitudes \( \psi_O \) and \( \eta_O \), respectively.

\( \psi_I \) and \( \eta_I \) themselves may be taken as the amplitude of signal and noise belonging to a reference input trace, for instance, the first (nearest-offset) trace of the CMP gather. Alternatively, \( \psi_I \) may be defined as the average of input signal amplitudes across the CMP gather; i.e.

$$
\psi_I = \frac{1}{N} \sum_{i=1}^{N} s_i,
$$

and \( \eta_I \) as the average of input noise RMS amplitudes

$$
\eta_I = \frac{1}{N} \sum_{i=1}^{N} n_i.
$$

Notice that if we define \( \psi_I \) according to equation (4.5) then \( \psi_O \) is equal to \( \psi_I \) in straight stacking, and consequently \( \psi_P \) is just 0 db, i.e., signal is "preserved."
note that expression (4.6) must not be interpreted as an averaging of noise traces as if in stacking; it is just an averaging of the values of the input noise RMS amplitudes across the CMP gather.

As with the time-misalignment problem, amplitude variations with offset can be of two kinds. When the amplitude variation is systematic, such as the decay of signal amplitude due to geometrical spreading, then the varying amplitudes can be approximated (e.g., using a geometrical-spreading equation) and substituted into the corresponding equations above to obtain an estimate for $\psi_P$.

On the other hand, when the amplitude variation is nonsystematic, e.g., when signal and random background noise amplitudes fluctuate in some unpredictable fashion, we may need to estimate $s_i$ and $n_i$ statistically. Incorporating the statistical distribution of the amplitude variations, we rewrite equations (4.3) and (4.4) as

$$\psi_0 \equiv \int_{s_{\text{min}}}^{s_{\text{max}}} s \, \tilde{p}(s) \, ds,$$

(4.7) and

$$\eta_0 \equiv \sqrt{\frac{1}{N} \int_{n_{\text{min}}}^{n_{\text{max}}} n^2 \, \tilde{p}(n) \, dn},$$

(4.8)

where $\tilde{p}(s)$ and $\tilde{p}(n)$ are the probability density functions for the signal and noise amplitudes, respectively. The expressions for the average of input amplitudes are

$$\psi_I \equiv \int_{s_{\text{min}}}^{s_{\text{max}}} s \, \tilde{p}(s) \, ds,$$

(4.9)

$$\eta_I \equiv \int_{n_{\text{min}}}^{n_{\text{max}}} n \, \tilde{p}(n) \, dn.$$

(4.10)

The systematic and nonsystematic classification of limitation mentioned here more or less coincides with the loose classification of limitations used in Table 3.2.
Limitations that are systematic are well estimated, while those that are nonsystematic are considered poorly estimated.

4.1.3 Remedial measures

In Chapter 3, we learned that in the presence of an NMO error (i.e., a time-misalignment problem), near-offset traces in a CMP gather contribute most to the stack of a primary or multiple event (see Figures 3.8 and 3.9). Since a multiple naturally has some moveout error when stacked using a primary velocity, we expect to obtain better attenuation of a multiple if, before stacking, we give smaller weights to the near-offset traces of the CMP gather and larger weights to the rest of the traces. This is the basic principle of offset-dependent weighting of traces prior to stacking, which is designed to attenuate multiples more effectively than when straight stacking is used. For our purpose, the objective of using offset-dependent weighting here is to obtain a reduction in $\eta_P$ so as to increase $\gamma_P$.

Let us say that $w_i$ is the offset-dependent weight applied to the $i$th trace of the CMP gather. The stacked output described in equation (4.1) now becomes

$$S_M(t) = \sum_{i=1}^{N} w_i f_M(t - \tau_i),$$

(4.11)

for a multiple event. Since the primary event, hopefully, has no residual moveout, then the stacked-output expression for the primary is just

$$S_P(t) = \sum_{i=1}^{N} w_i f_P(t).$$

(4.12)

Both equations (4.11) and (4.12) assume no variation of wavelet amplitudes across the CMP gather. To maintain the signal-averaging aspect of the stacking, $w_i$ must
satisfy a provision that

\[ \sum_{i=1}^{N} w_i f_p = \frac{1}{N} \sum_{i=1}^{N} f_p, \]  

(4.13)

or

\[ \sum_{i=1}^{N} w_i = 1. \]  

(4.14)

Let us first turn our attention to the amplitude-variation problem and its remedial measure before going further in our discussions about offset-dependent weighting. When both signal and noise amplitudes vary across the CMP gather (assuming no time misalignment of wavelets), how should we weight the traces so as to maximize \( \gamma_p \)? Do we equalize the amplitudes of the signal or do we equalize those of the noise? It turns out that the answer is neither of these; the optimum weights for this problem are (e.g., Brown et al., 1977)

\[ w_i = k \frac{s_i}{n_i^2}, \]  

(4.15)

where \( k \) is a constant proportionality factor. These weights are optimum in the sense that they minimize the expected noise power in the stack while preserving average signal amplitude. We can interpret these weights roughly as "rewarding" large signal amplitudes while "penalizing" large noise amplitudes. [Note that when signal and RMS noise amplitudes do not change from one trace to another, the optimum weights are uniform. Moreover, by equation (4.13), those weights are all equal to \( 1/N \).] For our purpose, let us call this type of optimum weighting "data-adaptive optimum weighting," owing to a need to estimate \( s_i \) and \( n_i \) directly from the unstacked data.

Using the above weights, \( \psi_O \) and \( \eta_O \) from equations (4.3) and (4.4) become [also see equations (2.11) and (2.13)]

\[ \psi_O = \sum_{i=1}^{N} w_i s_i, \]  

(4.16)
and
\[ \eta_0 = \sqrt{\sum_{i=1}^{N} w_i^2 \ n_i^2}, \quad (4.17) \]
and the provision for \( w_i \) adapted from equation (4.13) is
\[ \sum_{i=1}^{N} w_i s_i = \frac{1}{N} \sum_{i=1}^{N} s_i, \quad (4.18) \]
The output SNR from this optimum-weight stacking is (Brown, 1977)
\[ \gamma_0 = \sqrt{\sum_{i=1}^{N} \gamma^2_{i,i}}, \quad (4.19) \]
where \( \gamma_{i,i} \) signifies the SNR on the \( i \)th input trace.

Although stacking with weights as described in equation (4.15) in principle can improve stacking SNR, in practice the effectiveness is hampered by difficulty in estimating the amplitudes of signal and noise in field data. White (1977) suggested that inaccuracy in such estimations has resulted in generally marginal performance of the method in practice.

Now, what if the data contain multiples and at the same time have an amplitude variation problem? We would like to devise weighting that is optimized against both the multiple and the amplitude variation of signal and noise. At least one stacking method designed for this is what I shall call “offset-dependent optimum weighting” (Meyerhoff, 1966; Larner, 1994). Many other weighted stacking schemes can be regarded as either subcases or simplifications of offset-dependent optimum weighting, as we shall see below.

The set of weights used in offset-dependent optimum weighting is found by solv-
ing a set of normal equations

$$\sum_{j=1}^{N} M_{ij} w_j = k a_i,$$  \hspace{1cm} (4.20)$$

where \( k \) is a proportionality constant, \( i = 1, 2, 3, \ldots, N \), and

$$M_{ij} = \begin{cases} 
      b_i b_j E[\Phi_{mm}(t_i - t_j)], & i \neq j, \\
      b_i^2 E[\Phi_{mm}(0)] + c_j^2, & i = j.
   \end{cases} \hspace{1cm} (4.21)$$

Here \( a_i \), \( b_i \), and \( c_i \) denote relative strengths of signal, multiple, and RMS background-noise on the \( i \)th trace, respectively, and \( E[\Phi_{mm}(t_i - t_j)] \) represents the expected value of the autocorrelation function of the multiple wavelet at a time lag that is equal to the moveout difference between the wavelets on the \( i \)th and \( j \)th trace in the CMP gather. As before, to preserve the level of signal amplitude in the stack, \( w_i \) must satisfy equation (4.18). These weights \( w_i \) minimize the sum of the expected power in the stacked multiples and random background noise subject to the constraint that average input signal amplitude is preserved. For \( \Phi_{mm}(t_i - t_j) \), in practice we would estimate (roughly) the shape of the multiple wavelet based on its approximate frequency band. Also, in estimating the expected autocorrelation, we typically should allow for the likelihood that multiples have a range of moveout velocities.

Note that when we set the multiple strength \( b_i \) in expression (4.21) to zero, the weights reduce to

$$w_i = k \frac{a_i}{c_i^2}, \hspace{1cm} (4.22)$$

the same weights given previously in equation (4.15). The relative sizes of \( b_i \) and \( c_i \) provided for the computation of \( M_{ij} \) dictate how relatively much effort the design process puts into suppressing multiples as opposed to random noise. Any special effort to suppress multiples necessarily compromises to some extent the suppression of random noise, and vice versa. Also, the amplitudes \( c_i \) play the role of adding white
noise to the design process (e.g., they contribute just to the main diagonal of $M_{ij}$). The larger the judged importance of random noise relative to multiples as a problem, the closer the designed weights will come to satisfying equation (4.22).

### 4.1.4 Stacking methods in practice

Numerous CMP stacking methods have been suggested in the literature. Naess and Bruland (1985) presented an overview of many of these stacking schemes. In Table 4.1, I list some stacking methods, grouped according to their specialized objectives: attenuating either the coherent or incoherent noise components of the data. Brief descriptions for most of the listed methods, covering their basic procedures and characteristics, are given in Appendix A.

<table>
<thead>
<tr>
<th>Weighting that focusses on coherent noises</th>
<th>Weighting that focusses on incoherent noises</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Universal:</strong> uniform weighting</td>
<td></td>
</tr>
<tr>
<td><strong>Offset-dependent weighting</strong></td>
<td><strong>Data-adaptive weighting</strong></td>
</tr>
<tr>
<td>- inner-trace muting</td>
<td>- trace equalization</td>
</tr>
<tr>
<td>- square-root-offset weighting</td>
<td>- diversity stack</td>
</tr>
<tr>
<td>- offset-dependent optimum weighting</td>
<td>- data-adaptive optimum weighting</td>
</tr>
<tr>
<td><strong>Others</strong></td>
<td></td>
</tr>
<tr>
<td>- $N$-th root stack</td>
<td></td>
</tr>
<tr>
<td>- median stack</td>
<td></td>
</tr>
<tr>
<td>- alpha-trimmed mean stack</td>
<td></td>
</tr>
</tbody>
</table>

**Note:**
* this method also focusses on incoherent noise

Table 4.1. Stacking methods used in seismic data processing. Descriptions for most of the methods can be found in Appendix A.

Examining the various stacking schemes listed in Table 4.1, we can say that inner-trace muting and square-root-offset weighting are just crude simplifications of offset-dependent optimum weighting since the schemes have no real optimization basis.
and do not take into account amplitude variation in data. Likewise, the data-adaptive optimum weighting is a subcase of offset-dependent optimum weighting that ignores the need to suppress multiples. As we shall see later, trace equalization and diversity stack are special cases of data-adaptive optimum weighting. Median stack (Naess and Bruland, 1985), alpha-trimmed mean stack (Haldorsen and Farmer, 1989), and \( N \)-th root stack (McFadden et al., 1986) are alternative efforts at accomplishing goals of incoherent-noise suppression; I will not discuss them further.

The choice of weighting eventually depends on the particular limitation at hand (e.g., multiple, ground-roll, and background noise) and how much we know or can assume about the parameters of the limitation, plus how much effort we are prepared to pay for an anticipated resulting benefit. Modeling of data aimed at predicting SNR enhancement, such as will be illustrated in this chapter, might help in making such a choice.

### 4.2 Some notes on investigation methodology

We are primarily interested in analyzing the limitation on signal-to-noise ratio improvement, \( \gamma_{P(L)} \), in CMP stacking. This is accomplished by analyzing \( \gamma_P \) in the presence of the limiting factors and comparing it to the ideal \( \gamma_P \). Alternatively, the analysis can comprise investigations of limitation on signal improvement, \( \psi_{P(L)} \), and that of noise attenuation, \( \eta_{P(L)} \), through measurements of \( \psi_P \) and \( \eta_P \), respectively, and comparing these with their ideal values.

The analysis of limitations is divided into two groups. The first group involves analysis of time-misalignment problems such as limitations due to NMO-velocity error (here, I emphasize analysis on coherent noise attenuation, having discussed signal attenuation in detail in Chapter 3), nonhyperbolic moveout, and residual statics. The
second group consists of analysis of amplitude-variation problems such as limitations due to geometrical spreading, anelastic attenuation, SNR variation, and channel inequality.

In discussing time-misalignment problems, I typically present the analyses in terms of $\psi_P$ measurements (except for NMO velocity error which also involves $\eta_P$ measurements corresponding to multiples). Here the stacking of random background noise is not influenced by time misalignment, so $\psi_P$ directly reflects $\gamma_P$. On the other hand, amplitude-variation problems can influence the stacking of both signal and random background noise simultaneously; therefore, for convenience, for these problems I present the analyses in terms of $\gamma_P$ measurements.

The CMP gathers used throughout the following numerical experiments consist of 48 traces with a 4000-m spreadlength (except where wide-angle data are muted) and zero nearest-trace offset. Unless otherwise specified, the velocity used for NMO correction in the experiments follows the velocity function $v_{\text{stack}}(t) = 1500 + 600t$ m/s.

Throughout this thesis, to simulate realistic data processing I have applied muting of wide-angle data to limit stretching of wavelets due to NMO correction. Figure 4.1 shows example of muting patterns with a 30-percent NMO-stretch limit determined via equation (3.15).

With the application of trace muting, suppression of random background noise is reduced accordingly. As pointed out in Section 2.5, this reduces $\gamma_P$ by the same amount as the decrease in the noise suppression. The chart in Table 4.2 exemplifies such loss of $\gamma_P$ as a function of the number of traces lost (muted) from the full-fold of a 48-fold CMP gather.

Table 4.2 provides a useful reference for evaluating the severity of various stacking
Fig. 4.1. Muting of wide-angle data with a 30-percent NMO-stretch limit applied to a 48-trace, 4000-m CMP data. “A” is the mute pattern determined via equation (3.15) using \( v(t) \approx v_{stack}(t) \), where \( v_{stack}(t) = 1500 + 600t \) m/s. “B,” a more severe mute pattern, is determined using \( v(t) = 1500 \) m/s (a constant equal to water velocity).

<table>
<thead>
<tr>
<th>SNR loss (db)</th>
<th>Traces lost</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>45</td>
</tr>
<tr>
<td>10</td>
<td>43</td>
</tr>
<tr>
<td>8</td>
<td>40</td>
</tr>
<tr>
<td>6</td>
<td>36</td>
</tr>
<tr>
<td>5</td>
<td>33</td>
</tr>
<tr>
<td>4</td>
<td>29</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4.2. Loss of stacking SNR improvement, \( \gamma_{PL} \), as a function of the number of traces lost from the full-fold CMP gather. The CMP gather consists of 48 traces full-fold. Ideal \( \gamma_P \) from stacking with all 48 traces is approximately 17 db.
limitations discussed below. For example, from Table 4.2 a 6-db limitation is equivalent to the reduction in suppression of random noise when 75 percent of the total number of CMP traces are excluded from the stack. This is a significant limitation.

4.3 Stacking limitation due to time-misalignment problems

4.3.1 Systematic time misalignment

NMO velocity error. As mentioned above, offset-dependent weighting of traces prior to stacking can yield better attenuation of multiples than that producible by straight stacking. Imperfectly moveout-corrected primary, however, can also experience attenuation. To compare the performances on both primaries and multiples of the three offset-dependent weighting schemes listed in Table 4.1 with that of straight stacking, Figure 4.2 shows plots of attenuation obtained from numerical experiments for each of the four schemes as a function of moveout velocity (relative to the stacking velocity of 3000 m/s for zero-offset time of 2.5 s).

As seen in Figure 4.2, uniform weighting in general produces the least attenuation as a function of $v_{NMO}$ error, while offset-dependent optimum weighting yields the largest attenuation over ranges of moveout velocity for which the weights were designed to attenuate stacked events. The square-root-offset weighting produces results comparable to that of inner-trace muting. Square-root-offset weighting, however, should be the preferred method between the two because maximizing attenuation of multiple using inner-trace muting usually necessitates muting too many near-offset traces, thereby limiting random background noise suppression.

Offset-dependent optimum weighting also proves to be more robust compared to the other weighting schemes in the sense that it produces the least attenuation for events with small (e.g., less than 5 percent) moveout error, typical of imperfectly
Fig. 4.2. Stack performance as a function of moveout velocity for uniform weighting (straight stacking) and three offset-dependent weighting schemes for events at 2.5-s zero-offset time. For primaries, these curves denote $\psi_P$; for multiples, they denote $\eta_P$. The horizontal axis is true moveout velocities of either a primary or a multiple, and $v_{\text{stack}}$ is the velocity for NMO correction used throughout the experiment. For inner-trace muting, 18 near-offset traces have been muted to give the best attenuation of events. Opt.1 is offset-dependent optimum weighting designed for a range of moveout velocities between 1500 and 2700 m/s, while Opt.2 is that for moveout velocities between 1800 and 2400 m/s. The offset-dependent optimum weighting assumes a uniform SNR of 4. The wavelet for all events is Ricker zero-phase with 30-Hz dominant frequency and 0-db peak amplitude. The curves within the box are discussed in text and shown in more detail in Figure 4.3.
moveout-corrected primaries (see Figure 4.3).

Fig. 4.3. Enlargement of the curves within the box in Figure 4.2.

Following is an example of the use of the curves in Figure 4.2. Suppose we have a primary and a multiple with true moveout velocities of 2900 and 2000 m/s, respectively, and yet we stack the CMP gather with a velocity of 3000 m/s. Using uniform weighting, Figure 4.2 gives

\[ \psi_P = -8 \text{ db}, \]
\[ \eta_P = -20 \text{ db}. \]

Therefore,

\[ \gamma_P = \psi_P - \eta_P \]
\[ = -8 - (-20) \]
\[ = +12 \text{ db}. \]
On the other hand, if we use offset-dependent optimum weighting, $\eta_p$ for the multiple can be -39 db or more depending on the design of the optimum weighting, while $\psi_p$ for the primary has improved to -6 db (results for two offset-dependent optimum weighting designs are shown as Opt.1 and Opt.2 in Figure 4.2). As a result, $\gamma_p$ is 33 db for the optimum weighting, potentially a 21-db improvement over uniform weighting.

Plots similar to those in Figure 4.2 can be used to assess stacked output in processing of field data, as we shall see later in Chapter 5.

As mentioned in Chapter 3, the extent of the existence of coherent noise, such as multiples, in a data set is difficult to measure. For this reason, the limitation on coherent noises as an aggregate on the performance of CMP stacking is considered poorly estimated, although these noises may sometimes be dominant in the data.

Let us consider the moveout-corrected CMP gather shown in Figure 4.4a. This synthetic gather consists of 1000 multiples with various moveout velocities occurring at random zero-offset times. All multiples consist of wavelets with identical shape and amplitude, i.e., Ricker with a 30-Hz dominant frequency (26-ms dominant period) and -5-db RMS amplitude. Despite some evidence of coherency, the general appearance of the data resembles that of mixed random noise especially over much of the far-offset traces.

Figure 4.4b shows that the RMS amplitude of each trace of the CMP gather is approximately 14 db larger than that of the individual events that made up the trace. Now, since there are 1000 multiples within the 1000-ms long data, it follows that there are roughly 26 multiple wavelets randomly coexisting within any 26-ms (i.e., the dominant period of the multiples) time window on any trace. Interestingly,

---

1 Further discussion on the design and performance of offset-dependent optimum weighting is given in Appendix A.
**Fig. 4.4.** (a) Synthetic CMP gather (spreadlength = 4000 m) consisting of 1000 multiples with moveout velocities ranging between 1500 and 2400 m/s occurring at random zero-offset times. The data have been moveout corrected with a constant velocity of 3000 m/s. (b) The RMS amplitude from each trace of the CMP gather and that from individual multiple that made up the trace.
$\sqrt{26}$ is equivalent to about 14 db. Repetition of the experiment with different sets of 1000 multiples showed consistent results.

This data simulation suggests that the resultant of a large number of coherent noises existing together in a CMP gather and interfering at random with each other, may acquire the character of a random noise and therefore its RMS amplitude is larger by a factor of $\sqrt{M}$ relative to the amplitude of its individual constituents, where $M$ is the number of the coherent noise events coexisting within a dominant period of the data. These noises may therefore give the appearance of a background noise in a data set when in actuality the true random background noise has lower amplitude.

When the data in Figure 4.4a are stacked, however, the RMS amplitude of the stack trace, when compared to that of the prestack trace, still exhibits the PMR enhancement expected for multiple attenuation by CMP stacking, including the superiority of offset-dependent optimum weighting over the other weighting schemes in reducing the noise amplitude.

**Nonhyperbolic moveout.** The general use of hyperbolic traveltime equation (3.13) to approximate true reflection traveltime that is nonhyperbolic necessarily results in residual moveout, thus imposing a limitation on the performance of CMP stacking. The departure of the traveltime curve from being hyperbolic can readily be more severe when there is, for instance, (1) an anomalous high-velocity layer perturbing an otherwise smooth velocity trend as a function of depth (Brown, 1969), (2) lateral velocity inhomogeneity (Lynn and Claerbout, 1982), (3) anisotropy in the medium (Tsvankin and Thomsen, 1994), and (4) subsurface complexity (Tjan, 1995; Miller, 1974). While we do not normally use CMP stacking to process data from structurally complex areas, all of the other problems mentioned above can exist in a horizontally-layered medium, resulting in a limitation to CMP stacking.
To gain insight as to the size of the limitation due to nonhyperbolic moveout when we stack a CMP gather using the hyperbolic traveltime assumption, let us consider a medium with smoothly varying velocity as a function of depth only. Brown (1969) derived a correction factor that would approximate moveout errors resulting from the use of the hyperbolic approximation for the true traveltime to horizontal reflectors. Applying his correction factor, a closer approximation for nonhyperbolic moveout is expressed as (after Brown, 1969)

\[ \Delta T \approx \Delta T_s \left[ 1 - 1.4 \left( \frac{\Delta T_s}{T} \right) (\sigma/v_{r_{m_s}})^2 \right], \]  

(4.23)

with

\[ \sigma = KT/\sqrt{3}, \quad \text{for } \bar{v}_t = v_0 + KT, \]  

(4.24)

where \( \Delta T_s \) is the standard hyperbolic moveout correction [i.e., the \( \Delta t_{NMO} \) in equation (3.14)], \( T \) the zero-offset traveltime, \( v_{r_{m_s}} \) the RMS velocity, \( \bar{v}_t \) the time-average velocity\(^2 \), \( v_0 \) the velocity at the earth's surface and \( K \) a constant velocity gradient.

From equation (4.23), the residual-moveout correction is just

\[ \Delta \Delta T \approx 1.4 \left( \frac{\Delta T_s^2}{T} \right) (\sigma/v_{r_{m_s}})^2. \]  

(4.25)

From equation (3.14), if \( x << tv \) and if we assume \( v_{NMO} = v_{r_{m_s}} \), then \( \Delta T_s \approx x^2/(2v_{r_{m_s}}^2 T) \). Substituting \( \Delta T_s \) and \( \sigma \) in equation (4.25), we obtain a simplified

\[ \bar{v}_t \equiv \frac{1}{T} \int_0^T v_i(t) dt, \]

where \( v_i \) is the instantaneous velocity and the integral is over two-way zero-offset traveltime.

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expression for the residual-moveout correction:

\[ \Delta\Delta T \approx \frac{K^2 x^4}{8.57 v_{rms}^6} \]  

(4.26)

The quantity approximated by equation (4.26) is just the NMO error that would limit the amplitude of stacked signal. To get an idea of the size of the resulting stacking limitation, let us assume, for our purpose, that \( \bar{v}_t \) in equation (4.24) is equal to \( v_{rms} \approx v_{stack} \), where \( v_{stack}(T) = 1500 + 600T \) m/s. Figure 4.5 shows plots of \( \psi_p \) as a function of time and wavelet dominant frequency for a CMP gather with a spreadlength of 4000 m (except where wide-angle data are muted). The peak-

Fig. 4.5. \( \psi_p \) as a function of zero-offset time and wavelet dominant frequency. The horizontal axis denotes \( \psi_p \). The curves correspond to frequencies from 10 Hz (right), up to 80 Hz (left), in increments of 10 Hz. In the shallower portion of the data, spreadlength is reduced by an NMO-stretch limit of 30 percent, (a) using mute pattern "A" from Figure 4.1, and (b) using mute pattern "B".
amplitude attenuations are obtained through numerical experiments that simulated the straight stacking of a CMP gather with an event having a residual moveout described in equation (4.26). For a fixed spreadlength, limitation on signal improvement \( \psi_{P(L)} \) (i.e., the magnitude of \( \psi_P \) in Figure 4.5) increases as dominant frequency increases and as zero-offset time decreases. In Figure 4.5a, the maximum limitation is seen to range between 2 and 7 db for the generally useful seismic frequencies at approximately 2-s zero-offset time, the amplitude loss being mitigated by the trace muting for earlier times. To reduce the limitation even further, one might choose to define a more severe trace muting, such as exemplified in Figure 4.5b. Such practice may be justified since the use of long cables usually reflects our interest in deep targets. Now the limitation is under 3 db for all investigated frequencies.

To conclude, nonhyperbolic moveout is an intrinsic-processing error whose limiting power on the performance of CMP stacking can be large for high-frequency signals at moderate seismic depths. Moreover, the limitation can become more severe when the medium velocity varies erratically with depth or when the medium has a lateral velocity inhomogeneity and anisotropy. The limitation can be significantly reduced, however, by severe trace muting, with a trade off of reducing random-noise suppression.

### 4.3.2 Nonsystematic time misalignment

**Residual statics.** After field and residual static corrections, the remaining residual time shifts in CMP gathers are, it is hoped, small and randomly distributed within the gathers. When static-contaminated traces are stacked, amplitude attenuation and frequency filtering result on the stack of a primary event even when there is no moveout error associated with the event. A study by Marsden (1993) shows
that a 5-ms static time shift is enough to cause a 6-db loss of signal amplitude in the stacking of two traces having a 70-Hz signal wavelet. A CMP gather typically consists of more than just two traces; hence, the stacked output depends on many misalignments of wavelets, i.e., on the distribution of these misalignments (time shifts) among the traces in the gather. It follows that in order to quantify signal attenuation resulting from residual statics in CMP stacking, we need to know the distribution of the time shifts among traces across the gather. Since detailed information is usually unavailable from the data (if we knew the static time shifts, we would have removed them and therefore been rid of the problem), we will gain some insight by assuming a statistical distribution approximating the statics problem.

As an experimental illustration, consider event “A” in the NMO-corrected CMP gather in Figure 4.6. This event is a primary reflection plagued with static time

![Figure 4.6](image)

**Fig. 4.6.** Stacking an event with a maximum static time shift $\tau_{max}$ of 20 ms (event “A,” $-20ms \leq \tau \leq 20ms$) is akin to stacking a linear event with a maximum residual moveout of 40 ms (event “B”) when the static time shifts are distributed uniformly within the 40-ms time window.
shifts. The largest time shift ($\tau_{\text{max}}$) in this event is 20 ms (i.e., $\tau = \pm 20$ ms; the signal wavelets scatter within a 40-ms time window). Further, let us assume that the statics have a uniform probability density function, i.e., the time shifts are uniformly distributed within the 40-ms time window. Stacking of these misaligned wavelets is equivalent to stacking of wavelets reordered across the gather such that the event transforms into a linear event whose maximum residual moveout is 40 ms, as depicted by event "B" in the figure.

Knowing the probability density function of the static time shifts, we can readily predict signal attenuation via equation (4.2). Here, using numerical experiments employing a 48-trace CMP gather, and assuming a uniform probability density function, applying straight stacking, I compute signal attenuation as a function of maximum time shift and the dominant frequency of signal wavelet. The result is shown in Figure 4.7. The magnitude of signal attenuation is the limitation on stacking performance due to the statics. Not surprisingly, the larger the static time shift and the higher the dominant frequency of the wavelet, the greater the signal attenuation. (Equivalently, stacking of misaligned signal introduces a high-cut filtering of the input signal with the filter operator being a sampled version of the probability density function.) The event in Figure 4.6 has a maximum 20-ms static time shift and a dominant frequency of 30 Hz; hence, according to Figure 4.7, we could expect a limitation ($\psi_{P(L)}$) of about 19 db from stacking. This is clear justification for attempting to reduce the statics errors before stacking data, demonstrating the importance of field-statics and residual-statics corrections in seismic data processing. As mentioned above, however, after these corrections we usually still have some remaining residual time shifts in the data. Examining Figure 4.7, we see that even with a 5-ms maximum static time shift, the resulting $\psi_{P(L)}$ can still be large (i.e., $\geq 6$ db) for the higher-frequency signals.
Fig. 4.7. Signal amplitude response $\psi_P$ as a function of maximum static time shift and dominant frequency of signal wavelet. The top curve pertains to a 10-Hz signal. Subsequent curves are increments of 10 Hz, up to 80 Hz (bottom curve). The signal wavelet is Ricker, zero-phase. In all cases, the CMP gather consists of 48 traces. The box indicates signal attenuation corresponding to residual-statics problems remaining after conventional residual-statics correction (time shifts = ±5 ms).
Notice that signal attenuation due to residual statics does not depend on the spreadlength of CMP gathers. Expected signal attenuation depends only on the probability density function of the static time shifts.

4.4 Stacking limitation due to amplitude-variation problems

4.4.1 Systematic amplitude variation

Geometrical spreading. Amplitude loss due to geometrical spreading in a horizontally-layered medium is commonly given in the form of (Newman, 1973)

\[ A(t_i) = \frac{c}{t_i v_{rms}^2}, \]  

(4.27)

where \( t_i \) is the two-way traveltime to a reflection event on the \( i \)th trace of a CMP gather, and \( c \) is an arbitrary constant. In practice, stacking velocity is used for \( v_{rms} \).

Since \( t_i \) is a function of normal moveout, the variation of amplitude of a reflection event across the CMP gather is thus a function of normal moveout. Specifically, the signal amplitude decreases (as offset increases) relative to the nearest-offset trace with a factor of

\[ \alpha_i = \frac{c/(t_i v_{rms}^2)}{c/(t_1 v_{rms}^2)} = \frac{t_1}{t_i} = \frac{t_1}{t_1 + \tau_i}, \]  

(4.28)

where \( t_1 \) is the reflection time of the event on the nearest-offset trace and \( \tau_i \) is the moveout-difference of the event on the \( i \)th trace relative to \( t_1 \). Thus, SNR along a reflection can be expected to decrease with increasing offset.

On the other hand, we often apply a gain correction to data to equalize the signal amplitudes across the gather (e.g., for deconvolution purposes). This gain correction is just the reciprocal of the amplitude factor in equation (4.28). Equalization of the signal amplitudes, however, necessarily results in disequalization of
the background-noise amplitudes from trace to trace. Hence, either way we have an amplitude-variation problem that hinders our obtaining a $\sqrt{N}$ SNR improvement in the subsequent CMP-stacking of the data.

To gain an insight as to how much limitation one might suffer from geometrical spreading as well as from an application of gain correction to equalize the signal, I plot SNR improvement, $\gamma_p$, as a function of zero-offset time for a 48-trace, 4000-m CMP gather (Figures 4.8a and b).[^3]

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[^3]: The SNR improvements are computed using equations (4.3) and (4.4) with the variation of amplitudes governed by expression (4.28). Input signal and noise RMS amplitudes are referenced to the first trace of the CMP gather and set to 0 db. Here, we assume that noise amplitude is uniform in the original data, while signal amplitude becomes uniform only after gain correction.
In Figure 4.8a, limitation on SNR improvement, $\gamma_{P(L)}$ (i.e., the magnitude difference between the curve for $\gamma_P$ and that for ideal $\gamma_P$ of $\sqrt{N}$), is restricted to under 1 db at all times; a small limitation. Further, the augmentation of the limitation due to application of gain correction to data also proves to be insignificant (Figure 4.8b).

**Anelastic attenuation.** Anelastic attenuation influences CMP stacking much the same as does geometrical spreading. In this case, the amplitude loss is usually given in the form of (Hatton et al., 1986)

$$A(s, t_i) \approx e^{-\frac{ts_{ti}}{Q}}, \quad (4.29)$$

where $s$ represents the frequency of signal for which attenuation is being estimated, $t_i$ the two-way traveltime to the reflector on the $i$th trace, and $Q$ the quality factor averaged over the earth section above the reflector. Common $Q$ values are $50 \leq Q \leq 300$ (Hatton et al., 1986). Relatively high $Q$ values signify media that are not very "lossy," whereas a highly attenuative earth section will have a low $Q$.

Normalizing the attenuation factor of the $i$th trace relative to that of the nearest-offset trace (i.e., the 1st trace) in the CMP gather we obtain

$$A_{norm}(s, t_i) = \frac{e^{-\frac{ts_{ti}}{Q}}}{e^{-\frac{ts_{1(1)}}{Q}}} = e^{-\frac{ts_{(i-1)}}{Q}} = e^{-\frac{ts_{ti}}{Q}}, \quad (4.30)$$

the amplitude-variation factor that is comparable to $\alpha_i$ in equation (4.28). Hence, the offset-dependence of the attenuation filter is a function of NMO. The limitation on the stacking performance due to anelastic attenuation thus will be largest earlier in the data due to larger NMO, the limitation becoming negligible at depth.

Now, if we have a rough estimate for $Q$, we can estimate the stacking limitation
following the same approach used before in the analysis of geometrical spreading. Let us, for example, assume Q is 50. Also, let anelastic attenuation be the sole cause for amplitude variation in the CMP gather. Figures 4.9a and b show SNR improvement,

\[ \gamma_p \]

\(\gamma_p\) as a function of zero-offset time and frequency component of signal. The horizontal axis denotes \(\gamma_p\). In each figure, the solid/dotted curves are \(\gamma_p\) corresponding to frequencies from 80 Hz (left), down to 10 Hz (right), in decrements of 10 Hz; the dashed curve is the ideal \(\gamma_p\). \(\gamma_{P(L)}\) (i.e., the magnitude difference between the \(\gamma_p\) curves and the ideal-\(\gamma_p\) curve) as a result of signal amplitude decay with offset due to anelastic attenuation in (a) is significantly aggravated by application of an anelastic correction to data in (b). A Q value of 50 is used throughout the experiments. Muting pattern “A” from Figure 4.1 is applied.

\(\gamma_p\), estimated for signal frequency \(s\) between 10 and 80 Hz. The magnitude difference between the curves for \(\gamma_p\) and that for ideal \(\gamma_p\) of \(\sqrt{N}\) in the figures signifies the limitation on SNR improvement, \(\gamma_{P(L)}\).

In general, \(\gamma_{P(L)}\) increases as frequency increases and as zero-offset time decreases. In Figure 4.9a, the maximum limitation is seen to range between 1 and 6 db for the generally useful seismic frequencies at approximately 2-s zero-offset time,
the amplitude loss being limited by the trace muting for earlier times. As with the nonhyperbolic moveout problem, to reduce the limitation, one could define a more severe trace muting, such as shown in Figure 4.10.

![Amplitude vs Time](image)

**FIG. 4.10.** Same as Figure 4.9, but with a more severe trace-muting to reduce both the wavelet stretching due to NMO correction and $\gamma_{P(L)}$ due to anelastic attenuation. Here, muting pattern "B" from Figure 4.1 is applied.

It is easy to see from equation (4.30) that limitation will decrease rapidly with increase of $Q$ value, indicating that anelastic attenuation has meaningful influence only when $Q$ is small, i.e., for highly attenuative rocks. Otherwise its influence on CMP stacking is negligible.

In Figures 4.9b and 4.10b, augmentation of $\gamma_{P(L)}$ due to application of anelastic correction to data appears to be quite significant especially for the higher frequency components of the data. This is not surprising since, unlike in geometrical spreading, here the amplitude decay factor (i.e., the "weight") is exponential. This suggests that while the geometrical-spreading correction is safe to apply to any data, we might
refrain from applying an anelastic correction (e.g., inverse-Q filtering) prior to stacking when we know Q is very low and we are interested in preserving the high-frequency components of the data.

4.4.2 Nonsystematic amplitude variation

**SNR variation.** SNR variation is a data imperfection that influences $\gamma_P$ through the variation in amplitude of signal and random background noise from trace to trace. In this sense, the problems of variation in signal amplitude due to geometrical spreading and anelastic attenuation discussed in earlier sections are subsets of this problem. The difference is that the variation in amplitude from trace to trace discussed here is generally not systematic.

There are four possible cases of amplitude variation from trace to trace:

1. both signal and noise amplitudes vary independent of each other,
2. signal has the same amplitude on all traces, but noise varies,
3. noise has the same amplitude on all traces, but signal varies,
4. signal and noise amplitudes vary but their ratios are the same.

Cases (1), (2) and (3) are the subjects of SNR variation discussed in the present section, while case (4) is that of channel inequality addressed in the next section.

As mentioned in Section 4.1.3, to maximize output SNR in stacking a data set having SNR-variation problems we can use data-adaptive optimum weighting. In principle, if we apply the weights described in equation (4.15) to the data, then theoretically we will obtain a stacked output defined in equation (4.19).

Now, when the weights in equation (4.15) are adapted to case (2), they reduce to

$$w_i = k \frac{s}{n_i^2} = (constant) \frac{1}{n_i^2}.$$

(4.31)
Thus, the weights are inversely proportional to the noise power of the respective traces. This is just the basis of the diversity stack (e.g., Embree, 1968) commonly used in vertical stacking of vibroseis data.

As discussed in Section 4.1.2, to estimate $\gamma_{P(L)}$ due to nonsystematic amplitude-variation problems one needs to know the distribution of the amplitude variations, both in the signal and in the random background noise. Only then can one use equations (4.7) through (4.10) to arrive at an estimate of $\gamma_P$ and $\gamma_{P(L)}$.

For a demonstration, let us consider a problem of case (2) where the probability density function of the trace-to-trace amplitudes of the noise is uniform. Figure 4.11 shows estimates of $\gamma_P$ as a function of maximum amplitude variation of noise [i.e., $n_{\text{max}}/n_{\text{min}}$ in equations (4.8) and (4.10), the ratio between the largest and the smallest RMS noise amplitudes across the CMP gather]. Defining our $\gamma_I$ as the arithmetic mean of input SNR across the gather, we observe that stacking the data using uniform weighting results in a loss of $\gamma_P$ relative to $\sqrt{N}$ SNR improvement; this loss is approximately 2.5 db for a maximum amplitude variation of 5. On the other hand, using data-adaptive optimum weighting (e.g., the diversity stack), we can expect [theoretically, using equation (4.19)] a $\gamma_P$ that is a little better than $\sqrt{N}$, or about 3.5 db better than that for uniform weighting.

As exemplified above, the $\gamma_P$ limitation imposed by a uniformly-distributed amplitude variation in the random background noise does not seem to be large (i.e., generally, it is less than 6 db). Consider, however, a 48-trace CMP gather that contains one anomalously noisy trace. Assuming the RMS amplitudes of the noise on the rest of the traces are equal and the signal amplitudes on all traces are identical,

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4The choice of using case (2) from the three cases of SNR variation for the demonstration here is somewhat arbitrary. The fact that only one component of the SNR is allowed to vary, however, simplifies the design of the numerical experiment considerably.
Fig. 4.11. $\gamma_P$ as a function of maximum amplitude variation of noise. The top curve is the result of diversity stacking. The bottom curve is the result of uniform weighting. The CMP gather consists of 48 traces ($N = 48$) in all cases.
then, if we define $\gamma_I$ as that SNR belonging to the “good” traces (i.e., traces that are not noisy), $\gamma_P$ will be substantially limited when the bad trace is included in the straight stack (Figure 4.12). In Figure 4.12, loss of $\gamma_P$ relative to $\sqrt{N}$ exceeds 6 db for a noisy trace whose RMS amplitude is more than twelve times that of the noise on the good traces. The limitation, however, can easily be reduced to a negligible amount (a fraction of a db) by applying diversity stack as shown in the figure. Such noisy traces are common in vibroseis data (a prime example being passing vehicles near the survey area during data acquisition); therefore, diversity stack is routinely used in the vertical stacking of vibroseis data.

![Figure 4.12](image)

**Fig. 4.12.** $\gamma_P$ as a function of the strength of the noisy trace relative to the “good” traces. The vertical axis denotes $\gamma_P$. The dashed curve (“Optimum”) is the result of diversity stack. The bottom curve is the result of uniform weighting. In all cases, the CMP gather consists of 48 traces ($N = 48$) with one noisy trace.

Anomallyali high noise amplitudes can also occur for short durations on a CMP
trace. Such noise bursts might be caused by fish bites or objects hitting the streamer cable in marine data (Fulton, 1985). Unlike in diversity stacking, in this case the weights in equation (4.31) must be made time variant, i.e., the noise power used in the weighting must be measured in short time windows; since the weights that are optimum for the time levels where the noise bursts occur would not be optimum for other times where there is no burst on any of the traces. Figure 4.12, however, can still describe the limitation caused by a noise burst occurring on one of the 48 CMP traces at a particular time level. Here, the horizontal axis signifies the relative strength of the noise burst.

**Channel inequality.** When the weights in equation (4.15) is adapted to case (4), they reduce to

\[ w_i = k \left( \frac{s}{n} \right) \frac{1}{n_i} = (\text{constant}) \frac{1}{n_i}, \quad (4.32) \]

and the stacked output using such weights, as predicted by equation (4.19), has SNR given by

\[ \gamma_o = \sqrt{N} \left( \frac{s}{n} \right). \]

Here, the SNR improvement is exactly \( \sqrt{N} \).

If we take the viewpoint that channel inequality results in the weighting of the traces in a CMP gather by \( n_i \), then the weighting coefficients \( w_i \) are just those that cancel the "field" weights, thereby removing the trace-to-trace amplitude variation. If one can estimate \( n_i \) from the data, then \( w_i \) is just the inverse of \( n_i \), as above. In practice, this can be accomplished by normalizing the CMP traces by their RMS amplitude values. This is the basis of *trace equalization*, commonly used in routine data processing.

Figure 4.13 shows estimates of \( \gamma_p \) as a function of maximum variation in ampli-
tude in the CMP traces, assuming a uniform distribution in the probability density function of the amplitude variation. Using uniform weighting, the resulting limitation on $\gamma_p$ measured relative to the expected $\sqrt{N}$ improvement is less than 1 db for a maximum amplitude variation of 5. This limitation is relatively small.

### 4.5 Compound limitations on $\gamma_p$

It can be expected that, in practice, more than one type of limitation on $\gamma_p$ will be present simultaneously in the processing of seismic data. The example presented in Chapter 1 (Figure 1.2) combines problems of NMO error, static time shifts, and trace-to-trace amplitude variation of random background noise in one data set.

**Figure 4.13.** $\gamma_p$ as a function of maximum amplitude variation in the CMP traces with constant SNR. The top curve is the theoretical SNR improvement obtained when amplitude variation is successfully removed from the data. The bottom curve is the result of stacking with uniform weighting. The CMP gather consists of 48 traces in all cases.
Throughout this thesis we have presumed that limitations that influence the stacking of signal usually can be considered separately from those that influence the stacking of noise. This has been the basic premise of the separation of $\gamma_P$ into its components $\psi_P$ and $\eta_P$ in this thesis. However, there seems to be an interaction between limitations on the same component of $\gamma_P$. From the example in Chapter 1, had there been no such interaction, we would have observed that

$$\psi_{P(L)}(NMO + Statics) = \psi_{P(L)}(NMO) + \psi_{P(L)}(Static).$$  \hspace{1cm} (4.33)

Amplitude analysis in Figure 1.3 suggests that the above relationship approximately holds in the particular example; there, $\psi_{P(L)}(NMO)$ was 5 db and $\psi_{P(L)}(Static)$ was 2.5 db; meanwhile $\psi_{P(L)}(NMO + Statics)$ was 8 db. However, reproduction of the experiment in the figure with different distributions of static time shifts gave varying results, the variation being dependent primarily on the time shifts belonging to the near traces in the CMP gather, which contribute the most to the stacked output in the presence of the NMO error. Only if we could reduce the trace spacing within the CMP gather toward zero without changing the spreadlength would the compound signal loss $\psi_{P(L)}$ ultimately converge to the value given by the right-hand-side of equation (4.33) because in that limit, the distribution of the time shifts among the near-offset traces would resemble their total distribution across the gather (i.e., the statics distribution becomes spatially stationary). Nevertheless expression (4.33) can provide a fairly good estimate of the compound limitation. Hence, in general, the combined limitation due to NMO error and static time shifts in CMP stacking is approximately the product of limitations from the individual limiting factors.

Let us now examine the compound-limitation problem for the stacking of noise. Consider the following conceptual example. Suppose we have a 4-trace CMP gather
having a background-noise variation problem such that the noise amplitudes are

\[ n_i = 1, 2, 2, 1, \]  

(4.34)

where \( i \) is the trace index (i.e., \( i = 1, ..., 4 \)). Now, let us suppose we wish to gain-correct the data to compensate for the loss of signal amplitudes due to, let us say, geometrical spreading and anelastic attenuation, with the following gain coefficients,

\[ \alpha_i = 1, 1.5, 2.5, 4. \]  

(4.35)

The gained noise amplitudes are thus

\[ n_{i,\text{gain}} = 1, 3, 5, 4. \]  

(4.36)

The signal amplitudes that varied from trace to trace before the gain correction are now uniform. For the purpose of this example, however, we will not concern ourselves with the stacking of the signal; instead, we will study results of the stacking of the background noise only. The stacked-noise analysis from this experiment is summarized in Table 4.3.\(^5\) In the first row of Table 4.3, \( \eta_{P(L)} \) is computed before the application of gain correction [noise variation follows expression (4.34)]. In the second row, \( \eta_{P(L)} \) is computed with the noise variation as would be introduced by the gain correction alone, i.e., we assume there was no noise variation before the gain correction [noise variation follows expression (4.35)]. In the last row, \( \eta_{P(L)} \) is computed with the noise variation that results from the application of the gain correction on the original noise variation [noise variation follows expression (4.36)].

\(^5\)The reference \( \eta_l \) is 0 db, taken from trace 1 of the gather. Stacked output \( \eta_O \) is computed using equation (4.4). \( \eta_P(\text{Ideal}) \) is -6 db, i.e., \( 20 \log_{10}(1/\sqrt{4}) \). \( \eta_P = \eta_O - \eta_l \). \( \eta_{P(L)} = \eta_P - \eta_P(\text{Ideal}) \).
If there were no interaction between the influence of the original noise variation and that of the gain correction during the stacking, the following relationship would be true:

\[
\eta_P(L)(Variation + Gain) = \eta_P(L)(Variation) + \eta_P(L)(Gain). \tag{4.37}
\]

From the analysis in Table 4.3, the right-hand-side of equation (4.37) produces an estimate (12 db) that is only slightly larger than the actual limitation (11 db). Thus, equation (4.37) provides a close estimate for the compound limitation.

The above results suggest that compound limitations in CMP stacking approximately are given by the product of the limitations from the individual contributing factors; that is, the limitations are multiplicative to a sufficient level of approximation. As we shall see later in Chapter 5, accuracy of output predictions is generally of lesser importance than awareness of the presence and influence of limitations in the process of stacking.

<table>
<thead>
<tr>
<th>Background noise</th>
<th>(\eta_I)</th>
<th>(\eta_O)</th>
<th>(\eta_P)</th>
<th>(\eta_P(Ideal))</th>
<th>(\eta_P(L))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variation alone</td>
<td>0</td>
<td>-2</td>
<td>-2</td>
<td>-6</td>
<td>4</td>
</tr>
<tr>
<td>Gain alone</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>-6</td>
<td>8</td>
</tr>
<tr>
<td>Variation + Gain</td>
<td>0</td>
<td>5</td>
<td>5</td>
<td>-6</td>
<td>11</td>
</tr>
</tbody>
</table>

Table 4.3. Analysis of compound limitations in the stacking of random background noise.
Chapter 5

\( \gamma \) ANALYSIS FOR A FIELD DATA SET

In order to conduct signal-to-noise ratio (\( \gamma \)) analysis, in practice, it is necessary to estimate SNR and PMR from the data. This can be done either directly by measuring signal and noise amplitudes from the data (if they can be identified) and taking their ratio, or indirectly, e.g., by measuring the coefficient of coherence between adjacent traces in the CMP gather (e.g., Foster and Guinzy, 1967; White, 1973; White, 1984; and Walden, 1990). When SNR and PMR in the data are so low that visual distinction between signal and noise is impossible, however, an indirect approach is necessary. In the present experiments, I use direct measurements of signal and noise to assess the improvement of SNR and PMR achieved in CMP stacking of a field data set.

5.1 Field data example

Figure 5.1 shows a portion of a marine seismic line from the Northeast Java Sea in Indonesia, acquired in 1981. The target zone for improvement in this data set consists of weak seismic reflectors beneath a strong limestone reflector marker at approximately 2.2 s zero-offset time. Lithologically, this zone consists of sand and shale layers, which could constitute the source and reservoir rocks for hydrocarbon that might exist in the area. Reflections from boundaries of the high-velocity, high-density limestone layer, however, are so strong relative to those from the sand-shale sequence beneath them that reflections in the target zone are not only weak, but discontinuous. Moreover, as we shall see later, the target reflections are also masked
by strong multiple reflections that further complicate interpretation of the data.

![CMP stack section from a marine seismic line in the Northeast Java Sea, Indonesia. Exploration objectives in this section are the weak reflectors beneath the strong limestone reflector marker at 2.2 s. The box indicates a portion of the data that is used for processing demonstrations below.](image)

**Fig. 5.1.** CMP stack section from a marine seismic line in the Northeast Java Sea, Indonesia. Exploration objectives in this section are the weak reflectors beneath the strong limestone reflector marker at 2.2 s. The box indicates a portion of the data that is used for processing demonstrations below.

To improve the primary reflections in the zone of interest, I processed the data with several alternative approaches beyond the basic processing sequence (Figure 5.1 shows an output from the basic processing sequence).\(^1\) These added routines include moveout filtering in the shot and receiver domains (to attenuate linear noise), moveout filtering in the CMP domain (to suppress multiples), CMP stacking using various

\(^1\)The basic processing sequence here consists of gain recovery, deconvolution before stack, velocity analysis, and straight stacking.
offset-dependent weighting schemes, deconvolution after stack, time-variant spectral whitening, and migration, applied either individually or in combination. These extra processing efforts, however, have generally resulted in insufficient improvement of the target reflections relative to the output when only the basic processing sequence is used.

Disappointment with the generally small improvement resulting from these processing efforts motivated the present study, the goal of which is to understand limitations in performance of data processing imposed by the presence of possible data imperfections and processing errors.

5.1.1 Data overview

Figure 5.2a shows a representative CMP gather (CMP 530) from the line in Figure 5.1. The data are 48-fold and 5-s long with a 4-ms sample interval, and the near-offset receiver group is 200 m from the source location. With a trace spacing of 50 m, the CMP gather thus has a spreadlength of 2550 m. Fifty-one CMP gathers including CMP 530 (specifically CMP 510 - 560) will later be used in comparisons of results of various processings, as seen after the data are stacked. For our purpose, let us say that the zone of interest lies between 2.1 and 2.7 s zero-offset time; this zone includes the limestone marker and the target reflectors beneath it.

Amplitude analysis for the first trace of CMP 530 (Figure 5.2b) shows a background-noise amplitude of approximately -36 db (relative to the measurement scale used throughout the experiments in this chapter), measured in the top 150 ms of the trace, prior to the first arrival. Meanwhile, amplitudes of seismic events within the zone of interest range from -7 db for the limestone marker to as low as -25 db for
FIG. 5.2. (a) CMP 530 from the marine seismic data shown in Figure 5.1. (b) Amplitude analysis from the 1st trace of CMP 530. The box indicates the zone of interest in the data. Amplitudes are RMS values computed from 20-ms time windows.
the weaker events in the target zone.\textsuperscript{2} This suggests that we have sufficient input ratio of signal to random background noise ($\gamma_I$) to guarantee an output SNR ($\gamma_O$) $\geq 6$ db for these events, although it gives no information on the more substantial level of contamination from coherent noises, such as multiples and sidelobes of strong neighboring primaries.

Figures 5.3a and b are enlargements of Figures 5.2a and b over the zone of interest. In Figure 5.3a, we see the strong limestone reflector marker at 2.22 s zero-offset time. Figure 5.3b shows an amplitude of -7 db for this reflector ("A" in the figure). A second strong event at 2.47 s with an amplitude of -10 db is identified as the strongest multiple within the zone of interest ("B" in Figure 5.3b). The dominant frequency of the events in the zone of interest is approximately 25 Hz.

A velocity spectrum resulting from performing velocity analysis on CMP 530 is

\textsuperscript{2}In these experiments with field data, all amplitudes are RMS amplitudes measured within short time windows (20-24 ms). The use of RMS values results in some smoothing of the amplitudes. The short windows, however, guarantee that the smoothed amplitude would still retain local amplitude character, especially within the zone of interest, where events have a dominant frequency of 25 Hz.
shown in Figure 5.4. The limestone reflector ("A") is associated with a moveout ve-

Fig. 5.4. Portion of velocity spectrum resulting from performing velocity analysis on CMP 530. The coherency measure used in the velocity analysis is normalized cross-correlation (Yilmaz, 1987), using 32-ms windows at 16-ms time increments. "A" corresponds to the limestone reflector marker, "B" is the strong multiple at 2.47 s zero-offset time, and "\(v_{NMO}\)" is a velocity function drawn to represent the primary velocity function. The box indicates the zone of interest.

locity of 2400 m/s, and the strong multiple at 2.47 s ("B") appears to have a moveout velocity of approximately 2250 m/s. Further examination of the velocity spectrum does not seem to reveal any primary event that can be confidently interpreted beneath the 2.2-s limestone reflector marker. The velocity spectrum is dominated by multiples that are conspicuous for their low moveout velocities. Lacking better velocity picks
for primary events I draw the velocity function shown in Figure 5.4 to represent the primary velocity function that I will use to stack the data. Clearly, events in the target zone that were too weak to be in evidence on the velocity spectrum will suffer further because the chosen primary velocity function is bound to be in error.

5.1.2 Output expectation and limitation

The seismic events within the zone of interest are dominated by multiples. Since we expect that the amplitude of the random background noise will be sufficiently low in the stack, multiples are thus the main problem in the data; therefore stacking should be aimed primarily at attenuating the multiples as much as possible in the hope of recovering some hidden primary events.

Before discussing the multiple problem further, however, let us first consider other limiting factors that might apply to the stacking of the sample data set.

Residual time shifts are not normally associated with marine data. If we examine the moveout-corrected gather of CMP 530 (Figure 5.5), however, we observe some time deviations from perfect alignments of approximately ±5 ms, e.g., on the limestone reflector at 2.22 s between traces 26 and 27. Assuming that the hyperbolic moveout correction is as good as can be for the limestone reflector, these apparent time deviations could be due to a combination of inhomogeneity in the overburden and interfering noise. It is known that the sea bottom in the survey area in Northeast Java Sea contains an abundance of modern reefal buildups (now submerged under 120 to 180 m of water). These may have caused near-surface velocity anomalies that result in what may be approximated as time shifts in the data. Let us assume that these deviations exist in the target reflections. A ±5-ms maximum time shift can produce a limitation in stacked signal \(\psi_{P(L)}\) of approximately 1.5 db for a 25-Hz
Fig. 5.5. CMP gather from Figure 5.2a, moveout-corrected using the primary velocity function shown in Figure 5.4, and displayed over the zone of interest.

signal provided the time shifts have a uniform probability density function (recall Figure 4.7). Visual inspection of the signal wavelets along the limestone reflector in Figure 5.5, however, suggests a distribution of time deviations that is not uniform. The distribution is somewhat bi-modal, i.e., about half of the traces have a time shift of -5 ms and the rest +5 ms. A modeling of bi-modal distribution shows that, in this case, $\psi_{P(L)}$ is approximately 4.5 db.

As mentioned above, the RMS amplitude of the random background noise in the data is sufficiently low relative to those of the known signal and multiples so that this noise may not pose a problem in stacking. When SNR is low, however, it may be wise to inspect for possible noise-amplitude variation across the CMP gather that might cause some limitation. For example, as shown in Figure 5.6, there is a maximum of 10-db ($\approx$ a factor of 3) amplitude variation in the random background noise in CMP 530. By visual inspection (e.g., by rearranging the noise amplitudes in a descending
order, such as depicted in the figure) we can approximate the distribution of the amplitude variation and then estimate the limitation via equations (4.8) and (4.10). In our case, however, it turns out the limitation, \( \eta_{P(L)} \), would just be an insignificant 1 db. In Figure 5.6 the average RMS amplitude of the unstacked noise is -40 db.

![Graph showing amplitude variation](image)

**Fig. 5.6.** Trace-to-trace amplitude variation in the random background noise from CMP 530. Amplitudes are RMS values measured within the top 150-ms section of the CMP gather (prior to first arrivals). By rearranging the noise amplitudes in a descending order we can approximate the distribution of the noise variation. Here, the “rearranged” curve is close to being linear suggesting a probability density function that is approximately uniform.

If, in straight stacking of the data, the ideal noise attenuation is -17 db (i.e., \( 1/\sqrt{N} \) where \( N = 48 \)), then with 1-db limitation, we estimate that the stacked noise would be at about -56 db.

As yet, we do not know the strength of the target reflections within the zone of interest. Thus, at this point, although we expect that SNR after stack would in general be sufficiently large (\( \geq 6 \) db), we do not know whether PMR would also be sufficient for the target reflections. The data set within the zone of interest is
a good example of a large number of multiples interfering with each other to pose as a “pseudo” background noise with a much higher amplitude than that of the true random background noise in the data (recall Section 4.3.1). It is therefore more realistic to hope that, after stack, some of the target reflections would stand out above the stack of that pseudo background noise, rather than to hope for those reflections to have “sufficiently large” PMR as defined in Section 2.6.

Among the available stacking methods, offset-dependent weighting is particularly suited for the problem of multiples. Meanwhile, a particularly obvious target of attenuation would be the strong multiple at 2.47-s zero-offset time. Hence, for the purpose of our demonstration, in the following experiments I focus attention on attenuating this multiple using both straight stacking and offset-dependent weighting. The success of the effort can be measured in terms of multiple attenuation, $\eta_p$, which can be obtained from the data by measuring the amplitude of the prestack multiple, $\eta_I$, and that of the stacked multiple, $\eta_O$. We will assess this measured $\eta_p$ against the $\eta_p$ computed via modeling of the data, such as that shown in Figure 5.7.

From Figure 5.7, we expect that $\eta_p$ for the strong multiple will be as low as -29 db if we use an offset-dependent optimum-weighting scheme, the multiple having a moveout velocity of 2250 m/s (the other two weighting schemes would produce less attenuation). With an $\eta_I$ of -10 db, $\eta_O$ of the multiple would be at -39 db, still way above the amplitude of the stacked random background noise, which would be at about -56 db. It is yet to be seen whether the 29-db attenuation of the multiple will be sufficient to reveal any hidden primary that might exist in the data.
Fig. 5.7. Expected stack responses of the field data using uniform weighting and two offset-dependent weighting schemes. The response curves correspond to attenuation of events at 2.47 s zero-offset time when they are stacked after moveout correction with a primary velocity of 2900 m/s. The vertical axis denotes $\psi_p$ for primaries and $\eta_p$ for multiples. The optimum weighting is designed to suppress multiples with moveout velocity ranging between 2100 and 2400 m/s. The primary/multiple wavelet is modeled using a Ricker zero-phase wavelet with 25-Hz dominant frequency. The amplitude of each event is assumed to be uniform across the CMP gather.
5.1.3 Assessment of stacking results

In the following experiments, all CMP stacking of data involves weighted-averaging schemes to preserve the amplitude of signals. Unless otherwise specified, the velocity function used in the stacking is the $v_{NMO}$ curve drawn in the velocity spectrum in Figure 5.4.

**Random background noise.** Figure 5.8 shows the measured RMS amplitudes of the stacked random background noise from the 51 CMP gathers in the data set. The noise amplitude of the stacked trace for CMP 530 is about -53 db, close to the predicted $\eta_0$ value of -56 db.

**Primary and Multiple.** Figures 5.9a and b show the stacked results for CMP 530 over the zone of interest, for uniform weighting. The limestone reflector at 2.22 s now peaks at -10 db ("A" in Figure 5.9a). This means a loss of 3 db from the prestack amplitude ($\psi_I$ was -7 db). If we attribute this loss entirely to the nonhyperbolic time

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**Figure 5.8.** Amplitudes of stacked ambient noise from CMP 510 through CMP 560. Stacking is a uniform weighting scheme applied to the top 150-ms portion of the CMP gathers. No moveout correction is performed on the data before trace summation.
deviations (i.e., assuming no hyperbolic-moveout error for the limestone reflector), then the predicted $\psi_{P(L)}$ (4.5 db) misses the actual result only slightly (1.5 db).

-50 -40 -30 -20 -10

```
<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Amplitude (db)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2</td>
<td>2.4</td>
</tr>
<tr>
<td>2.4</td>
<td>2.6</td>
</tr>
<tr>
<td>2.6</td>
<td></td>
</tr>
</tbody>
</table>
```

**Fig. 5.9.** Stacked results of CMP 530 over the zone of interest for uniform weighting. (a) Amplitude analysis of the stacked output. (b) Stack trace (repeated three times). Amplitudes are RMS values in 24-ms gates. “A,” “B,” “C,” and “L” are discussed in text.

The strong multiple at 2.47 s (“B” in Figure 5.9a) stacks at -24 db; therefore, it has experienced an attenuation of 14 db from its prestack amplitude ($\eta_I$ was -10 db). This is within 1 db of our expectation as predicted in the plot of performance for uniform weighting shown in Figure 5.7. “C” in Figure 5.9a marks the approximate location of a primary reflection (as we shall see later) that is as yet indistinguishable due to the interfering multiple “B”. “L” is a negative lobe of the stacked multiple.

Figures 5.10a and b show the stacked results for CMP 530 over the zone of interest for an offset-dependent optimum-weighting scheme designed with the same parameters as that used for the prediction in Figure 5.7. Notice that now the strong multiple at 2.47 s (“B” in Figure 5.10a) stacks at approximately -37 db. This is a 27-db attenuation, short by 2 db from the prediction in Figure 5.7, but sufficient,
apparently, to allow the formerly indistinguishable seismic event ("C" in Figure 5.10a) to stand out.

![Amplitude (db)
-50 -40 -30 -20 -10
2.2
2.4
2.6

Time (s)

(a)

(b)

Fig. 5.10. Results of stacking CMP 530 over the zone of interest for an offset-dependent optimum weighting. (a) Amplitude analysis of the stacked output. (b) Stack trace (repeated three times). Amplitudes are RMS values in 24-ms gates. “A,” “B,” and “C” are discussed in text.

The 4-db loss in amplitude of event “A” (the limestone reflector) after optimum weighting compared to the result of uniform weighting is curious. In designing the optimum weights, I have assumed that the signal amplitudes are uniform across the CMP gather, while in actuality they are only approximately so (see Figure 5.5). In this sense, the weights are less than optimum. If desired, the optimum weighting scheme could have been designed to include that weakening of amplitude with offset. Optimum weights, however, need to be optimum in the sense of being robust and therefore not dependent on fine-detail characteristics of the signal and noise.

Figures 5.11a and b compare result of stacking using uniform weighting with that using optimum weighting, for 51 CMP gathers surrounding CMP 530. Notice especially in Figure 5.11b, the emergence of event “C” after the application of offset-
dependent optimum weighting.

Fig. 5.11. Stacking of 51 CMP gathers surrounding CMP 530, (a) using uniform weighting, and (b) using offset-dependent optimum weighting. For the optimum weighting, the set of weights that was used to stack CMP 530 in Figure 5.10 is applied for all CMP gathers. “A” is the limestone reflector marker. “B” is the strong multiple at 2.47-s zero-offset time. “C” is the event emerging after the application of offset-dependent optimum weighting.

So far we can only suspect that event “C” is a primary event formerly obscured by interference from the strong multiple “B.” If “C” is indeed a primary, the amplitude of “C” in Figure 5.10a (-27 db) is also suspicious since the primary velocity function that has been used to stack the data was not necessarily proper for this event. Erroneous stacking velocity would have reduced the amplitude of the stacked event.

To support the contention that event “C” is a primary, I vary the moveout velocity function used in the stacking of the data. Using velocity functions 1, 2, 3, 4, and 5 as shown in Figure 5.12, the resulting stacked output are shown in Figures 5.13a-e. In these figures, the amplitude of event “C” first increases and then decreases as the stacking velocity corresponding to the event decreases. Maximum amplitude of this event is reached when stacked using the third velocity function (Figure 5.13c;
\(\psi_o \approx -23\text{db}\). The behavior of the stacked amplitude of event "C" as a function of stacking velocity suggests that event "C" is likely a primary event. This event did not show any correlation peak in the velocity spectrum shown in Figure 5.4 most probably because of interference by the much stronger multiple "B." If we assume that the amplitude of event "C" in Figure 5.13c (-23 db) more or less reflects the event's amplitude before stack (i.e., stacking preserves amplitude of primaries), then "C" was 13-db (\(\approx\) a factor of 4) weaker than "B" prior to stacking (prestack amplitude of "B" was -10 db in Figure 5.3b). Since the two events have moveout velocities that are somewhat close to each other, the weaker event has been masked by the stronger in the velocity analysis (recall Section 3.1.1).

![Velocity spectrum figure](image)

**Fig. 5.12.** Reproduction of the velocity spectrum in Figure 5.4 over the zone of interest showing alternative stacking velocity functions (1 through 5) used to stack the data in order to test the validity of event "C" as a primary event. The velocity function labeled "\(v_{NMO}\)" is the stacking velocity used in the previous experiment. Event "C" is approximately at 2.51 s zero-offset time.
Fig. 5.13. Processing output of data stacked with five different $v_{NMO}$ functions using optimum weighting. Results in (a) through (e) correspond to stacking with velocity functions labeled 1 through 5 in Figure 5.12, respectively. "B," "C," and "L" denote the strong multiple, the suspected primary event, and a side lobe of the multiple, respectively. Notice that the optimum weighting has curiously made the side lobe "L" of the multiple increasingly larger [e.g., from result in (d) to that in (e)] relative to the amplitude of the multiple itself, probably due to interference with the stacking of some other event.
Figures 5.14a and b show a comparison of the result of stacking using uniform weighting against that using optimum weighting, for 51 CMP gathers surrounding CMP 530. Both are stacked with velocity function 3 of Figure 5.12. Although event "C" in Figure 5.14b appears to be stronger in amplitude compared to that in Figure 5.11b, multiple "B" here has a somewhat stronger amplitude also. If we compare amplitudes of "B" and "C" in Figure 5.13c with those in Figure 5.10, we see that amplitude of the stacked primary and that of the multiple have both increased by several db in Figure 5.13c. The slower stacking velocity used in Figure 5.13c has resulted in less NMO error (therefore less attenuation) for multiple "B" and (presumably) no NMO error for primary "C." On the other hand, the amplitude ratio (PMR) between "C" and "B" has not changed appreciably.

In Figure 5.15, I generate a stacking-performance chart similar to that in Figure 5.7 using a stacking velocity $v_{stack}$ prescribed by velocity function 3 shown in Figure 5.12. Multiple "B," having a moveout velocity of 2250 m/s, is predicted to be attenuated by as much as 24 db when we use the optimum weighting. With an $\eta_I$ of -10 db, I thus predict $\eta_O$ to be at -34 db. Examination of the actual $\eta_O$ for multiple "B" in Figure 5.13c, reveals that the prediction is close (within 3 db) to the actual result.

In the above optimum-weighting experiments I have used a moderate parameter for the noise-to-multiple amplitude ratio (i.e., 0.25) in the design of the weights. Now, having acquired a better stacking-velocity definition for the primary events within the zone of interest (i.e., velocity function 3 in Figure 5.12), let us use the noise-to-multiple ratio estimated from the data ($\approx 0.025$) as input into the design of the optimum weights. Figure 5.16 shows the stacked result for this weighting scheme. Notice the dramatic difference between the result of optimum weighting in
Fig. 5.14. Stacking of 51 CMP gathers surrounding CMP 530, (a) using uniform weighting, and (b) using offset-dependent optimum weighting. Both are stacked with velocity function 3 shown in Figure 5.12.

Fig. 5.15. Expected stack responses of the field data using uniform weighting and two offset-dependent weighting schemes. The response curves correspond to attenuation of events at 2.47 s zero-offset time when they are stacked after moveout correction with a primary velocity of 2650 m/s. The vertical axis denotes $\psi_P$ for primaries and $\eta_P$ for multiples. Other parameters are the same as those used to generate the response curves in Figure 5.7.
Figure 5.14b and that in Figure 5.16. Here, the weighting scheme may have further attenuated the multiples within the zone of interest such that now, beside event “C,” several other possible primary events stand out in the stack section (e.g., events “D,” “E,” and “F”). Event “D” could be a reflector from the base of the limestone layer, which, in a nearby well, has a thickness of approximately 90 m (with a sonic interval velocity of 3800 m/s).

The deterioration of the limestone reflector marker “A” at approximately 2.2 s (i.e., the reflection from the top of the limestone layer) is curious. This is essentially an exaggeration of the loss of amplitude that we observed in the result of the previous optimum weighting (i.e., the 4-db loss in amplitude of event “A” in Figure 5.10). In designing the weights for the optimum weighting, I have assumed that signal amplitudes are uniform across all CMP gathers. In actuality, these amplitudes not only vary across a CMP gather but also vary from one CMP gather to the next.

**Fig. 5.16.** Stacking of 51 CMP gathers surrounding CMP 530 using the offset-dependent optimum weights shown in Figure 5.17b and velocity function 3 shown in Figure 5.12.
Fig. 5.17. Optimum weights as a function of trace number designed for attenuating events at 2.47 s with moveout-velocity range between 2100 and 2400 m/s. The stacking velocity is 2650 m/s. Amplitudes of events are assumed to be uniform across the gather. The noise-to-multiple amplitude ratio allowed for in the design of the weights is a moderate 0.25 in (a), and a small 0.025 in (b).

This results in an uneven severity of limitation on the performance of the optimum weighting across the stacked data giving rise to the discontinuous appearance of the reflectors in the stack section. To demonstrate this contention, in Figure 5.18 I display the NMO-corrected CMP gather from CMP 516 to compare with that from CMP 530 (Figure 5.5). Notice that the amplitude variation of the wavelets for the limestone reflector at 2.2 s in CMP 516 is different from that in CMP 530 (look especially at the amplitudes on the first few traces and on the last 25 traces of the gathers). When optimum weights (such as those shown in Figure 5.17b) are applied to the two CMP gathers, the resulting stacked amplitudes are necessarily different. In Figure 5.16, the stacked amplitude of the limestone reflector at CMP 516 is strong, while that at CMP 530 is weak or distorted. If desired, the optimum weighting could have been designed to take into account this variability of amplitudes both within a CMP gather and from one CMP gather to the next. As mentioned above, however, optimum...
weighting needs to be robust so as not to be dependent on fine-detail characteristics of the signal and noise in order to be sufficiently practical in its implementation, as discussed in Appendix A.

Hence a weighting scheme designed with a small noise-to-multiple ratio, such as exemplified by the weights in Figure 5.17b, could have an unstable stacking performance in the presence of signal and noise variations in the CMP gathers, because the erratic weighting coefficients tend to over-emphasize the difference in characteristics of signal and noise from one CMP gather to the next. On the other hand, a weighting scheme designed with a sufficiently large noise-to-multiple ratio, such as that shown in Figure 5.17a, would be more stable in the sense that the variation of signal and noise on the individual traces tend to be averaged by the less erratically varying weights.

For the above reasons, I judge the stacked result shown in Figure 5.14b to be relatively more reliable than that shown in Figure 5.16. Nonetheless, the laterally
coherent events that were revealed in Figure 5.16 would still require further analysis to assess their validity as primary events instead of mere stacking artifacts (e.g., remnants of multiples or of sidelobes of primaries) produced by the erratic weights. Such analysis may include (1) moveout-velocity testing such as exemplified by the analysis in Figure 5.13, (2) modeling of events (primary and multiple) with amplitude variation to simulate the data, to be stacked with the erratic weights shown in Figure 5.17b, and (3) stacking of additional CMP gathers adjacent to the presently processed data set to look for consistency of results and continuity of events.

5.2 Conclusion from the field-data experiments

Results from these experiments suggest that analysis of signal-to-noise ratio improvements in CMP stacking can be used to estimate amplitudes of stacked output of field data to within a few db of accuracy.

Accuracy of output predictions, however, seems to be of lesser importance compared to awareness of the presence and influence of such limitations in the process of stacking. In the demonstration, I showed that expectations for the influence of NMO velocity errors in CMP stacking have been successfully exploited to recover a "hidden" event and investigate the validity of this event as a primary reflection.

Since the moveout-velocity differences between the primaries and the multiples in the zone of interest are likely small, moveout filtering may have difficulty in discriminating energies associated with the different events. Success in removing the multiples is also doubtful for predictive deconvolution or model-based multiple subtraction since there seems to be little periodicity in the multiples which, judging from their moveout velocities, are most likely of peg-leg type originating from either the limestone reflector or some slightly shallower reflectors. Offset-dependent optimum
weighting in this case appears to be a promising method to help recover the hidden primaries provided their amplitudes are not below the average of the stacked multiples and other coherent noise (lower-amplitude primaries will continue to be hidden behind the stronger coherent noise after stack). As shown in Figure 5.14b, at least one weak signal (event “C”) has been uncovered by stacking the data set with offset-dependent optimum weighting.
Chapter 6

CONCLUSION

6.1 Summary

Through analysis of the performance of CMP stacking, this research may serve as a prototype for gaining understanding of how various shortcomings in data, processing schemes, and processing parameters limit the amount of signal-to-noise ratio improvement in seismic data processing. In particular, improvement of signal-to-random-noise ratio for CMP stacking is generally less than the commonly quoted $\sqrt{N}$, where $N$ is the number of traces stacked. Moreover, since the limitations resulting from these shortcomings are quantifiable, one can reasonably predict the output of a CMP stacking (and, by extension, that of other processes) given any input seismic data. The quantification of stacking limitations can be done not only for signal-to-noise ratio related to random background noise, but also for that related to coherent noise in data. The result of these quantifications can then be used to assess the output of stacking of field data. Again, this thesis is intended to serve as a model for studies in quantification of signal-to-noise ratio enhancement and limitation for other data-processing techniques, such as moveout filtering, for which in principle signal-to-noise improvement can be larger than $\sqrt{N}$, but in practice time deviations and amplitude variations in the input data impose limitations.

To quantify stacking limitations, one can conveniently dissect the signal-to-noise ratio into its constituents, the signal and the noise. This separation is useful since,
in CMP stacking, the limitations on the signal differ from those on the noise.

Limitations of signal-to-noise ratio improvement in CMP stacking are essentially caused by time misalignment of signal and by amplitude variation of signal and noise across the CMP gather. Having identified a number of possible origins for these problems, I developed approaches to quantify the limitations owing to the individual limiting factors. I showed that a quantification may be accomplished via numerical experimentation using synthetic data that were modeled after field data or otherwise, when possible, through derivation of a stacking formula that would represent the stacking process.

Analyzing the extent of limitations attributable to each of the limiting factors using the above approach, I found that the severity of the limitation varied among the various limiting factors. In general, limitations that depend on moveout time (e.g., NMO velocity error, nonhyperbolic moveout, geometrical spreading, and anelastic attenuation) increase in significance for smaller zero-offset times since NMO is larger for smaller times. The customary application of stretch muting to limit stretching of wavelet due to NMO correction helps reduce these limitations. This, however, results in reduction of the number of traces stacked in the muted zone and consequently limits the power of the stack to suppress noise, our primary reason for employing CMP stacking. In addition, limitations with frequency dependence (e.g., NMO velocity error, nonhyperbolic moveout, residual statics, and anelastic attenuation) increase in significance for higher frequency signals.

Table 6.1 summarizes qualitatively results from analysis of the stacking limitations. "T" and "A" within the parenthesis indicate whether the limitation is a time-misalignment problem or an amplitude-variation problem (except for trace muting, which influences stacking signal-to-noise ratio improvement differently, as described in
Section 4.2). The “Extent of Limitation” listed in the table indicates, qualitatively, a relative limitation on signal-to-noise ratio improvement after CMP stacking. To

<table>
<thead>
<tr>
<th>Type of Limitation</th>
<th>Component Influenced</th>
<th>Extent of Limitation</th>
</tr>
</thead>
<tbody>
<tr>
<td>NMO velocity error (T)</td>
<td>signal, noise</td>
<td>large</td>
</tr>
<tr>
<td>coherent noise (T)</td>
<td>signal, noise</td>
<td>large</td>
</tr>
<tr>
<td>residual statics (T)</td>
<td>signal</td>
<td>large</td>
</tr>
<tr>
<td>subsurface inhomogeneity (T)</td>
<td>signal</td>
<td>large</td>
</tr>
<tr>
<td>nonhyperbolic moveout (T)</td>
<td>signal</td>
<td>large</td>
</tr>
<tr>
<td>noise spike/burst (A)</td>
<td>noise</td>
<td>large</td>
</tr>
<tr>
<td>SNR variation (A)</td>
<td>signal, noise</td>
<td>moderate to small</td>
</tr>
<tr>
<td>anelastic attenuation (A)</td>
<td>signal</td>
<td>moderate</td>
</tr>
<tr>
<td>geometrical spreading (A)</td>
<td>signal</td>
<td>small</td>
</tr>
<tr>
<td>channel inequality (A)</td>
<td>signal, noise</td>
<td>small</td>
</tr>
<tr>
<td>trace muting</td>
<td>noise</td>
<td>moderate</td>
</tr>
</tbody>
</table>

Table 6.1. Extent of limitations on signal-to-noise ratio improvement.

give a somewhat loose distinction between “large,” “moderate,” and “small” criteria in the table, let us say that a limitation is “large” when it can easily be more than 6 db, “moderate” when it is generally between 2 and 6 db, and “small” when it is most likely less than 2 db. Results in Table 6.1 suggest that limitations due to time misalignments are typically larger than those due to amplitude variations.

Most of the limitations listed in Table 6.1 can occur in combination in a data set.\(^1\) Combination of limitations can result in a relatively large compound limitation, even when the individual limitation is small from each of the contributing limitations. In such a case, the compound limitation would be approximately the product of the individual contributing limitations.

Accuracy of signal-to-noise ratio predictions through analysis of stacking limitations, as described above, depends largely on how well one can model the data and

\(^1\)Channel inequality, of course, cannot occur together with SNR variation.
approximate the data parameters (sometimes in a statistical sense). Some limitations, such as NMO velocity error, can be quantified fairly accurately, since the required parameters for the analysis (i.e., moveout velocity and signal waveform) generally can be estimated from the data. On the other hand, quantification of some other limitations, such as that of residual statics, may be less accurate, since one must estimate the distribution of the static time shifts statistically from the data. However, as demonstrated in Chapter 5, accuracy of prediction is perhaps less important than is our awareness of the presence and approximate extent of such limitations in the stacking process. Realizing the possibility of limitations in the data and in the processing scheme, a seismic data processor would do well to assess the stacked output against expectation, and when there is a discrepancy, investigate the possible causes through a more detailed signal-to-noise ratio analysis. Having discovered the problem, one can use results from the analysis to optimize the stacked output.

Analysis results from Chapter 4 suggest that one can optimize the stacked output by searching for the best stacking parameters, such as the stacking velocity (to minimize loss of amplitude of stacked-signal) and the spreadlength of the CMP gather (to minimize NMO error due to nonhyperbolic moveout and signal-amplitude decay with offset due to geometrical spreading or anelastic attenuation); by not applying inverse-Q filtering to correct for anelastic attenuation in data when Q is low\(^2\); by always applying residual statics correction even for marine data; and by employing offset-dependent optimum weighting to obtain the best multiple attenuation possible in CMP stacking, while maintaining optimum background-noise suppression.

To summarize, analysis of signal-to-noise ratio improvement and limitation in

\(^2\)Except when other processing steps, such as deconvolution, render such correction valuable. In this case, the decision should be based on the relative importance of CMP stacking as opposed to that of the other process in terms of the objective of the particular data processing.
the stacking of seismic data (or in other processes aimed at improvement of signal-to-noise ratio) can serve as a guideline to expectations for processing output. Such an effort can be a useful tool in careful data processing, particularly when struggling to enhance weak signal contaminated by various noises.

6.2 Future direction

Again, although it is the key one, CMP stacking is not the only way to improve signal-to-noise ratio in seismic data processing (recall Figure 1.1). The approach to analyzing signal-to-noise ratio improvement and limitation in CMP stacking as presented in this thesis could serve as a pattern for similar studies of other data-processing methods that also fail to produce satisfactory output in the presence of shortcomings in data, processing schemes, and processing parameters.

Consider moveout filtering for linear-noise attenuation. When amplitudes vary from trace to trace in the data gathers, the "linear" noise is no longer truly linear. Thus, the energy of the noise will be scattered in the frequency-wavenumber (f-k) domain representation of the data, thereby the noise is difficult to remove completely. It would be useful to quantify the extent of the limitation as a function of the amplitude variation (and possibly also as a function of time shifts) in the data.

Another example is predictive deconvolution before stack for multiple attenuation. Predictive deconvolution requires periodicity of multiples in data. Now, multiples (e.g., water-bottom multiples) are exactly periodic only on the near traces of a CMP gather and gradually loosing their periodicity with offset. It would be useful to know the quantitative performance of predictive deconvolution in attenuating multiples as a function of trace offset; that is, up to what offset would predictive deconvolution still perform satisfactorily.
Results of such analyses for a complete set of data-processing methods that aim at enhancing data signal-to-noise ratio (e.g., those of the methods listed in Figure 1.1) not only could help us choose processing sequences that are adapted to the noise characteristics of the data (e.g., those of linear noise, water-bottom multiples, peg-leg multiples, diffractions, varying random noise, ground-roll) but also could give us guidance as to when it is fruitless to keep trying alternative processing techniques in the hope of improving signal-to-noise ratio. In such a case, improving the quality of the field data (e.g., increasing source strength, or extending from 2D to 3D) may be the only option for obtaining better output in the data processing.
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Appendix A

RELATIVE CHARACTERISTICS OF SOME WEIGHTING METHODS IN CMP STACKING

A.1 Uniform weighting

In uniform weighting, moveout-corrected traces in a CMP gather are each given the same weight before being summed into a stack trace. If the traces are averaged (i.e., when the weights $w_i = 1/N$, where $N$ is the number of traces), then uniform weighting is equivalent to straight stacking, the most common form of CMP stacking in seismic data processing.

Consider the influence of CMP trace spacing on the stacked output of uniform weighting for events with some NMO error. We shall see that trace spacings commonly used in practice have little influence on the amplitude of the stacked output; that is, the asymptotic stacking formula developed in Chapter 3, which was based on continuous trace summation, can be regarded as generally adequate for output prediction. As seen in Figure A.1, the amplitude and shape of the stacked wavelet are stable when trace spacing is relatively small (e.g., 50 m); these, however, start to deteriorate when the trace spacing increases to approximately 200 m. Larger trace spacing results in oscillation of peak of the stacked wavelet. In this experiment, the input wavelet is a Ricker zero-phase with a dominant frequency of 80 Hz. For wavelets with lower dominant frequencies, the trace-spacing limit for which stacked output starts to exhibit output instabilities will be larger. For shallower events and events with slower
FIG. A.1. Stacked wavelets of a primary event in a 4000-m CMP gather stacked with various trace spacings. The event is at 2.5 s zero-offset time and has a true moveout velocity of 3000 m/s, but is stacked with a velocity of 3300 m/s. The input signal is a Ricker zero-phase wavelet with a 80-Hz dominant frequency. Output wavelet shapes and peak amplitudes become unstable when trace spacing $dx \geq 200$ m in (c) through (f).
moveout velocities, however, the trace-spacing limit will be reduced, since here, for an equivalent velocity error, residual moveout will be larger than that when the event is deeper or has a higher moveout velocity. In current practice, trace spacing within CMP gathers rarely exceeds 100 m (i.e., 50-m geophone interval in field data acquisition); therefore, in most cases we can take the CMP trace spacing as sufficiently small to merit the use of the asymptotic stacking formula [equation (3.11), and by extension, equation (3.12)] discussed in Chapter 3.

A.2 Inner-trace muting

Inner-trace muting is an offset-dependent weighting scheme that enhances suppression of multiples by applying zero weights to a number of near-offset traces of a CMP gather before stacking (these traces being the main contributor to output amplitude in the stacking of an event having a residual moveout). In land data, inner-trace muting is also used to exclude excessive source-generated noises, such as ground-roll, that dominate near-offset traces and late times.

Inner-trace muting yields larger multiple attenuation than does uniform weighting. Optimization of the attenuation, however, requires muting of a substantial number of near-offset traces (Figure A.2). In Figure A.2b, for instance, PMR improvement ($\gamma_P$) of at least 10 db larger than that obtainable through uniform weighting (Figure A.2a) can be achieved only by muting as many as 15 near-offset traces. At later times (Figures A.2d and f), even more traces (approximately 25) need to be muted to gain lesser $\gamma_P$ superiority over results of uniform weighting. This trace reduction necessarily results in limitation of background-noise suppression. Moreover, here accuracy of primary velocity is especially important, since when there is error, not only will the poorly-corrected primary be attenuated, this primary can also suffer a
Fig. A.2. CMP gathers having a primary event and a multiple event arriving at the same zero-offset time $t$ are stacked using uniform weighting (a,c,e) and inner-trace muting (b,d,f). Plots are cumulative-stack improvement of primary ($\psi_P$) and attenuation of multiple ($\eta_P$), with the resulting $\gamma_P$. For uniform weighting, trace 20 means result of stacking trace 1 through 20; for inner-trace muting, trace 20 means result of stacking trace 20 through 48. Here, the stacking velocity is given by $v_{\text{stack}}(t) = 1500 + 600t$ m/s. Multiples are 25-percent undercorrected in each case. The CMP gather has a spreadlength of 4000 m, and all wavelets are Ricker, zero-phase, 30 Hz.
significant shift in zero-offset time. This is demonstrated in Figure A.3.

The experiment in Figure A.3 compares $\gamma_P$ and the relative time shift of a stacked primary in CMP stacking, first using uniform weighting, and then using inner-trace muting. The CMP gather has a primary event (P) with a moveout velocity of 3000 m/s, and a multiple (M) with a moveout velocity of 2400 m/s, but the gather is stacked using a velocity of 3150 m/s. Displayed in Figures A.3a and b are cumulative stacked traces whose associated $\psi_P$ and $\eta_P$ are shown in Figures A.3c and d. Comparing Figures A.3c and d, we observe that maximum $\gamma_P$ achievable by inner-trace muting is about 10 db higher than that achievable by uniform weighting (compare $\gamma_P$ at trace 30 in Figure A.3c with that at trace 35 in Figure A.3d). Unfortunately, in order to accomplish this superiority, using inner-trace muting, we have to mute about 75 percent of the total CMP traces, resulting in an unacceptable 20-ms delay in zero-offset time of the stacked primary as shown in Figure A.3f. Meanwhile, the time shift is only about 5 ms for uniform weighting (Figure A.3e).

A.3 Square-root-offset weighting

As opposed to inner-trace muting, square-root-offset weighting enhances multiple attenuation by applying a set of weights that are proportional to the square-root of the source-to-receiver offsets of the traces. To maintain the averaging property of the stacking, the weights are constrained such that their sum equals unity.

In this weighting scheme, far-offset traces are increasingly emphasized. As with inner-trace muting, when there is error in the primary velocity used for the stacking, not only will the poorly-corrected primary be attenuated, this primary may also suffer some shift in zero-offset time. In this case, however, the shift is much less than that for inner-trace muting. For an experiment such as that shown in Figure A.3 but using
Fig. A.3. When the primary is poorly moveout corrected, inner-trace muting can result in a significant shift of zero-offset time of the stacked primary as well as some loss of stack amplitude. Here, experiments in Figure A.2 for t = 2.5 s are repeated; this time the data are stacked using a stacking velocity that is 5-percent higher than that of the primary velocity. The top figures (a,b) are stacked-trace displays; P signifies the primary, M the multiple. The middle figures (c,d) show cumulative amplitude attenuation for P (i.e., \( \psi_P \)) and M (i.e., \( \eta_P \)), and PMR improvement (\( \gamma_P \)). The bottom figures (e,f) show the shifts of zero-offset times for the stacked primary and multiple.
square-root-offset weighting for the method of stacking, the shift in the primary’s zero-offset time is only 7 ms (Figure A.4), as opposed to 20 ms for inner-trace muting (Figure A.3f).

![Graph](image)

**FIG. A.4.** Zero-offset times of the stacked primary event in an experiment such as that shown in Figure A.3 but using square-root-offset weighting for the method of stacking. The true zero-offset time of the event is 2.5 s, the maximum time shift is seen to be 7 ms.

Square-root-offset weighting is not based on any optimization scheme (it is, in fact, somewhat arbitrary). The weighting scheme, however, can yield multiple attenuation that is somewhat comparable to that of optimized inner-trace muting (see Figure 4.2). The scheme also performs quite satisfactorily in the suppression of random background noise (Figure A.5).

### A.4 Offset-dependent optimum weighting

Offset-dependent optimum weighting computes the stacking weights by minimizing the sum of the expected power in the stacked multiples and random background noise subject to the constraint that the stacked signal be preserved (Meyerhoff, 1966; Larner, 1994).
Fig. A.5. Square-root-offset weighting results in a slightly larger $\eta_P$ (vertical axis) than the ideal $N^{-1/2}$ attenuation of random background noise; therefore the limitation, $\eta_{P(L)}$, is small for all number of traces stacked. The attenuation curves correspond to the result of cumulative stacking with an increasing number of traces stacked up to 48 traces (increasing spreadlength out to a maximum of 4000 m).

The suppression of multiples has a direct trade-off with loss of SNR due to the non-uniform weighting of the noise traces. In Figure A.6a, background-noise attenuation ($\eta_P$) is plotted as a function of amplitude ratio of the noise to multiple (i.e., the relative importance of the noise as opposed to that of multiples as a problem in the data) assumed in the design of the optimum weights. The magnitude difference between the curve for $\eta_P$ and that for ideal $\eta_P$ of $N^{-1/2}$ is the limitation on background-noise attenuation ($\eta_{P(L)}$). Note that $\eta_{P(L)}$ is at worst when no background noise is allowed for in the design of the weights. When background noise is considered to be the dominant problem to be addressed, optimum weighting becomes uniform weighting and the limitation drops to 0. In contrast (Figure A.6b), optimum weighting performs best in attenuating multiple when no random noise is allowed for in the design of the weights. Again, when the noise is dominant in the design, the
performance of optimum weighting in suppressing the multiple is just that obtained with uniform weighting.

Comparing the stacking performances shown in Figures A.6a and b, we see that for any choice of noise-to-multiple ratio, offset-dependent optimum weighting could gain a large improvement in multiple attenuation (therefore gain in PMR) for a relatively small loss in background-noise attenuation (therefore loss in SNR) over the performance of uniform weighting: gain in PMR outweighs loss in SNR. This results suggests that one can always use offset-dependent optimum weighting to advantage.

**Fig. A.6.** (a) Attenuation of random background noise ($\eta_P$) as a function of noise-to-multiple ratio in offset-dependent optimum weighting. The magnitude difference between the “Optimum” curve and the $N^{-1/2}$ curve is the limitation on background-noise attenuation ($\eta_{P(L)}$). (b) The performance of offset-dependent optimum weighting in suppressing a multiple as a function of noise-to-multiple ratio allowed for in the design of the weights. The performance of uniform weighting is also shown for comparison. The vertical axis denotes $\eta_P$. In this experiment, prestack amplitudes of the primary, multiple and random background noise are assumed to be uniform from trace to trace. The CMP gather consists of 48 traces with a spreadlength of 4000 m. The multiple in (b) is at 2.5 s zero-offset time, has a moveout velocity of 2400 m/s, and is stacked with a velocity of 3000 m/s. The multiple wavelet is Ricker, zero-phase, 30-Hz.
When there is an error in the primary velocity used to stack the data, however, offset-dependent optimum weighting can produce a significant shift in zero-offset time of the stacked primary (Figure A.7). Here, for a similar experiment as that shown in Figure A.3 but using offset-dependent optimum weighting for the stacking method, the time shift is 12 ms. This timeshift depends on the wavelets of the event that are emphasized by the weighting scheme. For example, in Figure A.7 traces 30 to 35 are given the largest weights; therefore, the stacked wavelet is biased towards the wavelets on these traces and reflects their time shifts (NMO errors).

![Graph and diagrams](image)

**FIG. A.7.** (a) The offset-dependent optimum weights designed to suppress the multiple event M in the experiment in Figure A.3. The noise-to-multiple ratio here is assumed to be 0.25. (b) Moveout-corrected CMP gather with the optimum weights applied to the primary event P in the experiment in Figure A.3. The event is 5-percent undercorrected. (c) The stack trace of (b) showing a time shift of approximately 12 ms (true zero-offset time of the primary is at 2.5 s).

Offset-dependent optimum weighting produces the largest attenuation of multiple among offset-dependent weighting schemes (review Figure 4.2). It is, however, also the most difficult to implement since it requires estimation of several data parameters (i.e., the multiple waveform, the amplitudes of signal, multiples, and random background
noise) for the calculation of the weights. Rigorous implementation of the scheme may therefore be difficult and prohibitively costly, if, for instance, we wish to design a separate weighting scheme for each time level where multiples are present in the data, and also if we wish that the weight sets be allowed to vary from one CMP gather to the next, adapting to the data. Hence, in dealing with field data, while use throughout a data set of one set of optimum weights designed for a target depth at one CMP location along a line may be suboptimal from an ideal standpoint, it can be optimal in a practical sense. In this case, although the best attenuation naturally occurs for the multiples for which the weighting is especially designed, the weighting scheme can be expected to attenuate multiples at other depths and CMP locations better than does uniform weighting.

The use of one set of weights throughout the data means that the weighting scheme does not honor fine-detail variations in the amplitudes of signal and noise from one CMP location to the next. As exemplified in Chapter 5 (Figure 5.16), this could result in deterioration of primary events, especially when one uses an extremely small noise-to-multiple ratio parameter in the design of the weights. For this reason, it is better to allow a sufficiently large noise-to-multiple ratio in the weighting design even if the true ratio in the data is smaller, or at least to experiment with a moderate noise-to-multiple ratio parameter (e.g., 0.25) before attempting a more aggressive effort at multiple attenuation (by employing a smaller ratio) in order to detect any deterioration that might occur.

Also, the set of weights for a target depth is typically designed relative to multiples with a range of moveout velocities (since more than one multiple are usually present at any target depth). Since the method is most powerful when designed especially against a multiple with one moveout velocity, it follows that the narrower
the actual velocity range of the multiples the better the performance of the method. Figure A.8 illustrates this point: weighting designs associated with narrower velocity ranges typically result in superior attenuation of multiples within the corresponding design range. If, however, the multiples velocities are not perfectly known, it is wiser 

![Graph showing attenuation vs moveout velocity](image)

**Fig. A.8.** Performance of offset-dependent optimum weighting as a function of the range of moveout velocity of the target multiples incorporated in the design of the weights. The vertical axis denotes attenuation, $\eta_p$, of multiple. "Opt.1" corresponds to a weighting design that includes all multiples having moveout velocities uniformly distributed between 1500 and 2700 m/s. "Opt.2" is designed for multiples with moveout velocities between 1800 and 2100 m/s, "Opt.3" between 2100 and 2400 m/s, and "Opt.4" between 2400 and 2700 m/s. In this experiment, all multiples are at 2.5 s zero-offset time and have 30-Hz Ricker zero-phase waveforms. The CMP gather consists of 48 traces with a spreadlength of 4000 m. In all cases, the data are stacked with a velocity of 3000 m/s. Noise-to-multiple ratio is assumed to be 0.25. The attenuation curve for uniform weighting is shown for comparison.

...to choose a wider velocity range in designing the optimum weights. Consider a mul-
tiple with a moveout velocity of 2000 m/s in Figure A.8. Either “Opt.2” or “Opt.3” would attenuate this multiple by a few db more than the attenuation that would be produced by the wider velocity-range design “Opt.1.” But if we inadvertently use a wrong design, such as “Opt.4,” we would suffer a 10-db loss of attenuation of the multiple relative to the performance of “Opt.1.”

A.5 Data-adaptive optimum weighting

When offset-dependent optimum weighting ignores the need to address multiple suppression, the weighting scheme reduces to (Brown et al., 1977)

\[ w_i = k \frac{s_i}{n_i^2}, \]  

(A.1)

where \( s_i \) and \( n_i \) are relative strengths (amplitudes) of signal and random background noise on the \( i \)th trace of a CMP gather, respectively, while \( k \) is a constant proportionality factor.

CMP stacking using the weights described in equation (A.1) is called data-adaptive optimum weighting (owing to the fact that \( s_i \) and \( n_i \) are typically estimated from the unstacked data) and used particularly to maximize SNR in stacking of data afflicted with trace-to-trace signal and background-noise amplitude variations.

The effectiveness of data-adaptive optimum weighting depends very much on the accuracy of amplitude and SNR measurements from the data. Inaccuracy in these measurements, which is most problematic when SNR is low, could easily result in failure of the method to perform better than straight stacking (White, 1977).
A.6 Diversity stack

This weighting scheme is used when background-noise amplitude fluctuates (especially when anomalously high-amplitude noise exists on one or only a few traces) across the CMP gather while signal amplitude is constant.

This scheme is a special case of data-adaptive optimum weighting; i.e., if the signal amplitude $s_i$ in equation (A.1) is constant, independent of trace number, we obtain

$$w_i = k_1 \frac{1}{n_i^2}.$$

Thus, the weights are inversely proportional to the noise power in the respective traces. In practice, the weighting is accomplished by scaling of traces in the CMP gather by the inverse of their respective powers, where the powers are measured over short, sliding time windows (Embree, 1968).

From the example in Section 4.4.2 under SNR variation, the benefit of using diversity stack over straight stacking is only a few db gain in SNR improvement when the probability density function of amplitude variation in the background noise is uniform (see Figure 4.11). If spurious noise bursts afflict only a relatively few traces in a CMP gather, where the RMS trace amplitude of the noise is otherwise uniform, then the benefit of diversity stack can be significant (see Figure 4.12). This is why diversity stack is routinely used in vertical stacking of vibroseis data, where field data are often contaminated by noise bursts originating from passing vehicles during data acquisition.

Although diversity stack is typically used only in vertical stacking of vibroseis data, the underlying principle (i.e., attacking spurious, anomalously high-amplitude noise) can be applicable as well to CMP stacking (Lynn et al., 1987).
A.7 Trace equalization

Trace equalization is another special case of data-adaptive optimum weighting which is applicable when there is a fluctuation of amplitude levels across the CMP gather, while SNR is the same for all traces. In this case the weights from equation (A.1) reduce to

\[ w_i = k_2 \frac{1}{n_i}. \]

In practice trace equalization is accomplished by normalizing the CMP traces by their corresponding RMS amplitudes prior to stacking.

By applying trace equalization, SNR improvement will once again be \( \sqrt{N} \) of input SNR. From the example in Section 4.4.2 under *Channel inequality*, the benefit of using trace equalization over straight stacking, however, is only a fraction of 1 db of SNR gain. The main purpose of trace equalization is obviously not to improve SNR in the stack but to prepare the data for subsequent multi-channel processing schemes that require amplitude stationarity across the gather, such as moveout filtering.