OPTIMIZATION OF PHOSPHATE ORE BLENDING: A LINEAR PROGRAMMING APPROACH

By

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ABSTRACT

This thesis briefly summarizes the geology, extraction, beneficiation, processing, resources, economics and demand for phosphate rock. This background is helpful in developing a mathematical model which will maximize phosphate recovery subject to grade constraints through a blending of three separate ores. A case study is based on a problem submitted by the J. R. Simplot Company of Conda, Idaho.

While a general understanding of the beneficiation process is essential, it will be treated almost as a "black box" as far as the mathematical model is concerned. A study to optimize the slurry plant and/or mill is beyond the scope of this thesis and could be considered as future thesis work.

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My greatest appreciation goes to my family; Karen, Jamie, and Jeffrey for their support and forbearance during the period of this undertaking. Theirs was the greater burden.

INTRODUCTION

Phosphorite is an amorphous rock composed primarily of phosphatic minerals. It is found in sediments of marine origin and is the most important source of phosphorus for fertilizer materials, phosphoric acid, and elemental phosphorus. Phosphate rock is a commercial term generally applied to any rock containing more than 20 percent P_2O_5. Both terms are normally used interchangeably.

Of the twenty nutrients known to be either essential for plant growth or to cause beneficial growth, nitrogen, phosphate, and potassium are required in large quantities. In fact, agriculture uses over 84 percent of the domestic phosphate rock production (4). Future world demand for phosphate rock will be supported by the demand for food from a rapidly increasing world population.

The phosphate rock industry expanded mining and processing facilities in 1964-66, anticipating continued strong demand for phosphate products. The demand did not develop, and an oversupply condition affected the industry through 1970. By 1969, prices had declined to levels barely sufficient to cover direct costs of production and handling. In 1971, however, there was an increased demand for phosphate rock and prices began to rise. A worldwide shortage

of phosphate rock developed in 1972 and continued through 1973. From 1973 to 1974, the price per metric ton more than tripled, from $14.20 to $49.90 (3). Since then, the price and demand relationships have remained fairly stable. However, because there are no known substitutes for phosphate rock from which phosphate fertilizers can be produced in the quantity required to sustain world agriculture production, it is possible that increased demand coupled with reduced reserves could result in a situation similar to the current "energy crisis".

The United States, the U.S.S.R., and Morocco are the major world producers of phosphate rock (3). Within the United States there are three major phosphate rock producing areas: Florida and North Carolina, the Western states of Idaho, Montana, Utah, and Wyoming (Figure 1), and Tennessee. Phosphate deposits in the Western states comprise 58 percent of the total United States reserves (1).

For the short term, the United States has adequate supplies of phosphate rock to meet both domestic and international commitments and will continue to be a net exporter of phosphate rock and fertilizer derivatives. The industry's production capability far exceeds United States consumption and a surplus is available to export.

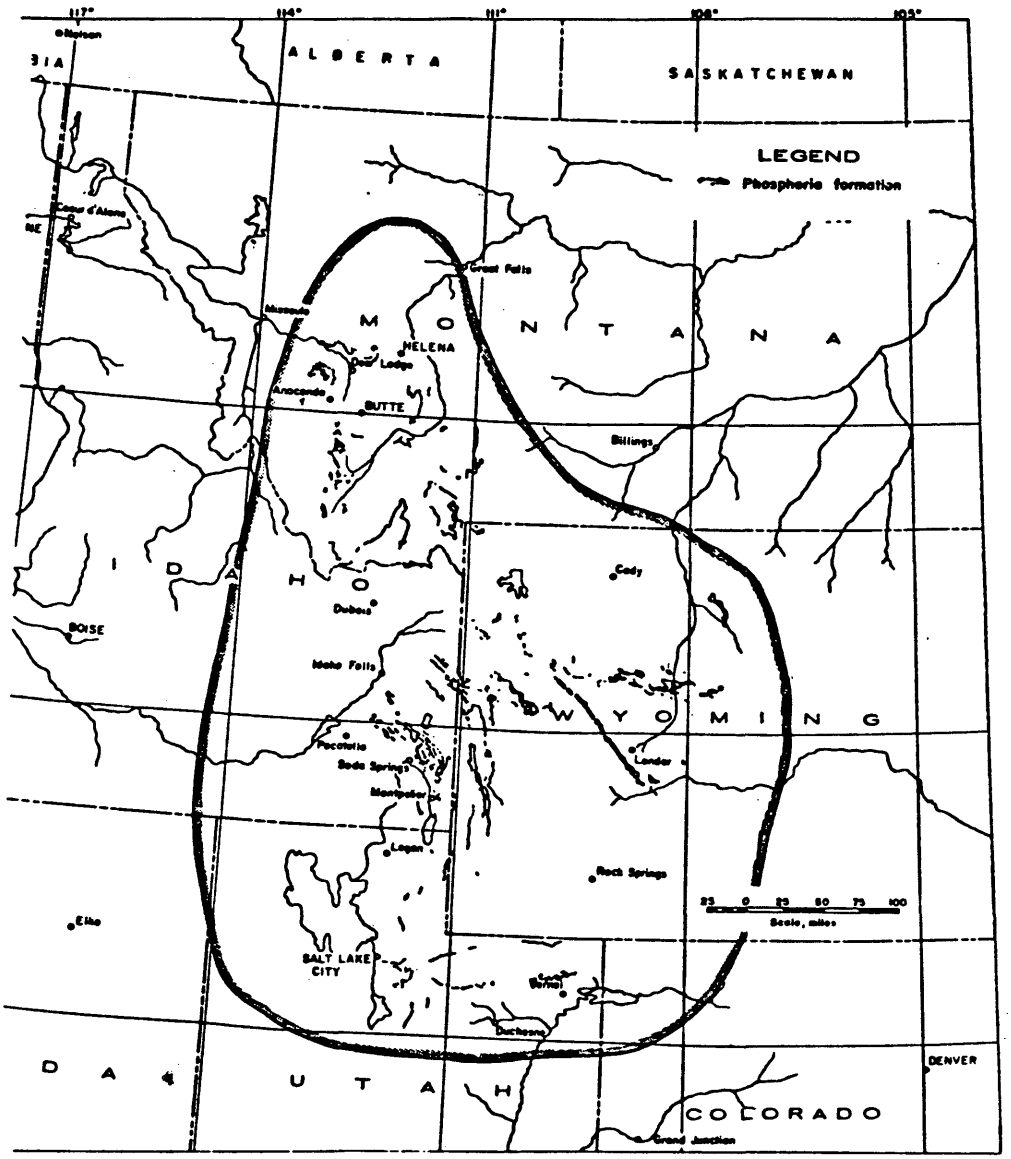


Figure 1 - Western Phosphate Field

Source: Bureau of Mines Report of Investigations #6485, p. 12.

It is estimated that about 45 percent of the phosphate rock produced in the United States is exported either as rock, a higher value fertilizer, or as an industrial chemical (3). The government does not maintain stockpiles of phosphate rock nor any derivative of phosphate rock.

Adequate fertilizer supplies to meet future demands of agriculture will become a serious strategic consideration during the last decade of this century (3). Programs to increase and conserve domestic reserves of phosphate rock will be necessary to assure adequate fertilizer supplies for the United States agricultural industry. Hopefully, the optimization models in this thesis will, in a very small way, extend the reserves through increased efficiency.

GEOLOGY

The earth's crust contains approximately 0.23 percent phosphorous pentoxide (P_2O_5) (4). However, minable deposits are found only in a relatively few places of the world. These are primarily in North America, Europe, and North Africa. Minable concentrations of phosphates are found in igneous rocks as sedimentary phosphorites, and as guano or deposits derived from guano. Because the geologic development, and consequently the mining and beneficiation methods vary so drastically from one area to another, this thesis will concentrate on the western phosphate field in the United States where J.R. Simplot Company's Conda Mine is located (Figure 2).

Phosphorite deposits occur in the marine Phosphoria formation of Permian age in the western field. These deposits crop out over an area of approximately 135,000 square miles in the four states of Montana, Idaho, Utah, and Wyoming and represent one of the largest available reserves of phosphate rock in the world.

The structural history of the phosphatic sediments is complex. Since deposition they have been compressed, folded, and exposed by erosion. As a result, their surface appearance is one of banded outcrops appearing at

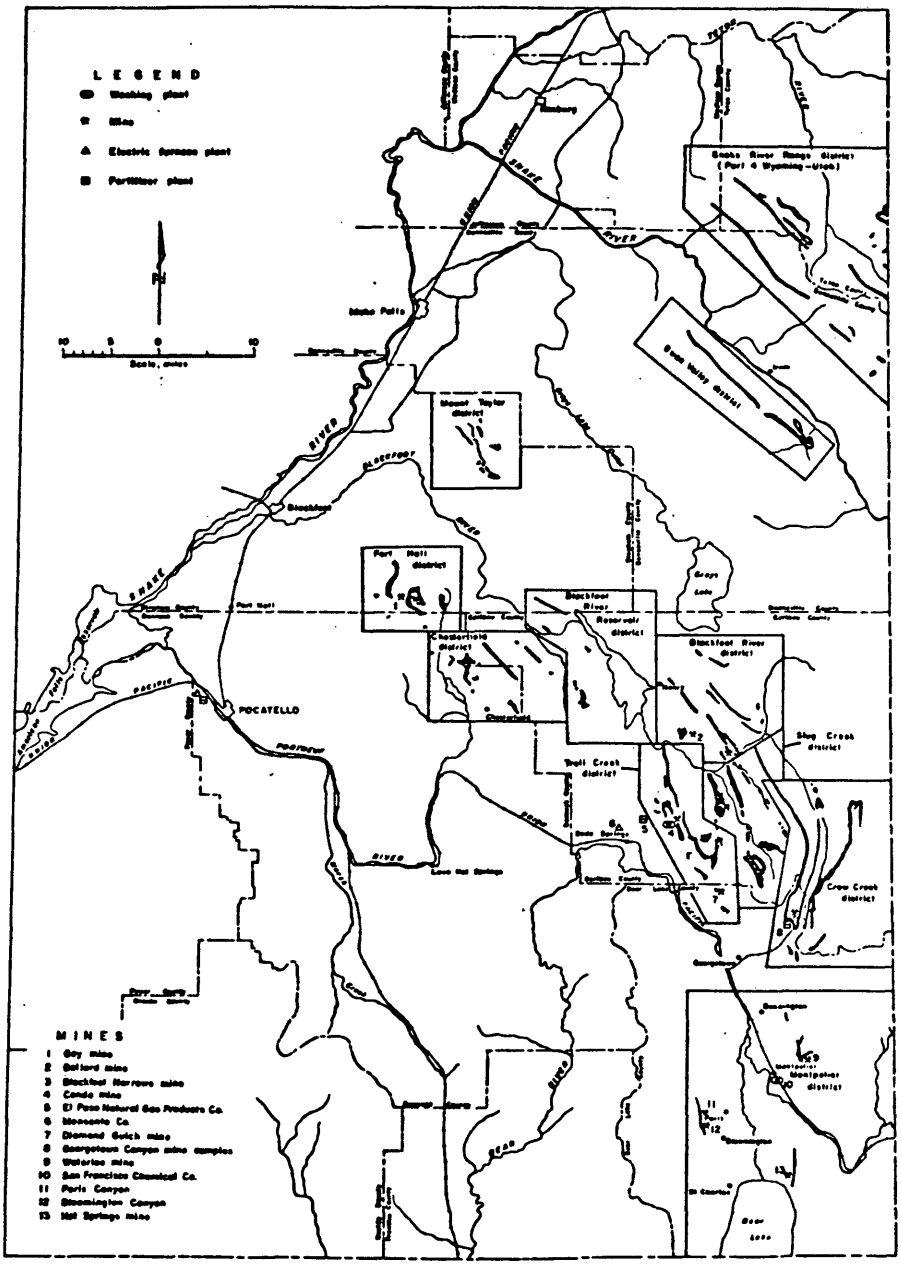


Figure 2 - Idaho Phosphate Districts

Source: Bureau of Mines Report of Investigations #6810, p. 14.

different elevations. Beds in the western part of the field have dips from nearly flat to vertical, and some are even overturned (1). Numerous faults of both large and small displacement disrupt the shale series in many places to an extent such that mining is almost impossible.

The phosphoria formation in the Conda area is composed of a lower phosphatic shale member (Meade Peak) and an upper chert member (Rex). The Meade Peak member averages about 150 to 200 feet in thickness and the chert about the same (1). The Meade Peak shales, in most instances, can be traced laterally by their surface expression and by the bordering chert and limestone outcroppings. This is especially true in the central part of the field where the terrain is rugged and the outcrops occur on the sides and crests of high ridges. However, in some places, talus slopes and overburden have hidden the phosphatic shale member. Tracing the outcrops in such locations requires projection from known stratigraphic formations.

It is not necessary within the scope of this thesis to provide a full, technical description of the phosphate deposits in the Conda area. There are many detailed reports available (1,2) for anyone wishing such additional information. Suffice it to say that the Conda phosphate deposits comprise some of the largest reserves of phosphorite available by surface mining in Idaho. Sufficient

minable reserves are available to continue the Conda milling operation for some years to come and its importance as a world phosphate producer will gradually increase over the next decade as other known reserves decrease.

EXPLORATION AND MINING

Preliminary exploration of a phosphate deposit for surface mining (the primary type of mining in the western field) makes extensive use of aerial photographs to provide an overall picture of the deposit. After the preliminary aerial photo maps are constructed, they are used to locate drill sites and trench locations. Trenches are located along the strike of the outcrop and spaced at intervals that will provide the grade thickness, depth of overburden, and dip of the formation (2). These intervals may range from a few feet to several hundred feet, depending on the continuity of the outcrop and the topography of the area. The trenches are usually laid out normal to the strike in order to provide a true measurement of the formation thickness.

Drilling is an additional phase in the exploration process. Rotary drilling is the standard, but deep diamond drilling is used to determine structure, faulting, grade, and thickness at depth. The J. R. Simplot Company drilled the Gay Mine on a grid pattern as part of the exploration process.

Exploration does not have to be particularly detailed since the required information is normally limited to

thickness and grade of minable deposits, and thickness of waste and overburden sections. This information can then be mapped to provide data for mine planning and development.

Phosphate rock mining is primarily a function of three conditions: the geology of the formation, the ore grades and thickness, and the depth of the overburden. In the western phosphate field, the two mining methods being used are open pit and underground. Of the seven operating mines in the field, only one uses underground methods (4). The remaining six use various modifications of open pit mining.

Open pit mining techniques vary from mine to mine as a function of the three conditions mentioned above. However, most mines follow the same general procedure of:

(1) removing and stockpiling the topsoil for reclamation use later;
(2) removal and disposal of the overburden and low grade waste shales; and
(3) mining of the phosphate rich beds and hauling the ore to blending stockpiles.

At J.R. Simplot's Conda Mine, the following mining procedure is used (4):

1) topsoil is removed and stored for reclamation use.
2) the chert overburden is drilled with rotary blast-hole drills and blasted using AFNO (ammonium nitrate and fuel oil) and high velocity boosters.
3) after blasting, the chert overburden is removed by a 25 cubic yard electric shovel and loaded onto 170 ton trucks.
4) the trucks haul the waste material to mined out pits for disposal.
5) the ore is then mined using hydraulic shovels.
6) after removal, the ore is hauled to the blending stockpiles by rear dump trucks (35 to 50 ton).
7) as a pit is mined out, the topsoil is recovered and used as the top layer for pits after backfilling. Where the quantity of topsoil is insufficient, waste shales with plant nutrients are substituted as a starter soil for new vegetation.
8) equipment for various support functions include; rubber tired dozers, graders, track dozers, service vehicles, and water trucks.

A typical, open pit phosphate mine has pit slopes varying from 45 to 70 degrees. Bench heights average around 40 feet with an average width of 17 feet (4). Pit depth is a function of the ore body formation and the impurities found in the ore at depth.

BENEFICIATION

Beneficiation is the milling process required to bring a mined ore to a specified grade. An honest metallurgical engineer will agree that the beneficiation process is as much an art as it is a science. Consequently, most mills are tailored to a specific mine or group of mines providing a relatively consistent product. The mill is usually able to alter certain internal sub-processes to accommodate variations in the ore feed in order to achieve the proper grade. It is with this understanding that the beneficiation process is described for the J. R. Simplot mill in Conda, Idaho.

First, it must be understood that not all of the phosphate rock in the western field must be beneficiated. Beneficiation is determined by the concentration of P_2O_5 present in the ore. Western phosphate rock is classified by the following grades (1):

1) Acid - (Fertilizer) Grade	+ 31%	P_2O_5
2) Furnace Grade	24 - 32%	P_2O_5
3) Beneficiation Grade	18 - 24%	P_2O_5
4) Low Grade Shale	10 - 18%	P_2O_5

The J. R. Simplot mill is presently blending and beneficiating all four grades in the form of three separate ores (main bed, shale, blacks). Ore analysis shows P_2O_5 ranging from 15.96 percent to 32.16 percent (5).

The beneficiation of western phosphates generally involves scrubbing, washing, grinding, screening, and desliming to remove clay and silt materials. Depending upon the ultimate use of the product, some may be calcined to remove the carbonaceous material. One plant (Stauffer Chemical at Vernal, Utah) uses a flotation process for upgrading the phosphate rock (4).

Beneficiation of the Conda phosphate ore occurs at both the slurry plant and milling facilities (6). Flowsheets of both plants are shown in Figures 3 and 4.

At the slurry plant, stockpiled ore is transported by front-end loaders and blended in an ore surge bin or hopper of 1,750 cubic feet capacity. All ore entering the hopper must pass through a 10" x 12" grizzly. Ore greater than 10" x 12" is rejected. All ore from the hopper is conveyed to a scrubber/trommel screen for primary size separation. The one inch material goes to the slurry pump for slurrying to the mill. The material passing the

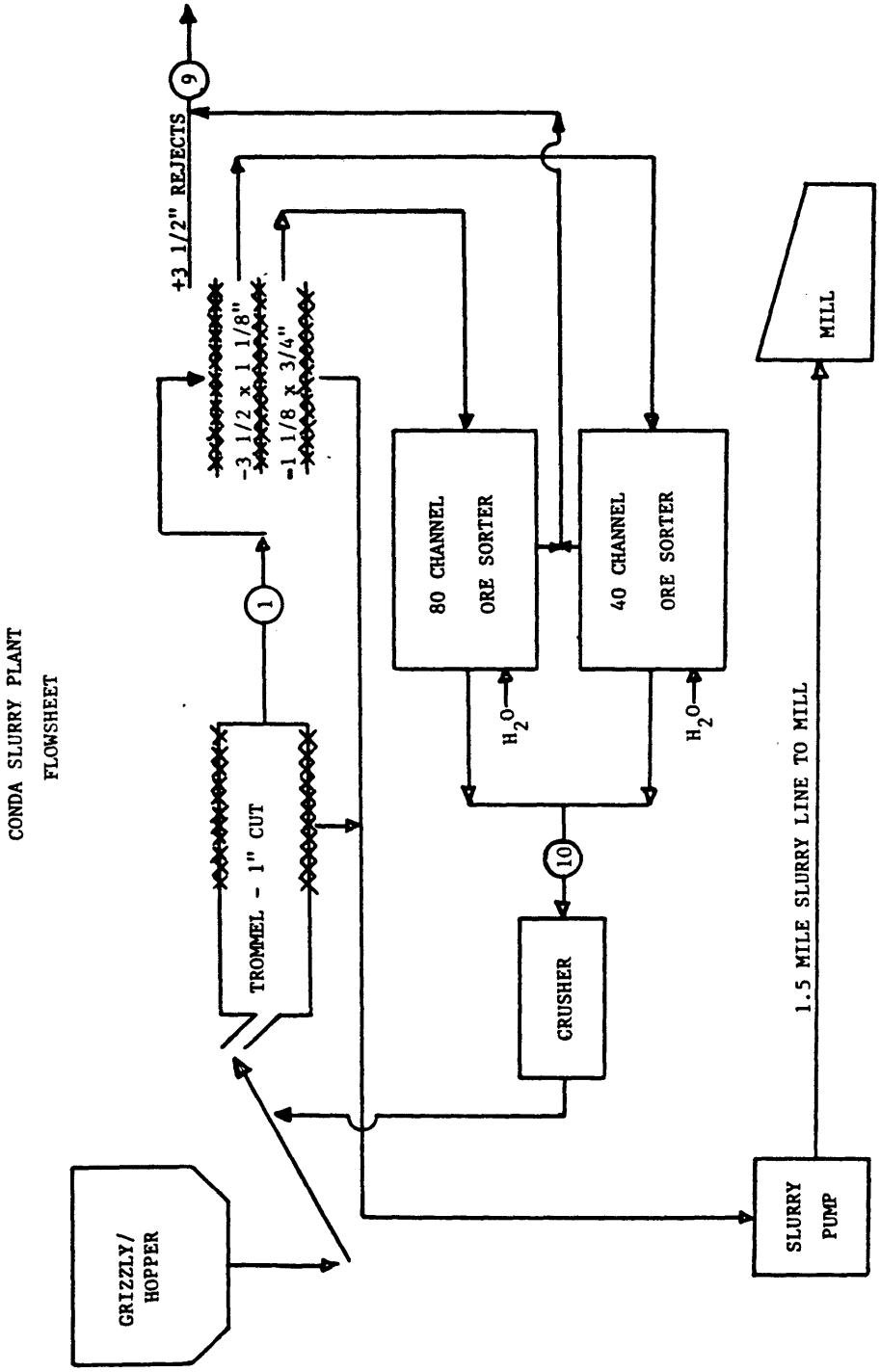


Figure 3 - Conda Slurry Plant Flowsheet

Source: Personal Communication from Mr. Wayne Perkins.

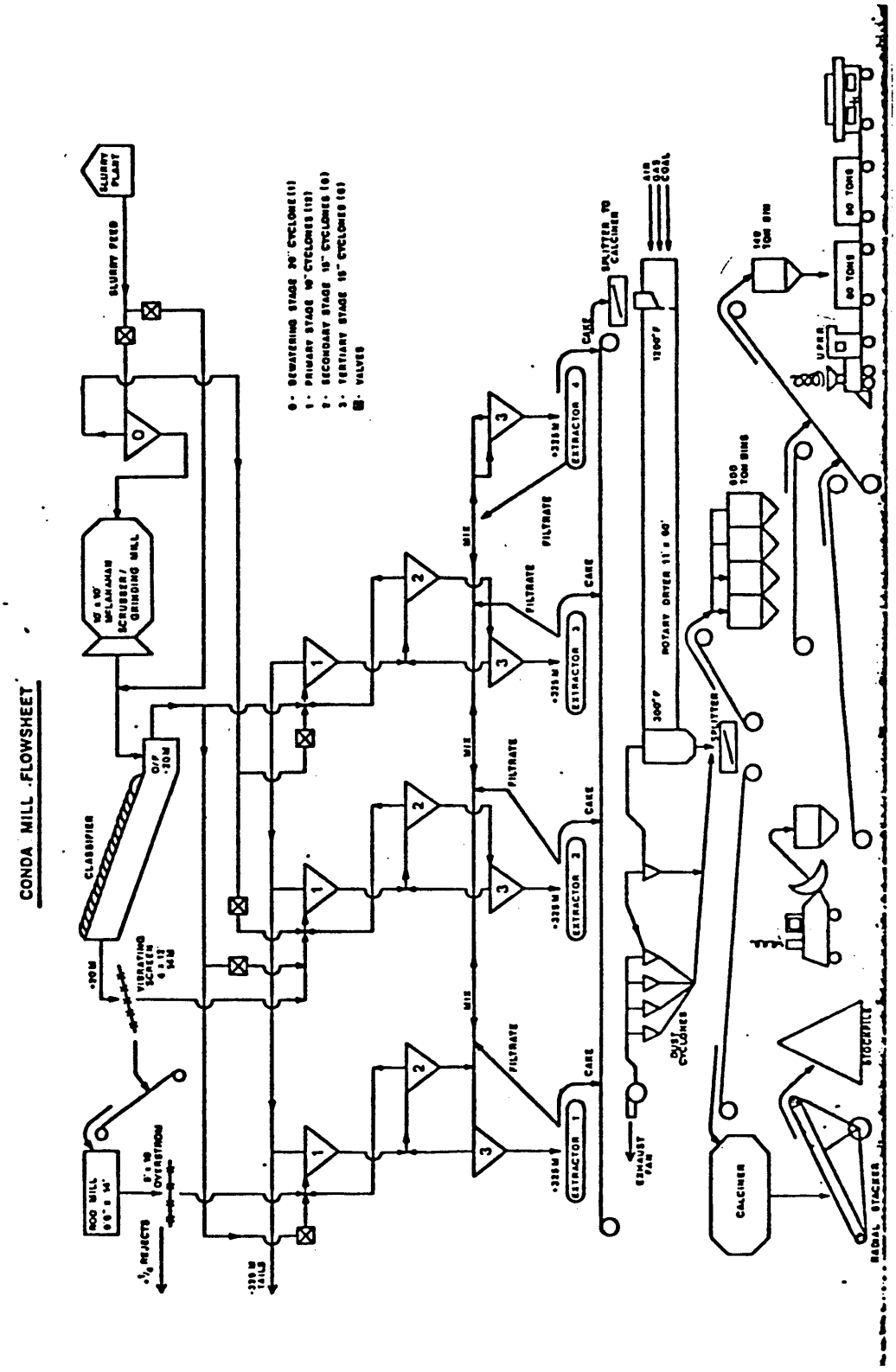


Figure 4 - Conda Mill Flowsheet

Source: Personal Letter from Mr. Wayne Perkins Dated 4 March, 1981.

trommel goes to a series of screens. The screens separate the material into 3½ inch + which is rejected, 3½ inch by 1 1/8 inch which goes to the 40 channel ore sorter, 1 1/8 inch by 3/4 inch which goes to the 80 channel ore sorter, and 3/4 inch which goes directly to the slurry pump. Rejects from the 40 and 80 channel ore sorters are rejected as tailings while that passing both is sent to a crusher. The crusher loops the material back to the scrubber/trommel and the process repeats itself until all ore is either rejected or sent to the slurry pump. The ore, in slurry form, is pumped through a six inch inside diameter, 1.5 mile, urethane lined steel pipeline to the mill.

Arriving at the mill, the slurried ore is sent either to the scrubber/grinding mill or the classifier (the option is available through a system of valves). Feed entering the scrubber/grinding mill is dewatered by a 20 inch hydrocyclone with the overflow reporting to the first stage hydrocyclone sumps of the screen and classifier sections. The underflow of the dewatering hydrocyclone reports to the scrubber/grinding mill and then is pumped to the classifier for size classification.

An approximate 20 mesh split is made in the material arriving at the classifier (whether from the scrubber/grinder mill or direct from the slurry plant) with the

overflow (-20 mesh) reporting to a primary stage of hydrocyclones. Classifier underflow (+20 mesh) reports to a set of vibrating screens to make a 14 mesh split. Screen undersize reports to another primary stage of the hydrocyclones and the screen oversize reports to the rodmill for further processing.

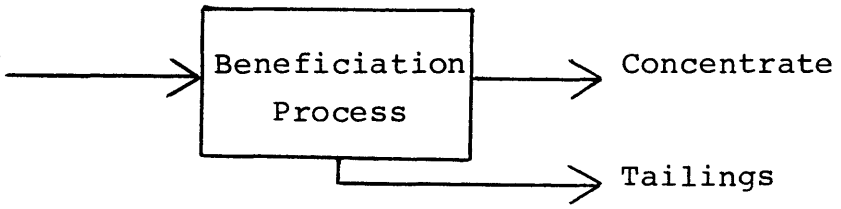
The open circuit rod-milling circuit grinds almost all material to -20 mesh. Rodmill product is screened with the -¼ inch material reporting to another stage of primary hydrocyclone and the +¼ material is rejected.

The three stage washing of the hydrocyclone system makes a rough 250 mesh split. The overflow (-250 mesh) of the primary stage reports to tails with the underflow reporting to the feed sump of the secondary stage. The overflow of the secondary stage reports to the feed sump of the primary stage and the underflow reports to the feed sump of the tertiary stage of hydrocyclones. The tertiary stage overflow reports to the feed sump of the secondary stage and the underflow reports to the belt filters.

Hydrocyclone underflow reporting to the filter is dewatered (most of the water passes through the filter) and the resulting product in a cake-like form is conveyed to the gas-fired rotary dryer. The dryer further reduces moisture content for transportation to four 600 ton storage bins for shipment or storage.

DEVELOPMENT OF THE LINEAR PROGRAMMING MODEL

A general understanding of the aforementioned beneficiation process is essential to the development of the linear programming model since there must be an appreciation for the somewhat erratic nature and adaptability of the process or sub-processes. Because of this, and the fact that the plant is fixed, at least in the short run, it is not within the scope of this thesis to consider the beneficiation process in the model. Instead, beneficiation will be treated as a "black box" where ore feed = concentrate + tailings; or as the schematic shows:



The objective of the model is to maximize the percent P_2O_5 in the concentrate. If this were the extent of the problem, however, it would be merely a matter of beneficiating the highest grade ore. Obviously then, there must be some constraints to contend with.

The ore is a mixture of many compounds (6); P_2O_5, MgO, CaO, SiO_2, Al_2O_3, Fe_2O_3, K_2O, Na_2O, V_2O_5, and some organic material and water which will be referred to as

LOI (loss on ignition). The only compounds we are concerned with in the concentrate are P_2O_5 which has a lower bound, and MgO and LOI which both have upper bounds.

MgO is generally considered to be the most undesirable impurity, in terms of expense, because of the formation of magnesium ammonium pyrophosphate sludge in the process (4). Further, magnesium precipitates fluorine in the reactor stage during the wet acid process, which causes plugging of the gypsum filters. For these reasons, the upper bound on MgO in the concentrate is 0.8 percent by weight (5). All percents referred to in the remainder of this thesis are considered to be weight percents.

While LOI is a valid constraint, it is generally not a significant problem. The problem with large amounts of LOI is that in the calcine plant, the LOI will combust in the pre-heat chamber causing an uncontrollable heating situation. Small amounts of LOI are desirable in that they can be used to fuel the pre-heat chamber of the calciner. The upper bound on LOI is 7.5 percent (5).

Even though P_2O_5 is to be maximized, there is a required lower bound on the concentrate because of product specifications. Since not all concentrate is calcined, the lower bound on P_2O_5 is set at 30.5 percent (5). The overall up-grading of P_2O_5 in the beneficiation process

averages approximately 6 percent depending upon the ore blend (6).

The next constraint to be considered is the reserves and/or mining ratio. It would be ideal if the optimal blending ratio exactly equaled the mining ratio, but this is very seldom the case. Therefore, a proportionality constraint is used to approximate the reserves/mining ratios within some tolerance factor, delta, which can be adjusted to allow more or less flexibility, or assigned such a large number so as to render the constraints ineffective. This allows the user to vary blends as he wishes.

The final constraints concern the upper and lower bound on the ore feed rate. The mill is able to efficiently process between 220 and 270 dry tons per hour. Since the ore feed is not dry, this is a calculated percentage based upon the assayed water content. This is necessary since the units of concentrate are in dry tons per hour.

The model then takes the general form:

Find the ore feed rates such that

$$P_2O_5 = (PR) \sum_{i=1}^{n} (PA_i)(F_i)$$

is a maximum. What this says is that tons of P_2O_5 in the concentrate equals the total of the phosphate assay of each blend times the phosphate recovery factor,

where:

PR = phosphate recovery in percent.

PA_i = phosphate assay of ore i in percent.

F_i = feed of ore i in tons per hour.

i = 1,2,....n.

n = total number of ores to be blended.

Subject to the following constraints:

1) That the total of assayed MgO in each blend times the MgO recovery factor be less than the allowable upper bound. Stated mathematically, $\underline{MgO}$: $(MR)\sum_{i=1}^{n}(MA_i)(F_i) \leq (UB_m)(RDT)\sum_{i=1}^{n} F_i$

where:

MR = MgO recovery in percent.

MA_i = Mgo assay of ore i in percent

UB_m = upper bound of MgO in percent of concentrate.

RDT = recovered dry tons (concentrated) as percent of total feed.

2) That the total of assayed LOI in each blend times the LOI recovery factor be less than the allowable upper bound. Stated mathematically, $\underline{LOI}$: $(LOIR)\sum_{i=1}^{n}(LOIA_i)(F_i) \leq (UB_{loi})(RDT)\sum_{i=1}^{n} F_i$

where:

LOIR = LOI recovery in percent.

$LOIA_i$ = LOI assay of ore i in percent.

UB_{loi} = upper bound of LOI in percent of concentrate.

3) That the total of assayed P_2O_5 in each blend times the P_2O_5 recovery factor be greater than the allowable lower bound. Stated mathematically,

P_2O_5: $(PR) \sum_{i=1}^{n} (PA_i)(F_i) \geq (LB_p)(RDT) \sum_{i=1}^{n} F_i$

where:

PR = phosphate recovery in percent.

PA_i = phosphate assay of ore i in percent.

LB_p = lower bound of phosphate in percent of concentrate.

4) That the maximum feed rate of the total blended ore be less that the mill processing capacity. Stated mathematically, Max Feed: $\sum_{i=1}^{n} F_i \leq UB_f$

where:

UB_f = upper bound of feed rate in tons per hour.

5) That the minimum feed rate of the total blended ore not be less than the lowest efficient operating level of the mill. Stated mathematically,

Min F: $\sum_{i=1}^{n} F_i \geq LB_f$

where:

LB_f = lower bound of the feed rate in tons per hour.

6) That the ore blends have a fixed relationship to one another, such as mining ratio, within some acceptable deviation or tolerance factor. Stated mathematically, $F_i = FR_i(\sum_{i=1}^{n} F_i) \pm \Delta$

where:

FR_i = Feed ratio of ore i as a percent of total feed. $\sum_{i=1}^{n} FR_i = 1$

Δ = tolerance factor (+ indicates upper bound, - indicates lower bound).

The P_2O_5 constraint is redundant in the sense that the objective function requires that P_2O_5 be maximized. The objective function value thus achieved will be the maximum amount of P_2O_5 obtainable under the given conditions. However, the value of the objective function has the units tons/hour of P_2O_5 in the concentrate. Since the grade specification value for P_2O_5 is in percent, the objective function value must be divided by the total amount of concentrate produced per hour. While this is a simple hand calculation, the P_2O_5 constraint eliminates this exogenous requirement. If the program is infeasible, with zero slack in the P_2O_5 constraint, the indication is that the lower bound of P_2O_5 may not be met. A quick calculation should verify it one way or another. Without this constraint, the program might report a feasible solution, but not meet minimum grade requirements in P_2O_5.

The proportionality constraint needs some further explanation. The constraint is actually i <u>pairs</u> of

constraints where delta, the tolerance factor, has an upper and lower bound. Further, there is a redundancy in the proportionality constraints if all i pairs of constraints are actually listed in the program. This is caused by the fact that if one has, for example, three ratios all summing to 1, then you need only describe two of the three and the third is constrained by default. In theory, any one of the constraints may be discarded and the result will be the same. However, all proportionality constraints are left in the program for clarity and to reduce the probability of mistakenly entering a wrong ratio in the constraint. It can be proven that the same results will be obtained either way.

Expanding the general form and rearranging terms to fit the standard LP form gives the following:

OBJECTIVE FUNCTION:

$$\text{Maximize: } Z = (PR)(PA_1)(F_1) + (PR)(PA_2)(F_2) + (PR)(PA_3)(F_3)$$

SUBJECT TO CONSTRAINTS:

$$\text{MgO: } (MR)(MA_1)(F_1) + (MR)(MA_2)(F_2) + (MR)(MA_3)(F_3) - (UB_m)(RDT)(F_1) - (UB_m)(RDT)(F_2) - (UB_m)(RDT)(F_3) \leq 0$$

Note: MR, MA_i, UB_m, and RDT are all data input coefficients which means that the constraint

can be reduced to three terms ≤ 0 if desired.

$$\text{LOI: } (LOIR)(LOIA_1)(F_1) + (LOIR)(LOIA_2)(F_2) + (LOIR)(LOIA_3)(F_3) - (UB_{loi})(RDT)(F_1) - (UB_{loi})(RDT)(F_2) - (UB_{loi})(RDT)(F_3) \leq 0.$$

Note: As in the MgO constraint, since LOIR, $LOIA_i$, UB_{loi}, and RDT are constant, input coefficients, the constraint can be reduced to three terms ≤ 0.

$$P_2O_5\text{: } (PR)(PA_1)(F_1) + (PR)(PA_2)(F_2) + (PR)(PA_3)(F_3) - (LB_p)(RDT)(F_1) + (LB_p)(RDT)(F_2) + (LB_p)(RDT)(F_3) \leq 0.$$

$$\text{MAX F: } F_1 + F_2 + F_3 \leq UB_F$$

$$\text{MIN F: } F_1 + F_2 + F_3 \geq LB_F$$

$$PROP1_{ub}\text{: } 1-(FR_1)(F_1)-(FR_1)(F_2)-(FR_1)(F_3) \leq \Delta ub$$

$$PROP1_{lb}\text{: } 1-(FR_1)(F_1)-(FR_1)(F_2)-(FR_1)(F_3) \geq \Delta lb$$

$$PROP2_{ub}\text{: } 1-(FR_2)(F_2)-(FR_2)(F_1)-(FR_2)(F_3) \leq \Delta ub$$

$$PROP2_{lb}\text{: } 1-(FR_2)(F_2)-(FR_2)(F_1)-(FR_2)(F_3) \geq \Delta lb$$

$$PROP3_{ub}\text{: } 1-(FR_3)(F_3)-(FR_3)(F_1)-(FR_3)(F_2) \leq \Delta ub$$

$$PROP3_{lb}\text{: } 1-(FR_3)(F_3)-(FR_3)(F_1)-(FR_3)(F_3) \geq \Delta lb$$

The model, while in general form, has been somewhat tailored to fit the case study problem of the J. R. Simplot Company in that n = 3. However, it can be expanded or reduced to fit the ore blending requirements of the given situation.

A case study using actual operating data and ore analysis values provided by the J. R. Simplot Company is presented below. Values chosen for the case study represent the most probable within a distribution range and are tempered with an experience factor.

Data input values are as follows:

$PR = 65.11\%$

$PA_1 = 30\%$

$PA_2 = 25\%$

$PA_3 = 19.6\%$

$MR = 28.38\%$

$MA_1 = 0.7\%$

$MA_2 = 1.9\%$

$MA_3 = 0.38\%$

$UB_m = 0.8\%$

$RDT = 52.81\%$

$LOIR = 4.06\%$

$LOIA_1 = 6.0\%$

$LOIA_2 = 7.5\%$

$LOIA_3 = 23.0\%$

$UB_{loi} = 7.5\%$

$LB_p = 30.5\%$

$UB_f = 270$ dry tons

$LB_f = 220$ dry tons

$FR_1 = 1/6$

$FR_2 = 4/6$

$FR_3 = 1/6$

$\Delta_{ub} = 1.0$

$\Delta_{lb} = -1.0$

Using the above data, the problem becomes:

Problem: Find the optimum blend ratio and maximum P_2O_5 subject to the given data.

Formulation:

Maximize: $Z = (0.6511)(0.30F_1 + 0.25F_2 + 0.196F_3)$

which simplifies to :

$$0.1953F_1 + 0.1628F_2 + 0.1276F_3$$

Subject to:

MgO: $0.2838\ (0.7F_1 + 1.9F_2 + 0.38F_3) -$

$(0.8)(0.5281)(F_1+F_2+F_3) \leq 0$

which reduces to:

$$-0.22382F_1 + 0.11674F_2 - 0.314636F_3 \leq 0$$

LOI: $(0.0406)(0.06F_1 + 0.075F_2 + 0.23F_3) - (0.075)(0.5281)(F_1 + F_2 + F_3) \leq 0$

Reduced to:

$$-0.03717F_1 - 0.03656F_2 - 0.03027F_3 \leq 0$$

P_2O_5: $(0.6511)(0.3F_1 + 0.25F_2 + 0.196F_3) - (0.305)(0.5281)(F_1 + F_2 + F_3) \geq 0$

Reduced to:

$$0.03423F_1 + 0.001730F_2 - 0.03347F_3 \geq 0.0$$

MAX F: $F_1 + F_2 + F_3 \leq 270$

MIN F: $F_1 + F_2 + F_3 \geq 220$

PROP1u: $0.8333F_1 - 0.1667F_2 - 0.1667F_3 \leq 0.1$

PROP1l: $0.8333F_1 - 0.1667F_2 - 0.1667F_3 \geq -0.1$

PROP2u: $0.3333F_2 - 0.6667F_1 - 0.6667F_3 \leq 0.1$

PROP2l: $0.3333F_2 - 0.6667 F_1 - 0.6667F_3 \geq -0.1$

PROP3u: $0.8333F_3 - 0.1667 F_1 - 0.1667F_2 \leq 0.1$

PROP3l: $0.8333F_3 - 0.1667F_1 - 0.1667F_2 \geq -0.1$

All of the above may be collected in a final form in program format given in Table 1, below.

TABLE 1 - LINEAR PROGRAM MODEL COMPUTER FORMAT

Maximize: $z\ 1/4/1 = 0.1953F_1 + 0.1623F_2 + 0.1276F_3$

Subject To:

$$\text{MgO:}\quad -0.22382F_1 + 0.11674F_2 - 0.314636F_3 \leq 0$$

$$\text{LOI:}\quad -0.03717F_1 - 0.03656F_2 - 0.030271F_3 \leq 0$$

$$P_2O_5\text{:}\quad 0.03423F_1 + 0.00173F_2 - 0.033470F_3 \geq 0$$

$$\text{MAX F:}\quad F_1 + F_2 + F_3 \leq 270$$

$$\text{MIN F:}\quad F_1 + F_2 + F_3 \geq 220$$

$$\text{PROP1u:}\quad 0.8333F_1 - 0.1667F_2 - 0.1667F_3 < 1$$

$$\text{PROP1l:}\quad 0.8333F_1 - 0.1667F_2 - 0.1667F_3 \geq -1$$

$$\text{PROP2u:}\quad 0.3333F_2 - 0.6667F_1 - 0.6667F_3 \leq 1$$

$$\text{PROP2l:}\quad 0.3333F_2 - 0.6667F_1 - 0.6667F_3 \geq -1$$

$$\text{PROP3u:}\quad 0.8333F_3 - 0.1667F_1 - 0.1667F_2 \leq 1$$

$$\text{PROP3l:}\quad 0.8333F_3 - 0.1667F_1 - 0.1667F_2 \geq -1$$

The problem, run on a Digital Equipment Corporation "DEC 1091", required a run time of 0.92 kilo-core seconds. The actual computer output results are shown below.

Simplex Solution

The Problem is Feasible

Number of Iterations = 6

Optimal Value for Z1/4/1 = 43.90218

	Variable		Value	Reduced Cost
	F_1	=	46.00900	0.3364403E-07
	F_2	=	179.9820	0.5587935E-08
	F_3	=	44.00900	-0.5401671E-07
Slack in	MgO	=	3.133451	0.0000000E+00
Slack in	LOI	=	9.622493	0.0000000E+00
Slack in	P_2O_5	=	0.4132757	0.0000000E+00
Slack in	MAXF	=	0.0000000E+00	0.1623499
Slack in	MINF	=	50.00000	0.0000000E+00
Slack in	PROP1u	=	0.0000000E+00	0.3250003E-01
Slack in	PROP1l	=	2.000000	0.0000000E+00
Slack in	PROP2u	=	1.027000	0.0000000E+00
Slack in	PROP2l	=	0.9730002	0.0000000E+00
Slack in	PROP3u	=	2.000000	0.0000000E+00
Slack in	PROP3l	=	0.0000000E+00	0.3520006E-01

The program optimized a 1-4-1 blend ratio (within the delta tolerance levels) giving a 43.9 ton per hour (30.8%) rate of P_2O_5 in the concentrate. Further, evaluating the slack values (the difference between the constraint value and the bound) gives some good information. Slack in Max Feed, upper bound proportionality constraint for feed 1, and lower bound proportionality constraint for feed 3, all give an indication of which way the program wants to go. That is, it wants to feed more of total feed and it wants to increase feed 1 and reduce feed 3. This would be nice to know in evaluating additional blends or which way to "safe-side" the blend for the loader operator.

As stated previously, one of the pairs of proportionality constraints is redundant. As proof of this, the same data were run in two additional programs identical in every respect except for the removal of one of the pairs proportionality constraints. The first removed the second pair of proportionality constraints and the second removed the third pair. The computer solutions are shown below.

Simple Solution

The Problem is Feasible

Number of Iterations = 6

Optimal Value for Z1/4/1 = 43.90218

Variable		Value	Reduced Cost
F_1	=	46.00900	0.3364403E-07
F_2	=	179.9820	0.5587935E-08
F_3	=	44.00900	-0.5401671E-07
Slack in MgO	=	3.133451	0.0000000E+00
Slack in LOI	=	9.622493	0.0000000E+00
Slack in P_2O_5	=	0.4132757	0.0000000E+00
Slack in MAXF	=	0.0000000E+00	0.1623499
Slack in MINF	=	50.00000	0.0000000E+00
Slack in PROP1u	=	0.0000000E+00	0.3250003E-01
Slack in PROP1l	=	2.000000	0.0000000E+00
Slack in PROP3u	=	2.000000	0.0000000E+00
Slack in PROP3l	=	0.0000000E+00	0.3520006E-01

Simplex Solution

The Problem is Feasible

Number of Iterations = 6

Optimal Value for Z1/4/1 = 43.90218

Variable		Value	Reduced Cost
F_1	=	46.00900	-0.2328306E-08
F_2	=	181.0090	-0.9313226E-09
F_3	=	42.98200	-0.6984919E-09
Slack in MgO	=	2.690428	0.0000000E+00
Slack in LOI	=	9.628952	0.0000000E+00
Slack in P_2O_5	=	0.4494261	0.0000000E+00
Slack in MAXF	=	0.0000000E+00	0.1623534
Slack in MINF	=	50.00000	0.0000000E+00
Slack in PROP1u	=	0.0000000E+00	0.6770000E-01
Slack in PROP1l	=	2.000000	0.0000000E+00
Slack in PROP2u	=	0.0000000E+00	0.3520000E-01
Slack in PROP2l	=	2.000000	0.0000000E+00

The minute differences in values can be accounted for by rounding error. In any case, the values are accurate beyond the ability to control the ore feed. The importance is the evaluation of the gross ore blend ratio, which any of the programs can do. It is interesting to note that the computer run times for the two modified programs above are 0.38 and 0.41 kilo-core seconds, respectively. This is less than half the run time for the program incorporating all three proportionality constraints. Further, the connect time, which is a function of the operator's ability, would be reduced using either of the modified programs since they have two less constraints to contend with. Since computer costs normally consider connect and computational times, the modified programs offer an economic advantage.

The second program was further modified omitting the P_2O_5 constraint. Values obtained were identical and one iteration was saved in the computer solution. However, for the reasons stated previously in the discussion of this constraint, it is believed that the advantages of leaving the constraint in the program more than offset the computational time saved.

Finally, assigning the proportionality factor (delta) a value of 1,000 in effect eliminated the proportionality constraints. The program gave an objective function value

of 52.73 tons of P_2O_5 which equates to 37 percent P_2O_5 in the concentrate, but it required 100 percent feed of ore 1. While this clearly optimizes P_2O_5 production, it is totally out of line with mining/reserve ratios. However, there may be some production requirement wherein it would be desirable to use this blend for a limited period of time. In such instances, the ability of the program to provide this information is valuable. As a minimum, it is interesting to compare the required blend value to the "best possible" value.

DEVELOPMENT OF A QUICK AND DIRTY MODEL

As in the linear programming model, an understanding of the beneficiation process is necessary, but the process will be treated as a "black box". As before, ore fed into the slurry plant and mill equals concentrate plus tailings (see page 19).

The purpose of the quick and dirty model is to provide the mill manager with a quick, somewhere close to optimal, solution to the question of what feed blend options are available subject to grade constraints. The differences between this model and the full linear program are listed below:

1) A true, "optimal", single solution is not given.
2) The LOI constraint is not considered. This is realistic since, as stated previously, it is not usually a significant problem and can be remedied in alternative ways.
3) The proportionality constraints are tight. That is, a given blend is fed into the model and solved for percent P_2O_5 and MgO.
4) The required calculations can be accomplished using the least expensive pocket calculator on the market or pencil and paper if necessary.

This quick and dirty model is probably more realistic in the sense that the ore blend decision on any given day is based more on such factors as stockpile size or mining operations than on maximizing P_2O_5 in the concentrate. This doesn't diminish the value of knowing the "optimum" blend since it is not desirable to stray too far nor for too long a time from this blend. However, this second model can evaluate all desired blend combinations quickly and a comparison of percent P_2O_5 and/or MgO will give a relative order-of-merit listing of the various blends.

The model begins with three linear equations in general form.

$$(1) \qquad \sum_{i=1}^{n} FR_i = 1.0$$

$$(2) \qquad \sum_{i=1}^{n} (PR)(PA_i)(FR_i) \geq (LB_p)(RDT)$$

$$(3) \qquad \sum_{i=1}^{n} (MR)(MA_i)(FR_i) \leq (UB_m)(RDT)$$

Equation (1) simply states that the summation of feed ratios (FR_i) equals one. Once the feed blend ratios to be evaluated meet the condition of this equation, it is no longer necessary.

Equation (2) says that the phosphate recovered (in percent) times the phosphate assayed (also in percent) times the feed ore ratio for each ore must be less than or equal to the acceptable lower bound of phosphate (in percent) times the concentrate as a percent of total feed.

Equation (3) is essentially the same as equation (2) except that it uses MgO assay and recovery factors, and the upper bound of MgO is allowable.

Note that dividing equations (2) and (3) through by RDT and expanding, results in the following:

$$(4)\quad \frac{(PR)(PA_1)}{RDT} F_1 + \frac{(PR)(PA_2)}{RDT} F_2 + \frac{(PR)(PA_3)}{RDT} F_3 \geq LB_p$$

$$(5)\quad \frac{(MR)(MA_1)}{RDT} F_1 + \frac{(MR)(MA_2)}{RDT} F_2 + \frac{(MR)(MA_3)}{RDT} F_3 \leq UB_m$$

Since PR, PA_1, MR, MA_i, and RDT are input data and thereby constant for a given blending operation, it only remains to calculate the actual values of percent P_2O_5 and percent MgO for the blend and compare them to their respective lower and upper bounds. The percent P_2O_5 calculated can then be used to rank order the desired blend options to choose the best, second best, etc. This rank ordering will work for the model provided the MgO constraint is not violated.

If, at this point, it is desired to make a quick check on LOI for the selected blend(s), it could be accomplished in a similar manner:

$$(6)\quad \sum_{i=1}^{3} (LOIR)(LOIA_i)(FR_i) \leq (UB_{loi})(RDT)$$

The procedure for evaluation of this quick and dirty method then has the following steps:

1. Collect data input for; phosphate recovery, phosphate assay for each ore, percent recovered dry tons, MgO-Assay for each ore, and MgO recovery.
2. Select all possible blend combinations desired for evaluation.
3. Check to insure that the feed ratios (FR_i) sum to one for each blend combination.
4. Substitute data into equations (4) and (5) for each blend combination and solve.
5. If the solutions satisfy their respective inequalities go to step 6. If not, select next blend and go to step 3. Repeat until all desired blend combinations are evaluated.
6. Rank order those blends with highest P_2O_5 at top, next highest as second, etc.
7. Select the blend with highest P_2O_5, or as other conditions dictate.
8. If desired, evaluate LOI for selected blend(s) using equation (6).

As in the linear programming model, this model has been somewhat tailored to the J. R. Simplot case study. Modification to accommodate other considerations would not be difficult and should be similar to those developed above.

As in the linear programming model, a case study using actual operating data and ore analysis values provided by the J. R. Simplot Company is presented below. The values are the same, most probable data as those used in the L.P. model. This is to allow for comparison between the two techniques.

PROBLEM: Given the data input values listed below, evaluate all possible integer blend combinations such that $\frac{Fi}{6} = FR_i$. Rank order feasible blends as a function of P_2O_5.

Data Input Values:

Main Bed Ore Feed Ratio = FR_1

Shale Ore Feed Ratio = FR_2

Black Ore Feed Ratio = FR_3

Lower Bound P_2O_5 (LB_p) = 30.5%

Upper Bound MgO (UB_m) = 0.8%

Recovered Dry Tons (RDT) = 52.81%

Phosphate Recovery (PR) = 65.11%

Phosphate Assay of Ore 1 (PA_1) = 30%

Phosphate Assay of Ore 2 (PA_2) = 25%

Phosphate Assay of Ore 3 (PA_3) = 19.6%

MgO Recovery (MR) = 28.38%

MgO Assay of Ore 1 (MA_1) = 0.7%

MgO Assay of Ore 2 (MA_2) = 1.9%

MgO Assay of Ore 3 (MA_3) = 0.38%

FORMULATION:

$\underline{P_2O_5}$: $(0.6511)(0.3\ FR_1 + 0.25\ FR_2 + 0.196\ FR_3) \geq (0.5281)(0.305)$

$\Rightarrow\ 0.3699\ FR_1 + 0.3082\ FR_2 + 0.2417\ FR_3 \geq 0.305$

$\underline{MgO}$: $(0.2838)(0.007\ FR_1 + 0.019\ FR_2 + 0.0038\ FR_3) \leq (0.5281)(0.008)$

$\Rightarrow\ 0.3762\ FR_1 + 1.021\ FR_2 + 0.2042\ FR_3 \leq 0.80$

TABLE 2: QUICK AND DIRTY MODEL CALCULATION RESULTS

BLEND	Value P_2O_5	Value MgO	Feasibility
0-0-6	.2417	.2472	Infeasible P_2O_5
0-1-5	.2528	.3403	Infeasible P_2O_5
0-2-4	.2639	.4765	Infeasible P_2O_5
0-3-3	.2750	.6126	Infeasible P_2O_5
0-4-2	.2860	.7487	Infeasible P_2O_5
0-5-1	.2971	.8849	Infeasible Both
0-6-0	.3082	1.021	Infeasible MgO
1-0-5	.2631	.1702	Infeasible P_2O_5
1-1-4	.2742	.3690	Infeasible P_2O_5
1-2-3	.2852	.5051	Infeasible P_2O_5
1-3-2	.2963	.6413	Infeasible P_2O_5
1-4-1	.3074	.7774	Feasible
1-5-0	.3185	.9135	Infeasible MgO
2-0-4	.2844	.2615	Infeasible P_2O_5
2-1-3	.2955	.3977	Infeasible P_2O_5
2-2-2	.3066	.5338	Feasible
2-3-1	.3177	.6699	Feasible
2-4-0	.3288	.8061	Infeasible MgO
3-0-3	.3058	.2902	Feasible
3-1-2	.3169	.4263	Feasible
3-2-1	.3280	.5625	Feasible
3-3-0	.3391	.6986	Feasible
4-0-2	.3272	.3189	Feasible
4-1-1	.3383	.4550	Feasible
4-2-1	.3493	.5211	Feasible
5-0-1	.3485	.3475	Feasible
5-1-0	.3596	.4837	Feasible
6-0-0	.3699	.3762	Feasible

TABLE 3

QUICK AND DIRTY MODEL RANK ORDERING OF FEASIBLE BLENDS

Blend*	$\%P_2O_5$	%MgO
6-0-0	36.99	0.3762
5-1-0	35.96	0.4837
4-2-1	34.93	0.5211
5-0-1	34.85	0.3475
3-3-0	33.91	0.6986
4-1-1	33.83	0.4550
3-2-1	32.80	0.5625
4-0-2	32.72	0.3189
2-3-1	31.77	0.6699
3-1-2	31.69	0.4263
1-4-1	30.74	0.7774
2-2-2	30.66	0.5338
3-0-3	30.58	0.2902

*Note: Blends listed in order; parts main bed-parts shale-parts black

Table 3, together with the knowledge of current stockpile levels, mining operations of each of the ores, or other shortrun considerations, enables the decision maker to choose the best blend for the day's operation under the given conditions. At the very worst, he is able to make something more than an arbitrary decision.

CONCLUSIONS

Two mathematical models have been presented to assist the phosphate mill manager in selecting the proper blend for the day's operation. Each model has its own set of assumptions and criterion.

The linear programming model is more complex and can either evaluate a selected blend or give an optimum blend for a given set of data and operating conditions. The quick and dirty model evaluates alternative blends for a given set of data providing a quick check of feasibility for the particular blend. The major differences between the two methods are computational requirements and costs. The linear programming model requires a computer capable of handling small linear programs. The costs are associated with this requirement. On the other hand, the quick and dirty model requires nothing more than a small pocket calculator and associated costs.

While each model has its respective advantages and disadvantages, as discussed previously, the values calculated by each for the same data and a given blend are essentially identical. For example, the P_2O_5 calculated using the same case study values and the same 1/4/1 blend differed by only five one-hundredths or approximately 0.1 percent. The MgO calculation difference between the two

methods is less than 0.1 percent. These differences can fairly be attributed to rounding error.

Even though the models treated the beneficiation process as a "black box", they are able to accommodate changes in the process to include major alterations or addition/deletion of process sub-systems. The only changes in the model are in the recovery factors for P_2O_5, MgO, and total concentrate. Since changes in the beneficiation process may affect these factors, the new factors must be calculated through testing and data collection. Once the new factors have been verified, substitution into the model as new input data automatically updates it. Since the recovery factors are routinely calculated, this does not require any additional effort. In fact, these factors may vary during normal operations and should be checked and updated periodically.

Finally, as with any model, the two presented in this thesis are only as good as the data input. This makes the ore analysis and mill operating data critical. The models will evaluate alternatives and provide an "optimal" solution well within the accuracy of the data. This statement of qualification is made so that anyone

using the models understands the limitations of them and will insure that the data used is the best obtainable under the circumstances.

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APPENDIX A

NOMENCLATURE

F_i = Feed of ore i in tons per hour.

PA_i = Phosphate assay of ore i in percent.

PR = Phospahte recovery in percent.

MR = MgO recovery in percent.

MA_i = MgO assay of ore i in percent.

UB_m = Upper bound of MgO in percent.

RDT = Recovered dry tons as percent of total feed.

LOIR = LOI recovery in percent.

$LOIA_i$ = LOI assay of ore i in percent.

UB_{loi} = Upper bound of LOI in percent.

FR_i = Feed ratio of ore i.

LB_p = Lower bound of phosphate in percent.

UB_f = Upper bound of feed in dry tons per hour.

LB_f = Lower bound of feed in dry tons per hour.

Δub = Upper bound mining/reserve ratio tolerance factor (Dimensionless).

Δlb = Lower bound mining/reserve ratio tolerance factor (Dimensionless).